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Delta-nucleon and pion-nucleus interactions

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The effect of the Δ - N interaction in elastic and inelastic pion scattering is discussed. In elastic scattering, the two-body Δ - N interaction generates both Δ self-energy and Δ - h vertex corrections. The relative importance of these two kinds of medium corrections is discussed analytically. Numerical calculations are presented which show in detail the role of the Δ - N interaction in elastic scattering. In pion-nucleus inelastic scattering, a new reaction mechanism appears due to the two-body nature of the Δ - N interaction. The qualitative characteristics of these processes induced by Δ - N collisions are discussed. Deviations from quasifree isospin ratios in knockout reactions and excitations of specific nuclear levels are signatures of the interference between processes generated by the π - N and Δ - N interaction, respectively. In the formalism presented, the order of magnitude of these modifications can be related to the size of the experimentally observed true pion absorption cross sections.

[NUCLEAR REACTIONS Pion-nucleus scattering, effects of Δ - N interaction, isospin ratios in knockout reactions and inelastic scattering.]

I. INTRODUCTION

The importance of the Δ -nucleus interaction in resonance π -nucleus scattering is well established. Phenomenologically it has been demonstrated for scattering on light nuclei¹ that total and integrated elastic cross sections require the presence of a strongly absorptive Δ -nucleus interaction. This absorptive part has been successfully related to the mechanism of true pion absorption. Similarly, analysis of elastic angular distributions provides evidence for a spin-orbit component in the Δ -nucleus interaction.² The dynamical origin of the spin orbit force comparable to the nucleon spin orbit interaction and of the central interaction with about half the strength of the nucleon shell model potential, has not yet received a generally accepted explanation. The interpretation of the π -nucleus interaction in terms of the Δ -nucleus interaction has found strong additional support from analyses of π -nucleus reactions. With the Δ -nucleus interaction parameters determined from elastic scattering, the

Δ - h formalism has been applied to both excitation of discrete states³ and quasifree scattering.⁴ In the quasifree peak, or the first cross section maximum in discrete state excitations, theory and experiment agree within, typically, 20–30%. Furthermore, rather detailed aspects of the Δ - h description, such as consequences of the expected isospin dependence of the Δ -nucleus spreading potential⁵ or predictions of magnetic substate populations in inelastic scattering,⁶ appear to be confirmed by experiment.

Theoretically the description of medium modifications to free π - N scattering via the Δ -nucleus interaction follows simply from Δ dominance of π - N scattering. This, however, does not imply that all the effects of the Δ -nucleus interaction can be incorporated into a Δ optical potential, i.e., a one body operator acting only on the Δ degree of freedom. In addition to “elastic” scattering of the Δ , the presence of the Δ -nucleus interaction inevitably leads to rearrangement processes, i.e., processes which affect not only the propagation of the Δ but also the propagation of the residual $A-1$ nucleus in the presence of a Δ .

For this reason, the spreading potential determined phenomenologically from elastic scattering cannot be directly interpreted from the theoretical point of view as a Δ -nucleus optical potential. The spreading potential implicitly accounts not only for Δ self-energies but also, in an as yet undefined way, for Δ -h vertex corrections.

Owing to the presence of these rearrangement processes, the Δ -h formalism cannot provide a complete phenomenological framework if based only on a Δ -nucleus optical potential. In any type of π -nucleus reaction, the replacement of the Δ -nucleus interaction by a Δ optical potential necessarily implies the neglect without justification of a whole class of Δ -nucleus interaction induced processes.

In this paper, we explore the importance of these rearrangement processes in both elastic π -nucleus scattering and in π -nucleus reactions. Our emphasis will be, on the one hand, on the relation between Δ -h vertex corrections and Δ self-energies in order to provide a satisfactory theoretical basis for the interpretation of the phenomenological spreading potential. On the other hand, we shall focus on the implications of the medium corrections observed in elastic scattering on π -nucleus reactions which are induced by the Δ -nucleus interaction.

The particular way in which we approach the problem is very much influenced by the previous phenomenological analysis of elastic scattering, based on the spreading potential (Hirata *et al.*¹). In the alternative approach using effective Lagrangians, developed by Brown, Weise, and coworkers^{1,7,8} and Klingenberg *et al.*,¹ both Δ self-energies and Δ -hole interactions have been considered and found to be important. In practice, different physical mechanisms have been taken into account in these two types of processes: pion absorption and "reflection terms" in the Δ self-energy, and short-range repulsion and rho exchange in the Δ -h vertex corrections. The criteria for this selection are not entirely clear to us. Furthermore, we are not aware of any attempt in the microscopic approach to study the role of vertex corrections in inelastic reactions.

Formally, the starting point for our discussion is to assume that the Δ -nucleus interaction can be gen-

erated by an effective two-body Δ - N interaction. This effective Δ - N interaction itself may contain medium modifications and we shall not use any specific model. We assume, however, that within a partial-wave expansion, the Δ - N interaction can be characterized by a few strength parameters only. The restriction to low Δ - N partial waves seems to be appropriate for Δ - N momenta typically of the order of 150 MeV/c. It is known that Δ - N interactions generated through coupling to the absorption channels are effective in the low Δ - N partial waves only. To what extent this holds for the pion rescattering generated Δ - N interaction is not known.

In Sec. II we calculate approximately the relevant nuclear matrix elements for pion scattering on two nucleons in the presence of the Δ - N interaction. In Secs. III and IV this expression is then used to discuss the consequences of the Δ - N interaction in elastic scattering and π -nucleus reactions.

Our calculations are largely qualitative in nature due to the unavoidably crude approximations of an analytical approach. We therefore will concentrate in our discussion on qualitative aspects like selection rules which are related to the quantum numbers of the Δ - N interaction. It will be shown, that specification of the quantum numbers only allows us to establish the relation between Δ -h vertex corrections and Δ self-energies; similarly, we do not attempt to calculate cross sections for any specific reaction but rather connect medium modifications occurring in various reactions and elastic scattering. In this context, the connection between isoscalar longitudinal excitations and elastic scattering will be of particular importance as well as the relation between processes which usually are related to each other by "quasifree ratios" like $(\pi, \pi N)$ knockout reactions or $\Delta T=0$ versus $\Delta T=1$ excitations.

Within our discussion of elastic scattering we shall present results of numerical calculations which are free of any of the technical approximations of the analytical approach. This puts our conclusions with regard to elastic scattering on firmer grounds and allows us to specify the validity of the analytical evaluations.

