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Local and Global Interactions in an Evolutionary Resource Game

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Abstract Conditions for the emergence of cooperation in a spatial common-pool resource game are studied. This combines in a unique way local and global interactions. A fixed number of harvesters are located on a spatial grid. Harvesters choose among three strategies: defection, cooperation, and enforcement. Individual payoffs are affected by both global factors, namely, aggregate harvest and resource stock level, and local factors, such as the imposition of sanctions on neighbors by enforcers. The evolution of strategies in the population is driven by social learning through imitation, based on local interaction or locally available information. Numerous types of equilibria exist in these settings. An important new finding is that clusters of cooperators and enforcers can survive among large groups of defectors. We discuss how the results

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contrast with the non-spatial, but otherwise similar, game of Sethi and Somanathan (American Economic Review 86(4):766–789, 1996).

Keywords Common property · Cooperation · Evolutionary game theory · Global interactions · Local interactions · Social norms

JEL Classification C72 · Q2

1 Introduction

The management of common-pool resources (CPRs), such as forests, fishing grounds, and groundwater basins, is characterized by a conflict between individual and social interests. While the collective interest of the group is to limit harvesting to a sustainable level, the combined actions of individual harvesters pursuing their own interest inevitably result in a suboptimal outcome, characterized by excessive exploitation of the resource (Dasgupta and Heal 1979, Chapter 3). This dilemma can take the form of a game with two strategies for each harvester, namely defection and cooperation (Ostrom et al. 1994). In the finitely repeated CPR game, unilateral defection is the unique Nash equilibrium, while privatization of the resource is often the suggested management solution.

Privatization is, however, not always feasible in the case of common-pool resources, or it may destroy the culturally evolved social norms. Moreover, while economic theory predicts defection, case studies and laboratory experiments have provided evidence that, in real life, cooperation can be sustained among harvesters (Ostrom 1990; Hackett et al. 1994; Chermak and Krause 2002). Accordingly, cooperative outcomes can be reached as long as norms or rules, such as trust, reward or punishment, prevail in the group. The imposition of sanctions on harvesters that adopt excessive exploitation levels has proved to be particularly effective in sustaining cooperation, even in the case of non-repeated interactions among unrelated individuals or in very large groups. Evidence from the field suggests that some agents voluntarily engage in ‘altruistic punishment’, that is, penalizing free-riders, even if this implies an individual cost. Often, a small proportion of these altruistic punishers is sufficient to enforce cooperation in the group (Fehr and Gächter 2002). Both laboratory experiments and field studies confirm that social norms can solve a substantial number of common-pool resources management problems (Ostrom 1990; Ostrom et al. 1994; Baland and Platteau 1996; McKean 1992; Hviding and Baines 1994).

A rare theoretical analysis of the role of altruistic punishers or ‘enforcers’ in solving CPR dilemmas is presented in Sethi and Somanathan (1996). They consider an evolutionary CPR game played by a fixed population of players with three strategies: defection, cooperation, and enforcement. Payoffs of defectors are lowered by a sanction that depends on the number of enforcers in the population, while enforcers bear a cost that depends upon the total number of defectors. Players experience social learning and imitate the strategy that yields the highest payoffs. The agent is thus not optimizing over a large number of options but follows more realistically a naive learning rule. The evolution of strategies is then captured by a replicator dynamics equation, according to which the share of the best-performing strategy in the

population increases due to agents imitating it. Sethi and Somanathan combine replicator and resource dynamics to show how changes in the resource stock affect harvesting behavior, and vice versa. They identify two equilibria in the system: a final population composed of only defectors, and a cooperative equilibrium with only cooperators and enforcers.

The model considered in this paper adds a major innovation to Sethi and Somanathan's work, namely spatial interactions between agents. While Sethi and Somanathan restrict themselves to aggregate population dynamics, our model more realistically emphasizes the role of locality. The approach is, therefore, unlike replicator dynamics, based on the explicit modeling of micro-interactions among individuals. This approach results in a combination of local and global interactions. The presence of local interactions means observation by agents of strategies and performance of other agents in their local neighborhood, with a possible result that a strategy of a neighbor is imitated. Global interactions include effects that run through aggregate harvest and the resource stock, and feed back to profits associated with harvesting strategies employed by individual agents.

Field studies show that locality and territoriality can play a role in the management of CPR resources. An illustration is the Maine lobster fishery (Acheson 1987, 1998). Although legally an open-access area, the lobster fishery is divided in two-dimensional spatial territories assigned to each fisherman to put his traps. Free-riders in the group have an incentive to expand their territories and to put their traps beyond the assigned space. If a fisherman notices the practices of a free-rider, he can decide to sanction him, for instance by destroying his traps. Another example is the monitoring of communal forests in Honduras (Tucker 1999). Due to the extent of the forest and the large amount of inaccessible areas, forest guards are not an efficient solution to monitor the forest. Instead, monitoring is locally enforced by each member of the community. Every resident of the forest bears the responsibility of signaling residents trying to expand their fences. Violations are brought in front of the local authorities and go from verbal warning to fines, compulsory labor and jail. Evidence shows that this monitoring system works particularly well when people are competing for the same land. Note that in the case of the forest example the global nature of payoffs is perhaps less evident than in the fishery example. Nevertheless, local performance and sustainability of a forest certainly depend on the size of (continuous) forested area as well as forest management practices at distance, since these will affect biotic and abiotic conditions, notably ecological connections (meta-populations and biodiversity), groundwater flows and microclimates.

