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Jumping for distance: control of the external force in squat jumps

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ABSTRACT

RIDDERIKHOFF, A., J. H. BATELAAN, and M. F. BOBBERT. Jumping for distance: control of the external force in squat jumps. *Med. Sci. Sports Exerc.*, Vol. 31, No. 8, pp. 1196–1204, 1999. **Purpose:** It was investigated whether control in jumps for distance is related to control in jumps for height. **Methods:** Five male subjects performed maximum squat jumps in the following conditions: VJ (vertical jump), LJ (long jump), and two conditions with inclination angles of the body relative to the horizontal of 75 and 65°, respectively. An inverse dynamics analysis was performed using measured kinematics and ground reaction forces. In addition, jumps were simulated with a forward dynamic model of the musculoskeletal system, comprising four segments and six muscles. First, VJ was simulated by finding the optimal stimulation pattern, i.e., the pattern resulting in a maximum height of the mass center of the body (MCB). Subsequently, LJ was simulated using a “rotation-extension” strategy, i.e., by applying the optimal stimulation pattern for VJ to the system after imposing an initial angular velocity. **Results:** In the experiments, no significant differences were found among jumps with different inclination angles in the magnitude of the peak ground reaction force. The same was true for the magnitude of the velocity of MCB and the distance of MCB from the center of pressure at the instance of take-off. As the inclination angle became smaller, i.e., jumps were directed more forward, the net knee joint moment increased whereas net hip and ankle moments decreased. Also, the peak angular velocity in the hip joint was higher and the joint was more extended at take-off. The opposite was true for the knee joint. In the simulation study, using the “rotation-extension” strategy for simulating VJ, these adaptations in kinematics and net joint moments were reproduced satisfactorily. **Conclusion:** By virtue of the stabilizing effect of intrinsic muscle properties, a jump for distance may be achieved using control of a vertical jump according to a “rotation-extension” strategy. **Key Words:** BIOLOGICAL MODELS, MUSCLE CONTRACTION, MUSCULOSKELETAL SYSTEM, MOVEMENT, BIOMECHANICS, COMPUTER SIMULATION

Humans can engage in many different actions. Some of these are called explosive. Examples are jumping, kicking, and throwing. Common features of explosive movements are the short duration and high angular velocities. Many recent studies have investigated the coordination of explosive leg extensions, using either a simulation approach (e.g., 8, 12), an experimental approach (e.g., 1, 2), or both (e.g., 3). All of these studies were mainly concerned with vertical jumping. Although jumping for distance also received a lot of attention from scientists, most of this work was either concerned with the long or triple jump in athletics (e.g., 5), with developmental issues (e.g., 6) or with the use of the standing long jump in motor skill tests (e.g., 10). As far as we know no attempt has been made so far to relate the insights on coordination in vertical jumping to jumping for distance. The purpose of this study was to fill this gap using a combined experimental and simulation approach to study differences and similarities in coordination of squat jumps with different inclination angles.

In general, the aim of any maximum effort squat jump can be defined mechanically as a combination of realizing a specific inclination angle of the take-off velocity and maximizing the distance jumped. To maintain a clear distinction between these two requirements, the motion of the mass center of the body (MCB) is most conveniently described in polar coordinates (Fig. 1a). To optimize a squat jump with respect to its mechanical aim, two important constraints must be taken into account. Obviously the desired values of the angles α (between the position vector and the horizontal) and β (between the velocity vector and the horizontal) at take-off impose a directional constraint on the squat jump. Furthermore, in maximizing vc (the magnitude of the velocity of the MCB) and lc (distance between the center of pressure and the MCB), the transfer between joint angles and angular velocities and these variables must be taken into account. This transfer function is generally referred to as the geometrical constraint (7). For example, this constraint implies that in a less extended joint the contribution of the joint angular velocity to the extension velocity vc is larger. Assuming that near optimal coordination with respect to the geometrical constraint is achieved in a vertical jump (13), we expected that all maximal effort jumps would exhibit a similar kinematical pattern of explosive leg extension. To achieve this at different inclination angles, the direction of the ground reaction force (GRF) relative to the body must be adapted (see Fig. 2, a and b). This implies that a systematic

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Figure 1—a (left panel), Description of the kinematics of the mass center of the body (MCB) in polar coordinates. Length of the position vector (lc) between the center of pressure (CoP) and MCB and its angle with the horizontal (α) as well as the length of the velocity vector of the MCB (vc) and its angle with the horizontal (β) are shown; b (right panel), Illustration of the rotation extension model: from its starting position (dotted), the body is rotated until the onset of the extension phase (dashed), and subsequently an explosive leg extension is carried out until take-off (solid).

adaptation of net joint moments to the inclination angle of the jump is anticipated.