II. PARAMETRIZATION OF THE Δ - N INTERACTION AND THE NUCLEAR TWO-BODY MATRIX ELEMENTS

We write the partial-wave expansion of the effective Δ - N interaction in the following way:

$$t_{\Delta N} = \sum_{\alpha} g_{\alpha}(p', p) O_{\alpha} \quad (1a)$$

with

$$O_{\mathcal{J},L',L,S',S,T} = \sum_{M,m',m} |(\frac{3}{2} \frac{1}{2}) S' m'\rangle Y_{L',M-m'}(\hat{\vec{p}}') \langle S' m', L' M - m' | \mathcal{J} M \rangle \\ \times \langle S m, L M - m | \mathcal{J} M \rangle Y_{L,M-m}^*(\hat{\vec{p}}) \langle (\frac{3}{2} \frac{1}{2}) S m | \cdot P_T \rangle. \quad (1b)$$

$t_{\Delta N}$ of Eq. (1a) describes scattering from the initial ΔN state of relative momentum $\vec{p}$ into the final state of momentum $\vec{p}'$. α characterizes the states in the partial-wave expansion as explicitly written in (1b). Angular momentum conservation together with parity conservation restricts the values of L' to

$$L' = L, L \pm 2, L \pm 4. \quad (2)$$

For the evaluation of nuclear matrix elements of $t_{\Delta N}$, it is convenient to classify the various components (1b) in each term of the partial wave expansion (1a) according to their transformation properties in spin space and configuration space separately. The operator O_α in (1b) is written as

$$O_\alpha = (-1)^{\mathcal{J}+S+L+L'} (2\mathcal{J}+1) \sum_K (-)^K \frac{(2K+1)}{(2S'+1)^{1/2}} \begin{bmatrix} S & L & \mathcal{J} \\ L' & S' & K \end{bmatrix} [Q^K(L',L) \otimes T^K(S',S)]_0 T_0^0(T) \quad (3a)$$

with

$$Q_M^K(L',L) = [Y_{L'}(\hat{\vec{p}}') \otimes Y_L(\hat{\vec{p}})]_{KM}, \quad (3b)$$

$$T_M^K(S',S) = \sum_\mu |(\frac{3}{2} \frac{1}{2}) S' \mu\rangle \langle S \mu - M, KM | S' \mu\rangle \langle (\frac{3}{2} \frac{1}{2}) S \mu - M |, \quad (3c)$$

and, according to (3c),

$$T_0^0(T) = T_0^0(T, T) = P_T \quad (3d)$$

is the projector onto the Δ - N isospin T .

The reduced matrix element of $T_M^K(S',S)$ (in the convention of Ref. 9) is

$$(\tilde{S}' || T^K(S',S) || \tilde{S}) = \delta_{\tilde{S}',S'} \delta_{\tilde{S},S} (2\tilde{S}'+1)^{1/2}. \quad (3e)$$

In the following we consider separately the matrix elements of the various components in the decomposition (3a). We first calculate the following spin matrix element:

$$\langle \tilde{S}' M' | O_\sigma | \tilde{S} M \rangle = \langle \tilde{S}' M' | (S_1 \otimes k')_0 (T^K(S',S) \otimes Q^K)_0 (S_1^\dagger \otimes k)_0 | \tilde{S} M \rangle \quad (4)$$

which contains the necessary spin-transition operators S and $S^\dagger$ for the $\pi N \rightarrow \Delta$ transition. $\vec{k}$ and $\vec{k}'$ are the initial and final pion momenta in the process $\pi - 2N \rightarrow \pi - 2N$, and $\tilde{S}$ and $\tilde{S}'$ are the initial and final two nucleon spins. One finds

$$\langle \tilde{S}' M' | O_\sigma | \tilde{S} M \rangle = \frac{1}{3} \sum_{\Lambda, \lambda} (-1)^{1+\tilde{S}+M'} \left[\frac{2\Lambda+1}{(2\lambda+1)(2K+1)} \right]^{1/2} \langle \tilde{S}' M', \tilde{S} - M | \lambda M' - M \rangle \\ \times X_{\lambda, M-M'}(\Lambda, K, L', L) (\tilde{S}' || R_\lambda(\Lambda, K, S', S) || \tilde{S}), \quad (5a)$$

with

$$X_{\lambda, M-M'}(\Lambda, K, L', L) = [(k' \otimes k)_\Lambda \otimes Q^K(L', L)]_{\lambda, M-M'}, \quad (5b)$$

and

$$(\tilde{S}' || R_\lambda(\Lambda, K, \tilde{S}, S) || \tilde{S}) \\ = (-1)^{\lambda+\tilde{S}+\tilde{S}'} 4(2S'+1) [(2\lambda+1)(2S+1)(2\tilde{S}+1)(2\tilde{S}'+1)]^{1/2} \begin{bmatrix} \frac{1}{2} & \tilde{S}' & \frac{1}{2} \\ S' & \frac{3}{2} & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \tilde{S} & \frac{1}{2} \\ S & \frac{3}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & S' & \tilde{S}' \\ 1 & S & \tilde{S} \\ \Lambda & K & \lambda \end{bmatrix}. \quad (5c)$$

The matrix element of the analogously defined isospin operator,

$$O_\tau = (T_1 \otimes t_{\pi'}^\dagger)_0 T_0^0 (T_1^\dagger \otimes t_\pi), \quad (6a)$$

with t_π the vector of the pion isospin, is obtained from (5) by replacing spins by isospins [$K=0$, cf. Eq. (3d)] and $X_{\lambda, M-M'}$ in (5a) by

$$\Theta_{\lambda, M-M'} = (t_{\pi'}^\dagger \otimes t_\pi)_{\lambda, M-M'}. \quad (6b)$$

With these expressions for the spin and isospin matrix elements we are in the position to evaluate approximately the relevant two body nuclear matrix elements. In the present section we derive the results for these matrix elements. This will serve as the formal basis for the later applications.

In Fig. 1 are shown the diagrams describing the basic processes to be evaluated. Note that antisymmetrization of the initial and final nucleons in general gives rise to four distinct contributions; in elastic scattering ($j_h m_h = j_p m_p$), exchange of initial or final nucleon lines are equivalent.

Analytic evaluation of these processes requires severe approximations. The crucial approximation in our evaluation of the configuration space part of the matrix elements consists in replacing the Δ and nucleon momenta appearing in both the Δ - N interaction [$g_\alpha(p', p)$ in (1a) and $X_{\lambda, \mu}$ in (5a)] and the Δ propagators (Fig. 1) by appropriate mean values depending on the external momenta only. The detailed choice of these average values will depend on

FIG. 1. Basic four processes for pion-nucleus particle-hole excitation in the presence of a Δ - N interaction. Dashed lines: pions; solid lines: nucleons; double lines: deltas; wiggly line: Δ - N t -matrix.

