A META-ANALYSIS OF $\beta$-CONVERGENCE: THE LEGENDARY 2%

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Abstract. The topic of convergence is at the heart of a wide-ranging debate in the growth literature, and empirical studies of convergence differ widely in their theoretical backgrounds, empirical specifications, and in their treatment of cross-sectional heterogeneity. Despite these differences, a rate of convergence of about 2% has been found under a variety of different conditions, resulting in the widespread belief that the rate of convergence is a natural constant. We use meta-analysis to investigate whether there is substance to the ‘myth’ of the 2% convergence rate and to assess several unresolved issues of interpretation and estimation. Our data set contains approximately 600 estimates taken from a random sample of empirical growth studies published in peer-reviewed journals. The results indicate that it is misleading to speak of a natural convergence rate since estimates of different growth regressions come from different populations, and we find that correcting for the bias resulting from unobserved heterogeneity in technology levels leads to higher estimates of the rate of convergence. We also find that correcting for endogeneity of the explanatory variables has a substantial effect on the estimates and that measures of financial and fiscal development are important determinants of long-run differences in per capita income levels. We show that although the odds of a study being published is not uniform for studies with different $p$-values, publication bias has no significant effect on the conclusions of the analysis.

Keywords. convergence; economic growth; meta-analysis

1. Introduction: The Legendary 2%

The notion of convergence has been at the heart of a wide-ranging debate in the growth literature for some time. Excellent surveys are those of Temple (1999), Durlauf and Quah (1999), and Islam (2003). Intuitively, the term ‘convergence’ suggests a process whereby poor countries catch up to richer ones in terms of income levels. The convergence literature is therefore concerned with an issue of vital importance in economics: it deals with the distribution of riches across the
world and its evolution over time. Arguably, this explains the sizeable efforts that the economic profession has devoted to the empirical study of convergence.

Empirical papers in the literature initially set out to investigate the convergence process using growth regressions, with the level of initial income as the pivotal explanatory variable. A negative correlation between growth and initial income implies a tendency for poor countries to catch up (Baumol, 1986). The convergence concept associated with these regressions is known as $\beta$-convergence. Over the years, an avalanche of empirical cross-sectional convergence studies dealing with economic growth differentials across countries or regions appeared, giving rise to the overall impression that a 2% rate of convergence is almost ubiquitous. It is occasionally suggested that the convergence literature has discovered a new ‘natural constant’ (Sala-i-Martin, 1996).

A slightly different but closely related literature deals with the distributional dynamics of per capita income levels and focuses on the cross-sectional dispersion of per capita income across countries or regions, and its evolution over time (Quah, 1993). Here, the key concept is $\sigma$-convergence, where $\sigma$ stands for the variation in the cross-sectional distribution of per capita income, measured either by the standard deviation of the distribution or by the coefficient of variation. The concepts of $\beta$- and $\sigma$-convergence are strongly related, and it has been shown that $\beta$-convergence is a necessary, though not a sufficient condition for the reduction in the dispersion of per capita income over time.

In this paper, we complement the excellent qualitative surveys of the convergence literature by providing a quantitative, statistical analysis of the empirical estimates of the rate of convergence recorded in the literature. Specifically, we address several unresolved issues of interpretation and estimation using a multivariate statistical technique known as meta-analysis (see Stanley, 2001, for an introduction). Meta-analysis constitutes a set of statistical techniques that can be used to compare and/or combine outcomes of different studies with similar characteristics, or, alternatively, with different characteristics that can be controlled for. Although each individual study may give a good indication of the sampling uncertainty of the convergence rate, meta-analysis opens up the possibility of investigating the relevance of non-sampling issues such as research design, model specification, and estimation technique, which are usually relatively constant within a study (Hedges, 1997). This can be accomplished by including non-sampling characteristics as moderator or predictor variables in a meta-regression model. An obvious advantage of a meta-regression framework is the multivariate setup that allows for an assessment of the ‘true’ convergence rate, concurrently accounting for differences within and between studies.

Meta-analysis was originally developed in psychology, and later on extended to fields such as biomedicine and experimental behavioral sciences, specifically education, but is now increasingly used in economics as well (see Card and Krueger, 1995; Smith and Huang, 1995; Ashenfelter et al., 1999; Görg and Strobl, 2001; Dalhuisen et al., 2003; Nijkamp and Poot, 2004, for recent applications of meta-analysis). Our study is not the first published paper to employ meta-analysis to analyze the income convergence literature. Dobson et al. (2003) use meta-regression techniques to assess...
the effect of study characteristics on the size of the estimated rate of convergence, using a sample of published and unpublished studies. We extend their analysis in several ways. First, we employ a random sampling technique to sample studies from the voluminous literature of convergence studies. Our sampling strategy guarantees an objective and representative selection of studies for an application where incorporating all available evidence is not feasible due to the size of the literature. Second, in our meta-regression analysis, we consider a wider set of variables potentially accounting for the observed variation in estimated speeds of convergence, and the selection of variables is firmly rooted in theory. Third, our sample contains over 600 estimates as compared to the 214 observations used in Dobson et al. (2003). Finally, we more fully exploit the variety of statistical techniques available for meta-analysis, including an extensive descriptive exploratory analysis. We also include information on the precision of the original estimates by weighting the original estimates in the meta-regression, so that more precise estimates are given more weight. Finally, we provide several tests for the presence of publication bias and correct for its occurrence in a multivariate regression setting.

Given the breadth of the empirical economic growth literature, we restrict the sampling of studies to a specific domain. We only consider studies employing the concept of $\beta$-convergence in a cross-country or panel data setting using growth or the level of income per capita as dependent variables. As a result, we do not consider studies focusing on the distribution of per capita income, pure time-series studies, studies analyzing local or club convergence, and studies using total factor productivity (TFP) as the dependent variable. We acknowledge that these approaches are related (George et al., 2003; Islam, 2003), but the domain restriction guarantees that the population of studies is sufficiently homogeneous to be comparable.

The remainder of this article is structured as follows. Section 2 shows how frequently used empirical models in the empirical convergence literature are related to theories of economic growth, and how theories have been translated into empirical models that can be estimated. Section 3 describes the sampling of studies and the key characteristics of our meta-database and provides several pooled estimates of the rate of convergence utilizing different assumptions about the underlying population effect and publication bias. Section 4 discusses the meta-regression results using differing assumptions regarding heterogeneity, dependence, and publication bias. Section 5 concludes.

2. Convergence: From Theory to Empirics

The parameter of interest in empirical convergence studies modeling economic growth as a function of initial income and possibly a set of conditioning variables is the estimated coefficient of the income level at the beginning of the growth period. A negative coefficient indicates that poor countries on average grow faster than richer ones, which not necessarily implies a shrinking distribution of per capita income because unexpected disturbances can take a country above or below its growth path. A crucial point, however, is that such inferences can be
drawn without explicit reference to a specific theoretical growth model. In order to clarify the issues surrounding the interpretation of the estimated rate of convergence, we next discuss the links between empirical research and theoretical studies of economic growth. We also dwell upon several operational issues, such as the specification of differences in technology and the definition of steady states.

2.1 *Theoretical Background*

A natural starting point for a theoretical discussion of economic growth is the neoclassical growth model developed by Solow (1956) and Swan (1956). The key assertion of this model is the existence of a unique balanced growth equilibrium, a result due to placing a number of restrictions on the characteristics of the production function. The two key restrictions are diminishing returns to scale with respect to reproducible factors (capital) and a constant and exogenous rate of Harrod-neutral (labor augmenting) technological progress.

In the steady state, both capital and output per worker grow at the constant exogenous rate of technological progress. Denoting total output as $Y$, physical capital $K$, labor augmenting technology $A$, and the size of the labor force $L$, we can define a Cobb–Douglas production function given by:

$$Y = K^\alpha (LA)^{1-\alpha}$$

(1)

with $0 < \alpha < 1$ for the share of output paid to the owners of capital, which satisfies the above conditions.

