A STRUCTURAL MODEL FOR THE COEVOLUTION OF NETWORKS AND BEHAVIOR

Chih-Sheng Hsieh, Michael D. König, and Xiaodong Liu*

Abstract—This paper introduces a structural model for the coevolution of networks and behavior. We characterize the equilibrium of the underlying game and adopt the Bayesian Double Metropolis-Hastings algorithm to estimate the model. We further extend the model to incorporate unobserved heterogeneity and show that ignoring this heterogeneity can lead to biased estimates in simulation experiments. We apply the model to study R&D investment and collaboration decisions in the chemical and pharmaceutical industry and find a positive knowledge spillover effect. Our model also provides a tractable framework for a long-run key player analysis.

I. Introduction

SINCE the seminal paper by Manski (1993), substantial progress has been made in the econometric analysis of networks following two research strands. The first strand studies the interdependence of individual behavior in a network under the assumption that the network structure is exogenously given. A popular model in this literature is the linear social-interaction model (Bramoulle, Djebbari, & Fortin, 2009; Lee, Liu, & Lin, 2010; Liu & Lee, 2010; Blume et al., 2015). The second strand focuses on the modeling and estimation of the network formation process, with some recent developments including Christakis et al. (2010), Snijders (2011), Graham (2015, 2017), Leung (2015), Boucher and Mourifié (2017), Mele (2017, 2018), Menzel (2017), Sheng (2017), Chandrasekhar and Jackson (2018), De Paula, Richards-Shubik, and Tamer (2018), Dzemski (2018), and Mele and Zhu (2019). To link these two research strands, we introduce a unified framework to model the coevolution of networks and behavior in this paper.

The microfoundation of our structural model is a network game where agents make decisions on actions and network links to maximize their utilities. The utility function is a generalization of the linear-quadratic utility function in Ballester, Calvó-Armengol, and Zenou (2006) by including direct payoffs from the network structure given by homophily/heterophily, congestion, and cyclic triangle effects. An important feature of the utility function is that it incorporates the two-way interdependence between networks and behavior (see figure 1 for an illustration). We show that under some mild assumptions, the utility function admits a potential function (Monderer & Shapley, 1996). The potential function aggregates individual incentives to change from the status quo and thus greatly simplifies the equilibrium analysis.

With a snapshot of the network and actions drawn from the stationary distribution, the structural parameters can be estimated based on the maximum likelihood principle. However, as Mele (2017) pointed out, the frequentist maximum likelihood method and Bayesian Metropolis-Hastings (MH) algorithm (Chib & Greenberg, 1995) are computationally infeasible due to the intractable normalizing constant in the Gibbs measure. To bypass the evaluation of the intractable normalizing constant, we adopt the Bayesian Double Metropolis-Hastings (DMH) algorithm (Liang, 2010; Mele, 2017) to sample from the posterior distribution of the structural parameters. Compared with Mele (2017), we face the additional complication as we need to simulate actions as well as network formation decisions. In Monte Carlo simulations, we find that ignoring unobserved heterogeneity leads to a biased estimate of the network spillover effect.

To illustrate the empirical relevance of our structural model, we apply it to study the interdependence of R&D investment and collaboration decisions in the chemical and pharmaceutical industry. Using a unique data set on R&D collaborations matched to firms’ balance sheets, we find a positively significant knowledge spillover effect on firms’ R&D investment decisions. We also find that an R&D collaboration is more likely to form between firms in the same subsector (the homophily effect), firms with different

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This paper has grown out of ideas in a manuscript circulated under the title “Network Formation with Local Complements and Global Substitutes: The Case of R&D Networks,” which is now split into two papers. In this paper, we propose a general econometric model for the coevolution of networks and behavior incorporating both network formation externalities and unobserved heterogeneity. In the second paper, we focus on the equilibrium characterization, estimation, and policy implications of dynamic R&D network models. We thank the editor, Bryan Graham, and three anonymous referees for valuable comments and suggestions. We also thank Vincent Boucher, Lung-fei Lee, Angelo Mele, Olivier Parent, Eleonora Patacchini, and the seminar participants at the Barcelona GSE summer forum (Networks: Information, Contracts, and Communities), the 2019 Asia Meeting of the Econometric Society, the 2019 China Meeting of the Econometric Society, the 29th Annual Meeting of the Midwest Econometrics Group, and the XIII World Conference of the Spatial Econometrics Association for helpful discussions. We further thank Christian Helmers for data sharing and Sebastian Ottinger for excellent research assistance.

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productivities (the heterophily effect), and firms with a common collaboration partner (the cyclic triangle effect). A firm is less likely to form a new link if it has many existing R&D collaborations (the congestion effect).

Finally, the proposed structural model has important policy implications as it allows the policymaker to identify the key player whose exit would have the largest impact on welfare in the long run. Conventional key player analysis assumes the links between the other agents are not affected by the exit of the key player (Ballester et al., 2006). This assumption makes sense if the key player analysis is considered as a short-run policy analysis, because it takes time for the other agents to adjust their links in response to the exit of the key player. However, in the long run, it is difficult to justify this assumption. Our structural model provides a tractable framework for a long-run key player analysis. In the empirical study, we find that the key player rankings in the short run and the long run do not coincide with each other. We also find that the key player ranking is correlated with both R&D expenditure and network centrality measures (including degree, betweenness, closeness, and eigenvector centralities) of a firm. Therefore, the key player ranking incorporates information on both R&D investment and network centrality of a firm.

Our paper is related to recent papers that study the identification and estimation of social interaction models with endogenous networks (Goldsmith-Pinkham & Imbens, 2013; Hsieh & Lee, 2016; Auerbach, 2019; Battaglini, Patacchini, & Rainone, 2019; Johansson & Moon, 2019; Lee et al., 2020). The focus of these papers is to consistently estimate social interaction effects controlling for the unobserved heterogeneity in the network formation process. In particular, Auerbach (2019) and Johansson and Moon (2019) take a fixed-effect approach and allow the unobserved heterogeneity to affect link formation nonparametrically. Since it is difficult to distinguish link externalities from a flexible form of unobserved heterogeneity (Graham, 2015), link externalities are excluded a priori from their network formation model. By contrast, this paper takes a random-effect treatment of the unobserved heterogeneity and develops a parametric framework that incorporates both link externalities and unobserved heterogeneity. Hsieh, Lee, and Boucher (2019) propose a two-stage network game, where the network is formed in the first stage, and in the second stage, individuals choose their actions taking the network as given. The equilibrium concept for the first stage of the game relies on a transferable utility assumption that allows agents to make side payments. The transferable utility assumption is reasonable for small networks but is difficult to justify when the network is large. The closest work to ours is Badev (2018), which proposes a network formation game where agents make decisions on binary actions and network links in the absence of unobserved heterogeneity. Our model considers continuous actions and allows for unobserved heterogeneity.