The objective of the present paper is to model local spatial interactions between boundedly rational agents in a CPR resource game. We aim to find conditions for the emergence of cooperative equilibria in the spatial evolutionary CPR game. The objective comes down to testing the robustness of Sethi and Somanathan's findings in an evolutionary game with spatial interactions. In particular, it will be examined whether other types of equilibria than found by Sethi and Somanathan emerge.

The present CPR game with spatial interactions has connections with two different bodies of literature. First, it relates to studies on local interaction games within the field of evolutionary game theory (Eshel et al. 1998; Lindgren and Nordahl 1994;

for an overview, see [Nowak and Sigmund 1999](#)). The structure of our game relates most closely to [Nowak and May \(1992\)](#). A well-known result of the literature on local games is that the introduction of local interactions between agents enhances the survival of cooperation. These insights have recently started to be studied in laboratory experiments ([Kirchkamp and Nagel 2007](#)). [Kirchkamp and Nagel \(2007\)](#), for instance, seem to reject our findings in a laboratory experiment. There are a few important differences with our model: (1) they have only two strategies and (2) in their models agents can learn from their own ‘history’ in a repeated game. Such experiments are still scarce, however, and further empirical testing is needed to validate the models. The game presented in this article differs from these approaches in that it combines both local and global interactions.

Second, the approach presented here also relates to the wide body of experimental and theoretical work on the evolution of social norms in CPR games. The role of social norms in these settings has been tested in laboratory experiments for instance by [Ostrom et al. \(1994\)](#) and by [van Soest and Vyrastekova \(2006\)](#). Only a few studies have formally analyzed an evolutionary CPR game with a variable resource stock ([Akiyama and Kaneko 2000](#); [Janssen 2001](#); [Sethi and Somanathan 1996](#)). Among these studies, Sethi and Somanathan’s analysis provides an attractive benchmark because of its simplicity, which is due to a combination of only three types of strategies and replicator dynamics. The present study adds insights to this literature as it illustrates how results from local interaction games can be applied to the problem of overexploitation of common-pool resources.

The problem of local interaction in a common-pool resource game is also studied by [Noailly et al. \(2007\)](#). But its setup differs substantially from the present one, in two respects. First, it studies interaction when agents are located on a circle, whereas here we consider interaction between agents situated on a torus. On the one hand, the circle allows for more theoretical insights, because of the fact that the structure is less complicated. On the other hand, the torus provides a richer and more realistic spatial structure of interactions and learning among agents. Most real world interactions, like in agriculture or in fisheries, occur in two-dimensional space. Second, [Noailly et al. \(2007\)](#) employ a different learning rule. In their setting, agents imitate the strategy yielding the highest average payoffs. For most parts of the present paper, we assume that agents are more ‘naive’ and simply imitate their most successful neighbor. We find that changing the learning rule does not qualitatively affect the results found in [Noailly et al. \(2007\)](#). We also find the emergence of equilibria in which cooperators and enforcers coexist in our settings. Yet, we find quantitative differences between the two models. A main new insight of the present study is that cluster equilibria of cooperators and defectors are favored when agents are learning in a ‘naive’ way. This study presents in addition innovative results regarding the role of certain parameters, such as population size and the productivity of harvesting technology.

The paper is organized as follows. Section 2 presents the benchmark neoclassical CPR game and its evolutionary spatial version. Section 3 identifies the equilibria of the system with a fixed resource level and shows how these contrast with Sethi and Somanathan’s findings. In addition, we test for the effects of changes in parameters and the incorporation of edge-effects. Section 4 concludes.

2 The Model

2.1 The CPR Game

The benchmark neoclassical CPR game (Dasgupta and Heal 1979, Chapter 3; Chichilnisky 1994) is presented briefly here. This game relies on the assumption of maximizing behavior and thus contrasts sharply with the evolutionary version of the game that will be discussed further on.

