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Are you sure? Stimulating Mathematical Thinking During Young Children's Play.

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SUMMARY This article deals with some questions related to the promotion of cultural learning processes in younger children (4-7 years old). In the study here reported, the promotion of mathematical activity was taken as the domain of study. Reasoning from a Vygotskian point of view, it was assumed that these learning processes should be embedded in the play activity of these children, and that these learning processes could be based on the children's ways of dealing with symbols/symbolic representations and meanings.

In this observational study (according to case study methodology) we tried to find out which teaching opportunities occur within a role play activity that could be considered valuable for the improvement of mathematical thinking activity. Furthermore, we tried to investigate in detail if mathematics-bound semiotic activity can be triggered within the play context by asking children if they were sure about their mathematical actions, and about the meanings and symbols they used. These queries required explanations and justifications from the children that prompted them to reflect on the meanings of their symbols (notations, words). In order to find answers to these research questions, we analysed eight video-taped sessions (with an average duration of 27 minutes) of role play-activity of small groups of children in a 'shoeshop-corner' at school.

It turned out that many mathematical teaching opportunities occur and that, if the teacher manages to make use of them, children can explicitly reflect on the relation between symbols and meanings within the play activity. Hence, I draw the cautious conclusion, that play activity can be a teaching/learning situation for the enhancement of mathematical thinking in children, provided that the teacher is able to seize on the teaching opportunities in an adequate way. To what extent this approach also leads to lasting **learning** results in all pupils is an issue for further study.

RESUMÉ Cet article traite de quelques questions relatives à la promotion des processus d'apprentissage culturel chez les jeunes enfants (de 4 à 7 ans). Dans l'étude rapportée ici il s'agit de la promotion des activités mathématiques. A partir de vue de Vygotskion, on a établi qu'il faudrait inclure ces processus d'apprentissage dans l'activité de jeu des enfants, et que ces processus d'apprentissage devraient s'appuyer sur la manière dont les enfants appréhendent les symboles ou représentations symboliques et les significations.

Dans cette étude (l'étude de cas), nous avons tenté de mettre en évidence les occasions d'apprentissage, au sein de l'activité de jeu de rôle, pouvant contribuer à l'amélioration de la pensée mathématique. Par ailleurs, nous avons essayé de savoir, dans le détail, si l'activité sémiotique liée aux mathématiques pouvait être déclenchée, dans un contexte ludique, en demandant aux enfants s'ils étaient sûrs de leurs actions mathématiques et des significations et symboles utilisés. Ces questions exigeaient des explications et des justifications de la part des

enfants, ce qui les a amenés à réfléchir sur les significations de leurs symboles (notations, mots). Afin de trouver des réponses à ces questions nous avons analysé huit séances, enregistrees en vidéo (d'une durée moyenne de 27 minutes), d'activité de jeu de rôle de petits groupes d'enfants dans un 'magasin de chaussures' à l'école.

De nombreuses occasions d'apprentissage mathématique sont apparues et il n'est avéré que si l'enseignant savait en tirer parti, les enfants pouvaient réfléchir de façon explicite sur la relation entre symboles et significations, dans le cadre de l'activité ludique. D'où ma conclusion prudente: l'activité ludique peut être une situation d'enseignement apprentissage permettant d'améliorer la pensée mathématique chez les enfants, à condition que l'enseignant apprécie les situations d'apprentissage de manière adéquate. D'autres études devraient permettre de savoir dans quelle mesure cette approche peut conduire également à des résultats d'apprentissage durables chez tous les élèves.

ZUSAMMENFASSUNG In diesem Artikel werden einige Fragen behandelt, die sich auf die Förderung kultureller Lernprozesse bei jüngeren Kindern (4-7 Jahre alt) beziehen. In der hier vorliegenden Studie wurde die Förderung der mathematischen Tätigkeit untersucht. In Orientierung an Vygotsky wurde davon ausgegangen, daß diese Lernprozesse in die Spieltätigkeit der Kinder eingebettet werden sollten, und daß sie verdientlichen können, wie Kinder mit Symbolen und symbolischen Deutungen umgehen.

In dieser Beobachtungsstudie (nach der Fallstudienmethode) versuchten wir herauszufinden, welche Lernmöglichkeiten innerhalb eines Rollenspiels entstehen, die für die Förderung des mathematischen Denkens wertvoll sein könnten. Darüber hinaus haben wir versucht zu klären, ob die mathematikgebundene semiotische Tätigkeit im Spielkontext dadurch ausgelöst werden konnte, in dem man die Kinder fragte, ob sie sich über ihre mathematischen Handlungen und die damit zusammen entian genden. Deutungen und Symbole im klaren waren. Diese Fragen machten Erklärungen und Rechtfertigungen von den Kindern erforderlich. Sie regten die Kinder dazu an, über die Bedeutung ihrer Symbole (Bezeichnungen, Wörter) nachzudenken. Um die Antworten auf diese Forschungsfragen zu finden, analysierten wir acht videoaufgezeichnete Spielsituationen (mit einer durchschnittlichen Dauer von 27 Minuten) mit kleinen Gruppen von Kindern in einer 'Schuhgeschäftsecke' in der Schule.