Savings can be a constant fraction $s \in (0,1)$ of income, as in the Solow model, or be determined by a consumer optimization problem, as in the Ramsey model. In both cases, a unique balanced growth equilibrium:

$$\frac{\dot{y}}{y} = \frac{\dot{k}}{k} = \frac{\dot{A}}{A} = g,$$

(2)

exists, where $y = Y/L$ and $k = K/L$ are expressed in per capita form and $g$ is the growth rate of technology.

In addition to having identical balanced growth equilibria, the Solow and Ramsey models also have identical implications for the transition toward the steady state. Denoting $\dot{y} = Y/AL$ and $\dot{k} = K/AL$ as output and capital per efficiency unit of labor, respectively, a Taylor expansion in log $\dot{k}$ around the steady state $\dot{k}^*$ results in

$$\frac{\dot{k}}{k} = \lambda (\log \dot{k}^* - \log \dot{k}),$$

(3)

for both the Solow and the Ramsey models. The implication therefore is that the growth rate of capital per efficiency unit of labor $\dot{k}$ is proportional to the distance between its current value and the steady state.

Although the interpretation of $\lambda$ as the rate of convergence to the steady state is the same in both models, the variable itself is a function of different parameters.
In the Solow model, it is given by \( \lambda \approx (1 - \alpha)(n + g + \delta) \), where \( n \) is the rate of labor force growth and \( \delta \) the depreciation rate. In the Ramsey model, the convergence rate \( \lambda \) is a function of both technology and preference parameters, such as the rate of inter-temporal substitution and the rate of time preference.

Solving the differential equation (3), and using the Cobb–Douglas function expressed in intensive form as \( \tilde{y} = \tilde{k}^\alpha \), we arrive at

\[
\log \tilde{y}(t) = (1 - e^{-\lambda t}) \log \tilde{y}^* + e^{-\lambda t} \log \tilde{y}(0). \tag{4}
\]

In order to see how equation (4) can be converted into an empirically testable form, one should note that the available data are defined in terms of per capita income, or \( y = \tilde{y}A \). Substituting into equation (4) and subsequent rearranging gives

\[
\log y(t) - \log y(0) = (1 - e^{-\lambda t}) \ln A(0) + gt - (1 - e^{-\lambda t}) \ln y(0) \\
+ (1 - e^{-\lambda t}) \ln \tilde{y}^*. \tag{5}
\]

The key proposition of the neoclassical growth model is convergence within an economy rather than across economies. This fundamental characteristic of neoclassical growth theory notwithstanding, the majority of papers in the empirical growth literature has estimated a cross-sectional version of the model. Assuming that the initial level and the growth rate of technology are constant across countries and \( x \) represents a vector containing the determinants of the steady state, equation (5) can be expressed as

\[
\log y(t) - \log y(0) = \xi + \beta \ln y(0) + x' \gamma, \tag{6}
\]

where \( \xi \) is a constant. The stochastic form of this equation is then typically estimated using simple ordinary least squares (OLS). However, for this approach to be valid, several strong assumptions have to be made. During the last two decades, the literature has been working on relaxing these assumptions, and this has resulted in a plethora of approaches to estimate the rate of convergence. In the remainder of this section, we discuss several of the issues involved in transforming equation (5) into an operational empirical model, since this is one of the main sources of heterogeneity across studies.

### 2.2 Treatment of Technology

In traditional neoclassical inspired approaches to empirical convergence, both the initial level of technology and its subsequent growth rate are assumed constant and identical for all countries, apart from random variation in initial technology that is subsumed in the error term (Mankiw et al., 1992). Specifically, it is assumed that the initial level of technology has a fixed and a normally distributed random component

\[
A_i(0) = a + \varepsilon_i \quad \text{with} \quad \varepsilon_i \sim N(0, \sigma^2), \tag{7}
\]

where \( i \) refers to the country. This is a rather strict assumption allowing for the estimation of equation (7) with OLS.

Extensions of the Mankiw et al. (1992) approach have moved from a cross-section to a panel data setting in order to relax the assumption of identical
technologies and to allow for country-specific differences in the level of technology by means of fixed or random effects (Islam, 1995). There is some discussion in the literature as to which type of estimator is more appropriate in the presence of endogeneity and omitted variable bias. The fixed-effects model (FEM) allows for individual effects, but the estimator is inconsistent in the presence of endogeneity. The random-effects model (REM) is not appropriate if the initial level of technology $A(0)$ is correlated with other explanatory variables, for instance, with the savings and population growth rates. Other variants, such as seemingly unrelated regression (SUR) estimation, allow for individual constants and correlated error terms, and the minimum distance (Chamberlain, 1982, 1983) and general method of moments (GMM) methods, allow for both individual effects and endogeneity of the explanatory variables.

Another issue centers on panel data estimates capturing short-run effects (e.g. business cycles) versus cross-sectional estimates depicting long-run transitional dynamics. Typically, panel data observations are 5-year averages, whereas cross-sectional observations are 25-year averages, or even longer in more recent applications. The empirical equation used to estimate the rate of convergence is derived from the neoclassical models using a first-order Taylor expansion. In a strict sense, this approximation is only valid in the neighborhood of the steady state. It is therefore difficult to defend the use of this equation to estimate a model using 25-, 50-, or even 100-year averages.

Apart from the level of technology varying across countries, it may also be that its growth rate differs across countries. Lee et al. (1997) allow for such variation and find a substantially higher estimate of the rate of convergence.

This discussion of the treatment of technology implies that potential heterogeneity in estimates of the rate of convergence within the convergence literature may be related to differences in the way technology is treated. In an operational sense, this yields a series of moderator variables to be considered in a meta-regression framework (see Section 4). Specifically, we investigate whether differences in the type of estimator employed in the primary studies, the data characteristics (cross-section versus pooled data), and the periodical frequency of the data affect the estimates of the rate of convergence obtained.

### 2.3 Definition of the Steady State

Another important potential source of heterogeneity deals with the definition of the steady-state per capita income level ($y^*$). The simplest identifying assumption amounts to steady states being identical, and this may very well be appropriate in studies considering convergence of states or regions within a country (Barro and Sala-i-Martin, 1992). In terms of equation (6), convergence in per capita income levels implies the term $x'\gamma$ is constant and that the coefficient of initial income should be negative for convergence to occur. This concept is known in the literature as absolute or unconditional convergence. The evidence on unconditional convergence is mixed. Negative estimates of $\beta$ in unconditional
convergence regressions have only been found for relatively homogenous samples such as OECD countries (Baumol, 1986).

The lack of evidence on unconditional convergence has prompted a wave of conditional convergence models in which steady states are allowed to differ across countries. In the simple Solow model, the steady state is given by

$$y^* = \left( \frac{s}{n + g + \delta} \right)^{\alpha/(1-\alpha)}.$$

Mankiw et al. (1992) extend the Solow model to allow for two forms of capital, viz. physical and human capital. The steady-state income level is then a function of the rates of investment in human and physical capital, the human and physical capital income shares, and the respective depreciation rates. If the rates of technological progress and depreciation are assumed to be the same across countries, the steady state can be uniquely defined in terms of the savings rate in physical and human capital and the population growth rate. This is the approach taken in the seminal Mankiw et al. (1992) paper. The dynamics of the Solow model imply that a country grows faster the further away it is from its steady state. Empirical conditional convergence results appear to support this notion in the sense that after controlling for steady-state differences (in population growth, savings, and human capital accumulation rates), poor countries grow faster than richer ones (Mankiw et al., 1992; Barro and Sala-i-Martin, 1995).

An alternative to this theory-based approach to conditional convergence is the less formal, data-driven approach of, amongst others, Kormendi and Meguire (1985), Grier and Tullock (1989), and Barro (1991). In this approach, extensive data sets are constructed, containing a host of variables potentially affecting economic growth. They are subsequently used to simply ‘try out’ regressions without a clear link to theory. These approaches are often criticized for testing without theorizing and for generating at best very restricted robust results (Levine and Renelt, 1992; Sala-i-Martin, 1997; Florax et al., 2002; Temple, 2003; Sala-i-Martin et al., 2004). Arguably, they can also be seen as attempts to investigate the empirical relevance of factors brought up in new endogenous growth theories (see Barro and Sala-i-Martin, 1995; Aghion and Howitt, 1998, for surveys). As such, they may result in better parameterizations of steady states as well as contribute to limiting the disturbing impact of omitted variables. The latter can also be achieved by restricting the sample to countries or regions that are similar in terms of technology and institutions (Barro and Sala-i-Martin, 1995).