the specific process. For the general considerations in this section it is not necessary to specify the procedure. Here, the only condition is that the fluctuations in the relevant momenta due to Fermi motion can be neglected. Thus we effectively treat the Δ propagator in closure and approximate the Δ - N interaction by a zero range interaction. Note, however, that we still allow for Δ - N interactions in partial waves different from zero. In this approximation we have to calculate the following matrix element:

$$M = \sum_{jm\tau} \int d\tau_1 \int d\tau_2 [\Psi_{j_p m_p}^{l*}(\vec{r}_2) \Psi_{j_m}^{l*}(\vec{r}_1) \langle (\frac{1}{2}\tau)_1 (\frac{1}{2}\tau_p)_2 | -1 \leftrightarrow 2] \\ \times O_\sigma O_\tau e^{i\vec{q} \cdot \vec{r}} \delta^{(3)}(\vec{r}_1 - \vec{r}_2) [\Psi_{j_h m_h}^l(\vec{r}_2) \Psi_{j_m}^l(\vec{r}_1) | (\frac{1}{2}\tau)_1 (\frac{1}{2}\tau_h)_2 | -1 \leftrightarrow 2] \quad (7a)$$

with nucleon shell model wave functions

$$\Psi_{jm}^l(\vec{r}) = \sum_{\mu} \langle \frac{1}{2}\mu, lm - \mu | jm \rangle | \frac{1}{2}\mu \rangle Y_{lm-\mu}(\hat{\vec{r}}) y_l(r) \quad (7b)$$

in which we have neglected spin-orbit effects. $\vec{q}$ is the momentum transfer to the nucleus

$$\vec{q} = \vec{k} - \vec{k}'. \quad (7c)$$

The appearance of the δ function in (7a) is a direct consequence of the approximation described above. We, furthermore, restrict our calculation to processes in L - S closed shell nuclei, where the sum over j for fixed l can be performed. We have omitted in the definition of M the form factors of the Δ - N interaction [g in Eq. (1a)], Δ propagators, and πN - Δ coupling constants which are all assumed to depend on external momenta only.

In order to relate the nuclear matrix element (7a) to the corresponding two-body spin-isospin matrix elements [(5) and (6)], we recouple spin and orbital angular momenta of the two nucleons involved and obtain, after summing over j , m , and τ in Eq. (5a),

$$\begin{aligned}
M = & \frac{\sqrt{\pi}}{18} \left[\frac{(2l_p+1)(2l_h+1)(2j_p+1)(2j_h+1)}{(2K+1)} \right]^{1/2} (-1)^{j_h+l_h+m_p+(\frac{1}{2})+\tau_p} \\
& \times \sum_{\Lambda', \lambda, \mathcal{J}, T_f} i^{\Lambda'} (-1)^{\lambda+T_f} \langle l_p 0, l_h 0 | \Lambda' 0 \rangle \langle j_p m_p, j_h - m_h | \mathcal{J} m_p - m_h \rangle \\
& \times \langle \frac{1}{2} \tau_p, \frac{1}{2} - \tau_h | T_f \tau_p - \tau_h \rangle \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \lambda \\ l_p & l_h & \Lambda' \\ j_p & j_h & \mathcal{J} \end{bmatrix} \Theta_{T_f, \tau_h - \tau_p} \tilde{R}_{\Lambda'}(q) \\
& \times \sum_{\Lambda} \sqrt{2\Lambda+1} [Y_{\Lambda}(\hat{q}) \otimes X_{\Lambda}(\Lambda, K, L', L)]_{\mathcal{J}, m_h - m_p} \\
& \times \sum_{\substack{\tilde{S} \tilde{S}' \\ \tilde{T} \tilde{T}'}} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \tilde{S}' \\ \tilde{S} & \lambda & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \tilde{T}' \\ \tilde{T} & T_f & \frac{1}{2} \end{bmatrix} [(2\tilde{S}+1)(2\tilde{S}'+1)(2\tilde{T}+1)(2\tilde{T}'+1)]^{1/2} \\
& \times [(-1)^{\tilde{S}'+\tilde{T}'} - (-1)^{\tilde{S}+\tilde{T}+\tilde{S}'+\tilde{T}'} + (-1)^{\tilde{S}+\tilde{T}-1}] \\
& \times (\tilde{S}' || R_{\lambda}(\Lambda, K, S', S) || \tilde{S})(\tilde{T}' || R_{T_f}(T_f, 0, T, T) || \tilde{T}) . \tag{8}
\end{aligned}$$

As in (5) and (6), the initial and final coupled two nucleon spins and isospins are denoted by $\tilde{S}^{(i)}$ and $\tilde{T}^{(i)}$; $\mathcal{J}$ is the total particle hole angular momentum, and T_f is the corresponding isospin. λ is, as in Eq. (5), the rank of the operator O_{σ} in spin space; the summation over the closed L - S shell (jm) restricts the values of λ to 0 or 1. Λ' is the particle-hole orbital angular momentum. The radial form factor in (8) is defined by

$$\tilde{R}_{\Lambda'}(q) = \frac{4(2l+1)}{4\pi} \int_0^{\infty} r^2 dr j_{\Lambda'}(qr) y_l^2(r) y_{l_p}(r) y_{l_h}(r) . \tag{9}$$

The four terms in parentheses correspond to the four diagrams M_0 to M_3 of Fig. 1. Owing to our approximation described above we force the initial and final two nucleons to be in a relative s state. Therefore both $S+T$ and $S'+T'$ have to be odd. This requirement of the Pauli principle is built into (8) by the sum over these four terms.

With the explicit form (5c) of the reduced matrix elements of R , the sum over the initial and final coupled two-nucleon spins and isospins involves three 6- j and one 9- j symbol and can be performed¹⁰ with the result

$$\begin{aligned}
M = & \frac{16}{81} \sqrt{4\pi} [(2l_p+1)(2l_h+1)(2j_p+1)(2j_h+1)]^{1/2} \\
& \times \sum_{\mathcal{J}, T_f, \lambda, \Lambda, \Lambda'} i^{\Lambda'} \langle l_p 0, l_h 0 | \Lambda' 0 \rangle \tilde{R}_{\Lambda'}(q) \langle j_p m_p, j_h - m_h | \mathcal{J} m_p - m_h \rangle \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \lambda \\ l_p & l_h & \Lambda' \\ j_p & j_h & \mathcal{J} \end{bmatrix} (-1)^{j_h+l_h+m_p} \\
& \times \langle \frac{1}{2} \tau_p, \frac{1}{2} - \tau_h | T_f \tau_p - \tau_h \rangle (-1)^{1/2+\tau_p} \Theta_{T_f, \tau_h - \tau_p} \sqrt{2\Lambda+1} \\
& \times [Y_{\Lambda}(\hat{q}) \otimes X_{\Lambda}(\Lambda, K, L', L)]_{\mathcal{J}, m_h - m_p} \frac{(2S'+1)(2T+1)}{64} \\
& \times \sum_{i=0}^3 (-1)^i \bar{\chi}_i(S', S, K, \lambda, \Lambda) \bar{\eta}_i(T, T_f) , \tag{10}
\end{aligned}$$

where

$$\bar{\chi}_i(S', S, K, \lambda, \Lambda) = 12 \left[\frac{(2S+1)(2\Lambda+1)}{(2K+1)} \right]^{1/2} \chi_i(S', S, K, \lambda, \Lambda). \quad (11)$$