The rest of the paper is organized as follows. Section II introduces the structural model, section III presents the estimation strategy, section IV provides an empirical illustration, and section V concludes. The proofs and technical details are collected in the online appendix.

II. Structural Model

A. Preferences

Consider a network $g \in G$ consisting of a set of agents $N = \{1, \ldots, n\}$, where $G$ is the set of all networks with $n$ nodes. The topology of the network is represented by an $n \times n$ adjacency matrix $G = [g_{ij}]$, where $g_{ij} = 1$ if agents $i$ and $j$ form a link and $g_{ij} = 0$ otherwise. The network links are reciprocal, that is, $g_{ij} = 1$ implies $g_{ji} = 1$. As a normalization, we set $g_{ii} = 0$ for all $i \in N$. Let $N_i = \{j \in N | g_{ij} = 1\}$ denote the set of agent $i$’s peers (or, loosely speaking, “friends”).

Agent $i$, with his exogenous characteristics given by a row vector $X_i$, makes decisions on network links $g_{ij}$ and effort of action $y_i$ to maximize utility. We assume $X_i$ can be observed by all the agents. To introduce unobserved heterogeneity in the econometric model, we allow some components of $X_i$ to be unobservable to the econometrician. Let $Y = (y_1, \ldots, y_n)'$, and let $Y_{-i}$ denote the effort levels of all agents but $i$. The utility of agent $i$ follows a linear-quadratic function given by

$$U_i(g, Y, X) = a_i(g, X) + b(X_i)y_i + \lambda \sum_{j \in N} g_{ij}y_j - \frac{1}{2} \gamma^2,$$

where

$$a_i(g, X) = \sum_{j \in N} g_{ij} \left\{ \delta_0 + h(X_i, X_j, \delta_1) + \delta_2 \sum_{k \in N \setminus \{i, j\}} g_{ik} + \delta_3 \sum_{k \in N \setminus \{i, j\}} g_{ik}g_{jk} \right\}.$$  

The first term of equation (1), $a_i(g, X)$, measures the direct utility from links. In particular, $\delta_0$ is the fixed cost of maintaining links, and $h(X_i, X_j, \delta_1)$ captures the (dis)similarity between agents $i$ and $j$ in exogenous characteristics, with the coefficient vector $\delta_1$ representing the homophily or heterophily effect depending on its sign. $\sum_{k \in N \setminus \{i, j\}} g_{ik}$ is the total

1 We consider undirected network links to be consistent with the empirical study of R&D networks. If network links are directed, $a_i(g, X)$ can be specified in a similar manner as equation (1) in Mele (2017).
number of links of agent $i$ excluding the link $g_{ij}$, with the coefficient $β_2$ representing the congestion effect. $\sum_{k \in N \setminus \{i, j\}} g_{ik} g_{jk}$ is the number of common "friends" between agents $i$ and $j$, with the coefficient $β_3$ representing the cyclic triangle effect.\footnote{It is possible to include additional terms in $a_i(g, X)$ as long as there exists a corresponding potential function under suitable restrictions.}

We impose the following assumption on $h(X_i, X_j, δ_1)$ to guarantee the existence of a potential function.

**Assumption 1.** $h(X_i, X_j, δ_1) = h(X_j, X_i, δ_1)$ for any $i, j \in N$.

The second term of equation (1), $b(X_i)y_i$, measures the direct utility from effort, with the marginal utility of effort given by $b(X_i)$. The third term, $λ\sum_{j \in N} g_{ij}y_j$, is the social utility, with the coefficient $λ$ representing the spillover effect. Finally, we assume the cost of exerting effort is given by the last term of equation (1), $4\gamma^2 γ$, which exhibits increasing marginal cost. Maximizing equation (1) with respect to $y_i$, gives the best response function for the effort choice

$$y_i = λ\sum_{j \in N} g_{ij}y_j + b(X_i),$$

which coincides with the one in Ballester et al. (2006).

**Remark 1.** In the network formation game considered in Mele (2017), agents only make decisions on links $g_{ij}$ to maximize the direct utility from links. In our model, agents make decisions on links $g_{ij}$ as well as effort $y_i$, taking into account the direct utility from links, the direct utility from effort, and the social utility. In the social interaction models with endogenous networks (see Auerbach, 2019; Johnsson & Moon, 2019), links are assumed to be pairwise independent (conditional on observed and unobserved individual attributes). In our model, links are interdependent with externalities given by the congestion and cyclic triangle effects in $a_i(g, X)$. Furthermore, Auerbach (2019) and Johnsson and Moon (2019) assume actions depend on links but not the other way around (conditional on observed and unobserved individual attributes). In our model, the social utility component in equation (1) captures the two-way interdependence between actions and links. Badev (2018) considers a network formation game where agents make decisions on binary actions and network links. In our model, the action space is allowed to be continuous, and the utility function implies a best response function given by equation (3) that underlies many well-known linear social-interaction models in the literature (see Bramoulle et al., 2009; Liu & Lee, 2010; Blume et al., 2015).

### B. Coevolution of Networks and Behavior

Let the realization of the network in period $t$ be denoted by $g_t$ with the adjacency matrix $G_t = \{g_{ij,t}\}$, and let the network including all the current links but $g_{ij,t}$ be denoted by $g_{-ij,t}$. Similarly, the effort profile of $N$ in period $t$ is given by the vector $Y_t = \{y_{i,t}\}$, and the effort profile of $N \setminus \{i\}$ is written as $Y_{-i,t}$. To simplify notation, we drop $X$ from $U_i(g, Y, X)$ henceforth.