Apart from omitted variable bias, endogeneity of the regressors has been identified as a major concern because it renders the OLS estimator biased and inconsistent. Cho (1996) convincingly argues that this is problematic for the Solow variables, population growth and the savings rate. However, many variables are potentially endogenous, even to the extent that Caselli et al. (1996) observe that: ‘[A]t a more abstract level, we wonder whether the very notion of exogenous variables is at all useful in a growth framework (the only exception is perhaps the morphological structure of a country’s geography)’. Barro and Lee
(1993) and Barro and Sala-i-Martin (1995) address the endogeneity issue by estimating a system of stacked equations, using lagged values of the explanatory variables as instruments, while Caselli et al. (1996), Hoeffler (2002), and others use a GMM estimator.

On the basis of the above, we once again identify a series of factors that may create heterogeneity in the empirical convergence literature. Specifically, we analyze the effects of including different sets of explanatory variables in the vector $x$, not only because omitted variable bias may be important when the specification is restricted to only a few variables, but also because the convergence rates estimated using different model specifications may, strictly speaking, be measuring different population parameters. The issue of endogeneity can be analyzed by specifying the type of estimator used in each primary study, and we also consider the effect of restricting the sample to countries or regions that are similar in the sense that they may share the same steady-state characteristics.

3. Literature Sampling and Combining Estimates

The empirical literature on convergence is large and rapidly expanding. On the one hand, this makes it prohibitive to sample all studies at a reasonable cost. On the other, it necessitates applying set, \textit{a priori} rules for sampling in order to safeguard the representative nature of the sample of studies.

We utilized the following sampling criteria. First, we searched the EconLit database for empirical studies on economic growth. Subsequently, we reduced the sample by considering only articles published in journals and in the English language, and excluded studies focusing exclusively on the time-series dimension, those using a growth accounting method, or employing TFP as the dependent variable. The total number of studies left after applying these criteria was 1650. As a final step, we randomly selected studies to be included in the meta-analysis from this listing of studies until the results of the meta-analysis were robust to including additional observations.

For each reported regression in the primary studies, we recorded an estimate of the rate of convergence and its associated standard error. In addition, we recorded publication details, characteristics of the original data set such as the number of cross-sectional and temporal units, the level of aggregation (countries or regions), whether or not purchasing power parity (PPP) exchange rates were used and their source, the initial year of the sample and the number of observations, and regression characteristics such as the type of estimator, and the type and number of conditioning variables included in the regression. The total number of observations in our database is 619, each corresponding to a regression, provided by 48 separate studies. An overview of the studies is given in Table 1, showing that with the exception of Taylor (1999), all studies provide multiple estimates, ranging from 2 to 54 per study. The average convergence rate is 4.3%, implying a half-life (i.e. the time span needed to cover half the distance to the steady state) of 16 years, and on average the rate of convergence ranges from 1.4 to 8.3%.
Table 1. Reference, number of estimates, convergence rate, and implied half-life of the studies included in the meta-sample.*

<table>
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<th>Study</th>
<th>Number of estimates</th>
<th>Convergence rate†</th>
<th>Implied half-life‡</th>
</tr>
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<td>6</td>
<td>3.25 17.52</td>
<td>20.71 4</td>
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<td>15</td>
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<td>4.31 25</td>
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<td>6</td>
<td>20.45 47.40</td>
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<td>2</td>
<td>1.79 1.83</td>
<td>1.86 38</td>
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<td>0.56 0.88</td>
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(continued)
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<td>1.43</td>
<td>4.30</td>
<td>8.34</td>
<td>16</td>
</tr>
</tbody>
</table>

*An extended table detailing, among other things, the source of the data, the spatial scale, the type of estimator, and control variables included in the study, is available on http://www.henridegroot.net/pdf/JES-database.xls.
†In percentage points.
‡For the mean convergence rate.
§Sum for the first column, average for the other columns.

![Convergence rate distribution](chart.png)

**Figure 1.** Within-study mean (■), median (□), and standard deviation around the mean (error bar) of convergence rates in percents per year, ordered according to increasing magnitude of the within-study mean. The numbers next to the references indicate the number of observations per study.
Figure 1 graphically provides descriptive statistics for the studies in our sample, including the mean, median, and standard deviation of the rate of convergence reported in each case. It also shows that most studies have a fairly homogeneous within-study distribution of estimates. Except for Henisz (2000), Savvides (1995), Abrams et al. (1999), and Arena et al. (2000), the mean and median estimates are fairly close, implying the within-study distribution is not skewed, and the within-study variance of the estimates is reasonably small.

Figure 2 presents the same data from a slightly different perspective. It shows a histogram of the convergence rates as a fraction of the total meta-sample \((n = 610)\). A small proportion of the estimates is negative, and there are a few (positive) outliers; approximately 9% of the estimated rates of convergence exceed 10%, implying a half-life of less than 7 years. A substantial number of observations is clustered around a convergence rate of 2%; the proportion of estimates that lies between a convergence rate of 1 and 3% is close to one-third.

Apart from information on the effect sizes, it is also desirable for the meta-analysis to take into account the fact that the standard errors of the respective estimates are different, among other things because the sample sizes of the primary studies differ. We can recover estimates of the rate of convergence and their associated standard errors from almost any regression of growth on the logarithm of initial income. Consider the following general model:

\[
\ln y_{it} - \ln y_{i,t-\tau} = \alpha + \beta \ln y_{i,t-\tau} + x_{it}'\gamma + \eta_t + \mu_i + \varepsilon_{it},
\]

where \(x_{it}\) is a vector of explanatory variables, \(\eta_t\) a time-specific effect, \(\mu_i\) a country-specific effect, and \(\varepsilon_{it}\) an error term that varies across countries and periods. A regression of this form will yield an estimate \(\hat{\beta}\) and a corresponding estimated standard error \(\hat{\sigma}_\beta\). The coefficient \(\beta\) and our variable of interest, the rate of convergence \(\lambda\), are related via
Estimates for the convergence rate $\hat{\lambda}$ can therefore be obtained as

$$\hat{\lambda} = -\frac{\ln(1 + \hat{\beta})}{\tau}$$

and the estimated standard error $\hat{\sigma}_\lambda$ can be approximated by

$$\hat{\sigma}_\lambda = \frac{\hat{\sigma}_\beta}{\tau(1 + \hat{\beta})},$$

where $\tau$ is the length of one time period. In our meta-analysis, we consider estimates of convergence rates and their associated standard error obtained directly using a non-linear estimation method, as well as those obtained through the transformations defined in equations (11) and (12).

### 3.1 Pooling Estimates

We continue describing the characteristics of our data set by using different methods to combine study estimates. In doing so, the estimated standard errors are taken into account, since they provide a measure of their precision. There are two common ways of combining study estimates, using either a ‘fixed-’ or a ‘random-’effects estimator. The fixed-effects method, also known as the inverse variance-weighted method, assigns to each estimate a weight inversely proportional to its variance. The crucial assumption of the fixed-effects method is that all studies measure the same underlying population effect. The random-effects method assumes that the studies are a random sample from a larger population of studies and that the population effect sizes are randomly distributed around a population mean. The weights in this case are the reciprocal of the sum of the between- and within-study variances (see Section 4).