A fixed population of m ($m > 1$) harvesters has access to a natural resource. The individual effort level of harvester i ($i = 1, \dots, m$) in period t ($t = 0, 1, \dots$) is denoted by x_{it} . Total effort of the population is simply the sum of all individual efforts:

$$X_t = \sum_{i=1}^m x_{it}. \quad (1)$$

Aggregate harvest H is a strictly concave and increasing function of total effort X_t and of the total stock of the natural resource N_t . In a first stage, we ignore resource dynamics to simplify the analysis and fix the resource stock N_t at N_0 . Suppressing N_0 , we can write the harvest rate as a function of X only and we can ignore time subscripts for all variables. We define F as the total harvest rate, which is strictly concave and increasing with $F(0) = 0$, $F'(0) > w$, $F'(\infty) < w$, where w is the constant cost per unit of effort employed.

$$H(X, N_0) = F(X). \quad (2)$$

Each agent receives a share of total profits in proportion to the amount of effort invested. Individual profits are then given by:

$$\pi_i(x_i, X) = \frac{x_i}{X} F(X) - wx_i, \quad (3)$$

where it is assumed that the harvested commodity is the numeraire. Individual payoffs are thus a function of a global factor, namely aggregate harvest. The larger aggregate harvest is, that is, the more defectors are present, the lower individual payoffs are. Thus, the level of aggregate profits is:

$$\Pi = \sum_i^m \pi_i(x_i, X) = F(X) - wX. \quad (4)$$

At X_P , which is the Pareto efficient level of effort defined by $F'(X_P) = w$, total profits are maximized and the resource is used efficiently. When access to the resource is open to everyone, entry of harvesters continues until X_O , where $F(X_O) = wX_O$, i.e., no harvester enjoys positive profits. In the case of a fixed population of $m(>1)$ identical agents, the Nash equilibrium X_C is defined by

Fig. 1 The CPR game

$$F'(X_C) - w = (m - 1) \left(w - \frac{F(X_C)}{X_C} \right) \quad (5)$$

Due to the assumptions made on F we have $F'(X_C) - w < 0$, implying that in the Nash equilibrium there is overharvesting with respect to the Pareto efficient outcome. This is illustrated in Fig. 1.

An evolutionary version of this standard framework has been studied by [Sethi and Somanathan \(1996\)](#). In an evolutionary game, harvesters are boundedly rational, which means that they do not solve any optimization problem. Instead, they rely on simple forms of social learning like imitation of the best-performing strategy. Diffusion of strategies occurs through the learning process and drives the evolution of strategies towards an equilibrium that falls between the benchmark equilibrium aggregate harvest rates X_P and X_O . In the remainder of the paper, we base our analysis on a similar evolutionary framework but introduce spatial interactions between the agents.

2.2 A Spatial Evolutionary CPR Game Without Resource Dynamics

The spatial evolutionary CPR game embodies the following four main features:

Space. A fixed population of m harvesters is distributed on a two-dimensional torus. A torus is a lattice whose corners are pasted together to ensure that all cells are connected, so that there are no edge-effects as illustrated on Fig. 2.

Each single cell of the torus is occupied by one, fixed player during the game. We define the neighborhood of each player as the set of the four closest neighbors, located North, South, East, and West of the player, as shown in Fig. 3. An alternative definition of neighborhood entails eight neighbors (adding North- and South-East and North- and South-West neighbors). We will not go into this.

Fig. 2 A torus **Fig. 3** A neighborhood, indicated by light-colored cells

Strategies. Just as in [Sethi and Somanathan \(1996\)](#), we consider three possible different types of strategy for every player: defection, cooperation, and enforcement. Enforcers punish defectors. Both cooperators and enforcers choose a low level of effort x_L to avoid overexploitation of the resource, while defectors choose a high level of effort x_H , which yields higher profits *ceteris paribus*.

As is usual in the evolutionary economics literature, it is assumed that players use rules of thumb rather than design their strategies in a sophisticated way. This, however, still leaves many options open. One could assume for instance that cooperators choose an effort level of $x_L = X_P/m$ such that the Pareto efficient outcome is reached when all agents are cooperators. Similarly, defectors could adopt the effort level $x_H = X_C/m$ such that when all agents are defectors the Nash equilibrium is reached. Alternatively it can be assumed that defectors play Nash given the actions, cooperation or defection, of all other players. In a more sophisticated way, one could also assume that agents observe actions taken in the past and condition their present decisions on them, i.e. play a best-response to the aggregate quantity in the last period. For example, they may expect other players to stick to what they did one period before. Finally, one could make assumptions regarding the set of players on which defectors condition their action. In the present paper we model behavior as being very simple following much of the literature. We assume that the variable effort level takes fixed and discrete values. So, at each instant of time, the strategy space of each player contains just two elements. Following the literature we assume that the individual effort levels satisfy.

$$X_P \leq mx_L < mx_H \leq X_C \leq X_O \quad (6)$$

with X_P , X_C and X_O as defined in the previous section, and mx_L (mx_H) the total harvest when all agents harvest low (high). In each round τ ($\tau = 0, 1, \dots$) of the game, the aggregate effort X_τ is calculated according to the distribution of strategies in the population:

$$X_\tau = m_{D,\tau}x_H + (m_{E,\tau} + m_{C,\tau})x_L, \quad (7)$$

where $m_{D,\tau}$, $m_{E,\tau}$ and $m_{C,\tau}$ are, respectively, the number of defectors, enforcers and cooperators present in the system in round τ .

Enforcers punish all defectors located in their neighborhood. Monitoring is thus conducted locally among close neighbors. This is a major difference with Sethi and Somanathan's model in which sanctions are imposed by the group of enforcers on the group of defectors at the aggregate level. Here, each enforcer punishing a defector bears a cost γ per defector, while each defector being punished by an enforcer pays a sanction δ . The maximum sanction falling on a defector is thus 4δ , when he is surrounded by four enforcers. Similarly, the maximum cost borne by an enforcer is 4γ .

