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Universal three-body parameter in ultracold $^4\text{He}^*$

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We have analyzed our recently measured three-body loss rate coefficient for a Bose-Einstein condensate of spin-polarized metastable triplet ^4He atoms in terms of Efimov physics. The large value of the scattering length for these atoms, which provides access to the Efimov regime, arises from a nearby potential resonance. We find the loss coefficient to be consistent with the three-body parameter (3BP) found in alkali-metal experiments, where Feshbach resonances are used to tune the interaction. This provides evidence for a universal 3BP outside the group of alkali-metal elements. In addition, we give examples of other atomic systems without Feshbach resonances but with a large scattering length that would be interesting to analyze once precise measurements of three-body loss are available.

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I. INTRODUCTION

When the short-range interaction between particles gives rise to (near-)resonant scattering, few-body properties are expected to become universal, i.e., irrespective of the precise nature of the interaction and therefore applicable to nucleons, atoms, or molecules [1]. Within universal few-body physics a hallmark prediction is the Efimov effect, in which three particles that interact via a resonant short-range attractive interaction exhibit an infinite series of three-body bound states, even in the regime where the two-body interaction does not support a bound state [2]. The first experimental evidence of Efimov trimers came from an ultracold trapped gas of atoms [3] by tuning the strength of the interaction via a Feshbach resonance [4]. In the context of ultracold atoms, the universal regime is realized when the s -wave scattering length a , characterizing the two-body interaction in the zero-energy limit, is much larger than the characteristic range of the interaction potential. Signatures of Efimov states are imprinted on trap loss caused by three-body recombination, which typically determines the lifetime of an ultracold trapped atomic gas or Bose-Einstein condensate. So far, observations of Efimov features have been made in ultracold quantum gases of bosons: ^7Li [5–7], ^{39}K [8], ^{85}Rb [9], Cs [3,10,11], a three-spin component mixture of fermionic ^6Li [12–14], and the Bose-Bose mixture $^4\text{K} + ^{87}\text{Rb}$ [15].

In addition to the scattering length, a three-body parameter (3BP) is needed to fully describe the spectrum of Efimov trimers. The 3BP accounts for all the short-range information that is not contained in the scattering length, including a true three-body interaction. It can be parametrized as the location of the first Efimov resonance, a_- , on the $a < 0$ side of a Feshbach resonance. Initially, the 3BP was thought to be very sensitive to details of the short-range interaction and therefore different for each (atomic) system [16]. However, experiments around different Feshbach resonances and with different alkali-metal atoms found the ratio $|a_-|/r_{\text{vdW}}$ in a narrow range between 8 and 10 [5,9,11], where $r_{\text{vdW}} = \frac{1}{2}(mC_6/\hbar^2)^{1/4}$ is the range of the tail of the two-body potential (also called the van der Waals length), with m the atomic mass and C_6 the long-range coefficient. There is a vivid theoretical

debate on the physical origin of this universal 3BP [17–22]. Most work points towards a three-body repulsive barrier that prevents the three atoms from probing the short-range interaction. An important question is how general the universal 3BP is. Experimental data outside the group of alkali-metal atoms could shed light on this issue.

In this paper we investigate the possibility of extracting the 3BP from our recently measured three-body loss rate coefficient in a Bose-Einstein condensate (BEC) of metastable triplet helium-4 (denoted as $^4\text{He}^*$) [23]. We will show that its value is consistent with those measured in alkali-metal systems, providing further experimental evidence of a universal 3BP. We will also discuss other atomic systems that can be analyzed in a similar fashion. The common feature is that in the absence of a Feshbach resonance, these atomic systems already have a scattering length that is much larger than the range of the potential. The mechanism for this is an almost resonant interaction potential, i.e., a bound state is almost degenerate with the collision threshold. This potential resonance is a simple single-channel effect. In contrast, a Feshbach resonance is a multichannel effect, where the width of the resonance introduces another length scale [4], which may give rise to nonuniversal physics. Therefore, potential resonances are more directly related to the universal description connected to a large scattering length than Feshbach resonances.