The χ_i have the following explicit representation:

$$\chi_0(S', S, K, \lambda, \Lambda) = (-1)^K \begin{bmatrix} \Lambda & \frac{3}{2} & \frac{3}{2} \\ \frac{1}{2} & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda & \frac{1}{2} & \frac{1}{2} \\ K & S' & S \\ \Lambda & \frac{3}{2} & \frac{3}{2} \end{bmatrix}, \quad (12a)$$

$$\chi_2(S', S, K, \lambda, \Lambda) = (-1)^{\Lambda+S+1} \begin{bmatrix} K & \frac{3}{2} & \frac{3}{2} \\ \frac{1}{2} & S' & S \end{bmatrix} \begin{bmatrix} \lambda & \frac{1}{2} & \frac{1}{2} \\ K & \frac{3}{2} & \frac{3}{2} \\ \Lambda & 1 & 1 \end{bmatrix}, \quad (12b)$$

$$\bar{\chi}_1(S', S, K, \lambda, \Lambda) = \bar{\chi}_0(S', S, K, \lambda, \Lambda) - \bar{\sigma}(S', S, K, \lambda, \Lambda), \quad (13a)$$

$$\bar{\chi}_3(S', S, K, \lambda, \Lambda) = \bar{\chi}_2(S', S, K, \lambda, \Lambda) - (-1)^\lambda \bar{\sigma}(S', S, K, \lambda, \Lambda), \quad (13b)$$

$$\bar{\sigma}(S', S, K, \lambda, \Lambda) = \delta_{S,1} 4\sqrt{3} (-1)^K \left[\frac{2\Lambda+1}{2K+1} \right]^{1/2} \begin{bmatrix} \Lambda & K & \lambda \\ S' & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & S' & \lambda \end{bmatrix}. \quad (14)$$

The functions $\bar{\eta}_i$ are special cases of the $\bar{\chi}_i$:

$$\bar{\chi}_i(S', S, K=0, \lambda, \Lambda) = \delta_{\lambda\Lambda} \delta_{S'S} h(\lambda) \bar{\eta}_i(S, \lambda) \quad (15a)$$

with

$$h(\lambda) = \frac{(-1)^{\lambda+1}}{2} \sqrt{6-5\lambda}. \quad (15b)$$

For later applications, we write down the explicit form of the $\bar{\eta}_i$:

$$\bar{\eta}_1 = \bar{\eta}_3 = (-1)^\lambda (1 - \frac{4}{3} \delta_{S,1}), \quad (16a)$$

$$\bar{\eta}_0 = \bar{\eta}_1 + \frac{4}{3} \delta_{S,1}, \quad (16b)$$

$$\bar{\eta}_2 = (-1)^\lambda. \quad (16c)$$

The expression for the $\bar{\chi}_i$ in terms of 6- j symbols reads

$$\begin{aligned} \bar{\chi}_2(S', S, K, \lambda, \Lambda) = & (-1)^{S+1+\Lambda} \left[\frac{2S+1}{2(2K+1)} \right]^{1/2} \begin{bmatrix} K & \frac{3}{2} & \frac{3}{2} \\ \frac{1}{2} & S' & S \end{bmatrix} \\ & \times [\delta_{\lambda,0} \delta_{\Lambda,K} \sqrt{(4+K)(3-K)} + \delta_{\lambda,1} \frac{1}{\sqrt{3(2K+1)}} (\sqrt{K(3+K)(4+K)} \delta_{\Lambda,K-1} \\ & - \sqrt{(K+1)(2-K)(3-K)} \delta_{\Lambda,K+1})] \end{aligned} \quad (17a)$$

and

$$\begin{aligned} \bar{\chi}_0(S', S, K, \lambda, \Lambda) - \bar{\chi}_2(S', S, K, \lambda, \Lambda) \\ = \delta_{\lambda,1} \frac{1}{3} \left[\frac{2(2S+1)(2\Lambda+1)}{2K+1} \right]^{1/2} \left[\delta_{S,1} [S'(S'+1)]^{1/2} \begin{bmatrix} S' & K & 1 \\ \Lambda & 1 & 1 \end{bmatrix} (-1)^{K+S'} - \delta_{S',1} \sqrt{S(S+1)} \begin{bmatrix} S & K & 1 \\ \Lambda & 1 & 1 \end{bmatrix} \right] \\ \times (-1)^{\Lambda+1} \equiv \bar{\delta}(S', S, K, \lambda, \Lambda). \end{aligned} \quad (17b)$$

For the interpretation of our result (10) we point out the following two selection rules valid independently of the specific process. First, angular-momentum conservation restricts $K \leq 3$ (triangular rules of the 9- j and 6- j in (12) and (13). The $K=4$ component of the Δ - N interaction [Eq. (3)], does not contribute here as a consequence of the closed S -shell assumption. Second, the $S=2$, $T=2$ and $S'=2$, $T=2$ Δ - N channels are not reached in the excitation. Formally this follows from the vanishing of the i sum in (10). For $T=2$ we have, according to (16),

$$\bar{\eta}_i(2, T_f) = (-1)^{T_f} \quad (18)$$

and, therefore,

$$\sum_i (-1)^i \bar{\chi}_i \bar{\eta}_i(2, T_f) \sim (-1)^{T_f} [1 + (-1)^\lambda] \bar{\sigma} = 0 \quad (19)$$

since

$$\bar{\sigma} = 0 \quad \text{for } S=2$$

and

$$\lambda = 1 \quad \text{for } S'=2.$$

Physically, this is a result of the Pauli principle together with our zero-range approximation described above. An $S^{(')}=2$, $T=2$ state requires the initial or final pair to be in an $\tilde{S}^{(')}=1$, $\tilde{T}=1$ state, since the πN - Δ transition is a vector operator in both spin and isospin. Within our approximation of neglecting Fermi momenta, this state is forbidden or, more generally, we expect contributions from this channel to be suppressed by $\langle p \rangle / k$ with $\langle p \rangle$ a typical nucleon momentum and k the pion momentum.