Simulations with a model of the musculoskeletal system have demonstrated that trajectory invariance in squat jumps can be achieved without changing the control signal of the model. Based on the work of van Soest et al. (12), in the present paper such a control signal is defined as a sequence of switch times at which the muscle stimulation changes from its minimum to its maximum value (STIM). In one study (11) intrinsic muscle properties counteracted small disturbances to such a degree that adaptation of STIM was not needed. In another study (13), a locally optimal STIM (specific to one starting position) was replaced by a globally optimal STIM that yielded good (albeit submaximal) results for many different starting positions.

Our first hypothesis was that the directional and geometrical constraints are satisfied in squat jumps by subdividing the execution of the jump into a *rotation phase* and an *extension phase* (Fig. 1b). At first the body is rotated relative to the horizontal from its initial squatted position, i.e., angle α decreases. Second, an explosive leg extension is carried out to realize as large a displacement of MCB as possible. The direction of the jump is determined by the rotation phase, which is absent or negligible in the vertical jump. Based on the results of simulation experiments, our second hypothesis was that the necessary adaptation of net joint moments during the extension phase occurs due to intrinsic muscle properties (i.e., stiffness and damping) alone.

METHODS

Subjects. Five physically fit male students participated in this study. The characteristics of this group were as

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follows: age 23.6 ± 2 yr, mass 67.3 ± 3.1 kg, height 1.81 ± 0.02 m (mean values $\pm$ SD). Written informed consent was obtained from all subjects according to the policy statement of the American College of Sports Medicine. All subjects followed a standard warm-up routine.

Experimental procedure. The subjects were instructed to perform the following types of jumps: VJ (maximum height jump), 75 J (spatial inclination angle of $75^\circ = 1.31$ rad), 65 J (spatial inclination angle of $65^\circ = 1.13$ rad), and LJ (maximum long jump). The order of the four conditions was randomized to minimize effects of fatigue and/or learning. The spatial inclination angle is the angle between the horizontal and the line through the subject's toes and the center of a ball (Φ , see Fig. 3a). In the 75 J and 65 J conditions, the subject's task was to hit the ball with the head if possible, without deliberately trying to make a header. The ball was hanging on a cord and its distance from the subject was increased whenever it was hit, so it remained barely reachable. All jumps were therefore assumed to be maximal jumps. Subjects were free to choose their initial squatted position on every trial to see whether it was systematically altered in relation to the type of jump. The subjects were instructed not to make a countermovement and to hold their arms on their back. The subjects were allowed three practice jumps in every condition. Subsequently, 10 jumps were registered per condition.

Kinematical and ground reaction force data were obtained during the sessions (see ref. 1 for a detailed description of data acquisition and apparatus). We registered the positions of four body segments: feet, lower legs, upper legs and

Figure 2—The effect of gravity (mg) on the direction of the resultant force (F_{res}) is illustrated in a (top left panel) and b (top right panel); c (bottom panel) displays the decomposition of the GRF in two components (F'_z and F'_y) that define its direction relative to lc (see Fig. 1a): F'_z runs parallel to lc , F'_y runs perpendicular to lc .

Figure 3—a (left panel), Experimental setup. Subjects jumped toward a ball in two conditions (75 J and 65 J). The ball was positioned at a distance from the subject where it remained barely reachable (two possible distances are shown). The inclination angle Φ with the horizontal was 75 or 65° for 75 J and 65 J, respectively; b (right panel), Definition of enclosed joint angles (θ) of hip, knee, and ankle.

head-arms-trunk (HAT) in 3-D coordinates. In total, 62 jumps were excluded from further analysis. Jumps were disregarded because of temporary occlusion of one or more markers, leaving a total of 138 jumps to be included in the data analysis. All positional data of the markers were transformed into the forceplate coordinate frame and filtered (zero lag low-pass Butterworth filter, cut-off frequency of 12 Hz). Further analysis was limited to the sagittal plane, thus taking left and right legs as one. The location of the segmental centers of mass, the mass distribution of the body, and segmental moments of inertia were calculated according to Winter (14). Although the subject was standing still with legs extended and a hip angle of approximately $\pi/2$ rad, the center of pressure (CoP) was calculated from the force-plate data and the location of the MCB was calculated from the joint angles and estimated mass distribution. Because of imperfect estimates, the calculated horizontal positions of CoP and MCB were not the same (as they should be in static situations). We chose to adjust the location of the center of mass of the HAT segment in such a way that the horizontal position of CoP and MCB were exactly the same. Net joint moments were calculated using a basic linked rigid segment analysis (see refs. 1 and 2). In the following, hip extending, knee extending, and plantarflexing moments are defined as positive. See Figure 3b for definitions of joint angles.