II. THREE-BODY LOSS IN ALKALI METALS

To relate our work to the alkali-metal experiments, we first summarize how the 3BP is extracted from three-body loss measurements around a Feshbach resonance [1,3]. In the limit of $|a| \gg r_{\text{vdW}}$ the three-body loss rate coefficient L_3 for identical bosons is given by

$$L_3 = 3C_{\pm}(a)\frac{\hbar a^4}{m}, \quad (1)$$

where $C_{\pm}(a)$ are dimensionless prefactors that depend on a . Here we assume that three atoms are lost from the trap in the event of three-body recombination. The scattering length a is tuned by a magnetic field from $a > 0$ to $a < 0$ through

resonance. The prefactors are given by

$$C_+(a) = 67.1e^{-2\eta_+} \{ \cos^2[s_0 \ln(a/a_+)] + \sinh^2 \eta_+ \} + 16.8(1 - e^{-4\eta_+}) \quad (2)$$

and

$$C_-(a) = \frac{4590 \sinh(2\eta_-)}{\sin^2[s_0 \ln(a/a_-)] + \sinh^2 \eta_-}, \quad (3)$$

respectively. On top of a strong a^4 scaling, L_3 shows, as a function of a , a series of resonances for $a < 0$ and minima for $a > 0$, and the locations of these Efimov features are determined by a_+ and a_- . The parameters $\eta_{\pm}$ are related to the decay of the trimers into atom-dimer pairs and provide a width to the Efimov features. Experimentally $a_{\pm}$ and $\eta_{\pm}$ are obtained by fitting Eqs. (2) and (3) to the measured L_3 spectrum as a function of a . For identical bosons $s_0 = 1.00624$, such that $C_{\pm}(a) = C_{\pm}(22.7a)$, and therefore a_+ and a_- are defined only within a factor 22.7^n , n being an integer. Universal theory requires a single 3BP and therefore the Efimov features for $a > 0$ and $a < 0$ are related, namely, via the relation $a_+/|a_-| = 0.96(3)$ [1]. A nonuniversal 3BP would manifest itself as random scatter of $|a_-|$ values in a range between 1 and 22.7 for different systems. However, the ratio $|a_-|/r_{\text{vdW}}$ was found in a narrow range between 8 and 10 for experiments with different alkali-metal atoms [5,9,11,18], indicating a universal 3BP [24].

III. ANALYSIS OF THREE-BODY LOSS IN $^4\text{He}^*$

Recently we have measured the three-body loss rate coefficient in a $^4\text{He}^*$ BEC, prepared in the high-field-seeking $m = -1$ Zeeman substate, and obtained the value $L_3 = 6.5(0.4)_{\text{stat}}(0.6)_{\text{sys}} \times 10^{-27} \text{ cm}^6 \text{ s}^{-1}$ [23]. For spin-polarized He^* Penning ionization is strongly suppressed [25] and three-body loss dominates the lifetime of a $^4\text{He}^*$ BEC. Scattering of spin-polarized He^* is given by the $5\Sigma_g^+$ potential, for which high-accuracy *ab initio* electronic structure calculations are available [26]. For $^4\text{He}^* + ^4\text{He}^*$ this potential supports 15 vibrational states. The highest excited vibrational state is weakly bound, which gives rise to a nearby potential resonance. Its binding energy is $h \times 91.35(6) \text{ MHz}$, measured by two-photon spectroscopy [27], from which a quintet scattering length of $141.96(9)a_0$ ($a_0 = 0.05292 \text{ nm}$) was deduced, consistent with the *ab initio* theoretical value of $144(4)a_0$ [26]. It is indeed much larger than the range of the potential, as $r_{\text{vdW}} = 35a_0$ [28], such that $a/r_{\text{vdW}} = 4.1$. The binding energy of this weakly bound two-body state corresponds to 4.4 mK , which is much larger than the trap depth of about $10 \text{ } \mu\text{K}$ and therefore both the formed dimer and the free atom leave the trap after three-body recombination. There are no broad Feshbach resonances in $^4\text{He}^*$ because of the absence of nuclear spin [29].

