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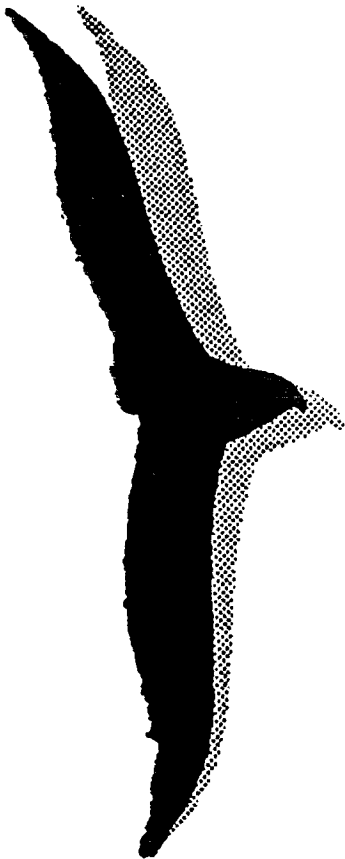
## Serie research memoranda

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Research Memorandum 1994-52

November 1994

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# **Structural and Frictional Unemployment in an Equilibrium Search Model with Heterogeneous Agents**

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November 1994

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# 1 Introduction

Numerous economists have studied the effect of mandatory minimum wages on wages and employment. Recently, there has been a new round in this discussion, as a result of evidence generated by natural experiments. The evidence shows that the effect of observed increases in the minimum wage has been small (see e.g. Card and Krueger (1994)). Yet there is still little agreement on the effect of minimum wages on employment. In a way, this is not surprising. To determine the effects, one must distinguish individuals who are unemployed because of the minimum wage from individuals who would be unemployed anyway. This distinction is that between ‘structural’ and ‘frictional’ unemployment.

In this paper we investigate whether the Burdett-Mortensen equilibrium search model can be used to distinguish between these two types of unemployment, so that we can obtain an estimate of the fraction of unemployment that can be attributed to the minimum wage. An interesting feature of this model is that it has a dispersed wage equilibrium in which the law of one price does not hold. This is because firms have monopsony power due to search frictions on the labour market, and, in equilibrium, there is a trade-off between the size of the firm and the wage paid at the firm. As a consequence, wages need not be equal to marginal value products and therefore it is possible that there is no effect of the minimum wage on unemployment. The fact that wage regressions with extensive controls for productivity characteristics of workers do not have an impressive explanatory power lends credence to a model that does not impose the law of one price. An additional advantage of the model is that it allows the shape of the whole equilibrium wage distribution to be sensitive to changes in the minimum wage. The latter has been regarded as a stylized fact of the labour market (see e.g. Kiefer and Neumann (1993)).

In Section 2 we discuss the Burdett-Mortensen model and the extension to heterogeneous agents. The Burdett-Mortensen model was developed in Burdett and Mortensen (1989) and Mortensen (1990), and has been discussed in a large number of papers. Van den Berg and Ridder (1992) present an extensive empirical analysis of this model. Because of this, the exposition in Section 2 will be brief. The data are described in Section 3. These are based on a survey conducted in the Netherlands from 1985 onwards. Again, because this survey has been used in numerous other studies (like Lindeboom and Theeuwes (1991), Van den Berg (1992), Van den Berg and Ridder (1992) and Van den Berg and Ridder (1993)), the exposition in Section 3 will be brief.

The likelihood function is derived in Section 4. Section 5 contains an ex-

tensive discussion of the identification of the model, and, in particular, of the different types of unemployment. The estimation results are in Section 6. We pay particular attention to the fit of the model to the wage data.

## 2 The Equilibrium Search Model

### 2.1 Equilibrium Search with Identical Agents

Consider a labour market with identical agents, i.e. a market in which all workers are equally productive at all firms. Even in this case, the Burdett-Mortensen model has a dispersed equilibrium wage (offer) distribution, i.e. the law of one price does not hold. Allowing for heterogeneity in the productivity of workers adds to the equilibrium dispersion of wages. The main theoretical contribution of this paper is the introduction of productivity heterogeneity in this basic model in such a way that frictional and structural unemployment can be meaningfully defined.

We assume that there are large fixed numbers of workers and firms participating in the labour market (formally a continuum of each). The measure of workers is denoted by  $m$ , whereas the measure of firms is normalized to one. Workers receive job offers at given Poisson rates,  $\lambda_0$  when unemployed and  $\lambda_1$  when employed, with  $0 \leq \lambda_0, \lambda_1 < \infty$ . Job offers are independent random drawings from the distribution  $F(w)$  of wage offers. When an offer arrives, the worker must decide whether to accept the offer or to reject it and continue searching for a better one. Workers become unemployed at the exogenous separation rate  $\delta$  ( $0 < \delta < \infty$ ). During unemployment, the worker receives unemployment benefits  $b$  ( $0 \leq b < \infty$ ). A firm posts a wage that is the same for all workers, and it does not bargain over this wage. In the basic model, the value product of any worker at any firm is the same. It is denoted by  $p$ . The firms have a linear production function, so that the average and marginal product are equal. Individuals and firms are assumed to maximize their expected wealth.

Assuming that the wage offer distribution is known and stationary over time and that wages are constant in jobs, the supply-side of this model is equivalent to the standard job search model with search on the job (see e.g. Mortensen (1986)). Thus, the optimal strategy of an unemployed worker has the reservation wage property. The reservation wage  $r$  is

$$r = b + (\lambda_0 - \lambda_1) \int_r^\infty \frac{\bar{F}(w)}{\delta + \lambda_1 \bar{F}(w)} dw \quad (2.1)$$

with  $\bar{F} = \mathbf{1} - F$ . Employed workers accept any wage offer that exceeds their current wage.