We calculate pooled estimates of the rate of convergence for our sample of 610 regressions. The pooled fixed effect estimate of the rate of convergence is 0.2% per year. The random-effects estimate is 2.4% per year. Both estimates are significantly different from zero with a $p$-value $<0.001$. The assumption of the fixed-effects method that there is one population effect size (one ‘true’ rate of convergence) is rather unrealistic given that we are combining estimates of studies with widely varying characteristics, and the rate of convergence is an average across different samples of countries and regions. Furthermore, both estimators assume that the observations are independent, which is probably reasonable if single measurements are taken from each primary study, but it is quite unlikely when multiple measurements are taken. The estimators are therefore not efficient, but Bijmolt and Pieters (2001) show that using multiple measurements is to be preferred in terms of detecting the ‘true’ underlying population effect size. Figure 3 shows a forest plot of the individual and pooled estimates using the random-effects model (REM). It is obvious that the results of Haveman et al. (2001), Abrams et al. (1999), Dixon
et al. (2001), and Arena et al. (2000) are furthest off the pooled estimate and especially the latter has a rather wide confidence interval.  

3.2. Publication Bias

A pivotal issue in meta-analysis is whether the meta-sample is subject to publication bias, implying a tendency for published papers to exhibit statistically significant results for the main variable of interest. This phenomenon may occur either because of self-censoring by authors or because editors of journals make publication decisions partly on the basis of statistical significance levels. One of the advantages of meta-analysis over a conventional literature review is precisely that its quantitative nature allows for testing and correcting for publication bias. Various tests have been developed, although some of them have been shown not to be overly powerful in detecting publication bias (Macaskill et al., 2001).

We proceed by using a test based on a visual representation known as the funnel plot (Egger et al., 1997). The funnel plot, presented in Figure 4, gives the convergence rate on the horizontal axis and its precision (as defined by the standard error) on the vertical axis. Figure 4 shows that as compared to statistical expectations, there is an apparent overrepresentation of studies showing convergence rather than divergence. Specifically, in view of the mean of 2%, there is an obvious imbalance between the occurrence of very large positive convergence rates and hardly any estimated rates that are smaller than zero.

Figure 3. Forest plot of 48 estimated convergence rates (in percents per year) with 95% confidence intervals based on random effects, including the pooled random-effects estimate as a dashed vertical line.
Moreover, the results of studies with smaller sample sizes (and therefore larger standard errors) scatter more widely, as expected, but are clearly underrepresented. Egger et al. (1997) suggest a test of funnel asymmetry in which the standardized effect size is regressed against the standard error. The test consists of evaluating whether the estimate of the intercept differs significantly from zero, which is taken to be indicative of publication bias. The estimated intercept for our meta-sample is 4.24, with a $t$-statistic of 19.01, indicating the presence of publication bias.

It has been suggested that the slope coefficient can provide a rough estimate of the effect size corrected for publication bias (Sutton et al., 2000a). In our meta-sample, this estimate is $-0.2\%$, with a 95% confidence interval ranging from $-0.28$ to $-0.11$. The evidence shown by the test and the funnel plot should, however, be interpreted with caution because it rests on a simple bivariate analysis and the effects may also be caused by other biases (see Egger et al., 1997; Sterne et al., 2001, for a discussion).

A method to correct for publication bias by combining estimates from the primary studies is due to Duval and Tweedie (2000a, b), who use a non-parametric ‘trim and fill’ method that estimates the number and outcomes of hypothetical studies that are missing due to publication bias, and adds the hypothetical study results to the meta-analysis so that in effect the symmetry in a funnel plot is recovered. The ‘trim and fill’ results for our meta-sample are very different depending on whether the fixed- or random-effects method is used to compute the combined estimates. The total number of observations (real and hypothetical) following the ‘trim and fill’ method is 901. When pooled, these
observations result in a convergence rate of −0.1% using fixed effects or 0.3% using random effects, both with \(p\)-values <0.001. These results should be contrasted with pooled estimates of 0.2% (fixed effects) and 2.4% (random effects), both with \(p\)-values <0.01, obtained using the traditional method that takes no account of publication bias.

From the above results, we infer the following preliminary conclusions. First, combining the estimated effect sizes attained in the empirical convergence literature by means of the fixed-effects estimator is overly restrictive. This is not all that surprising, because the fixed-effects estimate is simply an inverse variance-weighted average and the method assumes that there is a single, fixed underlying population effect size. This conclusion is also corroborated statistically by the results of the \(Q\)-test, which indicates the presence of heterogeneity.\(^{15}\) Second, the random-effects estimate provides some evidence to support the common perception that a ‘natural’ rate of convergence of about 2% exists. However, merely combining estimated convergence rates and assuming that all underlying differences are essentially unobservable and random is very restrictive as well. Specifically, some of the differences are easily observable (e.g. the estimation method, sample, and conditioning variables used in the primary studies), and this information should be used in order to reach a more efficient and informative conclusion. We therefore proceed by specifying a meta-regression in which differences are at least partly treated as observable. Finally, the results also show that one should be aware of the potential impact of publication bias; although in order to reach a definitive conclusion, it is necessary to apply a multivariate analysis, since the results of the publication bias tests could be driven by real underlying differences in the primary estimates.

4. Meta-Regression Model

We continue by presenting the results for a meta-regression specification with exogenous variables as indicated in Section 2 and described in more detail below. Before proceeding, however, we provide a detailed explanation of the different estimators that we apply.

The first estimator, which is becoming increasingly popular in recent meta-analysis applications (Smith and Kaoru, 1990; Boyle \textit{et al.}, 1994; Görg and Strobl, 2001), is the Huber–White estimator. This estimator simultaneously corrects for heteroskedasticity and cluster autocorrelation (Williams, 2000; Wooldridge, 2002, Section 13.8.2), and hence accounts for the multiple sampling data setup by allowing for different variances and non-zero covariances for clusters of measurements from the same study. Arguably, however, the Huber–White estimator is rather restrictive in assuming that all differences across observations and studies are observable and can entirely explain the empirical heterogeneity. In addition, the Huber–White estimator does not fully exploit all available information because it estimates the variance rather than taking it as given or recoverable from the primary studies.
The latter can be remedied by using a multivariate version of the fixed- or random-effects meta-estimator that we already employed in the bivariate case in the preceding section. We consider \( n \) growth regressions, indexed by \( i (= 1, 2, \ldots, n) \), and assume that deviations of the estimated convergence rate \( \hat{\lambda}_i \) from the true effect size \( \lambda_i \) are random:

\[
\hat{\lambda}_i = \lambda_i + \varepsilon_i, \quad \varepsilon_i \sim N(0, \sigma^2_i),
\]

\[
\lambda_i = \alpha + x_i^T \beta + \mu_i, \quad \mu_i \sim N(0, \tau^2),
\]

where \( \alpha \) is a common factor and \( x_i \) contains a set of design and data characteristics. We thus allow the true effect size and the precision of the estimated effect size to vary across regressions. The term \( \sigma^2_i \) is known as the within-variance and is usually taken as given and derived from the original regression. Any remaining heterogeneity between estimates is either explainable by the observable differences modeled through the moderator variables contained in \( x_i \) or is random and normally distributed with mean zero and variance \( \tau^2 \), the between-variance. If \( \tau^2 = 0 \), the model is referred to as the fixed-effects model (FEM), and it is assumed that all heterogeneity in the true effect size can be explained by differences in study characteristics. If the between-variance is not equal to zero, the model is a REM, which is usually referred to as a 'mixed-effects' model because it contains observable ‘fixed’ characteristics in \( x_i \) as well as a random unobservable component with mean zero and variance \( \tau^2 \). The unknown variance can be estimated by an iterative (restricted) maximum likelihood process or, alternatively, using the empirical Bayes method, or a non-iterative moment estimator (Thompson and Sharp, 1999). We use the iterative-restricted maximum likelihood estimator with weights

\[
\tilde{w}_i = \frac{1}{\sigma^2_i + \tau^2}
\]

(14)

to obtain estimates for the regression coefficients and \( \tau^2 \).

In comparison with the Huber–White estimator, the FEM is equally restrictive in assuming that the observed empirical heterogeneity is perfectly observable. It does, however, incorporate information on the estimated standard errors of the original regressions, although it does assume that all observations, including multiple observations taken from the same study, are independent. The mixed-effects estimator relaxes the assumption of fixed population effect sizes, but does not allow for dependence among the errors either. The latter may imply that the fixed- and mixed-effects estimators are not the most efficient estimators, and inferences regarding statistical significance should therefore be drawn with caution.