The coevolution of networks and behavior is specified as stochastic best-response dynamics (Blume, 1993). We assume time is discrete. Each time period is either a link-adjustment period (with probability $0 < p_0 < 1$) or an effort-adjustment period (with probability $1 - p_0$). In the following, we give details of these two adjustment periods and characterize the stationary distribution of the stochastic process.

**Link adjustment.** In a link-adjustment period, a pair of agents $i$ and $j$ is randomly selected from the population with probability $p(g_{-i,t}, X_i, X_j)$. To make the equilibrium analysis feasible, we impose the following assumption on the selection rule characterized by $p(g_{-i,t}, X_i, X_j)$.\footnote{See Mele (2017) for more discussion on the selection (or meeting) rule.}

**Assumption 2.** (i) $p(g_{-i,t}, X_i, X_j) = p(g_{-i,t}, X_j, X_i)$; (ii) $p(g_{-i,t}, X_i, X_j)$ does not depend on $g_{ij,t-1}$; and (iii) $p(g_{-i,t}, X_i, X_j) > 0$ for all $(i, j) \in N \times N$.

Conditional on being selected, agents $i$ and $j$ update the link $g_{ij}$ to maximize their utilities taking the rest of the network and effort choices as given. As in Mele (2017), we assume that agents do not take into account the effect of their decisions on the future effort choices and network evolution. To capture the uncertainty (from the perspective of the economist) in the link adjustment process, we introduce an idiosyncratic shock to the utility and assume that a link is formed if and only if it improves the average utility of agents $i$ and $j$ given by $U_{ij}(g, Y) = \{U_i(g, Y) + U_j(g, Y)\}/2$. More specifically, $g_{ij,t} = 1$ if and only if

$$U_{ij}(g_{ij,t-1}, Y_{-1}) + \epsilon_{ij,t}^{(1)} \geq U_{ij}(g_{ij,t-1}, Y_{-1}) + \epsilon_{ij,t}^{(0)},$$

where $\epsilon_{ij,t}^{(1)}$ and $\epsilon_{ij,t}^{(0)}$ are independent from each other, i.i.d. across links and time periods, and follow a Gumbel distribution with the distribution function $F(\epsilon) = \exp[-\exp(-\epsilon/\sigma^2)]$. The parameter $\sigma^2$ captures the level of “noise” in link adjustment decisions.\footnote{The parameter $\sigma^2$ can be identified because the coefficient of $\gamma^2$ is normalized to $-1/2$ in the utility function given by equation (1).}

**Effort adjustment.** In an effort-adjustment period, an agent $i$ is randomly selected from the population with probability $p(X_i)$. We assume any agent can be selected with positive probability in the following assumption.

**Assumption 3.** $p(X_i) > 0$ for all $i \in N$.

Conditional on being selected, agent $i$ updates the effort level $y_{it} \in \mathcal{Y}$ to maximize his utility, where $\mathcal{Y}$ is the set of all possible effort choices. We allow $\mathcal{Y}$ to be continuous and assume that, taking the network $g_{-i,t}$ and the effort levels of
the other agents $Y_{-it-1}$ as given, the probability that agent $i$ chooses an effort level in $Z \subset Y_i$ in period $t$ is given by

$$\Pr (y_i \in Z | g_{it} = g_{t-1}, Y_{-it-1}) = \frac{\int_Z \exp [\sigma^2 U_i (g_{t-1}, z, Y_{-i,t-1})] dz}{\int_{Y_i} \exp [\sigma^2 U_i (g_{t-1}, y, Y_{-i,t-1})] dy}.$$  \hspace{1cm} (5)

Similar to equation (4) in the link adjustment period, the probability given in equation (5) can be justified by an additive random utility model over a nonfinite choice set (McFadden, 1976), where the parameter $\sigma^2$ captures the level of noise in effort adjustment decisions. Equation (5) admits the probability density function,

$$p(y_i | g_{t-1}, Y_{-it-1}) = \frac{\exp [\sigma^2 U_i (g_{t-1}, y_i, Y_{-i,t-1})]}{\int_{Y_i} \exp [\sigma^2 U_i (g_{t-1}, y, Y_{-i,t-1})] dy}.$$  \hspace{1cm} (6)

**Equilibrium.** In the stochastic process described above, the coevolution of the network $g_t$ and effort choices $y_t$ follows a Markov chain. In the following proposition, we show that the Markov chain converges to a unique stationary distribution. Let $y$ denote the vector of all unknown parameters in the potential function defined in equation (7) and $\theta = (\gamma', \sigma^2'y')$.

**Proposition 1.** Let

$$Q(g, Y) \equiv Q(g, Y, X) = a(g, X) + \sum_{i \in N} b(X_i)y_i + \frac{1}{2} \sum_{i \in N} \sum_{j \in N} g_{ij}y_i y_j - \frac{1}{2} \sum_{i \in N} y_i^2,$$  \hspace{1cm} (7)

where

$$a(g, X) = \frac{1}{2} \sum_{i \in N} \sum_{j \in N} g_{ij} \left\{ \delta_0 + h(X_i, X_j, \delta_1) + \delta_2 \sum_{k \in N \setminus \{i, j\}} g_{ik} + \frac{2}{3} \delta_3 \sum_{k \in N \setminus \{i, j\}} g_{ik}g_{jk} \right\}.$$  

Under assumptions 1–3, the coevolution process of the network and behavior converges to a unique stationary distribution characterized by the Gibbs measure

$$\pi(g, Y | \theta) = c(\theta)^{-1} \exp [\sigma^2Q(g, Y | \gamma)],$$  \hspace{1cm} (8)

where $c(\theta) = \sum_{g \in G} \int_{Y_i} \exp [\sigma^2Q(g, Y | \gamma)] dy$.

**Remark 2.** $Q(g, Y)$ defined in equation (7) is known as the potential function (Monderer & Shapley, 1996). As the change in the utility of an agent (or the average utility of a pair of agents) from adjusting his effort level (or their link) is identical to the corresponding change in the potential function, the potential function keeps track of individual incentives to deviate from the status quo and thus greatly simplifies the equilibrium characterization of the coevolution process.

