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van Dijk, F., & Nijkamp, P. (1980). *Analysis of conflicts in dynamical environmental systems via catastrophe theory*. (Serie Research Memoranda; No. 1980-6). Faculty of Economics and Business Administration, Vrije Universiteit Amsterdam.

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ANALYSIS OF CONFLICTS IN DYNAMICAL ENVIRON-
MENTAL SYSTEMS VIA CATASTROPHE THEORY

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Researchmemorandum 1980-6

Paper presented at the First World
Conference on Regional Science,
Cambridge (Mass.), June 1980

Abstract

This paper addresses itself to long-term conflicting issues in environmental policy analysis. After a brief discussion of the "new scarcity" and its subsequent potential perturbations of our economic system in the long run, attention will be paid to some elements of catastrophe theory which may be helpful in gaining insight into the future impacts of inertia and conflicts in environmental management. Next, a fairly simple model for integrating natural resources and production will be constructed so as to illustrate the usefulness of catastrophe theory in economic-energy-environmental systems. The paper will be concluded with a brief judgement of using catastrophe notions in the social sciences.

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1. Introduction

The seventies have been marked by a drastic change in the attention from social scientists as well as from engineers. Environmental problems (pollution, decline in quality of life, exhaustion of energy and other natural resources, etc) have come to the fore. Moreover, in political sciences much attention has been oriented to conflictual equity problems (unequal distribution of welfare, unequal burden from environmental deterioration, unbalanced supply of energy sources, etc). The abundance of our affluent society seems to be accompanied by a new scarcity: lack of clean air and fresh water, lack of energy and raw materials, lack of an equitable distribution of welfare constituents both within and between nations, etc.

The economic and technological development of most industrial societies demonstrate a conflictual trend: any further increase in material economic growth evokes a counter-effect which neutralizes - or at least affects - the original economic progress. This "law of conservation of disaster" has its roots in the first and second law of thermodynamics, as will be shown in the following section.

The "new scarcity" and the conflicts involved may give rise to unexpected or at least unpredictable perturbations in the economic-environmental-technological systems of modern societies. Although, in the recent past, many dynamic models have been developed to analyse future (dis)equilibrium trends, only a few attempts have been made to study the potential dangers, causes and impacts of sudden shocks in our earthly system. In this respect, it may be useful to employ some elements from catastrophe theory in order to study the mathematical backgrounds of

sudden perturbations. Therefore, in section three a brief elementary exposition of catastrophe theory is given.

The notions of sudden break-downs can next be integrated in a formal model describing energy-economic-environmental interactions. In section four a simple model for dealing with perturbations in such dynamic systems will be described, while also some extensions toward more complete and complicated models will be discussed. The consequences of shocks and sharp conflicts in such models will be discussed in section five, while in a subsequent section also some policy implications will be outlined. A final section is devoted to a more general evaluation of the relevance of catastrophe theory for the social sciences.

2. New Scarcity: Conflicts and Inertia

The industrialized world is increasingly being confronted with the consequences of the new scarcity. The necessary inputs for our technological systems are no longer abundant, at least not for low prices. There are many risks and uncertainties in the supply of energy inputs and raw materials. Although the precise stock of energy sources and natural resources is unknown (cf. Commoner (1976) and Leontief et al (1977)), several authors argue that the world as a whole is not - in the nearby future - running out of its absolute natural resources (cf. a recent report sponsored by the Ford Foundation of "Energy: The Next Twenty Years"). A major problem, however, is the unequal distribution of these resources.

Thus, the new scarcity has three aspects:

- physical: the perpetual exhaustion of important natural resources in the long run.

- economic: the price increases due to the (perceived and sometimes purposely created, but not always really existent) scarcity.
- political: the unbalanced supply of resources due to their unequal geographical distribution.

Given the dependence of the industrialized world on energy and natural resources, it is clear that uncertainties and shocks in the supply of these productive inputs are harmful for the industrialized economies. A stable growth path is not very likely to occur in the future. Therefore, adjustments in technology and human behaviour may be stimulated in order to prevent the occurrence of unanticipated events which might induce unbalanced and crisis-oriented actions.

The necessity of such adjustments can be made plausible on the basis of the first and second law from thermodynamics (see Nykamp (1977)).