It is important to distinguish between the distribution of wages offered to individuals, which is the wage *offer* distribution  $F$ , and the distribution of wages received by workers who are currently employed. The latter is referred to as the earnings *distribution*, and we denote this distribution by  $G$ . In equilibrium, the flow of workers out of jobs with a given wage is equal to the inflow in such jobs. Similarly, the flows into and out of unemployment are equal. Firms that offer a wage lower than the reservation wage of the unemployed do not attract any worker and therefore cannot survive. The market is only viable if there is a positive gain from trade, i.e. if  $p > b$ . Under these assumptions we have

$$G(w) = \frac{\delta F(w)}{\delta + \lambda_1 \bar{F}(w)} \quad w \geq r \quad (2.2)$$

$$F(r) = 0 \quad (2.3)$$

$$\frac{u}{m} = \frac{\delta}{\delta + \lambda_0} \quad (2.4)$$

In (2.4),  $u$  is the number of unemployed workers. Thus,  $u/m$  is the rate of frictional unemployment in this market. It is determined by the rates  $\delta$  and  $\lambda_0$ . Note that with full information on the location of jobs, i.e. in the absence of search frictions,  $\lambda_0 = \infty$  and  $u = 0$ . *Frictional unemployment* should be distinguished from *structural unemployment*, which occurs if the unemployment benefits, or more generally the value of leisure, exceeds the value product  $p$ . This type of structural unemployment is voluntary, because workers are better off if they are unemployed. The model implies that the re-employment hazard does not depend on  $b$ . An increase in  $b$  increases  $r$ , but this only shifts the wage offer distribution. All wage offers remain acceptable to the unemployed. This is consistent with the findings of empirical studies of unemployment durations with Dutch and US data (see the surveys in Van den Berg (1990b) and Devine and Kiefer (1991)). Only if  $b$  exceeds  $p$  then the re-employment hazard falls to 0, and unemployment rises.

If there is a mandatory minimum wage, denoted by  $w_L$ , then wage offers must exceed this wage. If  $p < w_L$ , then firms do not employ any worker, and there is structural unemployment. This type of unemployment is involuntary if  $b < p < w_L$ , because workers would supply labour if the minimum wage would be lower than  $p$ . Hence, if  $p > \max(w_L, b)$  then there is frictional unemployment

equal to  $u$ , while if the reverse holds there is voluntary or involuntary structural unemployment equal to  $m$ .

For the moment, assume that  $p > \max(w_L, b)$ . The steady-state level of production is determined by the size of the steady-state work force  $l$  of the firm. That work force depends on the wage  $w$  set by the firm, the reservation wage  $r$  of the unemployed, and the distribution  $F$  of wages set by other firms competing for the same workers. Each firm chooses  $w$  to maximize its steady-state profit flow  $\pi$ , which, given  $r$  and  $F$ , equals  $(p - w)l(w; r, F)$ .

A non-cooperative steady-state equilibrium solution consists of a reservation wage  $r$  and a wage offer distribution  $F$  such that (i)  $r$  satisfies (2.1) given  $F$ , and (ii) every  $w$  in the support of  $F$  maximizes the steady-state profit flow  $\pi$ . Burdett and Mortensen (1989) prove that there is a unique equilibrium and they give closed-form solutions. The distributions  $F$  and  $G$  have probability density functions  $f$  and  $g$  with a support equal to  $[\underline{w}, \bar{w}]$ , with

$$\begin{aligned}\underline{w} &= \max(w_L, r) \\ \bar{w} &= \left[ \frac{\delta}{\delta + \lambda_1} \right]^2 \underline{w} + \left[ 1 - \left[ \frac{\delta}{\delta + \lambda_1} \right]^2 \right] p\end{aligned}\tag{2.5}$$

The equilibrium wage offer c.d.f. and p.d.f. are

$$F(w) = \frac{\delta + \lambda_1}{\lambda_1} \left[ 1 - \sqrt{\frac{p - w}{p - \underline{w}}} \right] \quad \text{on } [\underline{w}, \bar{w}]\tag{2.6}$$

$$f(w) = \frac{\delta + \lambda_1}{2\lambda_1 \sqrt{p - \underline{w}} \sqrt{p - w}} \quad \text{on } [\underline{w}, \bar{w}]\tag{2.7}$$

Substitution of (2.6) in (2.1) gives

$$r = \frac{(\delta + \lambda_1)^2 b + (\lambda_0 - \lambda_1) \lambda_1 p}{(\delta + \lambda_1)^2 + (\lambda_0 - \lambda_1) \lambda_1} \quad \text{i f } r \geq w_L.\tag{2.8}$$

If  $r > w_L$ , the equilibrium reservation wage is not given by (2.8). However in that case the reservation wage is not effective, because the lowest wage offer is  $w_L$ . If  $r \geq w_L$  then  $r$  and  $\bar{w}$  are weighted averages of  $b$  and  $p$ . Otherwise  $\bar{w}$  is a weighted average of  $p$  and  $w_L$ . Note that  $r$  is smaller than  $b$  iff  $\lambda_0$  is smaller than  $\lambda_1$ . In that case the unemployed accept a wage lower than  $b$ , because it is easier to find a higher paying job if employed. Allowing for this possibility is important given the empirical evidence on the relative size of  $b$  and  $r$  (see e.g. Narendranathan and Nickell (1985), Van den Berg (1990a) and Van den Berg (1990b)).

Using (2.2), the equilibrium wage (or earnings) density is

$$g(w) = \frac{\delta \sqrt{p - \underline{w}}}{2\lambda_1} \frac{1}{(p - w)^{3/2}} \quad \text{on } [\underline{w}, \bar{w}]. \quad (2.9)$$

Note that both  $f$  and  $g$  are increasing densities. The wage distribution is related to the income distribution, and there is abundant empirical evidence that the income distribution does not have an increasing density. We return to this issue below. For an employed individual earning a wage  $w$ , the exit rate out of that job equals  $\delta + \lambda_1 \bar{F}(w)$ . This rate decreases in  $w$ , which is consistent with a number of empirical studies on job durations (Lindeboom and Theeuwes (1991) and Van den Berg (1992)).

## 2.2 Heterogeneity in Value Products

In reality, workers and firms are obviously not identical. All parameters of the model, i.e.  $\lambda_0$ ,  $\lambda_1$ ,  $\delta$ ,  $b$  and  $p$  vary over workers and/or firms. As argued in Van den Berg and Ridder (1992), there are basically two ways to introduce heterogeneity: within the market and between markets. Heterogeneity within the market means that there is one labour market within which heterogeneous workers and firms interact. Heterogeneity between markets means that the labour market is segmented and consists of a large number of separate submarkets within which workers and firms are homogeneous. We follow the latter approach, and we assume that we observe a mixture of homogeneous markets. Conceivably we can stratify on all the parameters. In the present context, dispersion in  $p$  is particularly relevant, since it allows for the possibility of structural unemployment (namely when  $p < \max(w_L, b)$ ). Because we assume that  $p$  follows a continuous distribution, we have effectively a continuum of submarkets which differ in the value product of workers.