Payoffs can be formulated for each possible strategy, given aggregate effort X_τ and strategies located in the neighborhood:

$$\pi_{C,\tau} = \frac{x_L}{X_\tau} (F(X_\tau) - wX_\tau) \quad (8)$$

$$\pi_{Dk,\tau} = \frac{x_H}{X_\tau} (F(X_\tau) - wX_\tau) - k\delta \quad (9)$$

$$\pi_{El,\tau} = \frac{x_L}{X_\tau} (F(X_\tau) - wX_\tau) - l\gamma, \quad (10)$$

Since x_L and x_H are fixed and since $m_{E,\tau} + m_{C,\tau} = m - m_{D,\tau}$ we can write X_τ in (6) as a function of $m_{D,\tau}$ only, and, therefore, the profit levels defined above as a function of the number of defectors only. We refrain from doing so explicitly just for notational convenience.

Here $\pi_{C,\tau}$ denotes payoffs in round τ of a cooperator; $\pi_{Dk,\tau}$ ($\pi_{El,\tau}$) denotes payoffs in round τ of a defector (an enforcer) surrounded by k (l) enforcers (defectors). Thus, e.g., $\pi_{D3,2}$ refers to the payoffs of a defector surrounded by three enforcers in round $\tau = 2$. Obviously, $\pi_{Dh,\tau} > \pi_{Dh+1,\tau}$ and $\pi_{Eh,\tau} > \pi_{Eh+1,\tau}$ ($h \in \{0, 1, 2, 3\}$). In addition, from (6), (8), (9), and (10), we have $\pi_{D0,\tau} > \pi_{C,\tau} > \pi_{E1,\tau}$ and $\pi_{E0,\tau} = \pi_{C,\tau}$ for all τ .

We make one additional assumption regarding the level of punishment in the system, namely:

$$\pi_{D4,\tau} < \pi_{E0,\tau}, \quad \forall \tau. \quad (11)$$

This implies that a defector incurring the maximum sanction level, regardless of X , earns a lower payoff than any enforcer who does not punish ($E0$). This is to ensure that enforcers can actually win over defectors.

We further assume that $H(X, N_0)$ is a Cobb–Douglas production function:

$$F(X) = \alpha X^\beta N_0^{1-\beta} \quad \alpha > 0, \quad 0 < \beta < 1. \quad (12)$$

We can solve for the first-best, Nash and zero profit efforts:

$$X_P = \left(\frac{w}{\alpha\beta}\right)^{\frac{1}{\beta-1}} N_0, \quad X_C = \left(\frac{mw}{\alpha(\beta+m-1)}\right)^{\frac{1}{\beta-1}} N_0, \quad X_O = \left(\frac{w}{\alpha}\right)^{\frac{1}{\beta-1}} N_0 \quad (13)$$

Note that $X_P = X_C$ for $m = 1$.

Learning, i.e., updating of strategies, is driven by expectations of larger profits in the next period. In each round, every player updates his current strategy by imitating the strategy that yields the highest payoffs in his neighborhood. Similar learning rules, where agents simply pick up the strategy with the largest score, were used by Axelrod (1984, Chapter 8) and Nowak and May (1992). Eshel et al. (1998) instead impose a learning rule in which the on average best performing strategy in the observed neighbourhood is imitated. This rule is examined with a spatial evolutionary analysis in another paper (Noailly et al. 2007). When the best strategy in the neighborhood is identical to the player's current strategy, the player sticks to his current strategy. When multiple strategies other than the player's current strategy yield the largest (equal) payoffs in the neighborhood, that is, when there is a tie between two strategies, the player randomizes among these strategies with probability $p = 0.5$.

Time. We assume that agents exhibit synchronous behavior. In other words, interactions and learning occur simultaneously. Huberman and Glance (1993) have shown that asynchronous learning can lead to different outcomes than synchronous learning. In our model, seasonal harvesting of the resource suggests the existence of a 'global clock' that governs the learning process. Therefore, it is reasonable to assume that all harvesters modify their strategy simultaneously at the beginning of each new season. A season or a 'round' of the game can be described by the following sequence:

- i. Aggregate effort X_τ is computed given the number of defectors, cooperators and enforcers on the torus.
- ii. Aggregate harvest $F(X_\tau)$ is calculated.
- iii. Individual payoffs π_C , π_{Dk} and π_{El} are computed for all agents, given $F(X_\tau)$, the strategy chosen by each single agent and the distribution of strategies in his neighborhood.
- iv. Agents observe the payoffs of their neighbors' strategies and decide whether to stick to their current strategy or to adopt the strategy of their most successful neighbor. All agents update their strategy simultaneously. This updating process yields a new distribution of strategies in the population.