**Remark 3.** The Gibbs measure defined in equation (8) bears a resemblance to that in an exponential random graph model (ERGM) (Pattison & Wasserman, 1996). Nevertheless, there is an important difference. As the ERGM only models the network formation process without taking individual behavior into account, the corresponding Gibbs measure is a distribution of networks. By contrast, our framework jointly models the interdependent link and effort adjustment processes, and the resulting Gibbs measure is a joint distribution of networks and efforts. This poses a novel challenge for the estimation of model parameters.

**C. Key Player Analysis**

An advantage of the proposed structural model (compared to a reduced-form model) is that it can be used by policymakers to conduct counterfactual studies. In this paper, we focus on a particular counterfactual study: key player analysis (Zenou, 2016). The key player analysis measures the importance of a node according to the reduction in the total activity level (Ballester et al., 2006) or social welfare (König, Liu, & Zenou, 2019) were it to be removed from the network. This analysis has important policy implications. Take the interfirm R&D network as an example. The exit of a firm from the network could be due to either financial reasons, such as the recession experienced by the American automobile manufacturing industry during the global financial downturn of 2007–2008, or legal reasons, such as the emission-fraud scandal of Volkswagen in 2015. In the former case, the key player analysis can help the policymaker to know the overall welfare gain of bailing out a bankrupting firm, while in the latter case, the key player analysis can help the policymaker know the overall welfare cost by inflicting high penalties that might threaten the continued existence of a firm.

Conventional key player analysis assumes that the network is exogenously given and does not adapt to the removal of a node (henceforth referred to as the invariant network assumption). For a short-run key player analysis, this assumption is reasonable because it takes time for the network to rewire after a node is removed. However, to conduct a long-run key player analysis, it is desirable to relax this assumption and develop a model that allows the remaining network to evolve to a new equilibrium. Our model can be used for this purpose.

In the long run, the key player is the agent whose removal from the network leads to the largest reduction in expected social welfare. More specifically, the reduction in expected...
The computational problem still exists as $c(\theta)$ and $c(\tilde{\theta})$ in the acceptance probability do not cancel each other.

A way to bypass the evaluation of the intractable normalizing constant $c(\theta)$ is to use the exchange algorithm (Møller et al., 2006; Murray, Ghahramani, & MacKay, 2006) as follows.

**Algorithm 1** (Exchange Algorithm). At each iteration:

1. **Step 1:** Draw $\tilde{\theta}$ from the proposal distribution $q_0(\tilde{\theta}|\theta)$.

2. **Step 2:** Generate $(\tilde{g}, \tilde{Y})$ from the distribution $\pi(\tilde{g}, \tilde{Y} | \tilde{\theta})$ using a perfect sampler.

3. **Step 3:** Accept $\tilde{\theta}$ according to the acceptance probability

$$
\alpha_{0,EX} = \min \left\{ \frac{1}{\frac{p(\tilde{\theta}|g, Y)q_0(\tilde{\theta}|\theta)p(\theta|g, Y)q_0(\theta|\tilde{\theta})}{p(\theta|g, Y)q_0(\theta|\tilde{\theta})\pi(\tilde{g}, \tilde{Y} | \tilde{\theta})}} \right\}
$$

$$
= \min \left\{ \frac{1}{\exp[\sigma^{-2}Q(g, Y | \gamma)]p(\tilde{\theta}|g, Y)q_0(\theta|\tilde{\theta})\exp[\sigma^{-2}Q(\tilde{g}, \tilde{Y} | \gamma)]} \right\}
$$

**Proposition 2.** The unique stationary distribution of algorithm 1 is $p(\theta|g, Y)$.

**B. Double Metropolis-Hastings Algorithm**

In the second step of the exchange algorithm, we need to generate auxiliary data using a perfect sampler (Propp & Wilson, 1996), which is computationally costly. To overcome this issue, Liang (2010) and Mele (2017) propose a DMH algorithm, which uses a finite run of the MH algorithm initialized at the observed $(g, Y)$ to generate auxiliary data $(\tilde{g}, \tilde{Y})$. More specifically, at each iteration, the DMH algorithm follows the same steps as the exchange algorithm with the second step replaced by:

1. **Step 2**: Generate $(\tilde{g}, \tilde{Y})$ from the distribution $\pi(\tilde{g}, \tilde{Y} | \tilde{\theta})$ using a finite run of the MH algorithm initialized at the observed $(g, Y)$.

Compared with Mele (2017), one additional complication is that we need to simulate both networks $\tilde{g}$ and effort profile $\tilde{Y}$ in step 2* of the DMH algorithm. To generate auxiliary data $(\tilde{g}, \tilde{Y})$, one could design a sampler following the process described in section IIIB. However, the convergence of such a sampler could be slow in practice. To improve convergence and reduce computational burden, we propose the following MH algorithm to generate $(\tilde{g}, \tilde{Y})$:
Algorithm 2 (Auxiliary Data Generation). Given $\theta$, at each iteration:

Step 1: Draw $\tilde{g}$ from the proposal distribution $q_g(g|g)$. Let $\tilde{G}$ denote the adjacency matrix of $\tilde{g}$.

Step 2: Generate $\bar{Y} \sim N(\tilde{Y}^*, \Sigma_{\bar{Y}})$, where $\tilde{Y}^* \equiv (I_n - \lambda G)^{-1} B(X)$, with $B(X) = \{b(x_1), \ldots, b(x_n)\}'$, is the equilibrium effort vector derived from the best response function (3), and $\Sigma_{\bar{Y}} = \sigma_1^2(I_n - \lambda G)^{-1}$.

Step 3: Accept $(g, \bar{Y})$ according to the acceptance probability,

$$
\alpha_{(g; Y), \text{MH}} = \min \left\{ 1, \frac{\pi(g, \tilde{Y}^{*}|\theta)p_{Y}(Y|g)q_{g}(g|g)}{\pi(g, Y|\theta)p_{Y}(Y|g)q_{g}(g|g)} \right\}
$$

where $p_{Y}(Y|g)$ denotes the density function of $N(\tilde{Y}^{*}, \Sigma_{\bar{Y}})$.

In the following proposition, we show that the long-run stationary distribution of the proposed MH algorithm is the Gibbs measure defined in equation (8).