Es stellte sich heraus, daß es viele mathematische Lernmöglichkeiten gibt und daß Kinder genau über die Beziehung zwischen Symbolen und Deutungen innerhalb der Spieltätigkeit nachdenken können, wenn der Lehrer die Situation zu nutzen weiß. Ich ziehe daraus die vorsichtige Schlußfolgerung, daß Rollenspiel eine Lehr-/Lernsituation für die Förderung des mathematischen Denkens bei Kindern sein kann, Vorausgesetzt, daß der Lehrer in der Lage ist, die Lehrmöglichkeiten in geeigneter Weise zu erfassen und zu nutzen. Inwieweit diese Vorgehensweise zu bleibenden Lernergebnissen bei allen Schülern führt, bedarf weiterer Untersuchungen.

RESUMEN Este artículo trata de algunas cuestiones relacionadas con la estimulación de los procesos de aprendizaje cultural en niños pequeños (de 4 a 7 años). El estudio al que se hace referencia en este artículo, se centró en el campo de estimulación de la actividad matemática. Razonando desde un punto de vista Vygotskiano, se partió de la base de que dichos procesos de aprendizaje deberían estar integrados en la actividad de juego de estos niños, y que dichos procesos de aprendizaje podrían estar basados en la manera en que los niños comprenden los símbolos o las representaciones simbólicas y sus significados.

En este estudio basado en la observación (conforme a la metodología de estudio de casos) intentamos descubrir cuándo se producen oportunidades de aprendizaje durante una actividad de juego de imitación que puedan ser consideradas válidas para mejorar la actividad del pensamiento matemático. Asimismo intentamos investigar en detalle si se puede estimular la actividad semiótica relacionada con las matemáticas dentro de un contexto de juego,

preguntando a los niños si estaban seguros de sus acciones matemáticas, y sobre los símbolos y significados que utilizaban. Tales preguntas requirieron explicaciones y justificaciones por parte de los niños, lo que hizo que reflexionaran sobre el significado de sus símbolos (anotaciones, palabras). A fin de encontrar respuestas a las preguntas de la investigación, analizamos ocho sesiones grabadas en vídeo (con una duración media de 27 minutos) de una actividad de juego de imitación desarrollada por pequeños grupos de niños en el rincón de la 'zapatería' de la escuela.

De lo anterior se desprende que se producen muchas oportunidades para enseñar matemáticas, si el profesor sabe hacer uso de ellas, y que los niños pueden reflexionar explícitamente sobre la relación entre los símbolos y sus significados dentro del marco de la actividad de juego. Por consiguiente, me permito sacar la prudente conclusión de que la actividad de juego puede ser una situación de enseñanza-aprendizaje para el incremento de la capacidad matemática de los niños, a condición de que el profesor sea capaz de utilizar adecuadamente las oportunidades de enseñanza. Hasta qué punto este enfoque también lleva a resultados de aprendizaje duraderos en todos los alumnos es un tema que requiere más estudio.

Keywords: Play; Mathematics; Learning.

"Play is the main path to cultural development of the child, especially for the development of semiotic activity"

(L. S. Vygotsky, 1930/1984, p. 69).

Promoting cultural learning processes in early childhood?

Given the exigencies of modern society, it is often considered consistent that schools take care for the passing on of culture to all the members of the new generation, especially regarding the abilities of reading, writing, and arithmetic. In this article I want to deal with this question of how to start the promotion of relevant cultural learning processes in younger children (4 to 7 years old), focusing especially on mathematical learning. Starting from a Vygotskian frame of reference, I started from an action psychological approach to learning (see van Oers, in press), and set out with the assumption that learning processes must be rendered both meaningful for the pupils and promotive for their development. Hence, two criteria must be met for the promotion of cultural learning processes (like reading, writing, or arithmetic) in young children:

- the actions to be learned must be *functional for the child* (i.e. it must be clear for the pupil that the promoted actions contribute in a meaningful way to their current activities);
- the actions to be learned must be *developmentally productive* (i.e. it should be made plausible in an educational argument that the promoted actions are precursors or at least preparatory for a future developmental stage); this requirement is necessary in order to achieve continuity in the child's curriculum.