3 Spatial Evolutionary Dynamics

In this section we study the spatial evolutionary dynamics using numerical simulations. An equilibrium is a spatial distribution of strategies in which no player has an incentive to change strategy. It is important to realize that in our setting it is possible for a neighboring strategy to earn a larger profit than the player's current strategy even when these strategies are identical. For example, enforcers punishing one defector are in an equilibrium, even if they earn the lowest payoffs, as long as their neighbor with the highest payoff is a non-punishing enforcer. A similar reasoning applies to defectors. This contrasts with Sethi and Somanathan's model, in which agents belonging to the same (sub)group always earn equal payoffs, since sanctions and costs falling on defectors and enforcers are determined at the aggregate level. Such aggregation evidently leads to a loss of information and accuracy of description.

3.1 Notation and Parameter Values

We use D , C and E to refer to equilibria composed of only defectors, cooperators and enforcers, respectively. In addition, DE , CE and CDE refer to equilibria composed of the corresponding mixes of strategies. In contrast with D , C and E -equilibria, many diverse configurations can constitute DE , CE or CDE -equilibria. In addition, note that there cannot be any CD -equilibrium for the simple reason that this corresponds to the case where there is no local punishment between the agents. In this case, defectors are never punished and spread quickly over the lattice.

Which equilibrium emerges depends on three factors:

1. The initial spatial distribution of strategies. Initially, strategies either form clusters or are scattered irregularly. Section 3.3 studies how different equilibria can be reached by varying the initial spatial arrangement of the strategies.
2. The initial share of each strategy in the population. Strategy shares in round τ are denoted by $z_\tau = (\frac{m_{D,\tau}}{m}, \frac{m_{E,\tau}}{m}, \frac{m_{C,\tau}}{m})$, which corresponds to a population composed of a mix of m_D defectors, m_E enforcers and m_C cooperators in round τ . We will study how different initial shares (z_0) lead to diverse types of equilibria in Section 3.2.
3. Parameter values. Most of the simulations were performed on a 10×10 ($m = 100$) spatial grid. A simulation run corresponds to 50 rounds, which appears to be sufficiently long for the system to settle into an equilibrium. The following parameters were used:

$$\begin{aligned} \alpha &= 0.2 & \beta &= 0.2 \\ N_0 &= 500 & w &= 0.2 \\ x_H &= 4 & x_L &= 2 \\ \delta &= 0.4 & \gamma &= 0.1. \end{aligned} \tag{14}$$

Given the other parameter values, the levels of harvest x_H and x_L satisfy (6).¹ The level of sanction $\delta = 0.4$ satisfies condition (11).² A sensitivity analysis of critical parameters is conducted in Section 3.5.

¹ $X_P = 67$, $X_C = 495$ and $X_O = 500$ according to (13).

² Condition (11) is rewritten as:

$$x_H \left(\frac{\alpha X_\tau^\beta N_0^{1-\beta}}{X_\tau} - w \right) - 4\delta < x_L \left(\frac{\alpha X_\tau^\beta N_0^{1-\beta}}{X_\tau} - w \right).$$

After simplification, it follows that this is satisfied for all X_τ , i.e., in every round τ , whenever the following condition holds:

$$\left[\frac{1}{\alpha} \left(\frac{4\delta}{x_H - x_L} + w \right) \right]^{\frac{1}{\beta-1}} N_0 < m x_H.$$

3.2 The Effects of Initial Strategy Shares in the Population

In this section, we study the effects of variation in initial strategy shares. We consider all possible initial shares of cooperators, enforcers and defectors, i.e., the set of initial coordinates $z_0 = (\frac{m_D}{m}, \frac{m_E}{m}, \frac{m_C}{m})$. Further, we eliminate initial strategy shares composed of only one type of strategy as well as initial shares composed of only cooperators and defectors and of only cooperators and enforcers, as the outcomes can be easily predicted in these cases. We are left with 4950 coordinates in the set $Z = \{(0.99; 0.01; 0.01), \dots (0.01; 0.01; 0.99)\}$. For every $z_0 \in Z$, we compute 100 runs of 50 rounds, each run corresponding to a spatial configuration drawn from a uniform distribution, such that each player has a probability of 1/100, to be assigned to a particular position. In total, therefore, 495000 runs are necessary to cover the set Z . We use simplex representations to present the results. In a simplex, each point corresponds to a three-dimensional coordinate.

Figure 4 provides results about the frequencies of occurrence of each equilibrium for each initial strategy share, where every $z_0 \in Z$ is represented by a dot.³ The grey-black scale indicates the frequency of occurrence of each type of equilibrium out of 100 random spatial distributions. A black colored coordinate indicates that, starting with the respective z_0 , all runs converge to the given type of equilibrium.

As expected, Fig. 4a shows that *D*-equilibria are more easily achieved for initial populations with few enforcers. Inversely, *CE*-equilibria are more likely to be reached for initial populations composed of many enforcers (Fig. 4b). This is consistent with Sethi and Somanathan's findings. Second, *DE*-equilibria are most likely to be achieved for middle-range initial shares with a slight majority of defectors (Fig. 4c), while *CDE*-equilibria are most frequently achieved for middle-range initial shares with a slight majority of enforcers (Fig. 4d). This makes sense intuitively, since a large number of enforcers needs to surround cooperators to allow *CDE*-equilibria to emerge, as we will explain in Section 3.3. When there are more defectors in the system, cooperators find it difficult to survive, and *DE*-equilibria emerge.