Proposition 3. The unique stationary distribution of Algorithm 2 is $\pi(g, Y|\theta)$.

Remark 4. If step 1 of algorithm 2 adopts a local sampler, where only one randomly selected link is updated at each iteration, the convergence can be slow, as shown in Mele (2017). Therefore, we follow Mele’s suggestion (see appendix B of Mele, 2017) to allow for large steps, where multiple links are swapped at the same time, to improve convergence.

Remark 5. In step 2 of algorithm 2, we generate $\bar{Y}$ from a multivariate normal distribution because (a) it is computationally simple to sample from a normal distribution, and (b) it resembles the effort adjustment process described in section II.B. To see the second point, we assume that link adjustment periods arrive much less frequently than effort adjustment periods in the coevolution process. Given the network $\tilde{g}$, it follows a standard Gibbs sampler argument that the transition density defined in equation (6) converges to

$$
p_{\bar{Y}}(\bar{Y}|g) = \frac{\exp[\sigma^{-2}Q(\bar{g}, \bar{Y})]}{\int_{Y} \exp[\sigma^{-2}Q(\bar{g}, Y)]dY},
\tag{10}
$$

where

$$
Q(g, Y) = a(g) + \sum_{i \in N} b(X_i) y_i + \frac{\lambda}{2} \sum_{i \in N} \sum_{j \in N} g_{ij} y_i y_j - \frac{1}{2} \sum_{i \in N} y_i^2
$$

$$
= a(g) + B(X) Y - \frac{1}{2} Y'(I_n - \lambda G) Y.
\tag{11}
$$

Inserting equation (11) into equation (10), it follows by the Gaussian integral formula (Bronshtein et al., 2015) that

$$
p_{\bar{Y}}(\bar{Y}|g) = \frac{\exp[\sigma^{-2}B(X) \bar{Y} - \frac{1}{2} \sigma^{-2} \bar{Y}'(I_n - \lambda G) \bar{Y}]}{\int_{Y} \exp[\sigma^{-2}B(X) Y - \frac{1}{2} \sigma^{-2} Y'(I_n - \lambda G) Y]dY}
$$

$$
= (2\pi)^{-n/2} |\det \Sigma_{\bar{Y}}|^{-1/2} \exp \left[ \frac{1}{2} (\bar{Y} - \bar{Y}^{*})' \Sigma_{\bar{Y}}^{-1} (\bar{Y} - \bar{Y}^{*}) \right],
$$

which is the density function of $N(\tilde{Y}^{*}, \Sigma_{\bar{Y}})$.

Remark 6. In algorithm 2, we often need to evaluate $(I_n - \lambda G)^{-1}$ and $\det(I_n - \lambda G)$, where $G$ is the adjacency matrix of the network $\tilde{g}$ resulting from adding or removing a link to or from the network $g$. The computational cost of the inverse and determinant can be high when the network size is large. To alleviate the computational burden, we adopt a matrix perturbation technique detailed in the online appendix and derive a result that facilitates the computation of $(I_n - \lambda G)^{-1}$ and $\det(I_n - \lambda G)$ when $(I_n - \lambda G)$ is known.

C. Unobserved Heterogeneity

The structural model introduced in section II allows some component of exogenous characteristics $X_i$ to be unobservable to the econometrician. More specifically, let $X_i = [X_i^O, x_i^U]$, where $X_i^O = (x_{i1}, \ldots, x_{ik})$ is a $K$-dimensional vector of exogenous characteristics observable to the econometrician and $x_i^U \sim i.i.d. (0, \varsigma_i^2)$ is a scalar random variable capturing unobserved heterogeneity. Further, let $Z_i$ be a vector of dyad-specific exogenous characteristics based on $X_i^O$ and $X_i^U$. For example, one could define the $l$th element of $Z_i$ as $z_{ij,l} = |x_{il}^0 - x_{jl}^0|$ if $x_{il}^0$ is a continuous variable or $z_{ij,l} = 1(x_{il}^0 = x_{jl}^0)$ if $x_{il}^0$ is a binary indicator variable. In the empirical model, we assume that $b(X_i)$ in equation (1) is given by

$$
b(X_i) = \beta_0 + X_i^O \beta_1 + \beta_2 x_i^U
\tag{12}
$$

and $h(X_i, X_j, \delta_1)$ in equation (2) is given by

$$
h(X_i, X_j, \delta_1) = Z_i \delta_1 + x_i^U + x_j^U.
\tag{13}
$$

As the unobserved heterogeneity shows up in both the direct utility from links and the direct utility from effort, it introduces an additional layer of correlation between links and effort.

We regard $x^U = (x_1^U, \ldots, x_n^U)'$ as individual random effects with a density function denoted by $p(x^U)$. Instead of sampling $\theta$ from the marginal posterior distribution,

$$
p(\theta|g, Y, x^U) = \int p(\theta|g, Y, x^U, x^U) p(x^U) dx^U,
$$

which does not have a closed-form expression, we adopt the Bayesian data augmentation approach (Tanner & Wong, 1987; Albert & Chib, 1993) to sample $\theta$ together with $x^U$ from the joint posterior distribution $p(\theta, x^U | g, Y) \propto$
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TABLE 1.—MONTe CARLO SIMULATION RESULTS (PART I)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True Value</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.0300</td>
<td>0.0463</td>
<td>0.0280</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.5000</td>
<td>0.4516</td>
<td>0.5029</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.5000</td>
<td>0.4930</td>
<td>0.5009</td>
</tr>
<tr>
<td>$\delta_0$</td>
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<td>-2.8531</td>
<td>-2.4361</td>
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<tr>
<td>$\beta_1$</td>
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<td>0.5332</td>
</tr>
<tr>
<td>$\beta_2$</td>
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<td>-0.2860</td>
</tr>
<tr>
<td>$\delta_1$</td>
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<td>0.1927</td>
<td>0.1423</td>
</tr>
<tr>
<td>$\sigma^2$</td>
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<td>0.6562</td>
<td>0.4755</td>
</tr>
<tr>
<td>$\xi^2$</td>
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<td>1.1684</td>
<td>0.4755</td>
</tr>
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</table>

Model 1 ignores unobserved heterogeneity, and model 2 controls for unobserved heterogeneity. Standard deviations in parentheses.