The structure of Eq. (10) becomes transparent by comparing this process of π - N scattering modified by the Δ - N interaction with the standard single scattering term. At this point we specialize to the case of particle-hole excitations which comprises elastic scattering as well. It is trivial to perform the necessary projections of Eq. (10) and we find for the sum of the impulse approximation and second order term for a particle-hole excitation, characterized by the quantum numbers $(\mathcal{J}, M)$, (T_f, T_z) ,

$$\begin{aligned} M_{ph} = & g \frac{16}{81} \sqrt{4\pi} (-1)^{M+T_z+l_h} \Theta_{T_f, -T_z} h(T_f) [(2l_p+1)(2l_h+1)(2j_p+1)(2j_h+1)]^{1/2} \\ & \times \sum_{\lambda, \Lambda'} i^{\Lambda'} \langle l_p 0, l_h 0 | \Lambda' 0 \rangle \sqrt{2\lambda+1} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \lambda \\ l_p & l_h & \Lambda' \\ j_p & j_h & \mathcal{J} \end{bmatrix} \\ & \times \left\{ D_0^{-1}(\vec{k}', \vec{k}) R_{\Lambda'}^{\text{tr}}(q) (-1)^{\lambda+T_f} h(\lambda) [Y_{\Lambda'}(\hat{\vec{q}}') \otimes (k' \otimes k)_\lambda]_{\mathcal{J}, -M} \right. \\ & + \frac{V_\alpha}{4\pi} \tilde{R}_{\Lambda'}^{\text{tr}}(q) D_1^{-1}(\vec{k}', \vec{k}) D_2^{-1}(\vec{k}', \vec{k}) \frac{(2S'+1)(2T+1)}{64} \\ & \left. \times \sum_{i\Lambda} (-1)^i \bar{\chi}_i(S', S, K, \lambda, \Lambda) \bar{\eta}_i(T, T_f) [Y_{\Lambda'}(\hat{\vec{q}}) \otimes X_\lambda(\Lambda, K, L', L)]_{\mathcal{J}, -M} \right\}. \quad (20) \end{aligned}$$

$R_{\Lambda'}^{\text{tr}}$ is the standard particle-hole transition form factor

$$R_{\Lambda'}^{\text{tr}}(q) = \int_0^\infty r^2 dr j_{\Lambda'}(qr) y_{l_p}(r) y_{l_h}(r). \quad (21a)$$

$\tilde{R}_{\Lambda'}^{\text{tr}}$ is obtained from $\tilde{R}_{\Lambda'}$ [Eq. (9)] by summing over all the occupied orbits,

$$\begin{aligned} \tilde{R}_{\Lambda'}^{\text{tr}}(q) &= \int_0^\infty r^2 dr j_{\Lambda'}(qr) y_{l_p}(r) y_{l_h}(r) \times 4 \sum_l (2l+1) y_l^2(r) \\ &= \int_0^\infty r^2 dr j_{\Lambda'}(qr) y_{l_p}(r) y_{l_h}(r) \rho(r). \quad (21b) \end{aligned}$$

The D_i 's are the Δ Green's functions where we still allow for different closure energies, g is an overall energy and momentum dependent normalization factor, and V_α is the strength parameter of the Δ - N interaction in the relevant channel, related to the corresponding form factors g_α in Eq. (1) via the geometrical factors in Eq. (3). According to our approximation, V_α still depends on the external momenta.

Comparison of the first and second term in the curly brackets in Eq. (20) displays the modifications

of the impulse approximation due to the Δ - N interaction. The second order process involves two nucleons, and therefore the transition form factor contains an extra power of the density, giving rise to a smoother momentum transfer dependence. Furthermore, the Δ - N interaction changes the angular and isospin dependences of the impulse approximation which are characteristic for the 3-3 resonance.

In the impulse approximation, the 3-3 dominance implies factorization of isospin and angular dependences. This is not the case in the presence of a spin-isospin dependent Δ - N interaction, which in addition changes the typical p -wave angular dependence $(k' \otimes k)_\lambda$ to the more complicated angular dependence as described by X_λ [Eq. (20)]. As can be seen from the definition (5b) the angular dependence

X_λ results from both the π - N p -wave dependence and the angular dependence of the Δ - N interaction. The detailed study of these modifications will be the subject of the discussions on π -nucleus reactions. We now first turn to the discussion of the role of the Δ - N interaction in elastic scattering.

III. THE Δ - N INTERACTION IN ELASTIC π -NUCLEUS SCATTERING

We obtain from Eq. (20) for $\mathcal{J}=0$ and $T_f=0$ the elastic amplitude by multiplying M with $2(2j_h+1)$ and summation over (j_h, l_h) ; furthermore, in the second-order term the sum runs only over $i=0,1$. We find, after evaluation of $\bar{\chi}_i$ and $\bar{\eta}_i$, according to Eqs. (12) and (16),

$$\begin{aligned}
 M_{\text{el}} = & g_{81}^4 \left\{ D_0^{-1}(\vec{k}', \vec{k}) R^{\text{el}}(q) \vec{k}' \cdot \vec{k} + \frac{V_\alpha}{4\pi} \tilde{R}^{\text{el}}(q) D_1^{-2}(\vec{k}', \vec{k}) \right. \\
 & \times X_0(K, K, L', L) \frac{(2S'+1)(2T+1)}{64} (-1)^{S+K+1} \left[\frac{(2S+1)(K+4)(3-K)}{(2K+1)} \right]^{1/2} \\
 & \times \left[\begin{matrix} K & \frac{3}{2} & \frac{3}{2} \\ \frac{1}{2} & S & S' \end{matrix} \right] \left[1 - (1 - \frac{4}{3} \delta_{T,1}) \left[1 - \frac{16}{(K+4)(3-K)} \delta_{S,1} \delta_{S',1} \right] \right] \Bigg\} \\
 = & M^1(\vec{k}', \vec{k}) + M^2(\vec{k}', \vec{k})
 \end{aligned} \tag{22}$$

with

$$R^{\text{el}}(q) = \int_0^\infty r^2 dr j_0(qr) \rho(r), \tag{23a}$$

$$\tilde{R}^{\text{el}}(q) = \int_0^\infty r^2 dr j_0(qv) \rho^2(r). \tag{23b}$$

Here, we have identified the two Δ propagators in the second order process. The first (the 1) and second terms in the second set of large square brackets correspond to the terms $\bar{\chi}_0 \bar{\eta}_0$ and $\bar{\chi}_1 \bar{\eta}_1$ of the i sum in Eq. (20), respectively. The possible values for K in (22) are 0, 1, 2. $K=3$ does not occur in elastic scattering on a closed L - S shell nucleus. The vector operators S and $S^\dagger$ of the πN - Δ vertices allow for second or lower rank tensors only.

The modification of the impulse approximation (optical potential) has been derived to lowest order in the Δ - N interaction. For elastic scattering, this result can be interpreted nonperturbatively in terms of Δ -h doorway state expectation values. We recall that the three-dimensional doorway state approximation had originally been proposed by Kisslinger and Wang,¹ and subsequently applied by Saharia

*et al.*¹¹ in a phenomenological way. These authors have fitted three-dimensional doorway state expectation values to elastic scattering data, whereas we intend to use them mainly as a theoretical tool to relate vertex corrections and self-energies to each other, in a way relevant for elastic π -nucleus scattering. Within such a framework, the impulse approximation represents the normalization of the doorway states. We then interpret the first term (after normalization) in the square brackets as the doorway

FIG. 2. Diagrammatic representation of corrections to the elastic π -nucleus optical potential, to lowest order in the Δ - N interaction. M_0 : Δ self-energy; M_1 : Δ -hole interaction (vertex correction).

state expectation value of the Δ self energy, and the second as that of the Δ -h vertex correction. To illustrate this identification we show in Fig. 2 the Δ -h diagrams corresponding to M_0 and M_1 of Fig. 1.