The last estimator we use builds on the mixed-effects model but corrects for publication bias. The estimator for a simple univariate REM was developed in Hedges (1992), and later on extended to a mixed-effects model in Vevea and Hedges (1995).^18^ The approach is based on assuming that there is a step function for different classes of \( p \)-values, and subsequently estimating a model in which the sampling probability of the first class of \( p \)-values (e.g. \( p < 0.01 \)) is set to 1, and the sampling probabilities for the other critical classes of \( p \)-values (such as
0.01 < p < 0.05, 0.05 < p < 0.10, and p > 0.10) are estimated within the model. Intuitively one expects, in the case of publication bias, that the likelihood of sampling studies with increasing p-values will show a non-linear decline, or in other words, that studies with lower p-values are more likely to be published. The Hedges approach to modeling publication bias is based on weighted distribution theory, and the appropriate maximum likelihood estimator for a mixed-effects model incorporating a step function as well as tests for publication bias are derived in Vevea and Hedges (1995).

4.1 Empirical Results

Table 2 summarizes the results of the meta-regression model for the different estimators outlined above (Huber–White, fixed effects, mixed effects, and mixed effects with a step function). We use three classes of explanatory variables. One class deals with data characteristics, the second with estimation characteristics, and the third class refers to the inclusion of different conditioning variables in the primary studies.

One of the most striking results in Table 2 is that the results of the Huber–White estimator results in relatively few significant moderator variables. A formal comparison of the Huber–White estimator to the traditional fixed- and random-effects estimators is not yet available, but our results indicate that the Huber–White approach may not be very adequate because it does not utilize all available information (i.e. information on the precision of the estimates, as measured by the standard errors) and results in an overly conservative statistical assessment. It is well known that in the case where there is evidence of unobserved heterogeneity, the fixed-effects estimator is insufficiently conservative (Sutton et al., 2000a, pp. 83–84). Table 2 shows that the between-variance is relatively large and should not be ignored. Hence, the fixed-effects confidence intervals are likely to be too small.

In the remaining part of this section, we therefore focus on interpreting the results as provided by the mixed-effects model. Before doing so, we note that the estimation results with and without correction for publication bias are very similar. However, a likelihood ratio test on the null hypothesis of no publication bias is rejected, indicating that at least one of the estimated sampling probabilities assigned to the p-value classes is not equal to unity. It is easily verified (with the results provided in Table 2) that none of the estimated coefficients of the different p-value classes is significantly different from unity, with the exception of the class 0.05 < p < 0.10, which is different from unity at the 1% significance level.

We therefore find no evidence of publication bias in the traditional sense of the term’s meaning (i.e. insignificant results are under-sampled), although we do find that the sampling probability does not follow a uniform distribution for all the p-value classes. In particular, observations with p-values between 5 and 10% are oversampled. There could be several explanations for this result. Different authors and editors may follow different ‘publication’ rules, basing their submission or publication decisions on sample size, sample coverage, the standard deviation of the regression, or size and sign of the parameters. In addition, we
Table 2. Results of the meta-regression estimation for different estimators.†

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Huber–White (1)</th>
<th>Fixed effects (2)</th>
<th>Mixed effects‡ (3)</th>
<th>Mixed effects (corrected for publication bias) (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significance</td>
<td>$p &lt; 0.01$</td>
<td>$p &lt; 0.01$</td>
<td>$p &lt; 0.01$</td>
<td>$p &lt; 0.01$</td>
</tr>
<tr>
<td></td>
<td>$0.01 &lt; p &lt; 0.05$</td>
<td>$0.01 &lt; p &lt; 0.05$</td>
<td>$0.01 &lt; p &lt; 0.05$</td>
<td>$0.01 &lt; p &lt; 0.05$</td>
</tr>
<tr>
<td></td>
<td>$p &gt; 0.10$</td>
<td>$p &gt; 0.10$</td>
<td>$p &gt; 0.10$</td>
<td>$p &gt; 0.10$</td>
</tr>
<tr>
<td>Data characteristics</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Summers and Heston</td>
<td>$-0.789 (1.399)$</td>
<td>1.241*** (0.059)</td>
<td>0.124 (0.392)</td>
<td>0.130 (0.396)</td>
</tr>
<tr>
<td>Maddison data</td>
<td>$-0.317 (2.306)$</td>
<td>$-0.219 (0.153)$</td>
<td>0.109 (0.894)</td>
<td>0.113 (0.905)</td>
</tr>
<tr>
<td>Regional PPP × Regional aggregation</td>
<td>9.877 (8.000)</td>
<td>0.594*** (0.084)</td>
<td>1.847*** (0.594)</td>
<td>1.835*** (0.600)</td>
</tr>
<tr>
<td>Regional level of aggregation</td>
<td>6.246 (4.539)</td>
<td>$-0.217*** (0.078)$</td>
<td>1.098* (0.571)</td>
<td>1.124* (0.578)</td>
</tr>
<tr>
<td>Homogeneous sample</td>
<td>0.893 (0.948)</td>
<td>1.097*** (0.063)</td>
<td>0.851** (0.341)</td>
<td>0.848** (0.344)</td>
</tr>
<tr>
<td>Use of per capita income</td>
<td>$-5.016 (3.674)$</td>
<td>0.492*** (0.090)</td>
<td>0.602 (0.509)</td>
<td>0.611 (0.514)</td>
</tr>
<tr>
<td>Structure of the data</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of cross-sectional units§</td>
<td>0.000 (0.002)</td>
<td>$-0.001*** (0.000)$</td>
<td>$-0.001 (0.001)$</td>
<td>$-0.001 (0.001)$</td>
</tr>
<tr>
<td>Number of time units§</td>
<td>$-0.360 (0.218)$</td>
<td>$-0.098*** (0.010)$</td>
<td>$-0.125** (0.064)$</td>
<td>$-0.125* (0.064)$</td>
</tr>
<tr>
<td>Number of observations§</td>
<td>$-0.001 (0.002)$</td>
<td>0.001*** (0.000)</td>
<td>0.001 (0.001)</td>
<td>0.001 (0.001)</td>
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<td>Time span of the data§</td>
<td>0.010 (0.018)</td>
<td>0.012*** (0.002)</td>
<td>0.008 (0.009)</td>
<td>0.008 (0.010)</td>
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<td>Initial year of the sample§</td>
<td>0.004 (0.025)</td>
<td>$-0.007*** (0.002)$</td>
<td>$-0.002 (0.008)$</td>
<td>$-0.002 (0.008)$</td>
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<tr>
<td>Pooled data</td>
<td>7.821* (4.455)</td>
<td>0.307*** (0.065)</td>
<td>1.423*** (0.533)</td>
<td>1.404*** (0.538)</td>
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<td>Pooled data × Length of time units§</td>
<td>$-0.570 (0.356)$</td>
<td>$-0.051*** (0.005)$</td>
<td>$-0.172*** (0.0428)$</td>
<td>$-0.173*** (0.043)$</td>
</tr>
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<td>Estimation characteristics</td>
<td></td>
<td></td>
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<tr>
<td>Non-linear method</td>
<td>$-2.266 (2.153)$</td>
<td>0.125 (0.221)</td>
<td>$-1.027 (1.208)$</td>
<td>$-1.052 (1.226)$</td>
</tr>
<tr>
<td>Non-linear least squares</td>
<td>0.512 (1.982)</td>
<td>0.685*** (0.222)</td>
<td>0.897 (1.240)</td>
<td>0.932 (1.258)</td>
</tr>
<tr>
<td>Instrumental variables</td>
<td>$-0.218 (1.547)$</td>
<td>0.244** (0.108)</td>
<td>0.948** (0.481)</td>
<td>0.952* (0.489)</td>
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<tr>
<td>Seemingly unrelated regression</td>
<td>1.028 (1.620)</td>
<td>2.053*** (0.213)</td>
<td>1.888* (1.134)</td>
<td>1.946* (1.151)</td>
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<td>Fixed effects</td>
<td>3.754* (2.139)</td>
<td>2.404*** (0.112)</td>
<td>4.282*** (0.507)</td>
<td>4.350*** (0.510)</td>
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### Random effects