As noted above, there is abundant empirical evidence that the income distribution does not have an increasing density as is predicted by the model with identical agents. Allowing for heterogeneity in  $p$  may improve the fit to the observed wage offer and earnings distribution. To see this we consider the following transformation of  $w$  (note that we acknowledge the dependence of the support of  $w$  on  $P$ )

$$y = \frac{p - w}{p - \underline{w}(p)} \quad (2.10)$$

so that the excess wage  $w - \underline{w}(p)$  satisfies



$$w - \underline{w}(p) = (1 - y)(p - \underline{w}(p)). \quad (2.11)$$

If  $w$  is distributed according to  $F$  then the density of  $y$  is

$$f_y(y) = \frac{1}{2(1 - \eta)} y^{-1/2} \quad \eta^2 \leq y \leq 1 \quad (2.12)$$

while if  $w$  is distributed according to  $G$  then the density of  $y$  is

$$g_y(y) = \frac{\eta}{2(1 - \eta)} y^{-3/2} \quad \eta^2 \leq y \leq 1 \quad (2.13)$$

with  $\eta = \delta / (\delta + \lambda_1)$ .

Equation (2.11) describes the wage determination in the Burdett-Mortensen model. The excess wage  $w - \underline{w}(p)$  is a fraction of the excess productivity  $p - g(p)$ . This fraction is a random variable with a distribution that depends on  $\lambda_1/\delta$ , which is the expected number of wage offers during a spell of **employment** (i.e. a spell that starts with the acceptance of a job from unemployment and ends with a layoff). This ratio is a measure of the speed at which the worker climbs the job (and wage) ladder, with  $y = 1$  corresponding to the bottom,  $w = \underline{w}(p)$ , and  $y = \eta^2$  to the top,  $w = \bar{w}(p)$ , of this ladder. From (2.11) it follows that the moments of  $w - \underline{w}(p)$  in either the wage offer or the earnings distribution are the product of  $(p - \underline{w}(p))^n$  and an expression that only depends on  $\eta$ . By choosing an appropriate distribution of the productivity  $p$ , the moments of the observed wage offer or earnings distribution can be matched. Hence, we expect that an acceptable fit to the data depends on the allowance for sufficient heterogeneity in  $p$ . This is confirmed in Van den Berg and Ridder (1992).

### 3 The Data

The model is estimated with the OSA (Netherlands Organization for Strategic Labour Market Research) Labour Supply Panel Survey. This panel started in 1985. Presently four waves are available (April-May 1985, August-October 1986, August-October 1988, and August-November 1990).

In the OSA panel a random sample of households in the Netherlands is followed over time. Because the study concentrates on individuals who are between 15 and 61 years of age and who are not full-time students, only households with at least one person in this category are included. All individuals (and in all cases the head of the household) in this category are interviewed. The first wave consists of 4020 individuals (in 2132 households).

The data allow a reconstruction of the sequence of labour market states occupied by the respondents and the sojourn times and income levels in these states. Part of the information is retrospective. For example, the first wave (in 1985) contains information on the labour market histories from January 1, 1980 until the date of the interview. The following labour market positions are distinguished: employment (job-to-job changes are recorded), self-employment, unemployment, and not-in-labour-force (subdivided into being in military service, doing full-time education, and other activities not related to the labour market).

In this paper we restrict the attention to respondents who were participating in the panel as of the first wave. Individuals who were self-employed at certain dates of the time span covered by the survey were deleted, since it is likely that the behaviour of such individuals, at least in a certain period, deviates substantially from the behaviour as described by the model. For similar reasons, we do not use information on respondents who are observed to be nonparticipant in the labour market at certain dates. Finally, we delete observations for which the reported wage is smaller than the legal minimum wage.

As a result, we have a sample of 1767 individuals. Of these, 12% were unemployed at the date of the first interview. In our sample, 34% participates in all four waves of the panel, while 33% only participates in the first wave. The income changes at transitions occurring before the date of the first interview (April 1985) are only recorded to lie in one of a few broad intervals. This makes the information on spells ending before this date relatively inaccurate in comparison to spells ending after it, for which we observe exact income levels. Therefore, the first spell used is the spell which is ongoing at the date of the first interview. For computational reasons, information on subsequent spells is not used either.

The benefits level  $b$  is taken to be the mean in the sample. From the information in the survey, the mandatory minimum wage  $w_L$  can be calculated for each respondent. In the empirical analysis, we take the sample average.

## 4 The Likelihood Function

If we only allow for heterogeneity in  $p$ , then the parameters of the model are  $\lambda_0$ ,  $\lambda_1$ ,  $\delta$  and the parameters of the distribution of  $p$ . We estimate these parameters from observed labour market histories of a sample of individuals. In its simplest form, the model has no observed explanatory variables, and the dependent variables are aspects of the individual labour market histories. To be specific, the dependent variables are

*Position at time of first interview:*

$u = 0$ : unemployed

$u = 1$ : employed

*Elapsed and residual duration in position at first interview:*

$t_{0b}$  = elapsed unemployment duration

$t_{0f}$  = residual unemployment duration

$t_{1b}$  = elapsed job duration

$t_{1f}$  = residual job duration

*Paid and accepted wages:*

$w_0$  = re-employment wage of unemployed individual

$w_1$  = wage of employed individual at time of first interview

*First transition if employed at first interview:*

$v = 0$ : job-to-unemployment transition

$v = 1$ : job-to-job transition

The model implies a particular distribution for all these variables. For a given value of  $p > \max(b, w_L)$ , the employment and job durations are Exponentially distributed. Following Ridder (1984), this implies that the elapsed and residual durations are independent and also Exponentially distributed. If  $p < b$ , there is no gain from trade, and if  $p < w_L$ , the minimum wage is too high to employ the workers. In both cases the workers are permanently unemployed. Hence, the unconditional unemployment duration  $t_0$  in the population has the following defective distribution, in which  $k$  denotes the density for  $0 < t_0 < \infty$  and  $H$  is the c.d.f. of  $p$ :

$$\begin{aligned} k(t_0) &= \lambda_0 \exp(-\lambda_0 t_0) \bar{H}(\max(b, w_L)) & 0 < t_0 < \infty \\ \Pr(t_0 = \infty) &= H(\max(b, w_L)) \end{aligned} \quad (4.1)$$

The unconditional distribution of  $u$  is

$$\begin{aligned} \Pr(u = 0) &= \frac{\delta}{\delta + \lambda_0} \bar{H}(\max(b, w_L)) + H(\max(b, w_L)) \\ \Pr(u = 1) &= 1 - \Pr(u = 0) \end{aligned} \quad (4.2)$$

Conditional on  $p > \max(b, w_L)$ , the elapsed and residual unemployment durations are independent and Exponentially distributed with parameter  $\lambda_0$ .