For these "three-dimensional" doorway states we

with

$$\sigma(\vec{k}', \vec{k}) = \frac{V_a}{4\pi} \tilde{R}^{\text{el}}(q) X_0(K, K, L', L) \frac{(2S' + 1)(2T + 1)}{64} (-1)^{S+K+1} \left[\frac{(2S + 1)(K + 4)(3 - K)}{(2K + 1)} \right] \begin{bmatrix} K & \frac{3}{2} & \frac{3}{2} \\ \frac{1}{2} & S & S' \end{bmatrix} \quad (25a)$$

and

$$\gamma(T, S, S', K) = -\left(1 - \frac{4}{3}\delta_{T,1}\right) \left[1 - \frac{16}{(K + 4)(3 - K)} \delta_{S,1} \delta_{S',1} \right]. \quad (25b)$$

Similarly, we define partial wave projected doorway state expectation values:

$$\Sigma_l(k) = N_l^{-1}(k) S_l(k), \quad (26a)$$

$$\Gamma_l(k) = N_l^{-1}(k) S_l(k) \gamma(T, S, S', K), \quad (26b)$$

where $N_l(k)$ and $S_l(k)$ are the partial wave projections of N and S :

$$\left. \begin{aligned} N_l(k) \\ S_l(k) \end{aligned} \right\} = \int d\Omega_{\vec{k}'} d\Omega_{\vec{k}} Y_{lm}^*(\hat{\vec{k}}') Y_{lm}(\hat{\vec{k}}) \begin{bmatrix} N(\vec{k}', \vec{k}) \\ S(\vec{k}', \vec{k}) \end{bmatrix}. \quad (26c)$$

Within the three-dimensional doorway state approximation, the optical potential is expressed as

$$U(\vec{k}', \vec{k}) \sim g R^{\text{el}}(q) (\vec{k}' \cdot \vec{k}) D^{-1}(\vec{k}', \vec{k}) \quad (27)$$

with

$$\begin{aligned} D(\vec{k}', \vec{k}) &= D_0(\vec{k}', \vec{k}) - \Sigma(\vec{k}', \vec{k}) - \Gamma(\vec{k}', \vec{k}), \\ D_0(\vec{k}', \vec{k}) &\approx E^* - R + i\Gamma/2 - (\vec{k}' + \vec{k})^2/2M^*, \end{aligned} \quad (28)$$

where we have identified D_0 and D_1 in Eq. (22), and, for simplicity, we have assumed a Breit-Wigner energy dependence. We have neglected the fluctuations due to Fermi motion in the evaluation of the Δ kinetic energy doorway state expectation value.

As Eqs. (24) show explicitly the self energy Σ and the vertex correction Γ have within our approximation the same angle dependence if only one component in the Δ -N interaction (fixed T, K, S, S') is present. In other words, at this level vertex corrections simply renormalize the strength of the self-

therefore write

$$N(\vec{k}', \vec{k}) = (\vec{k}' \cdot \vec{k}) R^{\text{el}}(q), \quad (24a)$$

$$\Sigma(\vec{k}', \vec{k}) = N^{-1}(\vec{k}', \vec{k}) \sigma(\vec{k}', \vec{k}), \quad (24b)$$

$$\Gamma(\vec{k}', \vec{k}) = N^{-1}(\vec{k}', \vec{k}) \sigma(\vec{k}', \vec{k}) \gamma(T, S, S', K) \quad (24c)$$

energy and do not lead to an intrinsically new momentum dependence in the optical potential. Similarly, if more than one component is present, the effect of the vertex corrections can again be simulated by a redefined self energy, where each component has to be renormalized separately according to the values of γ (25b). Thus, from the phenomenological point of view, there is no way to disentangle on the basis of elastic scattering only Δ self energies and Δ -h vertex corrections. This in turn allows us to parametrize all the medium modifications related to Δ -N interactions in terms of Δ self energies only, without loss of generality. This result, which is of importance for actual analyses of π -nucleus elastic scattering, is based on the accuracy of the one doorway state approximation on the one hand and on the angle independence of the ratio of vertex corrections and self energies on the other hand.

Similar statements can be made with respect to the partial wave doorway state expectation values (26); here it is essential that vertex corrections and self energies are related to each other independently of the pion angular momentum.

Clearly, the simplicity of our analytical results rests heavily on the effective zero-range approximation, which we have done when replacing Δ and nucleon momenta by their average values. Within the one doorway approximation, the result is much more general for nucleon pairs in relative s states (e.g., ${}^4\text{He}$). For heavier nuclei, we have to rely on numerical calculations which strongly support our analytical results. We shall come back to these generalizations below.

We now discuss the relative importance of vertex

corrections for various situations. To set the scale we first consider the case of a spin-isospin independent Δ - N interaction. Spin independence implies $K=0$ and therefore $S=S'$ [Eq. (25a)]. Summing self-energies and vertex corrections over $S, T=1, 2$ we find

$$\bar{\gamma} = \bar{\Gamma} / \bar{\Sigma} = \sum_{S,T} \sigma \gamma / \sum_{S,T} \sigma = -\frac{1}{4}, \quad (29)$$

i.e., the standard reduction of the self-energy by vertex correction effects for such a case. For the Δ - N interaction, the assumption of spin-isospin independence is most likely not very realistic. It is known from experiment that pion rescattering and true pion absorption are the most important modes in π -nuclear systems. If interpreted as Δ - N interaction, pion rescattering contributes predominantly to the $T=2$ channel of the Δ - N interaction. The contribution to the $T=1$ channel is nine times smaller, while true pion absorption via Δ - $N \rightarrow NN$ trivially contributes to the $T=1$ channel only. From this point of view, spin-isospin independence of the Δ - N interaction could only be accidental.

For $T=2$ and either $S=2$ or $S'=2$, $\gamma = -1$, i.e., self energy and vertex corrections cancel each other as a result of the antisymmetrizations as discussed above. Thus, within our approximations, there are no medium corrections related to a $T=2$ and S or $S'=2$ Δ - N interaction. For the same reason, vertex corrections for a $T=2$ Δ - N interaction are larger in absolute value by a factor of 3 than the corresponding $T=1$ vertex corrections since the former require initial and final $T=1$ two nucleon states. For $T=2$ the values of γ are $\frac{1}{3}$, $\frac{3}{5}$, $\frac{5}{3}$, and -1 depending on the quantum numbers K, S , and S' .