The first law states that in a closed system the total quantity of
1)
material and energy is constant. This law implies that, as a whole, no matter and energy can be created or destroyed; only qualitative transformations are possible. Even production is essentially not a creation of new material goods, but only a qualitative change in inputs (cf. Boulding (1971)). The materials balance model, which is based on this first law, clarifies the fundamental constraints for our technological system: any rise in throughput will worsen the situation

- 1) The common denominator for material and energy can be derived from Einstein's well-known equivalence relationship.

at the input side through a decrease in resources, while it also affects the output side through an increase in waste discharges. Only recycling might be a means to get a more balanced supply of natural resources.

However, the second law of thermodynamics reduces the relevance of recycling. It states that in a closed system the entropy (a measure for the quantity of unavailable energy) tends to be at a maximum. Consequently, in the long run, all energy may be transformed into heat which will be distributed so regularly within the system that it is no longer usable (cf. Georgescu - Roegen (1976)). Therefore, without adding additional low entropy, the entropy of some amount of energy cannot be lowered, so that energy cannot be recycled; also even materials recycling requiring a low entropy energy input will be hampered through lack of energy resources. Only in the case of import of low entropy from outside the system (for example, solar energy), may a balanced situation emerge. This is also reflected by Prigogine's extended entropy relationship (see Bertalanffy (1972)):

$$(2.1.) \Delta E = \Delta E_i + \Delta E_t,$$

where ΔE , ΔE_i and ΔE_t represent the total change in entropy, the change in entropy due to import from outside and the change in entropy due to transformation processes in the system ($\Delta E_t > 0$).

It should be noted that the earth is an open system which possesses two kinds of energy, viz stock energy (embodied in the earth itself such as natural gas and oil) and flow energy (entering the earth from outside such as solar energy). Stock energy is essentially exhaustible and non-replenishable, whereas flow energy has an enormous potential (see section four).

The foregoing remarks have clarified the risks of our economic and

technological systems. The exhaustion of natural resources, the severe ecological and environmental deterioration and the international political responses may lead to many shocks and disturbances in the future. In the above-cited study, "Energy: The Next Twenty Years", it is even stated that serious shocks and surprises are certain to occur and that - due to instabilities and interruptions - the long-term outcome cannot reliably be predicted. In respect to this, Boulding's (1971) plea for a circular economy in which the production and consumption pattern is brought in harmony with the available resources and the constraints inherent in the limited availability of low entropy energy receives more perspective.

In light of the foregoing frictions in the functioning of an affluent welfare state, future prospects are not at all favorable. First, there are many conflicts among environmental protection, use of alternative energy sources, economic growth and depletion of raw materials (cf. Lakshmanan and Nykamp (1980) and Nykamp (1980)). All these conflicts pose serious threats to our societies and require a satisfactory policy response to what was once characterized as the "moral equivalent of war".

In addition, unpredictable shocks are more likely to occur as lack of flexibility in human behaviour leading to inertia (the tendency of a system to remain in its original position) is higher. Given the present risky situation regarding the supply of energy and raw materials, higher prices for these inputs cannot be avoided. However, there is no guarantee the price mechanism itself will lead to an adequate adjustment of technology and human behaviour. Instead, the transition from a period of low input costs of natural resources to high input costs may demonstrate several shocks and stepwise changes

affecting the stability of the total system. Therefore, the high transition costs may lead to severe perturbations and disturbances: a smooth and continuous adjustment is not likely to take place. In addition to monetary transition costs, such a discontinuous adjustment may also be caused by psychological adjustment costs due to lack of innovative abilities, social constraints, bandwagon effects, rigid institutional frameworks, etc. (cf. Leibenstein (1976)).

In conclusion, the occurrence of both conflicts and inertia in the energy-environmental-economic sector is a serious challenge to western societies. Conservation of natural resources and technological innovation may be a wise response to these frictions. As will be shown in the following sections, perturbations and shocks are more probable, as the degree of conflict and inertia in energy-environmental-economic systems is higher. The analysis of shocks and discontinuities in the state of a system may be based on some notions from catastrophe theory. Therefore, the following section will be devoted to some relevant elements from catastrophe theory. Thereafter, some discontinuous models describing sudden disturbances in energy-environmental-economic systems will be developed.

3. Some Elements of Catastrophe Theory

Dynamic processes - especially continuous and smooth transitions from one state to another - have often been studied by differential calculus. In the past, much emphasis has been put on the study of equilibrium states of dynamic systems, whereas the analysis of possibly discontinuous adjustments and shocks in such equilibrium states has only received minor attention. Catastrophe theory aims at filling

this gap by analysing discontinuous processes and abrupt transitions.