The likelihood function involves the joint densities of  $w_0, p$  and  $w_1, p$ . In these densities,  $w_0$  and  $w_1$  are only defined if  $p > \max(b, w_L)$ . Moreover, the distributions of  $w_0|p$  and  $w_1|p$  have a bounded (and identical) support, and the bounds  $\underline{w}$  and  $\bar{w}$  of the support depend on  $p$  (see Section 2). The support of the joint distribution of  $w_i, p$  differs according to the values of  $b, w_L, \lambda_0$  and  $\lambda_1$ . It is convenient to distinguish the following four regimes:

$$\text{I. } b \leq w_L, \quad \lambda_1 > \lambda_0$$

$$\text{II. } b \leq w_L, \quad \lambda_1 \leq \lambda_0$$

$$\text{III. } b > w_L, \quad \lambda_1 > \lambda_0$$

$$\text{IV. } b > w_L, \quad \lambda_1 \leq \lambda_0$$

Because  $b$  and  $w_L$  are known and constant within a segment of the labour market, we know whether the segment is in one of the first two regimes or in one of the last two regimes. In our sample we always have  $b \leq w_L$ . This implies that permanent unemployment is due to  $p < w_L$ , i.e. a high minimum wage, rather than  $p < b$ , i.e. a high level of unemployment benefits. In most cases, our estimate of  $\lambda_0$  is larger than  $\lambda_1$  (see below), so then regime II applies. In this regime the bounds on  $w$  are

$$\begin{aligned} \underline{w}(p) &= w_L & w_L \leq p \leq \underline{p} \\ \underline{w}(p) &= r(p) & \underline{p} \leq p < \infty \end{aligned} \quad (4.3)$$

$$\bar{w}(p) = \left[ \frac{\delta}{\delta + \lambda_1} \right]^2 w_L + \left[ 1 - \left( \frac{\delta}{\delta + \lambda_1} \right)^2 \right] p \quad (4.4)$$

if  $w_L \leq p < \underline{p}$ ; while

$$\bar{w}(p) = \frac{\delta^2}{(\delta + \lambda_1)^2 + (\lambda_0 - \lambda_1)\lambda_1} b + \left[ 1 - \frac{\delta^2}{(\delta + \lambda_1)^2 + (\lambda_0 - \lambda_1)\lambda_1} \right] p$$

if  $\underline{p} \leq p < \infty$

with

$$\underline{p} = w_L + \frac{(\delta + \lambda_1)^2}{(\lambda_0 - \lambda_1)\lambda_1}(w_L - b) \quad (4.5)$$

Low-productivity workers with  $0 < p < b$  are unemployed due to a high value of non-employment. Low-productivity workers with  $b < p < w_L$  are willing to work, but the high minimum wage makes firms unwilling to hire them. In markets with  $w_L < p < \underline{p}$  the lowest wage is equal to the minimum wage, and in markets with  $p > \underline{p}$  the lowest wage is equal to the reservation wage which is increasing in  $p$ . The equilibrium search model induces a positive correlation between  $p$  and the wage. Note that workers are always paid less than their marginal product.

Let  $I(\cdot)$  denote the indicator function of the event between parentheses. The likelihood for the case of complete information equals

$$\begin{aligned} L = & \int \left[ I(p < \max(b, w_L)) + I(p \geq \max(b, w_L)) \frac{\delta}{\delta + \lambda_0} \cdot \lambda_0 \exp(-\lambda_0(t_{0b} + t_{0f})) \right. \\ & \left. \lambda_0 \cdot \frac{\delta + \lambda_1}{2\lambda_1(p - \underline{w}(p))^{1/2}(p - w)^{1/2}} \cdot I(\underline{w}(p) \leq w \leq \bar{w}(p)) \right]^{1-u} \\ & \left[ I(p \geq \max(b, w_L)) \cdot \frac{\lambda_0}{\delta + \lambda_0} \cdot \frac{\delta(p - \underline{w}(p))^{1/2}}{2\lambda_1} \cdot \frac{1}{(p - w)^{3/2}} \right. \\ & \left. \exp \left( -(\lambda_1 + \delta) \cdot \frac{(p - w)^{1/2}}{(p - \underline{w}(p))^{1/2}} \cdot (t_{1b} + t_{1f}) \right) \right. \\ & \left. \delta^v \cdot \left( (\lambda_1 + \delta) \cdot \frac{(p - w)^{1/2}}{(p - \underline{w}(p))^{1/2}} - \delta \right)^{1-v} \cdot I(\underline{w}(p) \leq w \leq \bar{w}(p)) \right]^u dH(p) \end{aligned} \quad (4.6)$$

We do not have complete information on all observations due to censoring or missing data. On the assumption that censoring is uninformative and that data are missing at random, one can easily modify the likelihood to deal with these complications.

In the sequel we take a Log normal distribution for  $p$ , with parameters  $\mu_p$  and  $\sigma_p^2$ . Note that a discrete distribution for  $p$  with unknown points of support gives a likelihood function that is discontinuous in the parameters. Of course, one can choose any other positive (continuous) function. Some guidance on the choice of  $H$  is provided by a comparison of the empirical and fitted distribution of wages paid and offered. This raises the question of the identification of the parameters and the distribution of  $p$ , to which we turn next.

## 5 Distinguishing between Structural and Frictional Unemployment

The likelihood function combines two types of data: wage data and duration data. If there is permanent unemployment, then the distribution of the unemployment durations is defective. However, if there is also frictional unemployment then one can estimate the re-employment hazard  $\lambda_0$ . The job durations are distributed as a non-scalar mixture of Exponential distributions. The mixing distribution is that of  $p$  truncated at  $\max(b, w_L)$ . Whether the (truncated) mixing distribution can be identified from the mixture of Exponentials is an open question. Given the truncated mixing distribution one can identify  $\lambda_1$  and  $\delta$  from the job durations and the subsequent destination, i.e. the distribution of  $v$ .