The first question to be answered then, is how to promote forms of mathematical thinking within the context of young children's role play that are both functional (or personally meaningful) within that play and promotive for the future development of mathematical thinking as a domain specific form of learning activity (see Davydov, 1988; Van Oers, in press and b). Hence, the focus of this paper is on **teaching opportunities** for mathematical learning within the context of play (rather than the assessment of learning results). The concept of **teaching opportunities** is defined here as the occurrence of observable activity of children within the classroom in which the

teacher *in principle* could participate with the attempt to improve that activity. In this paper I want to describe how I tackled this problem and report some of the empirical results that we have obtained so far. More particularly, I tried to find answers to the following questions:

(1) which teaching opportunities for mathematics learning occur in a classroom within some play activity of 4 - 7 year old children?

(2) how can teachers take advantage of such teaching opportunities within play activity?

Play and mathematical thinking

With respect to the problem of the stimulation of mathematical thinking within play, first of all a clear conception of the relationship between mathematics and play is needed. In general we can distinguish between two different situations:

A *Mathematics made playful*: in this situation the primacy of mathematics is the starting point, and elements of the mathematical body of knowledge are transformed into some play activity. Examples of this are different kinds of the well know counting games, dipping rhymes, dominoes, lotto, Monopoly and different commercial games for children in which mathematical operations are embodied. A nice collection of such games is described by Delhaxhe & Godenir (1992).

This can be schematically represented:

B *Mathematising elements of play activity*: in this situation the primacy of the child's play is emphasised and the educator tries to introduce elements of mathematics in the child's play. This can be realised by taking up the child's spontaneous mathematical (or mathematics-like) actions, such as counting, comparing, relating, measuring; or by actively elicit new mathematics-like actions and subsequently trying to improve these actions.

Schematically:

In our investigation we started from the second conception of the relationship between mathematics and young children's play. Because we were working with children from 4 to 7 years old, we were actually dealing with the problem of mathematical actions within **role play** (as role play is considered to be the leading activity of children of this age, according to Leont'ev, 1973; El'konin, 1972; 1980). Furthermore, we assume that the meaning of the mathematical actions carried out within that context will be more easily preserved as a result of the functionality of those actions within the child's play. This maximises the chances for meaningful learning.

What is mathematics?

However, when teachers try to promote mathematical actions within play activities of the child, it is very important to clarify the notion of mathematics itself, because what the teachers conceive as mathematics in this context will strongly depend on what they actually consider to be a mathematical (or mathematics-like) action. Before starting our research in a classroom situation, I had to elaborate a conception of mathematics that is consistent with the action psychological view on human learning, and that could be communicated to the teachers involved. I will dwell briefly on this notion of mathematics from a psychological point of view in the next section (see also van Oers, 1990 for further elaboration of the relationship between mathematics education and the action psychological approach to learning).

I will not give an extensive argument for my view on mathematics here (I have discussed that issue elsewhere, see for example Van Oers, 1990; in press a). Following Freudenthal (1978;1991), I here conceive of mathematics as an activity of systematically organising a concrete or mental domain in terms of quantitative and/or spatial relationships, constructing methods for problem solving related to that activity, as well as finding good reasons for this method. Basically, mathematics is characterised by its pursuit of certainty regarding quantitative and spatial relations. According to my view, the question: "*Are you sure (about your answer/solution)?*" should be the leading question in the promotion of mathematical activity within the context of play, as well as later on, when mathematics has become an independent discursive activity¹.

Mathematics as semiotic activity

Mathematicians tend to take reflection on the meaning of symbolic expressions as an essential part of their mathematical activity. The influential mathematics educationalist Freudenthal, for example, described mathematics 'as an activity of discovering and organising in an interplay of content and form' (Freudenthal, 1991). Psychologically, this kind of activity can be interpreted as a special kind of semiotic activity, related to symbols that refer to quantitative and spatial relationships. That is, mathematics is here conceived of as a special form of the general human activity of constructing a qualified relationship between a sign and its meaning, or reflectively investigating the mutual relationship between sign and meaning in order to improve one's system of meaning (see for example van Oers, 1994)².

The next question then, is whether we can rouse this semiotic activity within young children's play without impairing the quality of this play, and whether it is possible to focus young children's semiotic activity meaningfully on mathematically relevant objects and thus get them involved in a form of mathematical semiosis (comparable to the view on mathematical activity as described in the foregoing section).

Several observations have now corroborated Vygotsky's idea (see Vygotsky, 1930/1984, p. 69), that play is the main path to cultural development, especially for the

development of semiotic activity'. As evidence has shown, it is possible to get young children involved in semiotic activity, related to diagrams (see among others Glotova, 1990; Van Oers, 1994), or related to everyday notions of language and maths (see for example Munn, 1994). Several investigators already have argued that the quest for the meaning of mathematical notions is encouraged by communication between adult and child, and the wish to understand one another and to be sure about the other's intentions (meanings). Communication (i.e. the activity of trying to convey meanings to another or to clarify them to oneself), is a driving force behind the development of mathematical thinking (see for example Kostjuk, 1949/1988; Hughes, 1986; Brissiaud, 1989). Or to put it another way: mathematical thinking is an emergent property of human communication in sociocultural contexts. I suppose that the question "Are you sure" (or its derivative embodiments) stimulates this kind of semiotic activity within the young children's play, eventually generating a new goal structure within the activity that underlies 'abstracted' mathematical activity (see also Saxe, 1984).

