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published in

Economics Letters
2018

DOI (link to publisher)

[10.1016/j.econlet.2018.07.023](https://doi.org/10.1016/j.econlet.2018.07.023)

document version

Publisher's PDF, also known as Version of record

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citation for published version (APA)

Angelini, G., & Gorgi, P. (2018). DSGE Models with observation-driven time-varying volatility. *Economics Letters*, 171, 169-171. <https://doi.org/10.1016/j.econlet.2018.07.023>

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DSGE Models with observation-driven time-varying volatility

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HIGHLIGHTS

- We allow for time-varying volatility in DSGE models.
- Variances evolve over time via an observation-driven updating equation.
- Time-varying volatility is particularly useful in forecasting.

ARTICLE INFO

Article history:

Received 5 December 2017

Received in revised form 7 May 2018

Accepted 13 July 2018

Available online 17 July 2018

JEL classification:

C32

C5

Keywords:

DSGE models

Score-driven models

Time-varying parameters

ABSTRACT

This paper proposes a novel approach to introduce time-variation in the variances of the structural shocks of DSGE models. The variances are allowed to evolve over time via an observation-driven updating equation. The estimation of the resulting DSGE model can be easily performed by maximum likelihood without the need of time-consuming simulation-based methods. An empirical application to a DSGE model with time-varying volatility for structural shocks shows a significant improvement in the accuracy of density forecasts.

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1. Introduction

There is an expanding literature on estimating DSGE models with time-varying parameters. Amongst others, [Fernández-Villaverde et al. \(2007\)](#) specify stochastic processes for a subset of the structural parameters and [Galvão et al. \(2016\)](#) propose a Bayesian method that introduces time variation in the estimation of the model. The inclusion of time-variation in parameters can be useful to improve the forecasts of DSGE models. [Diebold et al. \(2017\)](#), [Cúrdia et al. \(2014\)](#) and [Justiniano and Primiceri \(2008\)](#) show that accounting for time-varying variances in structural shocks plays a major role in enhancing the forecasting performance of DSGE models. The authors consider DSGE models where the specifications of the variances of the structural shocks are unobserved stochastic volatility processes.

In this paper we propose a new approach to introduce time-varying volatility in structural shocks of DSGE models. We allow the variances to follow an autoregressive process with innovation given by the score of the predictive likelihood. This method is based

on the Generalized Autoregressive Score (GAS) framework of [Creal et al. \(2013\)](#) and [Harvey \(2013\)](#).

We note that our method is very general and can accommodate time-variation in structural parameters of DSGE models. However, the practical implementation of the score-driven method is more challenging in this case because of identification problems for some structural parameters.

The proposed DSGE models with score-driven volatilities are easy to implement and they can be estimated by standard maximum likelihood without relying on simulation-based methods. This is in contrast to specifications of DSGE models with stochastic volatility. Finally, we show that our method is useful to enhance the accuracy of density forecasts. In particular, we implement a dynamic version of the DSGE of [An and Schorfheide \(2007\)](#) with time-varying variances for the innovation components of interest rate, supply and demand shocks. We report significant improvement in density forecasts compared to the DSGE model with constant variances.

2. DSGE with score-driven parameters

In this section we present a general approach to introduce time-variation in parameters of DSGE models. The approach is general

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Fig. 1. Estimation of the time-varying standard deviations $\sigma_{r,t}$, $\sigma_{g,t}$ and $\sigma_{z,t}$ based on U.S. quarterly data in the period 1984–2017. Shaded purple areas denote the 80% and 95% confidence bands computed using the approach of Blasques et al. (2016). Black dashed lines represent the estimated constant variances.

to accommodate time variation is structural parameters as well as volatility of structural shocks. We then focus on time-varying volatility for density forecasts in Section 3.

Let $Z_t = (Z_{1,t}, Z_{2,t}, \dots, Z_{n_z,t})'$ be an $n_z \times 1$ vector of endogenous variables and assume that, after a log-linearization, the economy is described by the following structural model:

$$\Gamma_{0,t}Z_t = \Gamma_{f,t}E_tZ_{t+1} + \Gamma_{b,t}Z_{t-1} + \Pi_t\varepsilon_t, \quad (1)$$

where ε_t is a $n_\varepsilon \times 1$ fundamental white noise term with covariance matrix $\Sigma_\varepsilon(\theta_t)$, and $\Gamma_{i,t} = \Gamma_i(\theta_t)$, $i \in \{0, b, f\}$ are matrices of dimension $n_z \times n_z$ and Π_t is a $n_z \times n_\varepsilon$, whose elements depend on the $n_\theta \times 1$ vector of time-varying parameters θ_t . The parameter vector θ_t can contain structural parameters as well as dynamic components of the volatility of ε_t . Assuming that there exists a unique stable solution of the system, the reduced form solution associated with the system (1) is:

$$Z_{m,t} = A(\theta_t)Z_{m,t-1} + B(\theta_t)\varepsilon_t, \quad (2)$$

$$y_t = C(\theta_t)Z_{m,t-1} + D(\theta_t)\varepsilon_t, \quad (3)$$

where y_t is the n_y -dimension vector of observed variables, $Z_{m,t}$ is the n_m -dimension sub-vector of Z_t that contains the candidate minimal states of the system, and $A(\theta_t)$, $B(\theta_t)$, $C(\theta_t)$ and $D(\theta_t)$ are time-varying matrices of parameters that depend non-linearly on θ_t through a set of Cross Equation Restrictions (CER), see Castelnovo and Fanelli (2015) for details.