In this example, we find that, on average, 43% of the runs (out of 495000) converge to a *D*-equilibrium, 16% converge to a *DE*-equilibrium, 16% to a *CE*-equilibrium, 25% to a *CDE*-equilibrium, 0% to a *E*-equilibrium.⁴ We find the same frequencies of occurrence of equilibria when we consider 200 instead of 100 random spatial configurations for each strategy share. This suggests that the randomization process of spatial configurations moves systematically through the space of initial spatial configurations. Finally, we can also set parameters such that all defectors play Nash and all cooperators play the Pareto optimum, i.e., $x_H = \frac{x_C}{m}$ and $x_L = \frac{x_P}{m}$ respectively.⁵ In this case, we find that on average 56% (out of 495000) of the simulation runs converge to a *D*-equilibrium, 29% to a *DE*-equilibrium, 14% to a *CDE*-equilibrium and 0% to *CE*- and *E*-equilibria.

³ For illustration purposes, we only map the results for initial shares multiple of 0.05.

⁴ *C*-equilibria can only occur when all agents cooperate initially.

⁵ This falls down to setting $x_H = 4.95$ and $x_L = 0.67$.

Fig. 4 Frequency of occurrence of each type of equilibrium for different initial strategy shares.

(a) Frequencies for initial coordinates converging to *D*-equilibria. (b) Frequencies for initial coordinates converging to *CE*-equilibria. (c) Frequencies for initial coordinates converging to *DE*-equilibria. (d) Frequencies for initial coordinates converging to *CDE*-equilibria

In order to compare our results with the ones obtained by Noailly et al. (2007), we perform additional simulations on the torus using their ‘sophisticated’ learning rule. This rule states that agents imitate the strategy that is the most successful on average. Using the same parameters as above, we find that with the sophisticated learning rule, 79% of the runs converge to a *D*-equilibrium, 23% converge to a *CE*-equilibrium and 3% to an *E*-equilibrium. There are no occurrences of *CDE*- and *DE*-equilibria. In other words, the main consequence of changing the learning rule is that such equilibria

Fig. 5 Evolution of strategy shares over time for four different initial spatial distributions of strategies, with $z_0 = (0.30, 0.40, 0.30)$. (a) D-equilibrium, $z_{50} = (1, 0, 0)$, $\sigma_D = 13$. (b) DE-equilibrium, $z_{50} = (0.91, 0.09, 0)$, $\sigma_{DE} = 10$. (c) CDE-equilibrium, $z_{50} = (0.37, 0.29, 0.34)$, $\sigma_{CDE} = 9$. (d) CE-equilibrium, $z_{50} = (0, 0.26, 0.74)$, $\sigma_{CE} = 3$

are more difficult to sustain. Qualitatively, these equilibria remain possible with the sophisticated learning rule.

3.3 The Effects of the Initial Spatial Distribution of Strategies

In this section, we study the impact of the initial spatial arrangement of strategies on convergence to equilibria. It turns out that the spatial distribution of strategies matters. The graphs in Fig. 5 show diverse possible dynamics in such a simplex, if the system starts near the center of the simplex at point $z_0 = (0.30; 0.40, 0.30)$, i.e., with an initial population composed of 30 defectors, 40 enforcers and 30 cooperators. Each simplex in Fig. 5 corresponds to a different random initial spatial arrangement of the strategies. Arrows indicate the evolution of strategy shares over time and σ denotes the number of rounds after which an equilibrium is reached.

Which equilibrium actually emerges depends crucially on the initial spatial location of the strategies on the lattice. In general, the coexistence of several strategies in the long run is favored by the formation of ‘clusters’. A cluster is a spatial configuration composed of a central agent and its 4 immediate neighbors. Any ‘cooperating cluster’ with a central cooperator surrounded by cooperators or non-punishing enforcers can survive because the central cooperator, not being punished, earns the highest possible

Fig. 6 Examples of clustering in CDE-equilibria, at $\tau = 50$, with $z_0 = (0.30; 0.40; 0.30)$. **(a)** Cooperating group in a large population of defectors. **(b)** Defecting group in a large population of cooperators and enforcers

payoff. This of course only holds as long as defectors located next to cooperators or enforcers are themselves punished and earn less than the central cooperator. An example is given in Fig. 6a. Similarly, a defecting group will survive when surrounded by cooperators and enforcers as long as the central defector is surrounded by 4 defectors, so that he earns the highest profit (see Fig. 6b).