Mathematical actions in a play context

In order to get more information to answer our questions, and to explore the validity of our theoretical suppositions, we decided to examine role-play situations in which the teacher participated in the children's play activity and supported and guided this activity into the desired direction. One of the aims of this 'guided participation' (compare Rogoff et al, 1993) would be to lead children towards semiotic activity that we supposed to be mathematically relevant.

The school that participated in this investigation was a school in Amsterdam with a mixed population of children, coming from a variety of socioeconomic backgrounds³. The school had adopted an educational concept in which the promotion of development by teaching was essential. For the lower grades (children from 4 to 7/8 years old) the school worked with the approach for Early Education in Primary Schools as elaborated by a Dutch Institute for School Improvement (APS)⁴. The cultural historical theory of Vygotsky (c.s.) is central in this approach (See Janssen-Vos & Pompert, 1993). The teachers were all well acquainted with the theory underlying the school concept and they had ample experience with the corresponding practice.

The investigation was carried out in May - June 1993. The basic idea was to encourage children in small groups (varying from 4 to 6 children; in one case 9 children) to start a role play in a shoeshop. Each group included children of different grades. Before the start of the project the headteacher discussed the general idea with her team, especially with regard to the importance of the "are-you-sure?" question and its possible realisations for the promotion and development of mathematical activity. Moreover, the teachers were encouraged to pay special attention to the actions of symbol use and to mathematical/arithmetic actions. The teachers did not get special guidelines apart from the above mentioned points of attention. They were all experienced in managing a group of playing children and promoting children's learning processes by guided participation in cultural activities (compare Rogoff et al, 1993).

Every role play session was guided by one teacher. Five teachers took part in the investigation, three of them acted twice in a role play session (with different groups). The teachers always participated in the children's play, sometimes as a member of the play (customer), sometimes as an organiser or innovator of the activity by asking play related questions, helping children, suggesting new ways of doing things etc.). In any case the teachers had a guiding role as to the course of development of the play. The basic structure of the shop activity setting was pre-given: there was a counter, an abacus, notebooks, cheques, many shoe-boxes, a lot of real second hand shoes, a cupboard, some chairs, a mirror and all kind of small artifacts that belong to a shoe shop. This material was given, but the children had to organise the details for

themselves. Playing in separate corners in the classroom in mixed age groups, with a participating teacher, was usual practice in the school.

The play sessions were recorded on a video tape for later analysis. We recorded eight sessions with a total amount of time of 215 minutes, with a mean duration of (approximately) 27 minutes (ranging from 6 to 55 minutes). The recording was carried out by an assistant teacher from a fixed position. The content of the role play changed in the different sessions, depending on the age of the pupils. For the younger ages the play was often manipulative (with money, shoes, boxes), but with the older children, there was more thematic role play in the shop (trying on shoes, buying, selling, etc), while the oldest children also played more “diverted” games, like stock keeping, using the shoe shop just as a meaningful background). The main formal characteristics of the sessions are summarised in the following table:

TABLE 1: Summary of characteristics of sessions

Session	N	Age range	Grade	Duration of session (in minutes)
A	9	4-5	1/2 (+4) ⁵	27
B	4	4-5	1/2	55
C	5	4-6	1/2 (+4)	6
D	4	4-6	1/2 (+4)	18
E	4	6-7	3/4	30
F	5	6-7	3	22
G	6	6-7	3	35
H	5	6-8	3/4	22

The sessions can be analysed from different angles. First of all we analysed the sessions A-B-C-D to find an answer to the following questions:

- if the teacher wants to participate in the children’s play with the objective of stimulating the children’s mathematical thinking, what kind of opportunities will occur ?
- is it possible to stimulate mathematical activity within the context of play by asking or acting out the “are you sure-question”, or equivalent kinds of behaviour (asking for reasons, agreement, explanations, or by explicating ambiguities etc)?

The data presented in this paper will be confined to these questions, analysed for the four groups mentioned.

Method of analysis

The analysis was carried out on the basis of the video-tapes. Before the actual analysis a few methodological problems had to be solved, regarding the observation frame (what will I be looking for?), as well as regarding how to use this frame.