Following the GAS framework of Creal et al. (2013) and Harvey (2013), the specification of the time-varying parameter vector θ_t in (2) and (3) is:

$$\theta_t = g(f_t), \quad f_{t+1} = \omega + \Phi f_t + \Lambda s_t, \quad (4)$$

where f_t is a $n_\theta \times 1$ vector, $g(\cdot)$ is a link function, ω , Φ and Λ are matrices containing static parameters to be estimated and s_t is the score innovation of the dynamic equation. More specifically, s_t is specified as

$$s_t = \frac{\partial \log p(y_t | \mu_t, \Sigma_t)}{\partial f_t}.$$

where $p(\cdot | \mu_t, \Sigma_t)$ denotes the density function of a multivariate normal with mean μ_t and covariance matrix Σ_t . The mean μ_t and covariance Σ_t are the conditional mean and covariance matrix of y_t obtained from the Kalman filter for the state space model in (2) and (3). We refer the reader to Dell Monache et al. (2016) and Bucchieri et al. (2017) for further applications of GAS time-varying parameters in the context of Kalman filtering. We note that in practice the score innovation s_t is typically not available in closed form and it can be computed using numerical differentiation. The estimation of the static parameters of the model can be performed by standard maximum likelihood through the Kalman filter.

3. Empirical illustration

The empirical analysis is based on the DSGE model of An and Schorfheide (2007) in which we allow for time-variation in the variances of structural shocks. The model is described by the following equations:

$$\tilde{x}_t = E_t\tilde{x}_{t+1} + g_t - E_tg_{t+1} - \tau^{-1}(r_t - E_t\pi_{t+1} - E_tz_{t+1}), \quad (5)$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa(\tilde{x}_t - g_t), \quad (6)$$

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)\psi_1\pi_t + (1 - \rho_r)\psi_2(\tilde{x}_t - g_t) + \varepsilon_{r,t}, \quad (7)$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}, \quad (8)$$

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \quad (9)$$

where $\varepsilon_{i,t} \sim WN(0, \sigma_{i,t}^2)$, $i = r, g, z$, (5) is a forward-looking output-gap equation where $\tilde{x}_t$ is the output gap, (6) is a forward-looking New-Keynesian Phillips Curve with inflation rate π_t , (7) is the monetary policy rule with policy rate r_t , while (8)–(9) define two autoregressive processes for the aggregate supply (g_t) and demand (z_t) disturbances (see An and Schorfheide (2007) for a discussion of the system in (5)–(9)). In the notation of (2)–(3), we have $Z_{m,t} = (r_t, g_t, z_t)'$ and $y_t = (y_t, \pi_t, r_t)'$. The variances $\sigma_{i,t}^2$, $i = r, g, z$, are given by

$$\sigma_{i,t}^2 = \exp(f_{i,t}), \quad f_{i,t+1} = \omega_i + \phi_i f_{i,t} + \lambda s_{i,t},$$

where the score innovation $s_t = (s_{r,t}, s_{g,t}, s_{z,t})'$ is specified as described in Section 2. The static parameters ϕ , λ and ω_i , $i = r, g, z$, are estimated by maximum likelihood, instead, the other parameters of the DSGE are calibrated to the values given in Komunjer and

Table 1

In-sample and out-of-sample results. The forecasting exercise is based on the last 5 years of the sample (from 2012 to 2017).

In-sample		
	Constant variances	Time-varying variances
Log-likelihood	115.35	130.54
AIC	−224.69	−251.09
Out-of-sample, log-score		
$\tilde{x}_t$	0.07	
π_t	0.08*	
r_t	−0.04	
Overall	0.07*	

*Denotes statistically significance at 5% level using the Diebold–Mariano test.

Ng (2011). In general, all the parameters can be jointly estimated by maximum likelihood but the calibration of some parameters may be preferred to avoid identification problems. The static DSGE model is obtained by setting ϕ and λ to zero.

We consider U.S. quarterly data in the period 1984:Q1–2017:Q3. Fig. 1 shows the estimated time-varying standard deviations $\sigma_{r,t}$, $\sigma_{g,t}$ and $\sigma_{z,t}$. The plots illustrate that the three variances are not constant over time assuming, for example, higher values during the recent financial crisis. The confidence bands are computed following the procedure in Blasques et al. (2016). We perform an in-sample and out-of-sample comparison between our approach and the static DSGE model. The out-of-sample forecasting study is based on the last 5 years of the sample (from 2012 to 2017) and the log-score criterion for density forecasts is employed as means of comparison, see Geweke and Amisano (2011). Table 1 reports the AIC criterion and the difference in log-score criterion between the dynamic and the static DSGE. The difference in log-scores is reported separately for density forecasts of inflation, output gap and interest rates but also jointly for the forecast of the joint density of the three variables. From Table 1, we can see that our model clearly has a better in-sample fit according to the AIC. As concerns the out-of-sample study, we note that our model is significantly better in forecasting inflation and the joint distribution of the three variables. Instead, the forecasts of output gap and interest rates are not significantly different. We can conclude that overall the empirical results underline how our time-varying DSGE model can outperform the static DSGE.

4. Conclusion

In this paper we have proposed a new way for incorporating time-varying variances in DSGE modeling. Our approach is easy to implement and the empirical results suggest a better in-sample and out-of-sample performance compared to DSGE model with

constant variances. Future research may focus on exploring the practical implementation of the proposed score-driven approach to introduce time-variation in structural parameters besides the variances of structural shocks.

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