Catastrophe theory is a recently developed mathematical language which - contrary to traditional calculus - is able to describe sudden space-time disturbances in dynamic systems. It was originated by Thom (1972), and further elaborated among others by Woodcock and Poston (1974), Lu (1976), Zeeman (1977), Woodcock and Davis (1978), and Poston and Stewart (1978). This mathematics of discontinuous adjustments has subsequently been applied to a wide range of phenomena from physics, biology and social sciences among others.

Applications of catastrophe theory can be found among others in:

- biology and medical science; for example, heartbeat problems (Zeeman (1972)); morphogenetic problems in embryology (Thom (1972)); fishery biology (Clark (1976)); developmental biology (Zeeman 1976b)); ecological systems (Jones (1975)).
- physics; for example, visual caustics (Thom (1972)); fluid mechanics and thermodynamics (Poston and Stewart (1978)).
- economics; for example, business cycle theory (Varian (1979)); environmental-economic modelling (Van Dyk (1979)); stock exchanges (Zeeman (1974)); fishery economics (Jones and Wolters (1976)); Nykamp and Sakashita (1977)).
- social and political science; for example, political decision-making (Coppock (1977)); censorship (Isnard and Zeeman (1976)); military decisions (Isnard and Zeeman (1976)); history (Iberall (1974)).
- geology and geomorphology; for example, fluvial systems (Graf (1979)); geological steady-state mechanisms (Cubitt and Shaw (1976)); geological modelling (Henley (1976)).

- regional science and geography; for example, urban facilities (Poston and Wilson (1977)); spatial interaction (Wilson (1979)); modal choice (Wilson (1976)); urban systems (Amson (1975)); city size (Isard (1979)); geographical modelling (Wagstaff (1976)).

The foundations of catastrophe theory were laid by Thom (1972) who provided a geometrical exposition of the theory based on differential topology;¹⁾ an algebraic presentation of the theory was given later by Stewart (1975). Thom argued that changes in many systems - despite their quantitative complexity and varying dimensions - are qualitatively simple and stable. Contrary to ordinary calculus where changes in the direction or quality of curvature define singularities on a graphic curve, in topology (space-time) singularities are geometrical projections from one surface to another which - despite distortions of the surfaces - retain their basic (qualitative) form, although they may change in size or magnitude.²⁾ Such changes in a system were called catastrophes.

Catastrophe theoretic models analyse the ways in which equilibrium states of a system may undergo shocks or jumps. Suppose that the behaviour of a system can be described by a set of state (endogenous) variables $\underline{x} = (x_1, \dots, x_I)'$ and external (control) variables $\underline{a} = (a_1, \dots, a_J)'$. These variables are related to each other by a dynamic or potential energy function f which has to be minimized:

$$(3.1.) \quad f = f(\underline{x}, \underline{a})$$

1) Differential topology analyses the behaviour of multidimensional geometric figures that demonstrate a stable structure after deformation (Zeeman (1976)).

2) Even dynamic systems - though they may change over time - retain basically their original shape after a distortion.

This function may be any kind of minimand, for example, a loss function, an entropy function, etc. Then for each value of $\underline{a}$ the possible equilibrium values of $\underline{x}$ are minimizing values for f ; in other words, the equilibrium values are defined by the solution of:

$$(3.2) \quad \frac{\partial f}{\partial x_i} = 0, \forall i$$

By varying $\underline{a}$ one will find new corresponding values of $\underline{x}$ from (3.2.). Thus these covariations of $\underline{a}$ and $\underline{x}$ determine a plane in $(\underline{x}, \underline{a})$ space. If the minimizing value of $\underline{x}$ is unique for a given $\underline{a}$, the corresponding surface is a single sheet. However, if a value of $\underline{a}$ produces multiple equilibrium values of $\underline{x}$, then the surface is convoluted by folds. Such a surface is a singularity that characterizes possible perturbations in the system; it is called a catastrophe, because the equilibrium value of $\underline{x}$ may suddenly jump to another value across the fold as the control variable $\underline{a}$ reaches smoothly and continuously a critical region. Thus, a catastrophe is a folded surface combining all $(\underline{x}, \underline{a})$ points satisfying (3.2.). Each state of the system - once it is in equilibrium - corresponds to a point on this surface, so that distortions from the one equilibrium point to another can be described by a movement of this point on the surface.