As to the observation frame, I decided to work with a list on which relevant mathematical actions of the children could be scored. On the basis of the research

literature regarding the development of the number concept, or mathematical thinking, I listed the psychological operations that are generally considered to be constituent in the development of the number concept. This resulted in the following 15 item-list:

TABLE 2: List of mathematical operations

classification
seriation
conservation
counting
1-1 correspondence
measuring
estimating
quantitative problem solving
simple arithmetic
concepts (more, less, add, etc.)
number words
space-time orientations (right, left, before, after, etc.)
schematising/notation
dealing with dimensions
dealing with money/paying

Our next and more complicated question was, how to use this observation frame as a system for the analysis of the video-tapes. It was clear from the analysis, that a formal counting of mathematical actions would not make sense as far as practical teaching opportunities are concerned. For instance, by analysis and re-analysis of the tapes I came across events of mathematical action that the teacher paid no attention to (while they were too brief to be noticed or get involved with it). I did not count these as teaching opportunities with practical relevance (although formally they could be interpreted as such). By taking these practical concerns into account, I decided to construct a 'conservative' measure of teaching opportunities, i.e. a measure that gives a reliable minimum estimation of practically relevant teaching opportunities and avoids quantitative overestimation.

A time sample analysis, then, would not make much sense, as we were not interested in characterising the sessions for their extent of mathematisation, neither were we interested in comparing the sessions among each other on the extent of mathematisation. Furthermore, just tallying events of the above mentioned actions (operations) would give an overestimation of the real amount of mathematical activities within play, because -as we could observe many times-it happens frequently that children start an activity (like measuring, classifying, or counting money etc) and keep on being engaged with it for some time, although with interruptions.

An example of such interrupted activity counted as just one event is the following. On a certain moment one child was trying on shoes in the shoe shop, then started to walk around on those shoes, while chatting and playing with other children,

helping other children with arranging boxes etc, and after a while return to fitting another pair of shoes. From a purely formal point of view, these moments could be counted as two events of shoe-fitting (or in more mathematical terms: measuring), but from a psychological point of view it makes more sense to consider this whole event as one (although interrupted) activity of measuring (shoe-fitting).

For the current investigation I decided to count only the major events of mathematical actions: those events that were extended enough to be seized on by the teacher as an educative opportunity. So I did not count incidental, fleeting occasions of counting, measuring, concept use, classification etc, for it would be impossible to employ these as teaching opportunities in a classroom⁶. I decided to count only actions that met one or more of the following conditions:

- actions that were functional within the activity of the children involved;
- actions that attracted common attention of more persons;
- actions that lasted for more than 3 seconds.

In order to be tallied as relevant, it was not necessary that the teacher indeed got involved in that action. In this way I expected to get a measure for the *teaching opportunities* within the play activities of the children. It will be clear that this measure does not coincide with the actual occurrence of the different actions at the individual level. For example: the children were often individually counting objects or using quantification terms (many, a million etc), but when these were used for private reasons, or just incidentally without any other getting involved, I did not consider these actions as teaching opportunities.

From the episodes that the teacher actually interacted with the children, I particularly analysed the occurrence of "are you sure" questions and tried to establish the effects of these questions on the course and results of the thinking process.

Description of some results

As to the occurrence of the various mathematical actions, we can summarise the results of our observations in the four younger groups (groups A, B, C, D from Table 1) in Table 3. It can easily be seen from his table that there are a lot of **teaching opportunities** within these children's play activity in the classroom, that could be used by the teacher as opportunities to promote the development of mathematical thinking (or the number concept). Actually, it is more than the teacher can even notice, and certainly more than she can ever seize on. We often saw children practising operations or exploring situations or relations on their own or together with other children, while the teacher was working with another child. She could not take advantage of that teaching opportunity then.

When the teacher was interacting with the children, she provided various kinds of assistance, like giving information, asking questions, correcting, providing new words, summarising or reformulating expressions of the children, suggesting solutions etc. One of the strategies of the teachers was to test the certainty of the children's answers, assertions, or solutions. They asked "how can we know....", "are you sure?", "is that possible..?", "Is this right?", "is this the right one", "how can we find the right one, and be certain about it?" and other equivalent expressions. These questions prompted the children to give explanations or to look for alternative solutions. The protocol given below (obtained in group B; the episode reported here lasted 35 minutes) is an example of how a teacher guided children in a problem solving process related to exhibiting shoes in the shoeshop, and finding means of registering shoes in order to be able to find the right (fitting and matching) shoe. It is reported here because it can be considered instructive for similar cases of teacher intervention.