Evaluation of experimental conditions. To evaluate the method used to manipulate the direction of the jump, we calculated means and SD of the angles α and β at the instant of take-off. As can be seen in Table 1, the experi-

mental design was successful in this aspect. To obtain a criterion for jumping performance, we calculated jumped height (Δz) and jumped distance (Δy) on each trial. These measures were defined by:

$$\Delta z = z_{to} - z_{ref} + \dot{z}_{to}^2/2g = lc_{to}\sin\alpha_{to} - z_{ref} + v_{to}^2\sin^2\beta_{to}/2g \quad (1)$$

and

$$\Delta y = y_{to} + 2\dot{y}_{to}/g = lc_{to}\cos\alpha_{to} + v_{to}^2\sin(2\beta_{to})/g \quad (2)$$

where y_{to} and z_{to} are the horizontal and vertical position of MCB at take-off, respectively, z_{ref} is the reference height of MCB, i.e., the height of MCB while standing upright, $\dot{y}_{to}$ and $\dot{z}_{to}$ are the horizontal and vertical components of the velocity of MCB at take-off, respectively, and g is the gravitational constant ($= 9.81 \text{ m s}^{-2}$). In the following, the index "to" denotes the value of a variable at take-off. Note that in Equation 2 it is assumed that the height of MCB at landing equals z_{to} .

Statistical analysis. The main objective of our statistical analysis was to determine whether or not a given variable was systematically covarying with the inclination angle of the jump, operationally defined by jumped distance (Δy). To compensate for the large interindividual differences between our subjects, every variable of interest (X) in trial i of one of the subjects was normalized as:

$$x_i = (X_i - M_{X,VJ})/M_{X,VJ} \quad (3)$$

where $M_{X,VJ}$ is the mean of variable X in the subject's VJ and x_i is the (dimensionless) normalized variable we used for statistical analysis. Subsequently all data x_i were combined for all subjects and two types of statistical analysis were performed:

1. paired Student t -test on difference in means between VJ and LJ ($\alpha \leq 0.05$);
2. test on independence between variable x_i and Δy ($F_{138,1}$, $\alpha \leq 0.05$).

The second test was included because a significant difference in means between VJ and LJ for one variable does not necessarily imply that this variable is systematically related to Δy . In the following, rescaled difference in means ($\Delta m = LJ - VJ$) and SD of both LJ and VJ will be given. These were obtained by multiplying normalized results with the variable's mean value in the VJ condition. Also the rescaled slopes of the regression lines and correlation coefficients will be presented. The importance of a significant result must not be overstated, given the small sample size. The tests included are mainly used to shed light on the main trends in the experimental data. The explanation that is offered for these trends is based on biomechanical model simulations (see below).

TABLE 1. Evaluation of experimental conditions.

Variable	α_{to} (rad)	β_{to} (rad)	lc_{to} (m)	vc_{to} (m s^{-1})	Δz (m)	Δy (m)
VJ	1.49 ± 0.05	1.54 ± 0.02	1.07 ± 0.02	2.77 ± 0.19	0.43 ± 0.04	0.08 ± 0.08
75J	1.27 ± 0.05	1.18 ± 0.05	1.08 ± 0.02	2.80 ± 0.13	0.38 ± 0.05	0.77 ± 0.10
65J	1.18 ± 0.05	1.01 ± 0.06	1.08 ± 0.03	2.75 ± 0.25	0.31 ± 0.04	1.00 ± 0.16
LJ	1.04 ± 0.07	0.78 ± 0.10	1.05 ± 0.03	2.80 ± 0.13	0.16 ± 0.08	1.18 ± 0.09

Task variables (mean $\pm$ SD). lc_{to} and vc_{to} are the magnitudes of the position and velocity vectors of the mass center of the body (MCB) at take-off and α_{to} and β_{to} their respective orientations relative to the horizontal (see also Fig. 1). Δz is the highest point reached by the MCB relative to standing upright. Δy is the horizontal distance traversed by the MCB (see Equation 2). The conditions are: VJ (vertical jump), 75J and 65J (inclination angles of 75 and 65°, respectively), and LJ (long jump).

Figure 4—Typical example of convergence in the velocity of the mass center of the body (vc). The time histories of one VJ and one LJ of one subject are displayed.

Model simulations. A model of the human musculo-skeletal system was used in this study to test our second hypothesis. The model is described in detail in Appendix A and B. The purpose of this model was not to reproduce all of the details observed in the experiment but to make a qualitative assessment about the strategy used to control the direction of squat jumps. Consequently, no attempt was made to fit model parameters to subject characteristics. The model consists of four segments (HAT, upper leg, lower leg, foot). Hill-type muscle models (11,15) are used to describe the behavior of the six muscles driving the model. These six muscles represent m. gluteus maximus, hamstrings, m. rectus femoris, mm. vasti, m. gastrocnemius, and m. soleus. The model performance is controlled by the six switch-times for muscle stimulation (STIM).

Two forms of optimization of model performance were carried out using a Simplex algorithm (4). First, the STIM that yielded a maximal Δz from a equilibrium squatting position was calculated. This optimal jump will be referred to as VJ_{sim} . The second optimization was carried out in the following manner:

1. A nonequilibrium starting position was specified by giving all the segments an (arbitrary) initial angular velocity of -0.6 rad s^{-1} .

2. Subsequently, the STIM obtained for VJ_{sim} was applied to the model. The timing of the start of STIM was optimized in order to maximize Δy .