Under certain assumptions, only a small number of different qualitative types of catastrophes do exist; in other words, perturbations through space/time proceed in only a limited number of distinct ways. Analogous to the fact that there are only three space-filling, equal-sized units for a plane (viz triangle, square, and hexagon) whose qualitative form is determined by the number of equal sides, the space-time continuum can also be subdivided by only a limited number of forms

which are determined by the number of control variables. Thom (1972) has demonstrated that, if the number of control variables does not exceed four (i.e., $J \leq 4$), there are seven types of elementary catastrophes, each defined by a certain specification of the minimand f from (3.1.). In other words, the state variables x can only change at most in seven qualitatively distinct, discontinuous ways through marginal changes of the control variables in a certain critical region. The mathematical specifications of the seven elementary catastrophes in the four-dimensional universe (viz., fold, cusp, swallowtail, butterfly, hyperbolic, elliptic and parabolic) can be found in Zeeman (1976). Despite the limited number of elementary catastrophes an extensive set of equilibrium state transitions can be described by it.

The simplest type of catastrophe is the fold. Such a system has only one control variable and one state variable; it can easily be represented by a two-dimensional graph. An example of the energy function f and its corresponding fold is represented in Fig. 1 and Fig. 2, respectively. Any change in the control variable a leads to a shift in the state variable x in order to reach a minimum value of f . Due to the existence of a local minimum, a jump may occur through a variation of a . The fold catastrophe, which is based on a single control variable, is a fairly limited phenomenon and, hence, often not so very useful.

Among the higher-dimensional catastrophes is the cusp catastrophe (with two control variable and a single state variable) very often being used. In this case, the catastrophe surface has the form of a fold that disappears at some point (the cusp point); see Fig. 3. The cusp is in fact

a surface with locally reverse slopes. Some changes of the control variables a_1 and a_2 may lead to smooth and continuous adjustments of the equilibrium, while others may lead to abrupt changes and shocks. Suppose that we have an equilibrium point P on the upper leaf of the cusp. Now there are two ways to reach a new equilibrium point on the lower leaf of the equilibrium surface. One would be to vary a_1 and a_2 so that the equilibrium path falls over the fold and jumps to the lower leaf. The other route from P to the lower leaf would be to move continuously around the cusp C . This implies, that both an abrupt perturbation and a continuous smooth adjustment are possible.

Fig. 3. demonstrates that for certain values of the control variables, three values of the state variable may exist. Two of these points are stable, as they correspond to minimum values of f , whereas the third point is unstable, as it corresponds to a local maximum of f . The set of unstable points is reflected by the shaded leaf of the equilibrium surface and will not be attained when the system is in equilibrium. The projection of the boundaries of this unstable leaf (i.e., of the outer edges of the fold) on the (a_1, a_2) plane (the control surface) is called the bifurcation set. Other important features of cusp catastrophes (such as divergence, bimodality, hysteresis and splitting factors) are described in the literature referred to above, and will only briefly be described here.

Divergence occurs when nearby starting conditions on the manifold evolve to entirely separated final states; for example, one on the lower leaf and another one on the upper leaf. Bimodal behaviour implies that, for certain values of the control variables, two stable (equilibrium)

values of the state variable may exist. Hysteresis reflects a certain asymmetry in system behaviour, viz. when the state of a system follows a different return path after a reversal of the direction of the control variable. A splitting control factor refers to a situation where only one control causes the discontinuity in system behaviour.

It has to be noted that in many situations the equilibrium surface of the cusp catastrophe from Fig. 3 cannot easily be interpreted in terms of the minimization of a certain energy function f . Therefore, in several applications and illustrations of catastrophe theory, the equilibrium surface is not regarded as the representation of a first-order derivative of a minimand, but as a structural model in itself with implicit equilibrium and disequilibrium structures (for example, a Lyapunov function). Thus, there are essentially two complementary interpretations possible, viz. either the existence of a minimand $f(\underline{x}, \underline{a})$ or the existence of a set of equations describing equilibrium trajectories. In the latter case, we may assume a set of differential equations:

$$(3.3.) \quad \dot{x}_i = g(\underline{x}, \underline{a}) \quad , \forall i$$

The system will move to an equilibrium when $\dot{x}_i = 0$, for a given $\underline{a}$. The equilibrium manifold is then defined by the latter condition. In such cases, for example, the cubic function from Fig. 1 may also be regarded as an equilibrium surface (with jumps).

The advantage of catastrophe theoretic interpretations of phenomena is that properties of the underlying structure can be studied in a mathematically convenient form without knowing explicitly the precise specification of the related functions, as was also demonstrated by the conclusion drawn from Figures 1 - 3. Even changes in dynamic systems can be described by means of fairly simple elementary singularities based on nonlinear relationships between state and control variables.