We now consider Eq. (2) to find the set of a_+ and η_+ values that explains our observed value of L_3 . Following the current convention, we present the 3BP in the form $|a_-|/r_{\text{vdW}}$ by using the universal relation $a_+/|a_-| = 0.96$. In the alkali-metal experiments typically $\eta_+ \approx \eta_-$ and therefore in the following we will only use η . In Fig. 1 we show two sets of solutions of Eq. (2) that match our measured L_3 value, namely, $|a_-|/r_{\text{vdW}} = 2.3$ (dashed lines) and 7.7 (solid lines),

FIG. 1. (Color online) Universal three-body loss curves [Eq. (2)] for $^4\text{He}^*$ with $|a_-|/r_{\text{vdW}} = 2.3$ (dashed lines) and $|a_-|/r_{\text{vdW}} = 7.7$ (solid lines), for different values of η , that match our measured L_3 value (see inset).

for different values of η . In both cases our data point is located far outside an Efimov minimum, giving rise to a weak dependence of η on L_3 . That is the reason why our L_3 value, obtained for a single scattering length, provides information about a_- .

In Fig. 2 we show the set of solutions to Eq. (2) in $(|a_-|/r_{\text{vdW}}, \eta)$ parameter space for our value of L_3 , represented by the black solid line, with the gray shaded area reflecting the experimental uncertainty in our measured L_3 value. Within the range of 1 to 22.7 for $|a_-|/r_{\text{vdW}}$, we indeed find two narrow regions of $|a_-|/r_{\text{vdW}}$ around 2 and 8, provided that η is not too large. For $\eta = 0.1$ we find $|a_-|/r_{\text{vdW}} = 7.7(7)$ and $2.3(2)$. If η becomes larger than 0.5 the Efimov minima are washed out and their location becomes undefined, giving rise to a broad range of possible $|a_-|/r_{\text{vdW}}$ values. For comparison, the 3BPs obtained from the different alkali-metal experiments are depicted by the colored symbols, with their numerical values

FIG. 2. (Color online) Graphic representation of the set of $|a_-|/r_{\text{vdW}}$ and η values for which Eq. (2) matches our observed value of L_3 , given by the black solid line, where the gray band corresponds to possible values based on our L_3 error bar. Also indicated are the $|a_-|/r_{\text{vdW}}$ values for the alkali-metal experiments: Cs, 8.6(2), 10.2(6), 9.5(8), 9.5(3) [11] (red diamonds), ^7Li 8.1(3) [5], 9.2(3) [6], 8.3(4) [7] (blue squares), ^6Li 9.3 [30] (green circle), ^{85}Rb 9.23(7) [9] (orange triangle), showing at the same time the observed η parameters.

given in the caption. We expect the value of η for $^4\text{He}^*$ to be similar to those found in the alkali-metal systems, since Penning ionization will play no important role in the decay mechanism of the Efimov trimers. Figure 2 shows that our value is consistent with the 3BPs found in the alkali-metal system, considering the scatter shown in the available data and our uncertainty in L_3 .

In our analysis we rely on two assumptions. The first assumption is that $a/r_{\text{vdW}} = 4.1$ is sufficiently large for application of Eq. (2). Here we notice that the three-body loss data around a Feshbach resonance fit well for $|a|$ larger than a few r_{vdW} . Effects beyond universal theory [31–33] may be present, but are small enough not to alter our conclusion. The second assumption is that three atoms are lost for each three-body recombination event. For $a > 0$ additional resonances on top of the a^4 scaling have been observed in three-body loss spectra [6,8,34]. Those features are explained by secondary atom-dimer collisions that are resonantly enhanced near $a = a_*$, where a_* is the atom-dimer Efimov resonance position [1], which effectively leads to an enhancement of the number of atoms lost in a three-body recombination event. The precise underlying mechanism, and therefore what to extract from these additional resonances, is still under debate [35–37]. Here we note that if we take $|a_-|/r_{\text{vdW}} = 8$, then $a_* = 300a_0$, which is far away from the actual value $142a_0$, such that secondary atom-dimer collisions are expected not to play a role for $^4\text{He}^*$.