TABLE 2.—MONTe CARLO SIMULATION RESULTS (PART II)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True Value</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
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<tbody>
<tr>
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<td>-0.0027</td>
<td>0.0316</td>
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<tr>
<td>$\beta_1$</td>
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<td>$\beta_2$</td>
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<tr>
<td>$\beta_2$</td>
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<td>$\delta_1$</td>
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<tr>
<td>$\sigma^2$</td>
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<td>0.4663</td>
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<tr>
<td>$\xi^2$</td>
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<td>1.1993</td>
<td>0.1549</td>
</tr>
</tbody>
</table>

Model 1 ignores unobserved heterogeneity, and model 2 controls for unobserved heterogeneity. Standard deviations in parentheses.

$\pi(g, Y|\theta, x^U)p(\theta)p(x^U)$ in the MCMC procedure. The details of the MCMC procedure are in the online appendix.

D. Monte Carlo Experiments

We conduct a Monte Carlo simulation with 100 repetitions to examine the performance of the proposed MCMC procedure. In each repetition, we generate a network of size $n = 100$ and the corresponding effort levels according to the Gibbs measure defined in equation (8). The detailed data-generating process (DGP) runs as follows. First, we generate exogenous individual characteristics $x_i^O$ and $x_i^I$ in equation (12) from log-normal distribution $\ln x_i^O \sim N(1.5, 0.5)$ and normal distribution $x_i^I \sim N(0, 1)$, respectively, with the coefficients $\beta_1 = 0.5$ and $\beta_2 = 0.5$.

The dyad variable $Z_{ij}$ in equation (13) is generated by $Z_{ij} = |x_i^O - x_j^O|$ using the variable $x_i^O$ previously produced. We set the spillover effect $\lambda = 0.03$, the fixed linking cost $\delta_0 = -2.5$, the homophily effect $\delta_1 = 0.5$, the congestion effect $\delta_2 = -0.25$, the cyclic triangle effect $\delta_3 = 0.15$, and the noise parameter $\sigma^2 = 0.5$.

Then we generate the network and effort levels by algorithm 2 in section IIIIB with 1 million iterations and treat the realization of the last iteration as the generated sample. On average, the generated network has the average degree of 3.306, the clustering coefficient is 0.028. The average effort level is 2.952.

We perform the MCMC procedure for estimation, with and without controlling for unobserved heterogeneity, for 20,000 iterations. We drop the first 10,000 draws for burn-in and use the rest of the draws for computing the posterior mean as the estimate of each parameter. The simulation results are reported in table 1, and the values reported are the mean and standard deviation of the 100 repetitions.

From the estimation results of model 1 reported in the left panel of table 1, we can see that the spillover effect $\lambda$ is overestimated by 54.3% when unobserved heterogeneity is not controlled for. The estimates of other parameters are also affected: the estimate of $\beta_1$ is downward biased by 9.7%; the estimates of the fixed linking cost $\delta_0$ and the homophily effect $\delta_1$ are downward biased by 14.1% and 38.5%, respectively; the estimates of the congestion effect $\delta_2$ and the cyclic triangle effect $\delta_3$ are upward biased by 58.6% and 28.4%, respectively; and the estimate of the noise parameter $\sigma^2$ is also upward biased by 31.2%. These numbers reveal that neglecting unobserved heterogeneity could cause severe biases in the estimation. On the other hand, the estimation results of model 2 reported in the right panel of table 1 show that the proposed MCMC procedure can successfully recover the true model parameters under the correct model specification that takes unobserved heterogeneity into account.

Furthermore, to see how ignoring unobserved heterogeneity affects the direction of estimation bias of the spillover effect parameter $\lambda$, we redo the simulation experiment with $\beta_2 = -0.5$ and other parameters unchanged. The simulation results are reported in table 2. Comparing tables 1 and 2, we find that the spillover effect parameter $\lambda$ is overestimated when $\beta_2$ is positive and underestimated when $\beta_2$ is negative. Our intuition for this result is the following. In equation (13), $x^U$ captures the unobserved degree heterogeneity (Graham, 2017). That is, an agent with a higher $x^U$ is likely to form more links. If $\beta_2 > 0$ in equation (12), then an agent with a higher tendency to form links is likely to spend more effort. Thus, ignoring $x^U$ would confound this effect with the spillover effect and lead to an upward bias for the estimated spillover effect. For the same reason, when $\beta_2 < 0$, ignoring $x^U$ would cause a downward bias on the estimated spillover effect.

IV. Empirical Illustration

To illustrate the empirical relevance of our model and estimation strategy, we apply it to study R&D investment and collaborations. R&D collaborations have become a widespread phenomenon especially in industries with rapid technological innovations such as the chemical and pharmaceutical industries (Hagedoorn, 2002; Roijakkers & Hagedoorn,
2006). Through such collaborations, firms generate knowledge spillovers not only to their collaboration partners but also to other firms that are indirectly connected to them within a complex R&D network (König et al., 2019). The network perspective is thus crucial to understand outcomes in R&D-intensive markets where collaborations can be frequently observed (Powell et al., 1996, 2005).

A. A Simple Model of R&D Collaborations

The microfoundation of our empirical illustration is a Cournot competition model with firms engaging in R&D investment and collaborations to lower production cost. This model has been adopted by d’Aspremont and Jacquemin (1988), Goyal and Moraga-Gonzalez (2001), Petrakis & Tsakas (2018), and König et al. (2019) to study R&D networks. More specifically, consider a set of firms $\mathcal{N} = \{1, \ldots, n\}$ with their characteristics given by $X_i$. Firms can reduce their production costs by investing in R&D as well as by benefiting from an R&D collaboration with another firm. The amount of cost reduction depends on the R&D effort $y_i$ of firm $i$ and the R&D efforts of firm $i$’s collaboration partners. The marginal production cost $c_i$ of firm $i$ is given by

$$ c_i = -b_1(X_i) - y_i - \lambda \sum_{j=1}^{n} g_{ij} y_j, \quad (14) $$

where $b_1(X_i)$ captures firm heterogeneity with regard to productivity and $g_{ij}$ indicates whether firms $i$ and $j$ have an R&D collaboration. The parameter $\lambda$ captures the knowledge spillover effect. We assume that the cost of R&D effort is given by $\frac{1}{2} y_i^2$. We further assume it is costly to maintain R&D collaborations with the collaboration cost given by $-a_i(g)$. With output $q_i$, firm $i$’s profit is given by

$$ \Pi_i = (p_i - c_i)q_i - \frac{1}{2} y_i^2 + a_i(g). \quad (15) $$

where $p_i$ is the price of the good produced by firm $i$. We assume firms are local monopolies with the inverse demand function $p_i = b_0 - q_i$, where $b_0$ represents the market size. Substitution of the inverse demand function and equation (14) into equation (15) yields

$$ \Pi_i = [b_0 - q_i + b_1(X_i) + y_i + \lambda \sum_{j=1}^{n} g_{ij} y_j]q_i - \frac{1}{2} y_i^2 + a_i(g). \quad (16) $$