Finally, in higher dimensions than the cusp catastrophe (i.e., $J = 3$ or $J = 4$), the system itself is still definable in a topologic and algebraic sense, but its equilibrium manifold cannot be depicted in a graphical way. Therefore, most applications of catastrophe theory confine themselves to the cusp catastrophe. This method has been applied now with varying success by many disciplines. Although its usefulness is still untested, the concepts of perturbations from catastrophe theory suggest that a closer examination of the potential of catastrophe theory for the analysis of the environmental conflicts discussed in Section 2 is meaningful. This will be the subject of the following sections.

4. Conflict and Inertia in Energy and Resource Modelling

It has been explained in section 2 that low entropy energy is a really scarce input. In the present section, both stock energy and flow energy will subsequently be dealt with in a catastrophe theoretic context by constructing a dynamic model for the total stock of available energy, and the use of stock and flow energy for the production of commodities.

A situation of limited energy resources implies the following possible constraints and conflicts:

- the supply of stock energy may never exceed the total available stock.
- a choice between a present and future use of resources.
- oligopolistic tendencies with price increases and rationing of resources.
- higher extraction costs as the stock of energy resources decreases.
- sharper conflicts between energy conservation and environmental management.
- sharper distributional frictions among energy users.

The analysis of equilibrium tendencies may start with a traditional supply-demand framework. The demand e^d for stock energy (oil, etc.) is negatively related to its price p :

$$(4.1.) \quad e^d = f^d(p),$$

while the supply e^s is positively related to the price and the available stock k^e of energy resources:

$$(4.2.) \quad e^s = f^s(p^e, k^e)$$

Clearly, the following condition holds:

$$(4.3.) \quad e^s \leq k^e$$

On the basis of (4.1) - (4.3) the supply-demand equilibrium ($e^s = e^d$) can normally be derived in a straightforward manner. Due to conflicts and inertia, however, an adjustment process in the use of energy resources toward a stable equilibrium state is not guaranteed. Reasons for the occurrence of a disequilibrium situation may be: high transition costs for continuous adjustments, asymmetric behaviour in the case of price increases and price decreases, slowly functioning institutional and democratic procedures, and so forth. This may imply that the demand function may demonstrate a stepwise shape with jumps (see also Leibenstein (1976)).

Such frictions and inertia in the use of energy resources require an adjusted model. The assumption will be made that the demand curve is S-shaped, implying a high inelasticity for low and high prices (see Pi. 4.) This assumption is plausible, because in a situation with relatively low prices, saturation phenomena may occur, whereas in a situation with relatively very high prices, the demand mainly covers the primary needs, so that there is hardly any flexibility in behaviour.

Next, one may assume a high degree of inertia in demand behaviour, so that adjustments take place only via shocks: prices of energy resources have to reach a certain level before a sudden jump in demand will occur. Furthermore, one may assume an asymmetry in behaviour, as far as a price increase and a price decrease is concerned. This gives rise to a demand function of the type sketched in Fig. 5 (see also Fig. 1). This demand curve demonstrates the above mentioned features of bimodality (multiple stable equilibrium points for a state variable), hysteresis (asymmetric adjustments for reverse directions of input variables) and discontinuity (shocks and perturbations). Divergence and splitting factors do not occur for this simple two-dimensional fold.

Next, one may combine Fig. 4 and 5 by assuming various degrees of inertia, so that inertia becomes a continuous phenomenon entering the model as a third variable i (see Fig. 6). It is easily seen that Fig. 6 represents the well-known cusp catastrophe surface, in which the inertia acts as a splitting factor. This implies that all kinds of perturbations, smooth adjustments, bimodal behaviour, and divergences can be reflected by means of Fig. 6.

By confronting next demand with supply according to (4.1.) - (4.3.), it is clear that the equilibrium states may also demonstrate discontinuities and jumps. This can easily be seen, for example, by assuming for the moment an S-shaped supply curve without inertia (see Fig. 7). The threshold level k^e in Fig. 7 results from condition (4.3.). It is clear that a stable supply-demand equilibrium is more likely to exist, as the inertia is lower (this is easy to see by confronting Fig. 4 with Fig. 7). On the other hand, a situation with much inertia may give rise to discontinuities

and disequilibria (as can be seen by confronting Fig. 5 with Fig. 7). Next, one may also determine a whole equilibrium trajectory in the (e^s, p^e) - space for varying values of k^e . Two illustrations of supply curves corresponding to two values of k^e are included in Fig. 8 (dashed areas). The lower dashed area corresponding to a lower stock of available energy resources intersects the catastrophe fold, so that in this part of the manifold perturbations in the supply-demand equilibria will occur. Consequently, shocks are more likely as the stock of energy resources decreases.