In terms of the first hypothesis: the extension phase (VJ_{sim}) and rotation phase are optimized consecutively. Because the optimized long jump consists of a rotation phase followed by an extension phase, it will be referred to as ROTEX.

RESULTS

Initial conditions. We used the index "0" to denote the value of a variable at the beginning of a trial (i.e., its mean value during the first 10 samples). In our experiment, we found that a difference between VJ and LJ existed in θ_0 for

hip and knee. Both joints were more extended at the start of the LJ (Δm of 0.07 and 0.11 rad, respectively). However, linear regression of these initial joint angles on Δy revealed no systematic relation between starting position and distance jumped ($r = 0.16$ and 0.21 for hip and knee, respectively).

Kinematics. High kinematic invariance was exhibited in the magnitude of both the position and velocity vectors of MCB. No significant differences in vc_{to} were found. This was due to convergence in the vc histories (see Fig. 4). A significant difference existed between the means of lc_{to} in VJ and LJ but this difference was very small (less than 0.02 m) and not systematically related to Δy (see Tables 1 and 2). In contrast, a clear relationship existed between both α_{to} and β_{to} and the inclination angle of the jump (see Table 2). Stick-figures of a vertical jump and a long jump are presented in Figure 5. To facilitate comparison between these jumps we rotated the stick-figures so that lc was vertical for each stick-figure shown (see Fig. 7a). Inspection of the displayed values for $\delta (= \pi/2 - \alpha)$ in Figure 7a reveals that differences in α were already present before any observable change occurred in joint angles. The trajectory of the MCB can thus adequately be described in polar coordinates by a rotation of lc around the CoP (i.e., a decrease in α), followed by an extension of the body (i.e., an increase in lc). Another aspect of this apparent division of the movement into two distinct phases is the increase in movement duration with Δy . This was mainly due to an increase in the duration of the rotation phase; the extension phase seemed to be hardly affected. Analysis of joint angles showed a significantly more extended hip and less extended knee at take-off in LJ when compared with VJ (see Fig. 7a and Table 2). Also, a significant trend was present for these joint angles with respect to Δy , whereas no differences were found for the ankle angle at take-off (see Table 2). We also found smaller peak angular velocities in LJ than in VJ for ankle and knee, but only the decrease in knee angular velocity was systematically related to Δy . In contrast, the peak angular velocity in the hip was significantly larger in LJ and also increasing with Δy .

TABLE 2. Experimental results: kinematics.

Variable	Analysis				
	MD			LR	
	Δm	SD_{VJ}	SD_{LJ}	Slope	r
$vc_{to} [\text{m}\cdot\text{s}^{-1}]$	0.07	0.05	0.15	0.07	0.16
$lc_{to} [\text{m}]$	-0.02*	0.01	0.02	-0.01	-0.16
$\alpha_{to} [\text{rad}]$	-0.44*	0.04	0.07	-0.37*	-0.94
$\beta_{to} [\text{rad}]$	-0.76*	0.02	0.10	-0.64*	-0.96
$\theta_{to} [\text{rad}]$					
Hip	0.17*	0.06	0.11	0.20*	0.65
Knee	-0.26*	0.04	0.17	-0.17*	-0.52
Ankle	-0.01	0.04	0.07	0.01	0.05
$\theta_{max} [\text{rad}\cdot\text{s}^{-1}]$					
Hip	0.63	0.43	0.79	0.95	0.50
Knee	-3.58	0.49	1.69	-2.43	-0.60
Ankle	-1.32	0.75	1.05	-0.93	-0.34

Kinematic variables (* denotes significant difference or correlation). MD designates the results of the analysis of the difference in means. $\Delta m (= LJ - VJ)$ and standard deviations of LJ (long jump) and VJ (vertical jump) are given. LR designates the results of linear regression analysis of variables on Δy : slope of the regression line and r (correlation coefficient) are given. See Table 1 and Fig. 1a for definition of lc_{to} , vc_{to} , α_{to} and β_{to} and Fig. 3b for the definition of joint angles θ . θ denotes joint angular velocity.

Figure 5—Stick figures of an experimental vertical jump (VJ: top panel) and long jump (LJ: bottom panel) of one subject. The body's configuration is shown with the ground reaction force (GRF) and the velocity of the mass center of the body (vc) at seven intervals. The GRF and vc vectors are scaled to fit the figure.