IV. OTHER SYSTEMS

There are more atomic systems with a nearby potential resonance, for which a similar analysis as that performed for $^4\text{He}^*$ can be done once a precise measurement of L_3 becomes available. Alkali-metal atoms prepared in a spin-stretched state (i.e., electron and nuclear spin maximally aligned) scatter only in the triplet potential. Therefore alkali metals with a large triplet scattering length provide the opportunity to extract the 3BP obtained from three-body loss in the presence of a potential resonance. Two candidates are ^{85}Rb [$a_T = -388(3)a_0$ [38], $r_{\text{vdW}} = 82a_0$] and Cs [$a_T = 2440(24)a_0$ [39], $r_{\text{vdW}} = 101a_0$]. An experimental challenge is to distinguish three-body loss from two-body loss processes, such as spin relaxation and hyperfine-changing collisions, especially in the case of Cs [40].

Another group of atoms that do not possess Feshbach resonances are the alkaline-earth-metal elements and Yb . In the electronic ground state the atoms have zero electron spin and therefore there is only a single two-body potential, which is of singlet character. Furthermore, the bosonic isotopes have zero nuclear spin and two-body loss processes are completely absent. An interesting example is Ca , for which potential

resonances show up for all the bosonic isotopes [41]. In the following we will discuss two isotopes of Sr and Yb , for which a is accurately known, $a \gg r_{\text{vdW}}$, and the first three-body loss measurements in BECs have already been reported.

For ^{86}Sr [$a = 798(12)a_0$ [42], $r_{\text{vdW}} = 75a_0$], Stellmer *et al.* [43] report an upper limit of $L_3 = 6(3) \times 10^{-24} \text{ cm}^6 \text{ s}^{-1}$, which is one order of magnitude larger than maximally allowed by Eq. (2). The authors indicate that secondary collisions, possibly enhanced by a resonance in the atom-dimer cross section, may explain this discrepancy. We note that if one tentatively assumes that the scattering length is indeed near the atom-dimer resonance, i.e., $a_* \approx 800a_0$, then $a_- \approx -750a_0$ and thus $|a_-|/r_{\text{vdW}} \approx 10$. This is a hint that three-body loss in ^{86}Sr is consistent with the universal 3BP.

For ^{168}Yb [$a = 252(3)a_0$ [44], $r_{\text{vdW}} = 78a_0$], Sugawa *et al.* [45] report an upper limit of $L_3 = 8.6(1.5) \times 10^{-28} \text{ cm}^6 \text{ s}^{-1}$. If we perform a similar analysis as for $^4\text{He}^*$ we find again two solutions of $|a_-|/r_{\text{vdW}}$. Taking the upper limit, one of the two solutions lies in a narrow range between 8 and 9. Here a smaller L_3 leads to a larger $|a_-|/r_{\text{vdW}}$, and a value between 10 and 11 is reached when the reported L_3 value is reduced by a factor of 2. This is a strong indication that three-body loss in ^{168}Yb is also consistent with the universal 3BP.

V. CONCLUSIONS

We find our measured L_3 coefficient in spin-polarized $^4\text{He}^*$ to be consistent with the 3BP that was recently found in comparing measurements using alkali-metal atoms. We give further examples of atomic systems without a Feshbach resonance but in the presence of a nearby potential resonance for which the 3BP can be extracted from an accurately measured L_3 , such as alkali-metal atoms in spin-stretched states and alkaline-earth-metal atoms. We find that the three-body loss measured in ^{168}Yb strongly indicates consistency with the universal 3BP.

We provide experimental evidence for a universal 3BP, outside the alkali-metal group and in the absence of a Feshbach resonance. A universal 3BP means that short-range three-body physics is not relevant for the Efimov spectrum. This implies that not only three-body observables in the universal regime are fully determined by two-body physics, but four-body [46–48] and N -body ($N > 4$) [49,50] observables as well.

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