In addition to the above mentioned figures, one may also calculate - for each value of k^e - the corresponding equilibrium values of p^e and e^s (or e^d) by means of (4.1.) and (4.2.). Of course, this will also lead to cusp manifolds which can be depicted in a similar way as in Fig. 6 (see Fig. 9 and 10).

Instead of inertia at the demand side, one might also assume inertia at the supply of energy (due to discontinuous technological transition processes in the exhaustion of resources, or due to sudden political perturbations in the oil countries). Such situations can also be described by means of catastrophe surfaces for the supply side and can be treated in a similar manner.

Furthermore, one may also assume discontinuities at both the supply side and the demand side. This would certainly increase the probabilities of the occurrence of perturbations for the equilibrium states of the system. All these situations can, in principle, be represented in a graphical way.

Finally, one may also introduce the existence of flow energy (e.g., solar energy). Then the model described by equations (4.1.) - (4.3.) can be extended with the following supply-demand relationships:

$$(4.4.) \quad z^d = f^d(p^z, p^e, i^z)$$

and:

$$(4.5.) \quad z^s = f^s(p^z, u),$$

where z^d and z^s reflect the demand for and supply of flow energy, p^z the price of flow energy, i^z the degree of inertia in the demand for flow energy, and u the state of technology regarding the development of flow energy (in other words, u determines the maximum usable amount of flow-energy). Equation (4.4.) allows a certain substitution between flow and stock energy: the higher the price of stock energy, the higher the demand for flow energy. The inertia factor i^z again reflects a resistance phenomenon which may lead to discontinuous adjustments.

The relationships (4.1.) - (4.5.) may be analysed sequentially. First, the relationships (4.1.) - (4.3.) have to be examined. The results (in terms of outcomes of p^e for varying values of k^e and i) may be substituted into (4.4.). This implies that (4.4.) may demonstrate discontinuities from two sources, viz. p^e and i^{z1} . The resulting pattern may be graphically represented in a way analogous to Fig. 6. Next, one may solve the equilibrium trajectory of relationships (4.4.) and (4.5.) in the same way as indicated before. Clearly, one may also analyse a situation where k^e forms a constraint on a further search for flow energy.

1) This implies that i^z is no (single) splitting factor.

Thus far, the spatial elements have hardly been dealt with in the foregoing analysis. It is clear, however, that the inertia factor i can easily be interpreted as a characteristic for a spatial entity (city, region, etc.). This would imply that all regions of a certain spatial system can be plotted on the i -axis according to their average degree of inertia. Then the same conclusions as before can be drawn. In this case, it is easily seen that regions with a low inertia (i.e. a high flexibility to adjust themselves to new situations such as price changes) will normally demonstrate a smooth and continuous development path, whereas regions with a high inertia may suffer from serious jumps and perturbations.

Clearly, the spatial picture is becoming more complicated when spatial interdependences are taken into account (for example, interregional energy and commodity flows, diffusion of pollution). In such cases the originally smooth trajectory of the one region may be affected by sudden disturbances from other regions, so that the total spatial system may become more unstable. On the other hand, the influence of a smooth trajectory from one region on a catastrophe in another region may reduce the impact of this singularity, so that a more stable pattern may arise. Analogously, two catastrophes from different regions may reinforce each other. The resulting trajectories are, however, very hard to describe in a formal analytical sense, because of the high number of control variables. In such situations, a butterfly or a parabolic would be a more appropriate analytical tool. These complex situations can, however, not be represented in a graphical way.

As explained in sections 1 and 2, the new scarcity relates also to raw materials. The demand-supply relationships as well as the conflicts and inertia in the use of raw materials can be analysed by means of analogous catastrophe theoretic notions (see for the use of dynamic models for raw materials, Van Dyk(1979)). Problems of recycling can be dealt with as well. The relationships between energy use and technical transformation processes of raw materials can also be analysed in a similar way, although one has to keep in mind the limited number (2) of control variables which are permitted in cusp catastrophes.

In the case of direct relationships between energy use and pollution discharges, the impacts of continuous adjustments or sudden jumps on environmental quality can, in principle, be dealt with in a similar way. The same holds true for a spatial subdivision of the system concerned into different energy users or consumers, so that distributional impacts can be analysed as well.