<table>
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<td></td>
<td>Mean</td>
<td>SE</td>
<td>Mean</td>
<td>SE</td>
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<tr>
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<td>$-0.317$</td>
<td>$1.050$</td>
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<td>$0.275$</td>
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### General method of moments

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<td></td>
<td>Mean</td>
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<td>Mean</td>
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<td>$2.853$</td>
<td>$2.998$</td>
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<td>$7.900$</td>
<td>$1.140$</td>
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<tr>
<td>$6.228$</td>
<td>$0.537$</td>
<td></td>
<td>$6.268$</td>
<td>$0.540$</td>
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### Conditioning variables

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<td></td>
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<td>Standard Solow</td>
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### $\tau$

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### $R^2$-adjusted\[†\]

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### $F$-statistic

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<tr>
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<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SE</td>
</tr>
<tr>
<td>$18.23$**</td>
<td>$37.45$***</td>
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### Root MSE

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<tbody>
<tr>
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<tr>
<td>$4.88$</td>
<td>$3.66$</td>
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### Log-likelihood

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<tbody>
<tr>
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<td>Mean</td>
</tr>
<tr>
<td>$-949.02$</td>
<td>$-937.77$</td>
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### Observations

<table>
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<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>$610$</td>
<td>$610$</td>
</tr>
</tbody>
</table>

†The results are provided with standard errors in parentheses. Statistical significance is indicated using ***, ** and * referring to the 1, 5, and 10% level. The dependent variable is the average rate of convergence per year in percentage points.

‡The estimates for the mixed-effects estimator have been generated using the routine provided by Oosterbeek (see footnote 18).

§Continuous variables. All other variables are dummies.

¶The $R^2$ results in columns (1) and (2) are not directly comparable, in particular because the usual domain is not applicable in the case of the adapted weighted least squares estimator for the fixed-effects model.
have sampled studies with different primary variables of interest (e.g. taxes, geography, or education). The decision to publish could be based on the significance level of these variables and not on the significance level of the estimated rate of convergence. Moreover, the decision to publish a study often depends on the sign, size, and significance levels of previous studies, or of other regressions within the same study. A more in-depth analysis of the mechanisms driving publication bias is an interesting area for future research. For the moment, we conclude by noting that oversampling of observations with p-values between 5 and 10% does not significantly affect the results of the meta-regression.

4.2 Results for Data Characteristics

The first set of variables included in the regressions is related to data characteristics. The variables ‘Summers and Heston,’ ‘Maddison’, and ‘Regional PPP × Regional aggregation’ refer to the source of the PPP rates used in the primary study. The Regional PPP × Regional aggregation term refers to studies at the regional level that make use of data adjusted for regional price differences. The reference category is data based on market-exchange rates. Our hypothesis is that the use of PPP rates leads to higher estimates of the rate of convergence. The intuition is that, after controlling for the steady state, the coefficient of income measures how fast countries approach their steady state. The use of market-exchange rates makes poor countries appear poorer than they actually are. After controlling for the steady state, it appears that countries are further away from the steady state than they really are, or in other words, that they are approaching it more slowly. Our hypothesis is supported in the case of the mixed-effects model, particularly for regional PPP rates. The coefficients for Summers and Heston and Maddison data are positive although not statistically significant. In the case of regional PPPs, their use raises the estimated rate of convergence by 1.8 percentage points.

We also investigate whether the use of regional data leads to different results. Our hypothesis is that regional data are more homogenous than cross-country data, particularly when it comes to the level and growth rate of technology. Omitted variable bias due to excluding a measure of technology from the original growth regression is expected to create a downward bias, since the coefficient of initial income is negative, and the effect of technology on growth is positive (for a discussion, see Caselli et al., 1996). The empirical results appear to confirm this hypothesis: the use of regional data (expected to be more homogenous in terms of technology and other omitted variables such as institutions) leads to a rate of convergence that is, on average, 1.1 percentage points higher.

We have also included a constructed variable to measure the effect of using a relatively homogeneous sample. ‘Homogeneous’ is a dummy variable that equals one if the sample comprises the OECD countries, a regional cross-country sample, or a regional sample (e.g. the provinces of Spain or the prefectures of Japan). The coefficient is also positive and significant in this case; the use of a
homogeneous sample leads to convergence rates that are, on average, 0.9 percentage points higher.

Finally, we included a dummy variable to record whether the dependent variable in the growth regressions is per capita income or per capita gross product, labeled ‘Per capita income’. Some theoretical models have predicted different results due to migration, particularly for regional data sets. Our regressions indicate that this distinction does not lead to significantly different estimates of the rate of convergence.

4.3 Results for Structure of the Data

The next set of variables included in the regressions is related to the dimensions and structure of the data. One hypothesis is that averaging over a larger number of countries (or regions) and time units leads to lower estimates of the rate of convergence. The reason is that it increases the heterogeneity in the sample, and therefore the likelihood of omitted variable bias. The regression results appear to confirm our hypothesis, although for the number of time units only and with a rather small effect of $-0.1$ percentage points. Surprisingly, the variable ‘Number of observations’ has a positive coefficient in all the weighted regressions, but it is not significantly different from zero in most cases.

Another hypothesis concerns the total time span of the data. Use of data spanning a larger number of years (50–100 years instead of the usual 25) could lead to higher estimates of the rate of convergence, since theory predicts that the rate of convergence decreases as a country approaches its steady state (for a discussion, see Barro and Sala-i-Martin, 1995, p. 53). The regression results, however, show that there is no significant difference.

We also included a variable to control for the initial year of the sample, labeled ‘Initial year of the sample’, hypothesizing that convergence patterns may have changed over time. The coefficient is negative but not significantly different from zero.

Finally, we include two variables to measure the effects of short frequency on panel data estimates. The variable ‘Pooled data’ measures the effect of simply breaking up the data into several shorter periods – regardless of the type of estimator used; there are even some instances of OLS estimation. There is a rather large effect of shorter frequency on estimates of the rate of convergence. The interaction variable ‘Pooled data × Length of time units’ measures the effect of increasing the length of the growth episode (in the case of pooled data) by 1 year. The coefficient in this case is negative and highly significant, perhaps capturing the effect of business cycles.

4.4 Results for Estimation Characteristics

This set of variables includes the type of estimator used, and whether the estimate was found directly using a non-linear method or indirectly through a transformation. We include the variable ‘Non-linear method’ in order to verify that our transformation of the coefficient of initial income does not systematically bias the
estimates of the rate of convergence. As expected, the coefficient is not significantly different from zero.

The next group of variables is included to test some of the arguments advanced by different authors in the convergence debate. For instance, Caselli et al. (1996), Hoeffler (2002), and many others have shown that GMM estimation can correct for omitted variable bias (of country-specific effects) and endogeneity, both of which could bias the estimate of the rate of convergence downwards. Our results indicate that using GMM leads to estimates of the rate of convergence that are higher by 6.3 percentage points, a substantial difference. In a recent paper, Bond et al. (2004) again challenge whether the traditional use of the GMM estimator is adequate. Their slightly altered version of the GMM estimator results in estimates that are much closer to the habitual 2% rate. The use of the fixed-effects estimator also leads to higher estimates of the rate of convergence, by 4.4 percentage points, whereas the use of the random-effects estimator does not have a significant effect.

The use of the SUR estimator can also be expected to correct for omitted variable bias, since it allows for country-specific constants while allowing correlation in the error term. Our results indicate that the use of SUR leads to estimates that are 1.9 percentage points higher. The use of instrumental variables estimation raises the estimate of the rate of convergence by, on average, 1.0 percentage points, while the use of non-linear least squares has no discernable effect.

4.5 Results for Conditioning Variables

We include this last set of variables in order to test the arguments of the unconditional versus conditional convergence controversy. The variables in this section refer to the explanatory variables included in the original regression. Although in many meta-analyses the specification of the conditioning variables is dealt with rather casually, the simulation experiments in Koetse et al. (2004) and Keef and Roberts (2004) show that for a meaningful and statistically unbiased comparison, it is crucial that the meta-specification contains a judicious account of the conditioning variables of the primary studies. Our reference category is the unconditional convergence model.