TABLE 3: Mathematical teaching opportunities during play in four groups (A, B, C, D). Each category is illustrated by one example of a question or activity that the children were together engaged in for some time

	A	B	C	D	
classification (e.g. "these are all mama's shoes")	1 ¹	1 ⁴	-	2	
seriation (e.g. "this one is bigger than that one")	-	-	-	-	
conservation	-	-	-	-	
counting (reciting a series of numerals)	-	2	1	1	
one-to-one correspondence (<i>"these shoes match"</i>)	2 ²	1 ⁵	-	-	
measuring (pupils compare shoes with regard to length)	14	1	1	4	
estimating (guessing the size of the teacher's shoes)	1	-	-	-	
solving number problems (how much is two times sixty guilders?)	-	-	-	1	
simple arithmetic (add one)	-	1	-	-	
quantitative concepts (<i>"two shoes are one pair"</i>)	11 ³	1	-	3	
number words (e.g. using numerals)	6	-	1	1	
space-time orientations (<i>"Who is first?" "Put it on top of the pile"</i>)	3	-	-	1	
schematising/notation (<i>"I use P for papa's shoes"</i>)	-	1 ⁶	2 ⁷	2	
dealing with dimensions (reference to length, colour, height etc.)	4	-	-	-	
dealing with money (<i>"how much do we have to pay?"</i>)	-	3	1	-	
number of events	42	11	6	15	74
total duration of session (minutes)	27 m.	55 m.	6 m.	18 m.	106 m.

TABLE 3 - NOTES

- 1 This was an extended activity for more than 5 minutes in which the children sorted out shoes according to colour, size, type. Sometimes even multiple classification was involved.
- 2 Partly embedded in the classification task, when the children were looking for matching shoes (shoes that go together; shoes that are one pair); one-to-one corresponding also occurred when they had to find a box to put **one** pair of shoes in, or when they had to find the fitting top for each box.
- 3 Concepts like more, less, the same, a pair, nothing, a lot, how much, measure.
- 4 Extended activity of more than three minutes in which the children sorted out mama-shoes, papa-shoes, child's shoes, baby-shoes.
- 5 Looking for a shoe that matches the one given/found.
- 6 Symbolic reference (with letters) to different classes of shoes.
- 7 Children made diagrams representing the piles of shoes, like in a histogram.

When the children started their play activity there was a jumbled heap of shoes and a heap of boxes. The children had put most of the shoes in the boxes and put them on the shelves of a rack. In one episode the activity of the children evolved into an activity of symbol formation (the invention of a notation system and reflection on the relation between symbol and its meaning). The teacher starts questioning:⁷

- T: 'If someone wants to buy a shoe, and he wants a sandal, how can we quickly find the shoes we need ?
 p's: "Take all the shoes, look for it"
 T: 'Well, that's a lot of work, isn't it?

The pupils start looking in the boxes for a sandal.

- p: 'Here it is !'
 T: 'That's not my size'
 p: 'No,...'
 T: 'Do you really have to check all the boxes ?'

In the meantime the children keep looking for the wanted shoe, by opening boxes and looking in them.

- T: 'This is a problem. How do they do it in a real shoe shop? Are there boxes in the racks?'
 p: 'No, they put the shoes on the shelves'
 T: 'Yes, that's a good idea, then you can see immediately what kind of shoe it is'

(children endorse and start telling experiences)

- T: 'Let's do it'

Children start placing shoes on top of their boxes on the shelves of the rack until one child discovers a problem:

- p: 'but how do we do it with the boxes at the bottom of those piles, then?'
 T: 'Yes, what are we going to do with the shoes at the bottom'
 p: 'You can never reach them'
 (...)

T: 'How about putting just one shoe on the shelf and leaving the other shoe in the box over here?'

p's: 'Yes! (Pupils immediately get to this work)

T: 'Now everyone who comes into our shop, can see what shoes we have'

pupils working, teacher walks around (...)

T: 'Roy, which box matches with this shoe?'

p: 'This one'

T: 'No that one is grey, it is not the same'

pupil starts looking for the right shoe; the teacher continues asking children for shoes that go together. Pupils are very busy putting shoes on the shelves (...)

T: 'Well, I think there is a problem here again. If someone wants to buy this shoe (teacher points to one shoe in the rack) how are we gonna find the other one?'

pupils start opening the boxes, looking for the matching shoe

T: 'That's a lot of work! Is there no other way we can find to do this more quickly, finding the right shoe? We must be sure that we find the right shoe easily'

p: 'We must leave the tops off the boxes!'

T: 'Yes, but then it is difficult to make piles'

'Maybe it is a good idea to make something to the outside of the box so that you can recognise what's inside?'

p's: 'Yes, I know!' (one pupils starts looking for a pencil or something to write with), I know!'

T: 'Tell me what you are looking for'

p: 'We're gonna put letters on them'

T: 'Which letters?'

pupil points to numbers on a number table

T: 'Maybe we can make a drawing of the shoe in the box, to be sure we recognise them'

pupils start making labels, to stick them on the boxes

T: (asking children) 'Which shoe are you drawing?' 'And you?'

one pupil shows his sticker with a drawing of a shoe on it

p: 'Look, miss, this is for that shoe'

T: 'Ah, you also put letters on it? Which shoe does it match to?'

pupils points to a shoe

T: 'But that one is already being done by Roy! You better take another one'

all pupils are very busy making labels for a long time (about 5 minutes)

T: 'Are you doing this shoe?' (Teacher clearly showing that she is not certain about that)
'What can we add to it? What kind of shoes do we have?'

p's 'Women's shoes, boy's shoes, girl's shoes, gentlemen's shoes, ...'