5. The Mechanism of Catastrophe Theoretic Models

In this section the working mechanism of models based on catastrophe theory will be explained in greater detail. Special attention will be paid to the question as to whether an equilibrating mechanism for the demand for, and supply of, natural resources is likely to exist, given the sharp conflicts between economic growth targets, energy and natural resource conservation targets, and environmental management targets.

Suppose for the moment a given stock of available energy resources k^e , so that there is no stock increase due to explorations of new energy resources. Then, at a given level of inertia, there will be a steady decrease in the stock of energy. Due to the increased scarcity, the

commodity production will also decrease (given a fixed technological efficiency). Sooner or later, as time is going on, the production will come to an end. This can easily be demonstrated by means of Fig. 10, which represents the relationships between energy use, energy stocks and inertia. Depending on the level of inertia i , this equilibrium trajectory will show a smooth path or sudden jumps. Thus, assuming a fixed technological relationship between the use of stock energy and production (q), the trajectory of production through time (t) can be represented by Fig. 11 (low inertia) or Fig. 12 (high inertia). If the technological efficiency in the energy sector increases (due to price increases of energy resources), the trajectory of q will show a less rapid decline (see the dashed lines in Fig. 11 and 12). The technological innovation might even neutralize the effect of the perturbations due to inertia in Fig. 12.

Next, one may also suppose that the price increases of stock energy induce the use of flow energy (see equation (4.4.)). This might prevent a rapid depletion of the quantity of stock energy and might even allow a rise in production through time (see the dotted lines in Fig. 11 and 12). In the latter case , the new scarcity problem of limited energy would essentially have been solved.

Next, one may also examine the repercussions of an increase in k^e (the quantity of stock energy) arising from the detection of new resources (for example, new oil fields). This temporary increase in k^e (denoted by Δk^e) would also induce a rise in production, but in the long run, this additional quantity of stock energy will also be depleted, so that production will again decrease after some time. Thus this situation may lead to a sequence of jumps through time (see also Fig. 10). This is

illustrated for the trajectory of production in Fig. 13 and 14 in the case of low inertia (see also Fig. 11) or high inertia (see also Fig. 12), respectively.

Finally, it is plausible to assume that the inertia regarding a decrease in energy use during a period of exhaustion of resources is higher than the inertia regarding the increase in energy use during a period of detecting new resources. This implies that the energy demand e^d is changing smoothly and continuously during a rise in resources (A -- B -- C), whereas it may display abrupt and discontinuous shifts during a decrease in resources (C -- D -- E) (see Fig. 15). This can also be illustrated by referring to the bifurcation set associated with Fig. 15.

It is clear that this analysis of catastrophes can easily be generalized for the use of flow energy and of raw materials. In this way also, sequences of energy crises and raw material crises can be successively described.

6. Policy Implications

The foregoing models take for granted that the energy problem is a long-term issue and will also concern the future generations. The new scarcity indeed requires a long-term time perspective. No doubt, perturbations in the supply of natural resources may be highly probable, as is demonstrated by the explosive situation in the Middle East. A policy of conservation is thus a wise response to such uncertainties.

One policy instrument in such a conservation policy may be the price of natural resources. Although most probably, this price will not reflect the real scarcity, it may be one of the important vehicles to

equilibriate demand and supply (in addition to rationing of resources), as was also shown by the foregoing models.

In general, several policies are possible. Apart from a conservation policy per se, it may also be extremely important to reduce inertia and to speed up the development of alternative resources (especially flow energy). With regard to pollution, an important issue may also be the preservation of environmental quality. It is also clear that some of these policy measures may be in conflict with the traditional economic growth issues (such as employment and investment targets). Thus, natural resource policy may be composed of several instruments. Examples are:

- A rationing of the supply of energy resources. This is of course an effective policy, but due to indirect price effects the occurrence of social efficiency is not guaranteed. A fixed annual supply of energy resources by a government may, however, prevent sudden jumps in the catastrophe surface.
- A tax on the use of energy resources. Such a shift in the supply curve of energy resources may influence the trajectory of the energy use, so that abrupt changes may even be induced by a sudden change in the tax system.
- Improvement of technological innovation. This may lead to a more efficient input-output coefficient of stock energy, to the exploration of new forms of stock energy or to the development of a feasible flow energy technology. Such developments do not necessarily avoid abrupt transitions.