A case of special interest is the interaction of a Δ - N pair in a relative s state induced by the intermediate coupling to the absorption channel. Since only the 1D_2 two-nucleon state can be reached, we have $S=S'=2$ and in this case $\gamma = \frac{1}{3}$; i.e., vertex corrections increase the effect of the self energy. There is experimental data which may serve as an indication that, more generally, absorption occurs predominantly out of $S=2$ Δ - N states. The (π^-, pn) reaction on ^4He which can proceed only through Δ - N $S=1$ states has been observed to be strongly suppressed.¹² In this case, the ratio $\gamma = \frac{1}{3}$ would be expected for Δ - N interactions induced by pion absorption in partial waves other than the s wave as well.

In contrast to Δ - N interactions induced by pion rescattering or absorption, direct Δ - N interactions as, for instance, generated by meson exchanges, in general do not single out a definite component in the

parameterization (3a) together with (1a) of the Δ - N interaction. For illustrative purpose we consider the following three-types of Δ - N interactions:

$$V_i = g_i \delta^{(3)}(\vec{r}_N - \vec{r}_\Delta) Q_i \quad (30)$$

with

$$\begin{aligned} Q_0 &= 1, \\ Q_1 &= \vec{S}_\Delta \cdot \vec{S}_N, \\ Q_2 &= (\vec{S}_\Delta \cdot \vec{S}_N)(\vec{t}_\Delta \cdot \vec{t}_N), \end{aligned} \quad (31)$$

where S and t are spin and isospin operators, respectively, of the Δ and nucleon. We rewrite these operators in terms of the operators T as defined in (3c):

$$\vec{S}_\Delta \cdot \vec{S}_N = \frac{3}{4} T_0^0(2, 2) - \frac{5}{4} T_0^0(1, 1). \quad (32)$$

For $Q_0=1$, both self-energy and vertex corrections are nonvanishing and related to each other by $-\frac{1}{4}$, cf. Eq. (29), and we have

$$\Sigma_0(\vec{k}', \vec{k}) = \frac{g_0}{4\pi} \tilde{R}^{\text{el}}(q), \quad (33a)$$

$$\Gamma_0(\vec{k}', \vec{k}) = -\frac{1}{4} \frac{g_0}{4\pi} \tilde{R}^{\text{el}}(q). \quad (33b)$$

For V_1 and V_2 the sum over the self-energies corresponding to the different spin isospin channels vanishes; this result is trivial since the closed shell average of the spin or isospin is zero. The vertex corrections, however, are nonzero, and we find

$$\Gamma_1(\vec{k}', \vec{k}) = -\frac{5}{16} \frac{g_1}{4\pi} \tilde{R}^{\text{el}}(q), \quad (34)$$

$$\Gamma_2(\vec{k}', \vec{k}) = -\frac{25}{64} \frac{g_2}{4\pi} \tilde{R}^{\text{el}}(q). \quad (35)$$

Obviously these medium corrections can be generated by a Δ -nucleus potential (effective Δ self energy) of the form

$$\tilde{U}_i(r_\Delta) = \tilde{g}_i \rho(r_\Delta) \quad (36)$$

with

$$\begin{aligned} \tilde{g}_0 &= \frac{3}{4} g_0, \\ \tilde{g}_1 &= -\frac{5}{16} g_1, \\ \tilde{g}_2 &= -\frac{25}{64} g_2 \end{aligned} \quad (37)$$

which shows that medium corrections which are generated by vertex corrections may be comparable in size to those which are dominated by self energies. In the former case, the sign of the effect, i.e., effective attraction or repulsion in the Δ -h channel, depends sensitively on the quantum numbers involved. This implies that in the presence of several

dynamical components in the Δ - N interaction, the effective Δ self-energy is only indirectly related to the basic interaction.

The derivation of our results rests heavily on two basic approximations. We have assumed that the effects of the Δ - N interaction can be calculated reliably within the one-doorway state approximation. Furthermore, the effective zero range approximation has been instrumental in deriving the simple relations between vertex corrections and self energies. In the following we shall investigate the validity of these approximations for some special cases of the Δ - N interaction.

The one-doorway state approximation has been shown to be a very good approximation for light systems and we therefore do not expect major corrections here either. This will be confirmed in the numerical evaluation reported below.

With respect to the effective zero-range approximation, we note that in the special case of π - ^4He scattering, our results for the ratio between vertex corrections and self-energies [γ in Eq. (25b)] are

valid independent of the details of the Δ - N interaction. The crucial approximation for deriving Eq. (25b) is to assume the two nucleons to be in a relative s state. With Gaussian single-particle wave functions this is trivially satisfied for ^4He , and in such a situation exchange and direct terms differ only by spin-isospin factors. Thus, our zero range approximation affects only the explicit form of self-energies and vertex corrections but not their ratio.

For heavier nuclei, the two nucleons can be in a relative p state for instance. In our approximation those contributions are assumed to be suppressed. We have numerically tested our approximation for the case of a Δ - N s -wave interaction by including in the Δ - h Hamiltonian both the corresponding Δ self energies and Δ - h vertex corrections.

For such a calculation we need an explicit expression for the Δ - h matrix element of the two body operator $t_{\Delta N}$ [Eq. (1)]. The Δ - h matrix elements of this interaction are

$$\begin{aligned}
 & \langle (j'_\Delta j'_h) \mathcal{J} M, II_3 | t_{\Delta N} | (j_\Delta j_h) \mathcal{J} M, II_3 \rangle \\
 &= \sum_{\substack{\mu\mu' \\ \tau\tau'}} (-1)^{j_h + j'_h - \mu - \mu' + 1 - \tau - \tau'} \langle j'_\Delta M + \mu', j'_h - \mu' | \mathcal{J} M \rangle \langle j_\Delta M + \mu, j_h - \mu | \mathcal{J} M \rangle \\
 & \quad \times \langle \frac{3}{2} I_3 + \tau', \frac{1}{2} - \tau' | II_3 \rangle \langle \frac{3}{2} I_3 + \tau, \frac{1}{2} - \tau | II_3 \rangle \\
 & \quad \times \left[\delta_{j_h j'_h} \delta_{\mu\mu'} \delta_{\tau\tau'} \sum_{\tilde{j}\tilde{\mu}\tilde{\tau}} \langle j'_\Delta M + \mu, I_3 + \tau; \tilde{j}\tilde{\mu}, \tilde{\tau} | t_{\Delta N} | j_\Delta M + \mu, I_3 + \tau; \tilde{j}\tilde{\mu}, \tilde{\tau} \rangle \right. \\
 & \quad \left. - \langle j'_\Delta M + \mu', I_3 + \tau'; j_h \mu, \tau | t_{\Delta N} | j_\Delta M + \mu, I_3 + \tau; j'_h \mu', \tau' \rangle \right] \\
 &= \Sigma_\Delta + \Gamma_{\Delta h} .
 \end{aligned} \tag{38}$$