These examples illustrate that the inclusion of a spatial dimension, or a more micro approach, leads to major differences when compared with Sethi and Somanathan's results. While in their model only two equilibria, *D* and *CE*, occur, in the spatial model additional equilibria can be observed: namely, *DE* and *CDE*-equilibria. In Sethi and Somanathan's game, enforcers always earn lower profits than cooperators as long as there are some defectors in the population, so that, ultimately, they will be completely eliminated. Instead, in the spatial game with micro-interactions, enforcers *E1* to *E4* can 'survive' in the long run by forming clusters. This is a crucial element for the occurrence of *DE* and *CDE*-equilibria.

3.4 Stability

In this section we investigate whether the existence of equilibria with cooperators surviving in groups is robust to players making mistakes in choosing their strategies. In evolutionary game theory the stability of equilibria is tied to mutations, i.e. agents with a small positive probability making mistakes or experimenting with a different strategy (Eshel et al. 1998). Consider, for instance, an equilibrium where all agents are cooperators. According to our imitation rule, no agent will ever change its behavior. Suppose, however, that we allow each agent to make a mistake with a small given probability. At some instant in time, a cooperator may then become a defector, who will earn more than his neighbors. Eventually, all agents will become defectors and isolated mutations may not be sufficient to restore a cooperative equilibrium. The *C*-equilibrium is therefore not stochastically stable. Sethi and Somanathan (1996) do not inquire into stochastic stability, arguing that stochastic perturbations are unlikely to occur within the time-scales relevant for the management of natural resources. If we just as in Sethi and Somanathan consider our model as applying to fisheries, the time scales can be interpreted as referring to fishing seasons, while updating fishing

strategy occurs once per season. If it takes thousands of seasons and thus years before a *CDE*-equilibrium has vanished, then from a practical perspective this should not be regarded as a serious case of instability. Many other directed factors would have ample time to exert their influence on the system, negating the relevance of the stochastic factors. We investigate the stability of the cooperative equilibria and in particular of *CDE*-equilibria using numerical simulations. We follow here the same approach as in Noailly et al. (2007). In a first step, we study the time period over which *CDE*-equilibria disappear. We start from a fixed spatial configuration of 25 defectors, 50 enforcers and 25 cooperators, spatially distributed in the following order (from the top-left corner to the bottom-right corner on the lattice): 25 cooperators, 25 enforcers, 25 defectors and 25 enforcers.

In the absence of mutations, this initial state ends up in a *CDE*-equilibrium. We now assume that in each round each agent has a probability of deviating from its strategy equal to $\alpha = 0.005$. We conduct 100 simulation runs for different time horizons and compute the average time spent in each possible population configuration. The results are reported in Table 1. After 10000 rounds, the system spent on average 25% of the time in a *CDE*-configuration. The frequency of *CDE*-equilibria decreases very slowly per additional 10000 rounds. After 20000 rounds the system still spends on average 24% of the time in a *CDE*-equilibria. This example illustrates that the time-scales over which *CDE*-equilibria disappear may be very long.

In a second step, we examine the stability of equilibria by randomizing initial shares of strategy and their spatial distribution on the torus, as in Section 3.2. For each initial share, we now compute 100 runs of 500 rounds. We find that on average 54% of the simulations runs converge to a *D*-equilibrium, 22% to *DE*-equilibrium, 0% to a *CE*-equilibrium and 24% to a *CDE*-equilibrium. This does not formally prove the stochastic stability of *CDE*-equilibria, but the relatively large proportion of *CDE*-equilibria after 500 rounds, suggests that the *CDE*-equilibria are not a mere coincidence. Cooperative equilibria are therefore likely to survive in a large number of cases.

3.5 Sensitivity Analysis

In this section, we perform a sensitivity analysis of some central parameters, namely the sanction level δ , the parameter α that reflects harvest technology or resource price, and the population size m . Parameters are varied as follows: the level of sanctions is varied between $\delta = 0.1$ and $\delta = 1.5$; the parameter α is varied between $\alpha = 0.1$ and $\alpha = 0.7$ and the population size is varied between $m = 36$ and $m = 121$, using 5 different grids of 6×6 , ..., 11×11 .

Table 1 Percentage of time spent in each equilibrium in the presence of mutations

	D-equil.	DE-equil.	CE-equil.	CDE-equil
100	0.00	0.00	0.00	1.00
200	0.01	0.07	0.00	0.92
1000	0.36	0.21	0.00	0.43
10000	0.55	0.20	0.00	0.25
20000	0.56	0.20	0.00	0.24

For each possible parameter configuration, we performed simulation runs as in Section 3.2. To save on computation time, we only look at initial strategy shares multiple of 0.05. The results are given in terms of the average frequency of occurrence of a given equilibrium. Contour lines in Fig. 7 delimit the set of parameters converging to D-equilibria at a given frequency. For instance, the set of parameter values for which D-equilibria are reached at an average frequency larger than 90% is located below or on the contour line 0.9. Figure 7a shows that D-equilibria are more easily achieved for low levels of sanctions. These findings make sense intuitively, and are similar to those of Sethi and Somanathan. Further, D-equilibria are more likely to be reached for a large α . An increase in the net return to harvesting, due to, for instance, an improvement in technology or an increase in the resource price, causes a rise in profits and gives an extra advantage to defectors. As a consequence, the set of D-equilibria expands. This is also in line with Sethi and Somanathan's findings.