- Development of selective development patterns. This implies the search for new activities which do less harm to the environment and the natural resource system, while at the same time, they try to fulfill employment and production options.
- Decrease of inertia. This implies a policy aiming at achieving a greater flexibility of human behaviour regarding the new scarcity of natural resources. Provision of information and education may be one of the tools to influence social inertia so as to avoid future shocks.

Clearly, similar instruments and policies can be identified for natural resource management.

It is also clear that the ultimate combination of instruments to be employed will very much depend on the institutional system at hand, the actual developments in the natural resource sector, the effectiveness of environmental policies and the future perspectives of our economies.

7. Relevance of Catastrophe Theory for Social Sciences.

Catastrophe theory is a new field in mathematics. Any new topic runs the risk of being overrated during its first stages. Catastrophe theory, however, has drawn a lot of attention because of its applications to several imaginative problems in the social sciences: national defense, nerve impulses, prison riots, and human aggression. On the other hand, catastrophe theory has also been severely criticised (see Croll (1976), Kolata (1977), and Zahler and Sussmann (1977)). Consequently, a general judgement of the relevance of the theory is a necessary complement to this paper. The following comments can be made.

- Catastrophe theory provides a new and attractive way of thinking about phenomena in the social sciences which may show perturbations and abrupt transitions. Especially in a less controlled world such jumps are more likely to occur, so that catastrophe models may provide a sound mathematical foundation underlying shock phenomena in social problems that would otherwise prevent analytic investigation because of the mathematical complexities.
- Catastrophe theory mainly provides insight into the qualitative structure of the phenomena at hand. It may give a description of the general properties of social science phenomena, but very often it does not provide an accurate assessment of the time periods and the transitions of the events in the system at hand. The scope of the theory is more oriented to the identification of possible jumps than on the prediction of such jumps. So catastrophe theory provides more a mode of thinking than a precise description. This is of course a severe limitation.
- Catastrophe theory (at least in the framework of elementary catastrophes) requires the definition of a system within a restrictive control-state framework limited by the number of variables. The foregoing models have shown that sometimes the catastrophe representation forms a severe limitation for an appropriate description of real-world phenomena. This may lead to oversimplified catastrophe models and may reduce its empirical meaning.
- A major advantage of catastrophe theory is its integration of the concepts of shifts and equilibrium. In this respect, catastrophe theory can be interpreted as a generalisation of control theory which is mainly focussing on smooth adjustments. Perturbation phenomena are a useful field of study in catastrophe theory, although alternative approaches (e.g. singularity theory) may be equally useful.

- the applications in the field of catastrophe theory, which have been carried out thus far in various disciplines, may hardly be called applied research, because the phenomena have very often been described in such simple terms that they only served as illustrations. Our model shows indeed that a test on the empirical validity of the catastrophe model for natural resource systems is very hard to carry out. The main conclusions that could be drawn were qualitative in nature. Given a set of simple hypotheses on the structure of an energy system, one may show that under certain conditions shocks in the energy system are inevitable.

Our final conclusion is that catastrophe models have to be employed carefully in the social sciences. They may be potentially useful, as they provide analytical insight into discontinuities in the behaviour of a social system, though other methods such as bifurcation theory and differential game theory may be used as well. Catastrophe models are not able to describe nor to predict the precise state of the system, but rather call attention for the presence of qualitative and often abrupt transitions. Hence, catastrophe notions in the social sciences may be meaningful to derive testable hypotheses, which have to be verified by means of serious empirical work.

Fig. 1 A cubic energy function for a given value of a_1 .

Fig. 2 Equilibrium surface of a fold catastrophe.

Fig. 3 Equilibrium manifold of a cusp catastrophe.

Fig. 4 An S-shaped demand curve.

Fig. 5 A stepwise S-shaped asymmetric demand curve.

Fig. 6 A demand relationship for energy resources with varying degrees of inertia.

Fig. 7 An S-shaped supply curve.

Fig. 8 Integrated supply-demand catastrophe surface for energy resources.

Fig. 9 A cusp manifold for energy prices,
stock of energy resources and inertia.

Fig. 10 A cusp manifold for energy use,
stock of energy resources and inertia.

Fig. 11 Trajectory of production
in case of low inertia.

Fig. 12 Trajectory of production
in case of high inertia.

Fig. 13 Illustration of growth path of production in case of detection of new energy resources (low inertia).

Fig. 14 Illustration of growth path of production in case of detection of new energy resources (high inertia).

Fig. 15 Illustration of trajectory of energy use during various stages of exhaustion of stocks.

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