Roy shows a sticker he made

T: 'What did you write to it?'

p: ??

T: 'You cannot read it your self? Do you think that you can find again which shoe is in which box? (Teacher again prompting for certainty!)

p: 'yes'

T: 'Well, then stick it onto your box'

Pupils are very busy making stickers and show them to the teacher. Some children show the following labels:

R D T I or R A I D

p's 'What does it say?'

T: 'Raid' (T. reads: [raid])

T: 'Let's make a pile over here. One shoe in the rack, the other one in a box over there'

children are busy making labels with letters on them

T: 'Maybe not everyone is able to read it, you can also make a drawing of the shoes'

p: 'What does it say, Miss?' (pupil showing her label with letters MAAR)

T: 'It says "Maar"'

p: (repeats) 'Mmmaar...these are mmmama-shoes'

T: 'What a good idea of you to put an M on it for mama-shoes! Do you think we could do that for all the mama-shoes? 'Do you think you could do that too (to another pupil)?'

p: 'No'

T: 'Well, if you cannot write the letter, you may stamp it' (teacher offers a box with stamping materials).

pupils take over this idea immediately and start making labels with stamped letter combinations: I D P B P B R I T m

T: 'If you have papa-shoes, which letter will you put on it, then?'

p's: 'P'

Pupils are involved in the activity of making labels with the correct letter on it; after a while they start to stamp whole words (mama, papa, baby) on it, sometimes asking the teacher first how to write the word; the scene ends with making piles of marked boxes, assorted according to the symbols of the different categories they distinguished (mama-shoes, papa-shoes, children-shoes, baby-shoes. After setting up these piles the children start playing shoe shop.

After this digression of 35 minutes into an 'abstracted' form of semiotic activity the children started playing in the shoe shop, in the way they normally do, but now using the categories they agreed on, using the situation they constructed (with the piles of shoe boxes with one shoe in it and the other shoe displayed in the rack). There is every reason to believe that the 'abstracted' semiotic activity did not disturb their play: the children were all the time working very much involved, and -more importantly- they integrated the results of this activity in their role play later on.

Discussion

The results of this study show that in a role play activity in the setting of a shoe shop, there are a lot of opportunities for the teacher to get involved with pupils' mathematical actions and, by doing so, try to improve the pupils' mathematical thinking. As valuable actions we did not see only straightforward mathematical operations (like counting, measuring, adding etc.) but also more basic actions of semiotic activity. It stands to reason, that the amount and nature of the occurring mathematical actions are probably dependent on the nature of the play setting involved.

However, the teachers could not take advantage of all the opportunities that could be registered. In many cases the teachers actually assisted the children in performing the actions that they were carrying out. Cases of special interest for us were those interventions where the teacher guided the children's thinking towards semiotic activity relevant for mathematical thinking by way of "are you sure questions" (or variants of such questions). Of course, these questions are to be related to *solutions* suggested by the pupils (not to the problems). These questions are supposed to prompt the children to give arguments for their suggested solutions.

Analysis of one paradigmatic case, shows the potential relevance of this kind of interaction with children. It shows also that (and how) the teachers indeed used the "are-you-sure" questions successfully.

The protocol of the episode of semiotic activity in the classroom, as analysed above, shows how the teacher guided the children within their role play activity to some 'abstracted' activity related to representational/notational problems. By constantly putting forward new problems and asking certainty-questions about the solutions, the teacher guided the children into an activity of reflecting on symbols and their meaning within the context of their play, they actually (re)invented a notational system to effectively deal with the problems that emerged in the activity. They reflected on how to improve the form of the symbols used in order to make their meaning as clear as possible so that they could be sure to find the right shoe. The children invented signs to refer to classes of objects and started reasoning and discussing about these classes (mamma shoes, papa shoes etc), instead of dealing with the real shoes. Most importantly, however, was that they reflected together on the relationships between sign (symbol) and meaning: **improving the signs used regarding the intended meaning** ('the right shoe'), but also structuring the meaning (mama, papa, children, baby shoes) and representing this in corresponding notational means. These actions are clearly manifestations of what I called semiotic activity. In the session described above the children were guided towards this activity by the teacher's permanent queries about certainty with regard to the answers/solutions given.