Profit maximization with respect to $y_i$ gives $q_i = y_i$. Substitution of $q_i = y_i$ into equation (16) gives

$$ \Pi_i = a_i(g) + b(X_i)y_i + \lambda \sum_{j=1}^{n} g_{ij} y_j - \frac{1}{2} y_i^2. \quad (17) $$

where $b(X_i) = b_0 + b_1(X_i)$. Equation (17) conforms to the general payoff function defined in equation (1). In the empirical study, we assume that $a_i(g)$ is given by equation (2) with $h(X_i, X_j, b_1)$ defined in equation (13) and that $b(X_i)$ is given by equation (12).

B. Data

In the empirical illustration, we focus on the sector Chemicals and Allied Products (with two-digit SIC code 28), as it is one of the most active sectors regarding R&D collaborations. Our data of interfirm R&D collaborations stem from two sources that have been widely used in the literature (Schilling, 2009). The first is the Cooperative Agreements and Technology Indicators (CATI) database (Hagedoorn, 2002). The database records only agreements for which a combined innovative activity or an exchange of technology is at least part of the agreement. The second is the Thomson Securities Data Company (SDC) alliance database. SDC collects data from the U.S. Securities and Exchange Commission (and its international counterparts) filings, trade publications, wires, and news sources. We include only alliances from SDC classified explicitly as R&D collaborations.\(^\text{10}\) We then merge the CATI database with the Thomson SDC alliance database.

For the matching of firms across data sets, we adopt and extend the name-matching algorithm developed as part of the NBER patent data project (Trajtenberg, Shiff, & Melamed, 2009).\(^\text{11}\) The systematic collection of interfirm alliances in CATI started in 1987 and ended in 2006. We take 2006 as the base year and assume that an alliance lasts five years (Rosenkopf & Padula, 2008). We construct the R&D collaboration network by coding $g_{ij}$ as 1 if an alliance between firms $i$ and $j$ exists in 2006 and 0 otherwise.

The combined CATI-SDC database only provides the names of the firms in an alliance. To obtain information about their balance sheets and income statements, we match the firms’ names in the CATI-SDC database with the firms’ names in Standard & Poor’s Compustat U.S. and Global Fundamentals databases, as well as Bureau van Dijk’s Orbis database (Bloom, Schankerman, & Van Reenen, 2013). For the purpose of matching firms across databases, we employ the name-matching algorithm. Compustat and Orbis databases contain only firms listed on the stock market, so they typically exclude small, private firms. However, they should include most R&D-intensive firms, as R&D is typically concentrated in publicly listed firms (Bloom et al., 2013).

We use a firm’s log-R&D expenditure to measure its R&D effort. Moreover, the firms’ productivities are measured by their log-R&D capital stocks (lagged by one year). As in Hall, Jaffe, and Trajtenberg (2000), Bloom et al. (2013), and

\(^{10}\)For a comparison and summary of different alliance databases, including CATI and SDC, see Schilling (2009).

\(^{11}\)See https://sites.google.com/site/patentdataproject. We thank Engin Atalay and Ali Hortacsu for sharing their name-matching algorithm with us.
König et al. (2019), the R&D capital stock is computed using a perpetual inventory method based on the firms’ R&D expenditures with a 15% depreciation rate. We drop firm observations with missing values on either R&D expenditure or R&D capital stock, which results in a sample of 347 firms and 139 R&D alliances in the SIC-28 sector. The SIC-28 sector has eight subsectors coded with three-digit SIC codes. Among them, the subsector Drugs (SIC-283) is the largest in our sample with 256 firms and 119 R&D alliances. Descriptive statistics of the sample are shown in table 3.

C. Estimation Results

Assuming the observed R&D expenditures and collaborations follow the stationary distribution defined in equation (8), we estimate the model parameters using the MCMC procedure described in section III. We run the MCMC algorithm for 35,000 iterations and drop the first 5,000 draws for burn-in and keep every 20th of the remaining draws to conduct the posterior analysis, that is, we compute the posterior mean (as a point estimate) and posterior variance for each parameter. To check the convergence of the MCMC algorithm, we provide the trace plot of draws for the spillover effect parameter $\lambda$ in figure 2. The trace plot of MCMC draws for $\lambda$, and its posterior distribution in the upper and middle panels shows that the MCMC draws are stable and have good variations. The autocorrelation function (ACF) plotted in the bottom panel indicates that the correlation among draws declines gradually over iterations. The draws pass the convergence diagnostic test of Geweke (1992) with a $p$-value of 0.4698.\footnote{The convergence diagnostic test results for an equal mean of the first 10% versus the last 50% of the draws. We also try different proportions (e.g., 30% versus 70%) and obtain similar results for the convergence of the MCMC algorithm.}

The estimation results are reported in table 4. To capture firm heterogeneity in the marginal production cost given by equation (14), we include a productivity measure defined as a firm’s one-year-lagged log-R&D capital stock and subsector dummies (defined at the three-digit SIC level). As expected, the estimate of $\beta_1$ shows that higher time-lagged R&D capital stock reduces the marginal production cost.