From (2.11), (2.12) and (2.13) one can recover the moments of the distribution of  $p$  truncated at  $\max(b, w_L)$  from the earnings distribution and the distribution of accepted wages. In regimes II and III,  $p = \underline{w}(p)$  is a piecewise linear function of  $p$ , and this complicates matters, but in regimes I and IV there is a simple correspondence between the moments of  $w$  and those of the truncated distribution of  $p$ .

From  $\lambda_0$  and  $\delta$  one can compute the frictional unemployment rate. A comparison of this frictional rate with the observed unemployment rate at the time of the first interview allows us to determine the rate of structural unemployment, i.e.  $H(\max(b, w_L))$ . In the Netherlands there generally holds that  $w_L \geq b$ . In that case the structural unemployment rate is  $H(w_L)$ .

Although the structural unemployment rate is identified, the effect of a decrease in the minimum wage on that rate is not nonparametrically identified. This is because the shape of the density  $h$  of  $p$  between 0 and  $w_L$  is not nonparametrically identified. The effect of a marginal change is equal to the value  $h(w_L)$  of the density of  $p$  in the neighbourhood of  $w_L$ . To quantify the effect of a marginal decrease of  $w_L$ , an additional assumption is needed, like continuity of  $h$  at  $w_L$ . Note that we can recover the untruncated  $h$  for  $p > w_L$ .

It is interesting to compare this method with that of Meyer and Wise (1983a, 1983b). They estimate the effect of the minimum wage by fitting a truncated Log normal distribution to a cross-section of observed wages. It is well-known that the probability mass below the point of truncation is only identified by making untestable assumptions on the wage distribution (Flinn and Heckman (1982)). In particular, the untruncated wage distribution must be *recoverable* from the wage distribution truncated from below. By embedding the problem in an equilibrium search model we can, at least in theory, find the probability mass below the

truncation point without the need to make such an assumption. If the probability mass of productivities below the truncation point is the difference between the observed unemployment rate and the estimated unemployment rate.

In the empirical analysis, we take a parametric Log normal distribution. The family of Log normal distributions satisfies the recoverability. This implies that the size of the probability mass below the point  $w_L$  (and, indeed, the exact shape of the density below that point) can be inferred from the shape of the density above that point. As a result, the model with a parametric distribution is overidentified in the sense that both the wage and the duration data provide information on the size of the probability mass below  $w_L$  (i.e., structural unemployment). We return to this issue below.

## 6 Estimation Results

Table 1 presents the parameter estimates for the whole sample and for sample subsets obtained by stratification on age and education as of the first interview. We distinguish between four age categories (16-29, 30-38 and 39-61) and three education categories, labelled in increments of education as "edu1", "edu2" and "edu3". One may argue that wage dummies at the first interview are inadmissible as stratification variables if they are not time-invariant personal characteristics. We adopt this restriction for the reason that the mandatory minimum wage is strongly age-dependent. The results based on age stratification may be more informative concerning wage effects than the results based on education stratification. The sample sizes for the stratified samples exceed the size of the over-all sample for the reason that respondents with a wage smaller than  $w_L$  are deleted. The results are taken to be the average over the (sub)samples at hand.

Table 2 presents the implications of the parameter estimates for the variables of interest. It is convenient to discuss the contents of both tables simultaneously. In almost all cases,  $\lambda_0$  exceeds  $\lambda_1$ . As a result, for any value of the reservation wage  $r(p)$  generally exceeds the benefits level  $b$ . On average,  $r(p)$  (which is the mean of  $r(p)$  over  $p$ , conditional on  $p > w_L$ ) exceeds the minimum wage. All arrival rates are monotonically decreasing in age and productivity monotonically increases in age. The variables  $E_f(w)$  and  $E(w)$  note the expectation over  $p$  of the average wage offer given  $p$  and the average wage given  $p$ , all conditional on  $p > w_L$ . These also increase in age. The difference between  $E(p|p > w_L)$  and  $E(w)$  is small. This is primarily

The average unemployment rate is a bit higher than it is in the results, i.e.,  $b$  (within the range of Section 2). The data.

Table 1 presents the decomposition of the variation in the variables from the first interview.

It turns out that the attribute of education by stratification as a population offer varies more than are less dependent on the

Now : data at the first interview served as a small. For the first interview be interesting. Assume that the mean that is a bit small in the probability distribution. Information then the wage data

the expected numbers of job offers in employment is quite large, so that most individuals may move swiftly from a job with a moderate wage to a job with a wage close to the productivity level. For teenagers, an additional reason is that  $w_L$  is generally close to  $p$ , so that the range of wages is strongly affected by the minimum wage.

On average, frictional and structural unemployment equal 3.4% and 10.2%, respectively. Thus, unemployment is mainly structural. Frictional unemployment is largest for teenagers. This is mainly due to the disproportionately large layoff rate for these individuals. Structural unemployment is also largest for teenagers. For this group, the unconditional mean of  $p$  is relatively close to  $w_L$ . Indeed, most probability mass of  $p$  is in the neighbourhood of  $w_L$ . It turns out that for almost 20% of all teenagers their mandatory minimum wage exceeds their productivity, even though their minimum wages are smaller than those for older individuals.

The amount of structural unemployment is not a monotone function of age. The group with the second largest amount of structural unemployment is the age group 39-61. This may be somewhat surprising in light of the fact that this group has the highest mean productivities and wages of all age groups. However, the variance of the productivities for the age group 39-61 is much larger than the corresponding variance for the other groups. Because of this, the left tail of the productivity distribution is relatively fat, and almost 10% of the individuals has  $p < w_L$ . This age group presumably contains a relatively large number of individuals whose skills are not in demand anymore, and/or individuals who have lost their skills, and/or individuals who are disabled.

The estimated model can be used to examine the effect of a 10% increase of the mandatory minimum wage. It turns out that on average this would increase structural unemployment from 10.2% to 17.0%. This is substantial, and exceeds the estimated effects in other Dutch studies like Van Soest (1989), although it is only marginally larger than the estimated effect in Van den Berg and Ridder (1992). The increase of structural unemployment implies that the number of individuals who are not structurally unemployed decreases, so that with invariant arrival rates  $\lambda_0$  and  $\delta$  the absolute percentage of frictional unemployment decreases. However, the latter change is negligible in comparison to the change in structural unemployment. Note that the estimated model can also be used to examine the effects of decreases of  $w_L$ . Given the continuity of the productivity density, these effects will roughly be symmetrically opposite to the effects of increases of  $w_L$ . Recall however from the previous section that such results are sensitive to an untestable assumption.