According to my definition of mathematical activity, reflecting on notational means in order to make their shared meaning as unequivocal as possible is regarded here as a basic element of mathematical thinking (as well as of literacy or forms of scientific thinking). The findings of this study underline the fundamental importance of semiotic activity in early education as a developmental basis out of which different kinds of cultural learning emerge. Learning mathematics, then, can be seen as a process of becoming a participant in a mathematical community in which there is agreement on the rules of how to behave and to communicate. As Forman (1992) has argued, different conversational maxims are to be strictly observed in order to become a legitimate participant in this community. One of the maxims says that every participant in a communication ought to be as non-ambiguous, consistent and as relevant as possible (beside all kind of other conventions to be followed). The teacher's queries for certainty and unequivocally are actually prompts for being clear and non-ambiguous as to the symbols used. By doing so, the teacher gets the children involved in a style of conversation which might be fundamental to the development of mathematical thinking.

Additionally, it is important to note here, that the semiotic activity (and conversation) described was not an isolated event. In other sessions pupils spontaneously used notational means to indicate the respective classes of shoes; on other opportunities (in group C) children started to make drawings of the classified and coded piles of shoes and indeed started to operate 'mathematically' on these representations (counting amounts of boxes in each pile). Their drawings looked like histograms and were used as such by the children to find out how many boxes of shoes were in the store. This was one step into the direction of a more abstract dealing with the quantities involved (situated arithmetic), but still meaningfully related to the context of the shoe shop play activity⁸.

Basically, an important question here to ask is of course: have the children learned something of value from our interventions? However, the study was not conducted as an assessment of the learning results of children within a particular kind of play. I believe that these studies have to be carried out in the future, but first of all we have to make sure that mathematical activity (or precursors of mathematical activity) can be promoted at all within play. The present research tried to provide information for this argument.

Conclusions

The data of this observational study show that many mathematical teaching opportunities occur within a role play activity in a shoe shop setting. If the teacher manages to participate in the children's activity and can guide their thinking towards semiotic activity by way of "are you sure questions", it turned out to be possible to get the children involved in reflections on the meaning of symbols. According to my assumptions about the nature of mathematical thinking, this is an important aspect of the development of mathematical thinking. Finally, I cautiously tend to conclude, that play activity can be a teaching/learning situation for the enhancement of mathematical thinking in children, provided that the teacher is able to seize on the teaching opportunities in an adequate way. To what extent this approach also leads to lasting **learning** results in all pupils cannot be inferred from this study. This must be an issue for further research.

NOTES

- 1 It is important to note here that we are talking about the "are you sure questions" about answers or solutions given and not about statements of facts (like "are you sure that this glue will hold?" etc). Moreover, the "are you sure questions" lead to mathematical thinking as far as they promote non-empirical arguing with respect to the answers given. The history of physics can be taken as an example here (see for instance Dijksterhuis, 1969; see also Freudenthal, 1991, ch.1). It is however, impossible to deal with this epistemological question here.
- 2 Note that I am talking about semiotic activity and not about the semiotic function (in the Piagetian sense); by recognising mathematical activity as one kind of semiotic activity (beside others) does, of course, not suggest that mathematical activity is the only possible form of semiotic activity. Actually, there are reasons to suppose communalities between for example literacy and numeracy (see Munn, 1994) that come close to what I understand here as semiotic activity.
- 3 I am very grateful to the principal teacher (Trudy Pas) and her colleagues of this School "De Achthoek" in Amsterdam for their willingness to participate in the investigation and for their dedication to implement the concept in their daily classroom practices.
- 4 Primary school in Holland starts at the age of 4
- 5 Some groups from grades 1/2 also included one pupil from grade 4.
- 6 Of course these events could also be counted as teaching opportunities (in a formal sense).

It would raise the quantity of observations in table 3, but, however, this would also contaminate the estimation of practically useable teaching opportunities. Indeed our estimation entails some subjectivity because it may not be unambiguous what is counted as a major event or as just a fleeting, incidental moment. I decided for an estimation that runs the risk of incorrectly not recognising a "major event", instead of one that runs the risk of falsely including events with no *practical* relevance. Nevertheless, further elaboration of the operationalisation of (practically relevant) teaching opportunities is definitely needed.

- 7 The translation of the protocol is somehow stylised and abbreviated, in order to make the protocol not too repetitious as to the children's utterances; some utterances of the children could be understood only from the situation and their accompanying gestures etc.; literal translation would result in incomprehensible utterances; I give these utterances here in a somewhat improved version, but of course, preserving their meaning. Moreover, some of the children's remarks, interjections, half-completed or crumbled sentences etc turn out to be intrinsically in a literal way, as this would obscure their meaning. The adaptations of the children's utterances, however, have affected the original flavour of the children's conversations.
- 8 The older children (group F) for example played a stock keeping game related to the shoe shop, with cards (drawn in rotation from a batch) indicating how much shoes were sold, and how much have been purchased.

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