Kinetics. No differences were found in the magnitude of the peak ground reaction force (GRF_{max} , see Table 3). Obviously, the peak horizontal force increases significantly with Δy (the slope of the regression line was 243.5 N m^{-1}). Although the peak vertical force is higher in VJ than in LJ (a mean difference of 63.2 N was found), this variable was not significantly correlated to Δy . However, it is more convenient to look at the differences in components of the GRF relative to lc (F'_{z} and F'_{y} , see Fig. 2c), because these are related more easily to net joint moments and to our kinematic description in polar coordinates. The main difference between VJ and LJ existed for $F'_{y,max}$, which is larger for LJ and this increase is also significantly correlated to Δy (see Table 3). $F'_{z,max}$ is independent of the direction of the jump (see Table 3). These findings clearly show two aspects of the direction of the GRF in our experiment. First, the increase in F'_{y} with Δy indicates that the direction of the

GRF relative to the body is adapted to the direction of the jump. Second, the large increase in $F'_{y,max}$ with Δy and the absence of significant directional effects in either $F'_{z,max}$ or GRF_{max} indicates that the maxima for both components occur at different times. Indeed, we found F'_{y} always to reach its maximal value between 100 and 200 ms before the peak in F'_{z} . The changes in the direction of the GRF relative to lc must be caused by adaptation of net joint moments, given the considerable kinematic invariance in the experiment. Data clearly demonstrates that such an adaptation did occur. The hip and ankle moment decrease with Δy whereas the knee moment increases (see Table 3 and Fig. 6). The adaptation of moments was mainly a scaling in amplitude although also timing effects could be observed for some subjects (see Fig. 6).

Model simulations: vertical jump. In our simulation experiments, we obtained an optimal STIM for a vertical jump. As in the experiment, the vertical jump serves as a reference movement for the long jump simulations. The optimal solution calculated by the model yielded a relative height $\Delta z = 0.394 \text{ m}$ and a take-off velocity of 2.53 m s^{-1} . Because we are mainly interested in controlling the direction of the jump, our focus should be on α_{to} and β_{to} . The simulations yielded 1.55 rad for α_{to} and 1.49 rad for β_{to} (for comparison with experimental take-off angles, see Table 1). The total time needed for a simulated vertical jump was 0.335 s , which is similar to the duration of an experimental jump.

Model simulations: ROTEX. With optimally timed start at STIM, the ROTEX-model yielded a jumped distance $\Delta y = 1.11 \text{ m}$ and a take-off velocity of 2.54 m s^{-1} . In the ROTEX simulation the angles α_{to} and β_{to} were, respectively, 1.15 rad and 0.71 rad and lc_{to} was found to decrease only 0.02 m relative to VJ_{sim} . The difference between the duration of the extension phase in the optimized ROTEX-model and the total duration of the VJ_{sim} was only 0.043 s . The optimal duration of the rotation phase was calculated to be 0.188 s for the ROTEX-model, yielding a total execution time of 0.480 s . In Figure 7b we present rotated stick-figures of VJ_{sim} and ROTEX. Comparison of net joint moments in ROTEX and VJ_{sim} clearly shows an adaptation to the direction of the jump (see Fig. 8). Comparison of the simulation results with experimental data indicates that in model simulations the major differences and similarities between

TABLE 3. Experimental results: kinetics.

Variable	Δm	Analysis			
		MD	LR		
		SD_{VJ}	SD_{LJ}	Slope	r
$GRF_{max} [N]$	-27.8	33.1	90.3	-21.4	-0.12
$F'_{y,max} [N]$	25.5*	15.5	32.8	24.7*	0.41
$F'_{z,max} [N]$	-30.1	32.8	89.2	-23.3	-0.13
$M_{max} [N \cdot m]$					
Hip	-57.2*	12.7	17.3	-46.1*	-0.73
Knee	68.0*	16.8	44.6	60.5*	0.62
Ankle	-74.8*	13.3	16.8	-66.0*	-0.86

Dynamical variables (* denotes significant difference or correlation). MD and LR: see Table 2. GRF_{max} is the peak ground reaction force, $F'_{y,max}$ and $F'_{z,max}$ are the maxima in the two force components of the GRF relative to lc (see Fig 2c). M_{max} is the peak joint moment.

vertical jumps and long jumps are reasonably well reproduced.

DISCUSSION

To evaluate our findings, it is helpful to combine and discuss the experimental and simulation results on three different levels, i.e., the levels of MCB, joint, and muscle dynamics.

Whole body dynamics. The magnitudes of velocity and position vectors of the MCB (v_c and l_c) have rather stereotypical time histories regardless of the inclination angle of the jump. Also, no significant differences were found in relation to Δy for the peak GRF. On this level of analysis, we can therefore conclude that an invariant extension phase exists for all squat jumps. This suggests that an optimal performance with respect to the geometrical constraint is relatively independent of the directional constraint. The inclination angle of the jump is determined by a rotation phase before the explosive extension. This rotation phase can be initiated by perturbation of the static equilibrium starting position and, to gain forward velocity, by taking advantage of the body's mechanical situation as a pendulum inverted against gravity. In model simulations, we found that the MCB trajectory of a simulated vertical jump was easily converted into the trajectory corresponding to a long jump when such a perturbation was added through specification of nonequilibrium initial conditions.

In conclusion, we found good evidence in support of our first hypothesis, i.e., the execution of a long jump may indeed be subdivided in a rotation and an extension phase.