The age group 23-29 has the largest relative and absolute increase in structural unemployment following a 10% increase in  $w_L$ . The reasons that these increases are a bit larger than they are for teenagers are, first of all, that the absolute level of  $w_L$  is larger than it is for teenagers, so that a 10% increase entails a larger increase in absolute terms, and secondly that the variance of  $p$  is a bit smaller than it is for teenagers. Before we move on to examine other implications of the results, it may be worth noting that changing the unemployment benefits level  $b$  (within reasonable bounds) does not affect unemployment at all. As noted in Section 2, this is in accordance to the previous empirical research based on Dutch data.

Table 3 presents decompositions of the total variation in wage offers (i.e. decompositions of the variance of wage offers over the whole population). The variation due to frictions is defined as the mean over  $p$  (and over the stratification variables) of the variation in wage offers conditional on  $p$ . This variation stems from the fact that wage offers are dispersed in the homogeneous model.

It turns out that most wage offer variation is due to productivity variation. Note that most of the variation due to the stratification variables can also be attributed to productivity variation. The improvement in the fit that is obtained by stratifying the data partly reflects the inadequacy of the Log normal distribution as a description of the productivity heterogeneity distribution over the whole population. In any case, from an empirical point of view it is clear that the wage offer variation due to search frictions is not extremely important. This is even more true for the variation in paid wages, since in the homogeneous model these are less dispersed than wage offers.

Now let us compare the estimated total unemployment rates to those in the data at the first interview. Except for individuals with low education, the observed rates are over-estimated. However, the size of the difference is generally small. For example, the over-all unemployment estimate is 13.5%, whereas at the first interview 11.8% of the respondents was unemployed. Nevertheless, it may be interesting to examine this more closely, since this is a parameter of interest. Assume that the rates  $\lambda_0$  and  $\delta$  are estimated correctly. Then these differences mean that the fraction of structurally unemployed individuals in the data is a bit smaller than it is according to the estimated model. Now recall the remarks in the previous section on the overidentification of the model with Log normally distributed  $p$ . Basically, here both the wage data and the duration data provide information on the size of structural unemployment. If the model is misspecified then these pieces of information may seem to be in conflict. Apparently, the wage data suggest a larger fraction of structurally unemployed individuals than

the duration data. Now one may think of two explanations for this.

First of all, it may be that the sample is not random in the sense that structurally unemployed individuals are a bit under-represented. This may well be true. Structurally unemployed individuals will never find a job, so they may classify themselves as being a nonparticipant when being questioned on their labour market state. In that case they are not included in the sample. However, the state of nonparticipation also includes individuals who are not active on the labour market but would not be structurally unemployed if they would be active. For example, it includes all mothers who are at home full time, and it includes individuals who were formally classified as being disabled in an era in which these classification rules were rather lenient. The data we use do not enable making a distinction between these different groups of nonparticipants. Therefore we cannot deal with this any further.

Now consider a second explanation of the phenomenon above. As shown in the previous section, the information in the wage data on the size of structural unemployment originates from the fact that the Log normal distribution satisfies the recoverability condition. This information is therefore crucially dependent on the Log normality assumption. It may therefore be that the productivity distribution (over-all as well as per stratum) is misspecified in the sense that in reality it is not Log normal.

To shed more light on the specification of the productivity distribution, we examine the fit of the model to the data on  $w_1$  (the wage at the first interview of employed individuals). Note that the distribution of  $w_1$  itself is not Log normal, but instead follows from the joint distribution of  $w_1, p$  (see Section 4). However, almost all variation in  $w_1$  originates from the variation in  $p$ , and there is a positive correlation between  $w_1$  and  $p$ . Therefore there is a strong similarity between the distribution of  $w_1$  and the distribution  $p$  truncated from below at  $w_L$ . More importantly, we can obtain information on particular types of misspecification of the distribution of  $p$  from the fit of the model to the  $w_1$  data.

Figures 1-5 present graphs comparing the data on  $w_1$  to the fitted distribution, for the over-all sample as well as for the subsamples obtained by stratification on age. In all cases the model is able to explain the fact that the sample distribution of  $w_1$  is skewed to the right. Also, the left tail of the latter distribution is explained reasonably well. However, in all cases the model fails to capture the size of the peak at the mode of the sample distribution. Moreover, the extreme right-hand tail of the fitted distribution is too thin in comparison to the sample distribution, whereas in the area between the mode and the extreme right-hand tail, the fitted density is too high. It seems therefore that the productivity distri-

bution is not well specified on the interval from the mode up to infinity. Formal Chi-square goodness-of-fit tests result in rejection for all groups except teenagers. To what extent this affects the other parameter estimates and the estimated size of structural unemployment can only be inferred by estimating models with other distributions for  $p$ . This is beyond the scope of the paper.

## 7 Conclusion

In this paper we have constructed and estimated an equilibrium job search model that allows for a distinction between frictional and structural unemployment. This model is based on the equilibrium search model as developed by Burdett and Mortensen. Modelling the distinction is basically established by introducing heterogeneity in the productivity of individuals over the population. In the homogeneous search model, unemployment is only frictional, originating from the fact that people have to wait for some time to find a job. In the heterogeneous model, individuals are permanently unemployed if their productivity is smaller than their unemployment benefits or the mandatory minimum wage.

We pay particular attention to the identification of the model. We show that the model is nonparametrically identified. In contrast to ad hoc models for minimum wage effects, the magnitude of structural unemployment is well-identified. This is basically because we exploit the fact that the equilibrium search model explains both wage data and duration data in terms of the same deep structural “parameters”. If we parameterize the productivity distribution in our model then, for certain choices for the family of this distribution, the model becomes overidentified. This can be exploited to test the specification.

The results show that structural unemployment is empirically more important than frictional unemployment. Both frictional and structural unemployment are largest for teenagers. It turns out that for almost 20% of all teenagers their mandatory minimum wage exceeds their productivity, even though their minimum wages are smaller than those for older individuals. The group with the second largest amount of structural unemployment is the age group 39-61. The productivity dispersion within this group is relatively large. This probably reflects the fact that there are many older individuals whose skills are not in demand anymore or who do not have many skills at all.