Figure 7b shows that D-equilibria are best reached for a small population size. As population size falls, that is, as there are fewer harvesters with the given amount of resource, total effort decreases. This effect translates into a rise in net return to harvesting, causing the size of the set of D-equilibria to expand. Sethi and Somanathan suggest that population growth tends to reduce the importance of sanctions. Their motivation is that as the group gets larger, it becomes more difficult for enforcers to detect defectors. They coin this as an 'anonymity' effect which reduces the impact of individual sanctions. Since monitoring occurs locally in the spatial model, defectors cannot possibly 'hide' from their direct neighbors. As a consequence, there is no anonymity effect in our model. The explanation for the positive effect of population size on cooperation is that in a larger population and associated large space there are more opportunities for protected groups of cooperators and defectors to survive. This conclusion is opposite to the one by Sethi and Somanathan.

Finally, we test the robustness of our results to the inclusion of edge-effects. This means that we do not consider the game on a torus anymore but instead model a

Fig. 7 Range of convergence to D-equilibria (contour lines denote average frequencies of occurrence over all spatial configurations). (a) Parameter space (α, δ) , $m = 100$. (b) Parameter space (m, δ) , $\alpha = 0.2$

two-dimensional lattice. This might in some CPR applications be regarded as a more realistic approach. When we include edge-effects in our analysis and conduct the simulations with the parameters as in Section 3.2, we find that, on average, 27% of the runs (out of 495000) converge to a *D*-equilibrium, 11% to a *DE*-equilibrium, 24% to a *CE*-equilibrium, 38% to a *CDE*-equilibrium, and 0% to a *E*-equilibrium. In other words, it seems that edge-effects reinforce *CDE*-equilibria. The reason is that boundaries make it more difficult for *D*-equilibria to spread.

4 Conclusions

A spatial common-pool resource game has been studied, where space is defined as a two-dimensional torus. The combination of evolution, space and resource combines in a unique way local and global interactions. This implies a complex system, which requires numerical simulation to be understood well.

The main objective was to see if the findings in a non-spatial, but otherwise similar, game of Sethi and Somanathan (1996) are robust against introducing spatial or micro-level interactions among agents. This turns out not to be the case. An important new finding is that clusters of cooperators and enforcers can survive among large groups of defectors. In addition, all other strategy equilibria found by Sethi and Somanathan were assessed here as well.

Because Sethi and Somanathan restrict themselves to aggregate population dynamics, all agents following the same strategy face the same punishment or sanctioning costs, and the same profits. The incorporation of local interactions in our approach means that the replicator dynamics in Sethi and Somanathan is replaced by the explicit modeling of micro-interactions among individuals. A combination of local and global interactions results, where global interactions include feedback through aggregate harvest to profits associated with harvesting strategies. A difference with Sethi and Somanathan is that in our setting it is possible for identical strategies to have different profits, due to the fact that local neighborhoods can be different. As a result, the evolutionary dynamics become difficult to predict. Such disaggregation evidently increases realism and accuracy of description, at the cost of analytical tractability.

The occurrence of equilibria with clusters of cooperators and enforcers, and in which all three strategies appear, can be explained as follows. Enforcers punishing defectors can survive in the long run, as long as no cooperators are located in their neighborhood. Second, cooperators are protected by the formation of clusters of cooperators and enforcers around enforcers who do not punish any defectors. As a result, the imitation effect of the high profit of enforcing can be regarded to diffuse through space. These results stress a general insight, namely that spatial interactions favor diversity.

Conditions for the emergence of cooperative equilibria are identified by performing a range of sensitivity analyses. For example, population growth is shown to enlarge the set of cooperative equilibria, because it negatively affects the net return to harvesting. Edge-effects also enlarge the set of cooperative equilibria as they eventually limit the spread of defectors.

Several implications for the design of CPR institutions can be drawn from these results. Firstly, the findings confirm previous theoretical and empirical work on the role of sanctioning institutions to maintain cooperation in CPR communities. Secondly, the model suggests that spatial interactions between the agents are likely to play a central role in the management of common-pool resources. Spatial interactions may make a difference compared to spaceless structures, notably by enhancing cooperation and by facilitating diversity. We showed the role played by an initial cluster of cooperators and enforcers, implying that a core group of agents can spread the good example to the whole community. As a result, CPR institutions could be designed in such a way that they introduce explicit local interactions. There are many CPR communities in which agents are actually interacting with one another through a local network. This is the case in irrigation systems for instance.

More empirical work is needed to validate the insights of our model in practice. Further research should focus on collecting real-world examples of decentralized monitoring systems involving spatial interactions. Field studies could help to examine whether communities in which monitoring involves local interactions are more successful than others in sustaining the resource. Further empirical testing could also be provided by laboratory experiments reproducing the settings of our model. The literature looking at local evolutionary games and spatial networks in experiments provides a good starting point and could be reproduced in the context of a common-pool resource. Both field studies and experiments necessarily complement each other.

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