Joint dynamics. We found that joint moments were adapted to the direction of the jump. In combination with the differences in inclination angles, these adaptations result in (nearly) optimal coordination during the extension phase, as suggested by the invariance of $l_{c_{to}}$ and $v_{c_{to}}$. Although changing the direction of the jump was accompanied by changes in joint kinematics, these effects apparently cancel out on the level of the MCB. This is of course possible in a kinematically redundant skeletal system, where many different combinations of joint angles and angular velocities will lead to the same l_c and v_c . An infinite number of possible combinations of joint angles and joint angular velocities exists, given l_c and v_c , which makes it impossible to relate v_c and l_c invariance to events on the joint level, but not *vice versa*. Nevertheless, the changes in joint kinematics were very stereotypical, both across subjects and in the simulations. This stereotypy suggests that the changes in joint kinematics are a consequence of the differences in the mechanical conditions of the jump.

Muscle dynamics. Important intrinsic properties of striated muscle are the length and velocity dependence of muscle force (see ref. 15 for a review of Hill-type models of these properties). Using a model of the musculoskeletal system, we varied the initial conditions of the extension phase for a vertical jump by addition of a rotation phase to simulate a long jump. The control signal STIM that determines the timing of muscle activity during the extension

phase was not adapted to the direction of the jump. Thus, in the simulations the observed adaptation in joint moments can only be explained by intrinsic muscle properties. In the case of squat jumps, this effect of intrinsic muscle properties on the movement is apparently stabilizing. Although this stabilizing effect cannot be proven analytically because of the redundancy of the musculoskeletal system, it can be inferred from the simulations. The stabilizing effect of muscle properties can be illustrated by a simple example of two antagonist muscles controlling a reference (explosive) extension of a single joint. If a perturbation results in a higher angular velocity, it leads to a higher shortening velocity of the agonist muscle fibers. Consequently, the agonist muscle force decreases due to intrinsic muscle damping (concentric contraction). Furthermore, the higher lengthening velocity of the antagonist muscle fibers will lead to an increase in antagonist muscle force (eccentric contraction). Because the resulting decrease in the net joint moment counteracts the deviation in the angular velocity, the effect of intrinsic muscle damping can be qualified as stabilizing with respect to the reference movement.

With respect to the length dependence of muscle force (or muscle stiffness), the resultant effect is not so clear because it depends strongly on the distribution of optimum muscle

Figure 6—Net joint moments of a vertical jump (VJ), a long jump (LJ), and a jump with an approximate inclination angle of 75° (75 J) of one subject.

Figure 7—Rotated stick figures. All segments were rotated over an angle $\delta (= \pi/2 - \alpha)$, to facilitate comparison between vertical and long jumps (VJ and LJ, respectively). This yielded a vertically oriented lc (position vector of the mass center of the body, see Fig. 1a) for every configuration shown; a (top panel), experimental VJ and LJ; b (bottom panel), simulated VJ_{sim} (thin line) and ROTEX (bold line). In both figures the corresponding rotation angles are shown (bold numbers for LJ). In b, the starting position of VJ_{sim} is displayed three times for comparison with ROTEX during the rotation phase.

lengths. However, because of the high movement velocity, it is likely that the effects of intrinsic damping will dominate the muscle's response.

This line of reasoning can be used to explain the adaptations of net joint moments in the simulated squat jumps by the differences in joint kinematics and hence by the changes in the mechanical configuration of the system. This interpretation of the results of the simulation experiments supports our second hypothesis, that the adaptation of the joint moments is due to intrinsic muscle properties alone.

Model validity. A general analytical proof of the aforementioned relations between muscle properties and movement invariance cannot be given because of the redundancy of the musculoskeletal system and the nonlinear dynamics of skeletal muscle. Thus, the effects of intrinsic muscle properties on net joint moments can only be studied effectively with a musculoskeletal model as presented in the Appendix. The main mechanical properties of the muscu-

loskeletal system have been identified by others and incorporated in our model (cf. (8,13) for the application of similar models in the analysis of squat jumps). Because we succeeded in reproduction of the main trends in the experimental data, we conclude that the hypothetical rotation-extension control strategy is supported by our simulation experiments. However, some serious limitations of the musculoskeletal model must be addressed.

Because *in vivo* optimum lengths of muscle-tendon complexes and muscle fibers are unknown (and probably not the same for different individuals) this is a major uncertainty concerning our (and similar) models. Also, maximum forces of muscles cannot be measured directly. Furthermore, we did not use anthropometric characteristics and starting positions obtained from our subjects to define the skeletal parameters in the model simulations. These limitations make detailed quantitative comparison between simulations and experiment difficult. We anticipate, however, that a more sophisticated model (e.g., fitted to the experiment, less constraints imposed on STIM) would yield essentially the same results but at the cost of much more computational work and effort.

Finally, the obvious question of how the rotation phase is initiated in reality remains unanswered. Because *any* forward directed perturbation of the static equilibrium starts a

Figure 8—Net joint moments of the simulated vertical jump (VJ_{sim}) and long jump (ROTEX). The vertical line indicates the onset of the extension phase in ROTEX.

rotation phase, there exists a virtually infinite number of possibilities. Because we neither standardized the starting position nor recorded electromyograms (EMG), we were unable to analyze this aspect of the jumps. Also the absence of EMG records makes it impossible to verify the assumed invariance in muscular activation during the extension phase experimentally. Further research into this aspect of jumping for distance is required.