The variable ‘Standard Solow’ equals one if the Solow model variables (the savings and population growth rates) are included in the original regression, and zero otherwise. Our hypothesis is that inclusion of the Solow variables results in higher estimates of the rate of convergence, since they control (at least to some degree) for differences in steady-state levels. The coefficient is positive and significant in all the regressions and has a magnitude of 2.1 percentage points.

The variables ‘Enrollment rates’ and ‘Human capital stock’ are included to test the hypothesis that the steady state is partly determined by human capital (Mankiw et al., 1992), and our hypothesis is that the rate of convergence estimates are higher when human capital is included in the regression. The coefficients of both variables are, however, not statistically different from zero.
We base the categories of the other conditioning variables on the distinctions made in Levine and Renelt (1992), who study the robustness of coefficients in growth regressions. The fiscal policy variables are related to taxes and government spending. Trade and price distortions include openness, tariffs, and the black market premium. The financial markets variables are related to financial market development, such as the market capitalization ratio (the value of listed shares divided by GDP), and the value traded ratio (total value of traded shares divided by GDP). The monetary indicators cover variables related to monetary policy, specifically inflation and the interest rate. Political indicators include coups and revolutions, civil war dummies, and the democracy index. Social variables include health indicators, such as life expectancy, and demography variables. Sectoral composition refers to variables such as the number of people employed in agriculture or in manufacturing. Geography variables refer to variables such as latitude, landlocked dummies, distance to the nearest coast, and the average temperature.

Apart from the Solow variables discussed above, the only other variables that systematically affect the estimated rate of convergence are the dummies related to fiscal and financial conditions. In both cases, the effect of including them raises the estimate by approximately 1.7 percentage points. Our hypothesis is that sound fiscal and financial conditions contribute to the rate at which poor countries reach their development potential (their steady states) and the rate at which they catch up to more developed countries, perhaps through technology diffusion.

Finally, the variable ‘Regional dummies’ is included to measure the effect of using country, region, and continent dummies to proxy for broad technology (and steady-state) differences in cross-sectional data. Our hypothesis is that regional dummies serve part of the same purpose as country-fixed effects: they control for unobserved heterogeneity. We therefore anticipate a positive coefficient. The results indicate that including regional dummies leads to higher estimates of the rate of convergence, in the order of 1.1 percentage points.

5. Conclusions

The aim of this article was to analyze the results of the empirical literature on the rate of convergence and investigate potential sources of heterogeneity in the estimates. We start by computing a pooled (or combined) estimate and find a value close to a 2% rate of convergence using a model allowing for random differences across measurements. This result coincides with the legendary ‘natural constant’ of 2% suggested in the convergence literature. Our analysis shows as well, however, that the adjective ‘legendary’ should be interpreted as pointing to the ‘fabled’ status of the 2% rather than to the status of a popularly accepted ‘factual’. The rate of convergence varies systematically according to various observable differences between studies, even if one accounts for unobservable sources of variation and the potential impact of publication bias.

We use several weighted regression models to further explore the sources of between-estimate heterogeneity. Control variables included in our analysis are...
partly motivated by theoretical differences in the literature, related to the treat-
ment of technology, the difference between short-run effects and long-run transi-
tional dynamics, and differences in modeling the steady state in conjunction with
potential endogeneity of the regressors. The main control variables in our study
allow for differences in data characteristics such as the source of PPP rates, the
level of aggregation, the use of homogeneous samples, and structural character-
istics such as the number of observations. Furthermore, we include the time span
and frequency of the data, estimation characteristics such as the type of estimator
and whether a non-linear method was used, and the type of explanatory variables
included in the original regression to control for differences in the steady state.

We find that correcting for the omitted variable bias resulting from unobserved
heterogeneity in technology levels leads to higher estimates of the rate of conver-
gence. For example, the use of regional data (in which technology differences are
less pronounced) leads to a 1.1 percentage point higher estimate of the rate of
convergence. The use of a homogeneous sample of countries or regions leads to
higher estimates in the order of 0.9 percentage points. The inclusion of regional
dummies to control for unobserved heterogeneity in cross-sectional samples
increases the estimate by an average of 1.1 percentage points. The inclusion of
explanatory variables to control for differences in the steady state or, alterna-
tively, parameterize the unobserved level of technology, also leads to significantly
different estimates of the rate of convergence. The use of estimators such as
LSDV and GMM that control for country-specific effects has a substantial
impact on estimates of the rate of convergence of around 4.4 and 6.3 percentage
points, respectively. We also find that correcting for endogeneity in the explana-
tory variables results in higher estimates, as argued by Cho (1996) and Caselli
et al. (1996).

Finally, our analysis reveals that significant differences in convergence rates
exist for models deviating from the standard unconditional convergence model.
Specifically, models using a standard Solow specification as well as models
incorporating fiscal and financial variables typically lead to convergence rates
that are significantly higher than the legendary 2% rate.

Acknowledgements

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this paper.

Notes

1. The assertion of a constant ‘natural rate of convergence’ of 2% does not preclude
finding variation in empirical estimates. In a statistical sense, it implies that the estimates
are drawn from a single population distribution with a mean of 2%. The differences in
reported estimates are then solely due to estimation variance. The natural rate of
convergence in a panel data setup is generally believed to be substantially higher at a level of 4–6%, among other things because a panel data setup allows for modeling (unobserved) technological differences across countries (Islam, 2003, pp. 325–326). Caselli et al. (1996) even report convergence rates as high as 10% for panel data studies.

2. This can be illustrated using the concept of regression toward the mean (Galton’s fallacy). Galton noticed that the sons of exceptionally tall fathers tended to be shorter, or in other words that the sons of tall fathers tended to have a height that was closer to the population mean, and erroneously concluded that the distribution of heights was shrinking over time. The reason for Galton’s observation is that exceptionally tall fathers are outliers or rare occurrences, and it would therefore be extremely rare for their sons to also be outliers. Friedman (1992) applied Galton’s fallacy concept to the study of income convergence, by noting that in the presence of non-persistent random fluctuations in income, a regression of growth rate of income in period \( t \) on income in period \( t - 1 \) would result in a negative coefficient, even in the absence of a shrinking variance. See also Bliss (1999) for an extensive discussion on the relevance of Galton’s fallacy for the study of income convergence.

3. Strictly speaking, there is some variation in the empirical operationalization of the dependent variable. Some studies use income; others use the gross product, and the standardization ranges from per worker, to per capita, and per person aged 25–65 years.

4. This is not intended to suggest that combining, for instance, cross-section, time series, and panel data studies, or factor productivity and income/production studies is not feasible. Their combination would, however, require a careful account of the theoretical relationship between the different concepts, which should also be incorporated in the specification of the meta-regression equation. See Smith and Pattanayk (2002) for a similar line of reasoning with respect to non-market valuation.

5. Note, however, that a negative estimate of \( \beta \) is possible even in the absence of any form of convergence, due to Galton’s fallacy of regression toward the mean.

6. Today, EconLit contains references to articles in over 750 journals. Its history goes back to 1969 when it contained references to 182 periodicals. Less than 3% of the articles are in a foreign language (meaning other than English). See http://www.econlit.org, for details. In the search we used the search string ‘growth’ and/or ‘convergence’ not ‘ARCH, GARCH, Markov,…’.

7. In order to ensure that we obtain a random sample of studies, we first assign a unique ID to each study, based on an alphabetical ordering by author, year, and title. Subsequently, we generate a series of random numbers in Stata 8.0 using the command ‘uniform, ( )’, which returns uniformly distributed pseudorandom numbers on the interval \([0,1)\). Finally, we order the study ID series according to the random series, and we follow this new ordering in selecting the studies to be included, starting with the first one.