The distinction between frictional and structural unemployment is particularly relevant for policy analysis. The estimated model can be used to examine the effect of changes in the mandatory minimum wage on the magnitude of structural unemployment. It turns out that on average a 10% increase in the minimum

wage would increase structural unemployment from 10.2% to 17.0%.

Several topics for further research emerge. The most restrictive aspect of the theoretical model concerns the assumption that there is only between-market heterogeneity. It is clear that in reality individuals have the option to move from low-productivity jobs to high-productivity jobs. A synthesis with the equilibrium-search model allowing for within-market heterogeneity (see the pivotal paper by Eckstein and Wolpin (1990)) seems desirable.

On a more modest level, the results indicate that the Log normality assumption for the productivity distribution is too restrictive. Even the models that are estimated separately for different age and education categories are not able to explain the right-hand tail of the wage distribution very well. It is not clear to what extent this affects the estimates of the quantities of interest. Nevertheless, it may be interesting to estimate models with more flexible families of productivity distributions.

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# Appendix: Estimates, equilibrium implications, variance decomposition and fitted distributions

Table 1: Parameter estimates for the whole sample, and for subsamples stratified on age and education. Rates are per week. Standard errors in parentheses.

Parameters	$\lambda_0$	$\lambda_1$	$\delta$	$\mu_p$	$\sigma_p$	N	$\log L$
all	.075 (.005)	.057 (.011)	.003 (.0001)	7.62 (.008)	.30 (.005)	1767	-20697
educ.level 1	.063 (.001)	.049 (.016)	.003 (.0002)	7.50 (.011)	.23 (.006)	601	-6463
educ.level 2	.090 (.013)	.055 (.016)	.003 (.0002)	7.60 (.011)	.26 (.006)	753	-8823
educ.level 3	.080 (.011)	.071 (.021)	.003 (.0003)	7.82 (.018)	.33 (.012)	433	-550
age 16-22	.127 (.019)	.156 (.085)	.012 (.002)	7.13 (.022)	.25 (.015)	196	-1978
age 23-29	.121 (.010)	.067 (.018)	.007 (.0007)	7.51 (.001)	.17 (.006)	449	-5191
age 30-38	.092 (.011)	.028 (.010)	.003 (.001)	7.67 (.012)	.27 (.007)	577	-7138
age 39-61	.064 (.007)	.012 (.004)	.002 (.0002)	7.72 (.015)	.34 (.010)	635	-7032

Table 2: Eq ilibrium Implications; amounts in guilders per month									
		All	16-22	23-29	30-38	39-61	edul	edu2	edu3
$b$		1192	807	1120	1248	1320	1177	1189	1219
$E(r)$		1424	696	1432	1922	2242	1339	1539	1380
$w_L$		1401	1000	1450	1450	1450	1379	1399	1440
$E(p)$		2137	1295	1852	2219	2393	1856	2072	2622
$E(p p > w_L)$		2241	1395	1903	2289	2518	1943	2148	2688
$sdev(p)$		650	340	321	598	845	438	549	878
$E_f(w)$		1968	1253	1745	2103	2355	1751	1927	2256
$E_g(w)$		2203	1366	1863	2234	2461	1915	2115	2632
Frictional unem.		0.034	0.071	0.048	0.033	0.025	0.036	0.030	0.039
Structural unem.		0.102	0.192	0.091	0.070	0.099	0.123	0.084	0.083
Estimated unem.		0.135	0.263	0.140	0.103	0.123	0.159	0.114	0.120
Observed unem.		0.118	0.224	0.111	0.081	0.113	0.181	0.093	0.072
<u>min. wage +10%</u>									
Frictional unem.		0.031	0.061	0.041	0.031	0.023	0.031	0.027	0.037
Structural unem.		0.170	0.308	0.218	0.132	0.156	0.226	0.155	0.130



Table : Decomposition of variation in wage offers

Model:	no stratification	stratified on age	stratified on educ.level
due to productivity	71%	57%	41%
due to frictions	29%	10%	18%
due to age		33%	
due to education			41%