Theoretical relevance. Our simulation results indicate that trajectory stability of the MCB by joint moment adaptation can be achieved through intrinsic muscle properties (intrinsic stiffness and damping) in the ROTEX-model. A movement strategy that would rely on those properties would have the advantage of requiring only one muscle activation sequence for an explosive leg extension (STIM in our model) for many different jumps without sacrificing much in achievement. Such a strategy has been put forward by van Soest et al. who coined the expression "control that works" (13). In short, this means that the use of a locally

optimal STIM, specific to one particular starting position or direction (in the present study) can successfully be replaced (at least in principle) by near optimal "control that works" in all of these circumstances, thereby avoiding the storage and novelty problem associated with the motor program approach to movement coordination (9).

Finally, the striking invariance of vc_{to} and lc_{to} calls for an explanation. Although the material presented in this paper suggests that this invariance is a consequence of the structural and functional characteristics of the musculoskeletal system, at present a complete proof for this cannot be offered. A clear understanding of this invariance is one of the most challenging and important goals for future research.

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APPENDIX A. The musculoskeletal model.

The Hill-type muscle model is based mainly on the work of Van Soest et al. (12). The muscle is modeled with a contractile element (CE) and an elastic element (PE) parallel to it, both in series with another elastic element (SE). The CE describes the active force-length-velocity relationship of muscle fibers. The PE and SE describe the passive force-length relationship for muscle fibers and tendon, respectively. All the parameters of the musculoskeletal model are given in Appendix B. In the following, boldface characters denote column vectors or matrices, and normal characters are used to define operations on elements of vectors. Element indices are usually omitted for convenience of notation.

In the simulations of the musculoskeletal model the set of nonlinear differential equations Γ_{mus}

$$v_{ce} = d\mathbf{l}_{ce}/dt = \Gamma_{mus}(t, \mathbf{l}_{ce}, \mathbf{l}_{mus}, \mathbf{STIM}) \quad (\text{A0a})$$

defined by Equations A1-A5b is integrated simultaneously with the equations of motion of the linked segment model Γ_{seg}

$$d\Phi/dt = \Gamma_{seg}(\Phi, \mathbf{M}) \quad (\text{A0b})$$

derived from Equation A6. The state variables of the model are the CE lengths $\mathbf{l}_{ce}$ and $\Phi = [\varphi^T (d\varphi/dt)^T]^T$ (φ denotes transposed) where φ is the vector of angles of the four segments with respect to the horizontal. The input of the model $\mathbf{STIM}$ is the vector of six switch-times for muscle stimulation (see Eq. (A2)). Musculoskeletal interaction is modeled with the vectors $\mathbf{l}_{mus}$ and $\mathbf{M}$. The vector of muscle lengths $\mathbf{l}_{mus}$ depends on the vector of joint angles $\theta = [\theta_{hip} \theta_{knee} \theta_{ankle}]^T$ as

$$\mathbf{l}_{mus} = \mathbf{l}_0 - \mathbf{R} \cdot \theta \quad (\text{A0c})$$

where the $\mathbf{R}$ is a constant (6×3) moment arm matrix and $\mathbf{l}_0$ is a vector of muscle reference lengths. Joint angles θ are

defined according to Fig. 3c. The other interaction variable is the vector of net joint moments $\mathbf{M} = [M_{\text{hip}} M_{\text{knee}} M_{\text{ankle}}]^T$. The net joint moments are a function of muscle forces $\mathbf{F}$ defined by

$$\mathbf{M} = \mathbf{R}^T \mathbf{F} \quad (\text{A0d})$$

where $\mathbf{F} = [F_{\text{max},1} F_{\text{se},1} \dots F_{\text{max},6} F_{\text{se},6}]^T$. F_{max} is a force scaling parameter.

The normalized muscle force F_{se} is equal to the normalized tendon force and depends on l_{se} as

$$F_{\text{se}} = k_{\text{se}} l_{\text{se}}^2 \quad (\text{A1})$$

where $l_{\text{se}} = \max(0, l_{\text{mtc}} - l_{\text{ce}} - l_{\text{slack}})$ and $k_{\text{se}} = 1/((l_{\text{max}} - 1)l_{\text{slack}})^2$; l_{max} and l_{slack} are SE parameters. The normalized force of PE (F_{pe}) is defined similar to that of SE with $l_{\text{pe}} = \max(0, l_{\text{ce}} - (1 + \text{cwidth}/2)l_{\text{ce0}})$ and $k_{\text{pe}} = 0.75/((\text{cwidth}/2)l_{\text{ce0}})^2$; cwidth and l_{ce0} are CE parameters. In the optimal solutions, F_{pe} remained zero throughout the entire simulation.