The estimated spillover effect $\lambda$ is statistically significant. As the estimated coefficient $\beta_2$ of unobserved heterogeneity in the marginal production cost is statistically insignificant, the spillover effect is only slightly overestimated when unobserved heterogeneity is ignored. On the other hand, unobserved heterogeneity still plays an important role in the estimation of the collaboration cost given by equation (2). When unobserved heterogeneity is controlled for, we find that the collaboration cost is lower between firms in the same subsector (reflected by $\delta_{11}$; the homophily effect), firms with different productivities (reflected by $\delta_{12}$; the heterophily effect), and firms with a common collaboration partner (reflected by $\delta_3$; the cyclic triangle effect). A firm is less likely to form a new link if it has many existing R&D collaborations (reflected by $\delta_2$; the congestion effect).

Finally, we evaluate the model’s goodness-of-fit following Hunter, Goodreau, and Handcock (2008). We generate 1,000 networks with the estimates of models 1 and 2 reported in table 4, respectively. The model’s goodness-of-fit is examined by comparing the 1,000 generated networks with the observed network in terms of three network statistics: the degree (the number of links of a firm), the minimum geodesic distance (the number of links in the shortest path between two firms), and the number of edge-wise shared partners (the number of shared partners of two connected firms). The degree is included because it is a fundamental measure of network structure, and it often does a reasonably good job of explaining other higher-order network statistics (Faust, 2007; Graham, 2015). The geodesic distance is included because it is relevant to the speed of knowledge diffusion, which is especially important for R&D networks. The geodesic distance is also the basis of some well-known network centrality measures (Wasserman & Faust, 1994). The number of edge-wise shared partners is included based on the work by Snijders et al. (2006). The three network statistics are related to different aspects of network structure and provide independent criteria for goodness-of-fit.

We plot the distributions of the three network statistics of the observed network (in solid lines) and the corresponding means and 95% confidence intervals of the 1,000 generated networks (in dashed lines) for both models in figure 3. From the figure we can see that model 2 provides a better fit to the observed network than model 1. We also calculate the spectral goodness-of-fit (SGOF) proposed by Shore & Lubin (2015) for both models. The SGOF is analogous to the standard $R^2$ in a linear regression. It measures how well a network model explains the structure of an observed network based on the spectrum of the graph Laplacian. We calculate the SGOF based on the 1,000 networks previously generated and find that model 2 improves the goodness-of-fit of model 1 by 51% and this improvement is significant at the 5% level.\footnote{The details on how to calculate the SGOF are in the online appendix.}
TABLE 4.—Estimation Results

<table>
<thead>
<tr>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spillover effect $(\lambda)$</td>
<td>0.0125 (0.0027)**</td>
</tr>
<tr>
<td>Production cost</td>
<td></td>
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<tr>
<td>Productivity $(\beta_1)$</td>
<td>0.8599 (0.0284)**</td>
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<tr>
<td>Unobserved heterogeneity $(\beta_2)$</td>
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<tr>
<td>Subsector dummies</td>
<td>Yes</td>
</tr>
<tr>
<td>Collaboration cost</td>
<td></td>
</tr>
<tr>
<td>Constant $(\delta_0)$</td>
<td>-4.6716 (0.6963)**</td>
</tr>
<tr>
<td>Same subsector $(\delta_{11})$</td>
<td>0.8086 (0.2420)**</td>
</tr>
<tr>
<td>Diff-in-productivity $(\delta_{12})$</td>
<td>0.0618 (0.0461)**</td>
</tr>
<tr>
<td>Congestion $(\delta_2)$</td>
<td>0.0319 (0.0355)**</td>
</tr>
<tr>
<td>Cyclic triangle $(\delta_3)$</td>
<td>0.3151 (0.1386)**</td>
</tr>
<tr>
<td>Noise parameters</td>
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</tr>
<tr>
<td>Noise in decisions $(\sigma^2)$</td>
<td>0.4833 (0.0744)**</td>
</tr>
<tr>
<td>Unobserved heterogeneity $(\varsigma^2)$</td>
<td>0.7697 (0.0773)**</td>
</tr>
</tbody>
</table>

Model 1 ignores unobserved heterogeneity, and model 2 controls for unobserved heterogeneity. Standard errors in parentheses. ***, **, and * indicate that the highest-density range does not cover 0 at 99%, 95%, and 90% levels.
D. Key Player Analysis

With the estimates of model 2 reported in the right panel of table 4, we can conduct the key player analysis described in section IIC. We consider both short-run and long-run key player analyses. For the short-run key player analysis, we assume the network does not rewire after a firm is removed. For the long-run key player analysis, we simulate the coevolution process of R&D investment and collaborations for the remaining $n - 1$ firms using algorithm 2 in section IIIB after a firm is removed. We run the simulation for $n^2$ iterations and use the observation of the last iteration to calculate the welfare loss. We then repeat this procedure 200 times and report the average welfare loss of removing that firm. The results for the key player analysis are reported in table 5. Some main findings are summarized as follows:

- In general, the welfare loss is lower in the long run because firms can mitigate the welfare loss by forming new links with other remaining firms.

- The short-run and long-run key player rankings do not always coincide with each other. However, there is a high correlation (0.9277) between short-run and long-run welfare losses. This suggests the key player analysis has some robustness with respect to the invariant network assumption.

- The long-run welfare loss is highly correlated with the log-R&D expenditure, degree centrality, betweenness centrality, closeness centrality, and eigenvector centrality with correlation coefficients 0.7378, 0.8829, 0.8222,
0.7974, and 0.8368, respectively. Therefore, the key player ranking incorporates information on both R&D investment and network centrality of a firm.

V. Conclusion

This paper proposes a structural model for the coevolution of networks and behavior. We provide a microfoundation for the model and characterize the equilibrium of the coevolution process. We show the model can be estimated using an MCMC algorithm and investigate the finite sample performance of the estimation procedure in a Monte Carlo simulation experiment. We then apply the model to study R&D investment and collaboration decisions in the chemicals and pharmaceutical industry and find a positive knowledge spillover effect. We also demonstrate how to use the model estimates to conduct a long-run key player analysis.

Due to the generality of the utility function we consider, we believe that our structural framework, from both theoretical and empirical perspectives, can be applied to a variety of related contexts, where externalities can be modeled in the form of an adaptive network. Examples include peer effects in education, crime, risk sharing, and scientific coauthorship (Jackson & Zenou, 2014).

REFERENCES