8. In subsequent analyses, we discarded nine observations for which the estimated coefficient of initial income is smaller than \(-1\), because in those cases we cannot recover the rate of convergence; see equation (11) in the main text. Estimates smaller than \(-1\) imply that there is leapfrogging in the distribution, so that the rate of convergence becomes undefined. Note that this is different from divergence, which occurs when the estimated value of initial income is greater than zero, implying that the rate of convergence is negative.

9. See the Appendix for the derivation of equation (12).

10. We note that the meaning of the adjectives ‘fixed’ and ‘random’ in the meta-analysis literature is very different from the usual interpretation for panel data models in
standard econometrics, because they refer to assumptions about the underlying population effect size (Hedges and Olkin, 1985; Sutton et al., 2000a). In standard econometric terms, the fixed-effects meta-estimator is equivalent to the weighted least squares estimator using the estimated variances (derived in the primary studies) as weights and re-scaling the standard errors of the meta-regression by means of the square root of the residual variance. The random effects estimator is akin to a random coefficient model in which the within- and between-study variances are used as weights (Florax and Poot, 2005). Thompson and Sharp (1999) provide an excellent overview of various estimators that allow for random-effects variation.

11. Some people would maintain that in this field of study, the independence assumption may also be violated for single-sampling measurements, because many studies use the same underlying data (e.g. the Summers and Heston database).

12. Their conclusions should, however, be taken judiciously because their Monte Carlo simulation experiments are based on only two replications. In their experiments, they use randomly sampled single measurements of each study as well as the within-study average and median. Given the relatively large number of studies in our meta-sample, using the average, the median, or a randomly selected measurement of the primary studies is largely irrelevant, although small sample differences exist.

13. All estimations are performed with Intercooled Stata 8.0, including user-written routines for meta-analysis provided by Stephen Sharp, Jonathan Sterne, Thomas J. Steichen, and Roger Harbord. See the Stata website (http://www.stata.com) for details and references to the Stata Technical Bulletin.

14. Similarly, for the sample with 48 observations, the estimate for the constant is 3.26, with a t-value of 8.06. Several alternative tests are available. A regression of the effect size on the estimated standard errors (Card and Krueger, 1995) shows significant results as well, as does a weighted fixed effects meta-regression that includes the standard errors (see Stanley’s suggestion to use this approach in this issue). The rank correlation suggested by Begg and Mazumdar (1994) uses the association between the standardized effect and the sampling variance, measured by Kendall’s tau, to detect publication bias. The latter does not indicate publication bias in our samples, but the test is not very powerful (see Macaskill et al., 2001, for simulation experiments). Detailed results for all tests are available upon request.

15. The $Q$-test is given by

$$Q = \sum_{i=1}^{k} w_i \hat{\sigma}_i^2 - \left( \frac{\sum_{i=1}^{k} w_i \hat{\sigma}_i}{\sum_{i=1}^{k} w_i} \right)^2 \sim \chi^2_{k-1},$$

where $k$ is the number of study results and $w_i$ the inverse estimated variance, and tests the null hypothesis that the true effect size is the same for all studies, versus the alternative hypothesis that at least one of the effect sizes differs from the remainder. Note that the test assumes independent study results, and it is therefore not fully adequate in the case of multiple sampling (Sutton et al., 2000a, pp. 38–40; Florax and Poot, 2005). The $Q$-test results are highly significant in both the full data set and the restricted data set using single sampling. Detailed results are available from the authors on request.


17. See Sutton et al. (2000b) for a useful overview of various techniques to modeling publication bias. Recent applications of the Hedges approach in economic meta-analyses include Ashenfelter et al. (1999) and Florax (2002). We thank Hessel Oosterbeek for making available his Stata routine to estimate the publication bias model.
19. The test statistic (−2 times the difference between the unrestricted and the restricted log likelihood) follows a $\chi^2$ distribution with three degrees of freedom. A bivariate version of the Hedges approach also rejects the null hypothesis of no publication bias at the 1% significance level. The bivariate estimate of the rate of convergence corrected for publication bias is 2.9%, with a confidence interval of $2.55 < \lambda < 3.20$.

20. The results of the Hedges publication bias approach crucially depend on the number of $p$-value classes used in the analysis, and on the cut-off points used to define these categories. Increasing the number of categories improves the fit of the unrestricted regression, resulting in a higher log likelihood for the unrestricted model (relative to the restricted model), and therefore increasing the size of the likelihood ratio test statistic. Choosing a larger number of categories also implies that the weighting function is smoother, with fewer ‘kinks’. However, these advantages have to be balanced against the difficulties in interpreting the results when the number of categories is large, particularly in interpreting the estimated sampling probabilities for the different $p$-value classes. The analysis presented in Table 2 therefore makes use of cut-off points based on the socially salient $p$-values of 0.01, 0.05, and 0.10. A sensitivity analysis indicates that the estimated coefficients of the meta-regression corrected for publication bias do not vary significantly when a larger number of $p$-value classes is used, although the likelihood ratio test for the presence of publication bias is no longer significant when the number of categories is 3, with cut-off points at 0.01 and 0.05 only. See also Hedges (1992) for an extended discussion on the choice of values for the discontinuities in the weight function.

21. The simulation experiments in Koetse et al. (2004) show that the use of dummy variables to account for differences in the set of conditioning variables used in the underlying studies goes a long way toward removing bias in the meta-estimator. Keef and Roberts (2004) also point to a comparability problem for primary studies using different specifications, although their perspective is slightly different. They observe that for effect sizes scaled by a measure of variance to ensure a dimensionless metric of the effect size, a potential problem occurs. Since the variance becomes smaller the more conditioning variables a model comprises, the interpretation of differences between effect sizes across studies may be ambiguous. As such, this point is not relevant for our meta-analysis because the effect size, defined as the convergence rate in percents per year, is homogeneous across studied, and there is hence no need to scale it by its variance. However, the variance is used in determining the weights for the fixed- and random (or mixed)-effects estimator. As a result, measurements taken from primary studies with a ‘broader’ specification automatically receive more weight, since the variance of these measurements is given by $\sigma^2(x' \lambda)^{-1}$, and the residual variance $\sigma^2$ is smaller when the specification contains more conditioning variables. This is, however, not problematic since the chance of omitted variable bias occurring is smaller the ‘broader’ is the specification. Obviously, one does not know what the actual data-generating process is, and one may therefore be overcompensating. However, given that the inclusion of irrelevant conditioning variables does not have a detrimental effect on the properties of the estimator, the weighting process is in accordance with the quality of the estimates. See Koetse et al. (2004) for more details.

References


Appendix

For a random variable $X$ with mean $\mu_X$ and variance $\sigma_X^2$, we can approximate the mean and variance of $Y = g(X)$ using a first-order Taylor expansion of $g$ about $\mu_X$ (Greene, 2000, pp. 49–53):

$$Y = g(X) \approx g(\mu_X) + (X - \mu_X)g'(\mu_X).$$  \hfill (A1)

Recalling that for a linear function $U = a + bV$, the mean and variance are given by $E(U) = a + bE(V)$ and $\text{Var}(U) = b^2 \text{Var}(V)$, we obtain $\mu_Y \approx g(\mu_X)$ and $\sigma_Y^2 \approx \sigma_X^2 \left(g'(\mu_X)\right)^2$. Applying this result to $Y = \log(X)$ leads to $\mu_Y \approx \log(\mu_X)$ and $\sigma_Y^2 \approx \sigma_X^2 / \mu_X^2$. Correspondingly, for the convergence rate given in equation (11), we approximate the mean as

$$\hat{\mu}_i \approx -\frac{\ln(1 + \hat{\mu}_\beta)}{\tau}$$  \hfill (A2)

and its associated variance as

$$\text{Var}(\hat{\lambda}) \approx \frac{1}{\tau^2} \text{Var}(\ln(1 + \hat{\beta}))$$
$$= \frac{1}{\tau^2} \text{Var}(1 + \hat{\beta}) \frac{1}{(1 + \hat{\mu}_\beta)^2},$$  \hfill (A3)

$$= \frac{\text{Var}(\hat{\beta})}{\tau^2 (1 + \hat{\mu}_\beta)^2}$$

from which the estimated standard error given in (12) follows directly.