The activation q is modeled as a sigmoid function of time t and STIM

$$q = q_0 + (1 - q_0)/(1 + \exp(a_q + b_q T)) \quad (\text{A2})$$

where $T = \max(0, t - \text{STIM})$; a_q and b_q are shape parameters of the activation function; q_0 defines the (non-zero) initial value of q . The initial value of q was 0.05 for the three biarticular muscles HAM, REC, and GAS. The values for the three monoarticular muscles GLU, VAS, and SOL were chosen to generate isometric joint moments necessary for an equilibrium starting position.

The normalized force-length relationship of CE is modeled with a parabola

$$F_l = \max(10^{-6}, c(l_{\text{ce}}/l_{\text{ce0}})^2 - 2c(l_{\text{ce}}/l_{\text{ce0}}) + c + 1) \quad (\text{A3})$$

where $c = -1/\text{cwidth}^2$. The offset of 10^{-6} was necessary to avoid division by zero in Equation A4. According to Winters (16), the normalized force of CE ($F_{\text{ce}} = F_{\text{se}} - F_{\text{pe}}$) is related to q , F_l , and F_v by a product relationship. F_v represents the force-velocity relationship:

$$F_v = \max(\sigma F_{\text{ecc}}, F_{\text{ce}}/(qF_l)) \quad (\text{A4})$$

where F_{ecc} is the maximum normalized force in eccentric contractions. The scale factor σ ($=0.999$) is included to avoid division by zero in Equation A5b. The contraction velocity v_{ce} in Equation A0a is found with the inverted force-velocity relationship. The following equations de-

scribe this relationship for concentric and eccentric contractions, respectively:

$$v_{\text{cc}} = -f_q l_{\text{ce0}} b_{\text{rel}} ((1 + a_v)/(F_v + a_v) - 1) \quad \text{if } 0 \leq F_v \leq 1 \quad (\text{A5a})$$

$$v_{\text{ce}} = -c_{\text{slope}} f_q l_{\text{ce0}} b_{\text{rel}} ((1 - F_{\text{ecc}})/(1 + a_v))$$

$$* ((1 - F_{\text{ecc}})/(F_v - F_{\text{ecc}}) - 1) \quad \text{if } 1 < F_v < F_{\text{ecc}} \quad (\text{A5b})$$

where $a_v = a_{\text{rel}}/F_l$ and $f_q = \min(1, 3.33*q)$; a_{rel} , b_{rel} and c_{slope} are the parameters for the normalized force-velocity relationship.

The equations of motion of the linked segment model are written in the form

$$\mathbf{x} = \mathbf{A}(\Phi)^{-1} \mathbf{b}(\Phi, \mathbf{M}) \quad (\text{A6})$$

where $\mathbf{x} = [F_{\text{ry}}^T F_{\text{rz}}^T (d^2 \phi / dt^2)^T]^T$. $\mathbf{A}(\Phi)$ is a configuration dependent matrix.

The vector $\mathbf{b}(\Phi, \mathbf{M})$ depends on configuration and interaction variable $\mathbf{M}$. The derivative $d\Phi/dt$ in Equation A0b is obtained from $\mathbf{x}$ and Φ . The vectors F_{ry} and F_{rz} denote the horizontal and vertical components of the net reaction forces acting at the distal end of each segment, respectively. The vertical component of the ground reaction force ($F_{\text{rz},4}$) is used to determine the instant of take-off. A variable step size, variable 4th/5th order Runge-Kutta-Fehlberg method was used to perform numerical integration. The error tolerance level was 10^{-3} , which resulted in an average discretization time step of approximately 0.001 s.

APPENDIX B. Model parameters

All parameters of the linked segment model were taken from Van Soest et al. (13). The average moment arms for the six muscles reported in (13) were used for the constant moment arm matrix $\mathbf{R}$. The following parameters had the same value for all muscles: $l_{\text{max}} = 1.04$, $\text{cwidth} = 0.56$, $c_{\text{slope}} = 0.5$, $F_{\text{ecc}} = 1.3$, $a_q = 3$ and $b_q = -40$ [s^{-1}].

For SOL the force-velocity parameters a_{rel} and b_{rel} were 0.2 and 2 [s^{-1}], respectively. For the other 5 muscles $a_{\text{rel}} = 0.41$ and $b_{\text{rel}} = 5.2$ [s^{-1}]. The rest of the muscle parameters are given in Table 4.

TABLE 4. Muscle parameters.

Muscle	F_{max} [N]	l_{ce0} [m]	l_{slack} [m]	l_0 [m]
GLU	5000	0.200	0.150	0.439
HAM	4000	0.104	0.370	0.636
REC	3000	0.081	0.340	0.297
VAS	9000	0.093	0.160	0.184
GAS	4000	0.055	0.382	0.540
SOL	8000	0.055	0.246	0.380

F_{max} , l_{ce0} and l_{slack} are equal to the values reported in ref. 12, Soest, A. J. van, and M. F. Bobbert. The contribution of muscle properties in the control of explosive movements. *Biol. Cybern.* 69:195-204, 1993.