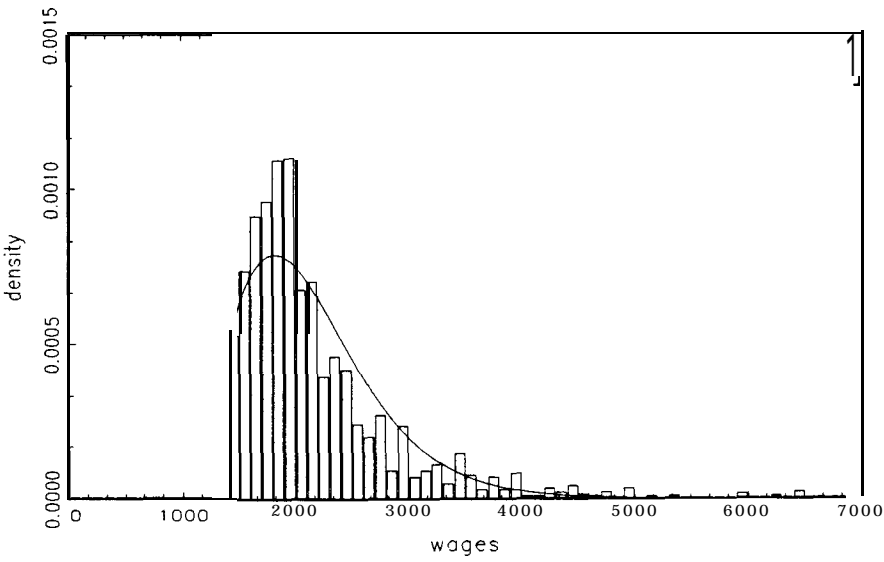


Figure 1: Fitted and observed wage earnings distribution for whole sample

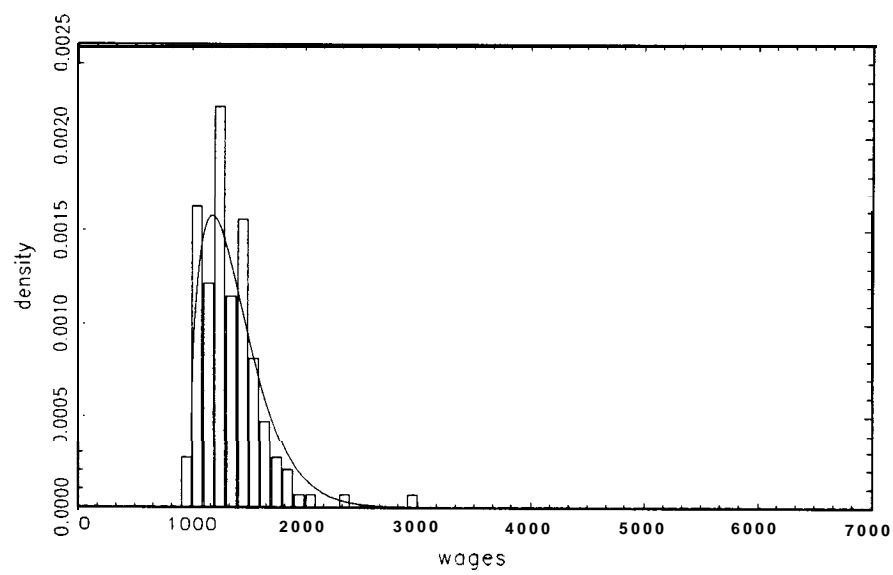


Figure 2: Fitted and observed wage earnings distribution for age 16-22

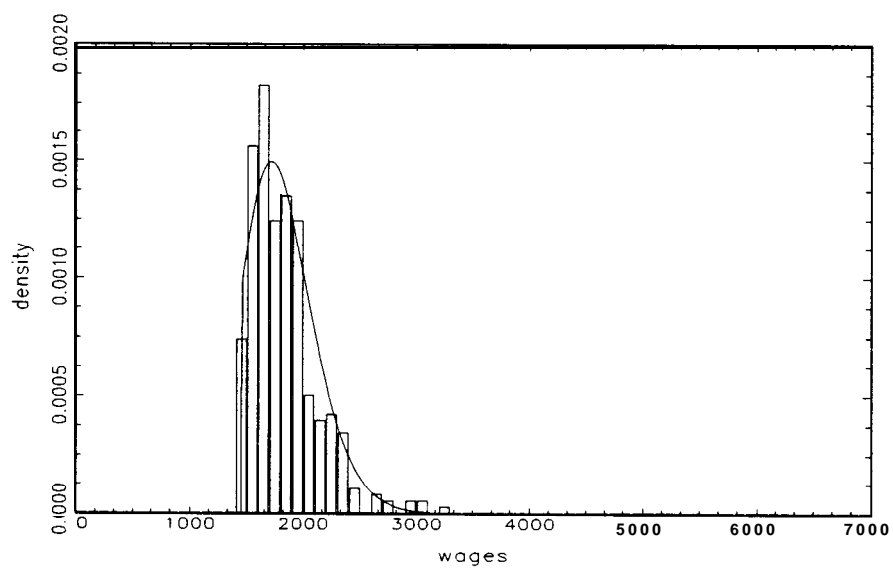


Figure 3: Fitted and observed wage earnings distribution for age 23-29

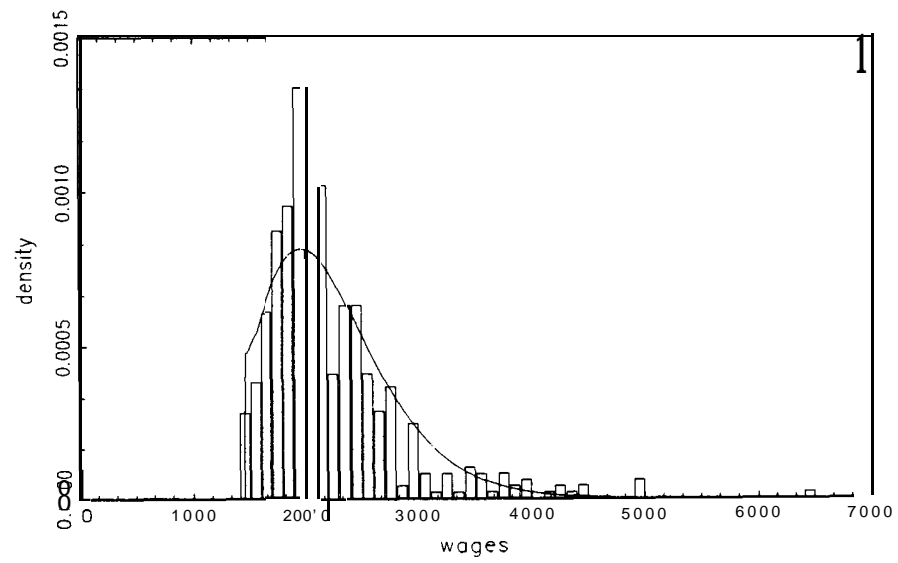


Figure 4: Fitted and observed wage earnings distribution for age 30-38

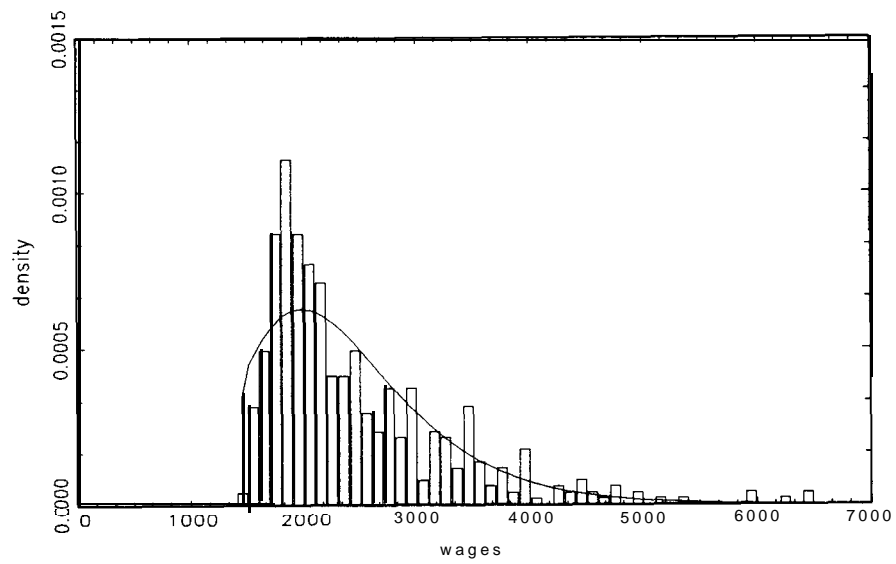


Figure 5: Fitted and observed wage earnings distribution for age 39-61