Performing the sums over magnetic quantum numbers we find

$$\Sigma_\Delta = \frac{\delta_{j'_\Delta j_\Delta} \delta_{j'_h j_h}}{4(2j_\Delta + 1)} \sum_{\mathcal{J}', I', \tilde{\mathcal{J}}} [(2\mathcal{J}' + 1)(2I' + 1)]^{1/2} ((j_\Delta \tilde{j}) \mathcal{J}', (\frac{3}{2} \frac{1}{2}) I' || t_{\Delta N} || (j_\Delta \tilde{j}) \mathcal{J}', (\frac{3}{2} \frac{1}{2}) I') , \tag{39}$$

$$\Gamma_{\Delta h} = - \sum_{\mathcal{J}', I'} [(2\mathcal{J}' + 1)(2I' + 1)]^{1/2} \begin{bmatrix} j'_\Delta & \mathcal{J}' & j_h \\ j_\Delta & \mathcal{J} & j'_h \end{bmatrix} \begin{bmatrix} \frac{3}{2} & I' & \frac{1}{2} \\ \frac{3}{2} & I & \frac{1}{2} \end{bmatrix} ((j'_\Delta j_h) \mathcal{J}', (\frac{3}{2} \frac{1}{2}) I' || t_{\Delta N} || (j_\Delta j'_h) \mathcal{J}', (\frac{3}{2} \frac{1}{2}) I') . \tag{40}$$

To further evaluate this expression, we use the decomposition (3) of the Δ - N interaction and calculate the Δ - h matrix elements separately for each component,

$$t_i = g_i(p', p) [Q^K(L', L) \otimes T^K(S', S)]_0 T_0^0(T) . \tag{41}$$

The general two body matrix element needed for (39) and (40) is

$$\begin{aligned}
& ((j'_\Delta j'_h) \mathcal{J}', (\tfrac{3}{2} \tfrac{1}{2}) I' || t_i || (j_\Delta j_h) \mathcal{J}', (\tfrac{3}{2} \tfrac{1}{2}) I') \\
& = \delta_{T, I'} (2I' + 1)^{1/2} \sum_{\lambda \lambda'} [(2j_\Delta + 1)(2j'_\Delta + 1)(2j_h + 1)(2j'_h + 1)(2\lambda + 1)(2\lambda' + 1)(2S + 1)(2S' + 1)]^{1/2} \\
& \quad \times \begin{bmatrix} \frac{3}{2} & l'_\Delta & j'_\Delta \\ \frac{1}{2} & l'_h & j'_h \\ S' & \lambda' & \mathcal{J}' \end{bmatrix} \begin{bmatrix} \frac{3}{2} & l_\Delta & j_\Delta \\ \frac{1}{2} & l_h & j_h \\ S & \lambda & \mathcal{J} \end{bmatrix} \\
& \quad \times ((l'_\Delta l'_h) \lambda', (\tfrac{3}{2} \tfrac{1}{2}) S', \mathcal{J}' || g(Q^K \otimes T^K)_0 || (l_\Delta l_h) \lambda, (\tfrac{3}{2} \tfrac{1}{2}) S, \mathcal{J}) \quad (42)
\end{aligned}$$

and

$$\begin{aligned}
& ((l'_\Delta l'_h) \lambda', (\tfrac{3}{2} \tfrac{1}{2}) S', \mathcal{J}' || g(Q^K \otimes T^K)_0 || (l_\Delta l_h) \lambda, (\tfrac{3}{2} \tfrac{1}{2}) S, \mathcal{J}) \\
& = (-)^{\lambda + \mathcal{J}' + S' + K} \left[\frac{(2\mathcal{J}' + 1)(2S' + 1)}{(2K + 1)} \right]^{1/2} \begin{bmatrix} \mathcal{J}' & S' & \lambda' \\ K & \lambda & S \end{bmatrix} ((l'_\Delta l'_h) \lambda' || gQ^K || (l_\Delta l_h) \lambda) . \quad (43)
\end{aligned}$$

For the evaluation of the isospin matrix element we have taken $I = 1$ appropriate for pion scattering on a closed shell nucleus.

After numerical evaluation of the “radial” integrals in (43) the matrix elements of the self-energy Σ_Δ and the vertex corrections $\Gamma_{\Delta h}$ are calculated. Once these rather complicated matrix elements are evaluated, the Δ -h calculation can be carried through. In this way inclusion of Δ -h vertex corrections in addition to the Δ self energy does not lead to any additional complications which would not be present already due to pion rescattering.

In the actual calculation we have used a separable s -wave interaction

$$t = t_0 g_0(r') g_0(r) P_{S=2} P_{T=1} , \quad (44a)$$

where, for specificity, we have chosen the quantum numbers corresponding to the absorption channel and parametrized g_0 as a Gaussian:

$$g_0(r) = \left[\frac{1}{r_0 \sqrt{\pi}} \right]^3 e^{-r^2/r_0^2} . \quad (44b)$$

We have performed calculations for several values of the range parameter of the interaction.

We have kept in our calculations the binding potential and the spin-orbit part of the spreading potential. The effects of the central part of the spreading potential of a standard Δ -h calculation are here accounted for by explicitly including vertex corrections and self-energies corresponding to the Δ - N interaction (44). As in a standard Δ -h calculation, we fit the total and integrated elastic π -nucleus elastic cross sections by variation of the strength parameter

t_0 [Eq. (44)]. We have performed the calculations for π - ^{12}C and π - ^{16}O scattering. The results are very similar for the two cases and we restrict the discussion to π - ^{12}C . We first discuss the zero range results. The coupling constant t_0 ($r_0 = 0$) has been determined to be

$$t_0 (r_0 = 0) = (140 - i980) \text{ MeV fm}^3 . \quad (45)$$

The angular distributions calculated in this way are practically identical to those of the standard Δ -h calculations; i.e., vertex corrections corresponding to this interaction (45) influence elastic scattering in the same way as self-energies. To illustrate this numerical confirmation of our analytical calculations we show in Table I detailed results for two partial waves. The first line shows the calculated imaginary part of the doorway state expectation value of the Δ -h Hamiltonian, the exact partial-wave amplitude a , and the one doorway state approximation to a , a_0 , for pion angular momenta $l = 0, 3$. To separate the effects of vertex corrections and self-energies, we have repeated the calculations by including only self energy (second line) and vertex corrections (third line), respectively. According to our analytical calculations [Eq. (25b)] we have renormalized t_0 by factors of $\frac{4}{3}$ and 4, respectively. The one doorway state results differ for the three cases by at most a few percent. Thus, nonstatic corrections to the πN - Δ vertex, inclusion of background scattering or effects coming from the non-closed L - S shell in ^{12}C do not change significantly the conclusions of our analytical calculations. Higher doorway states are of little importance as far

TABLE I. Comparison of 162 MeV $\pi^+ - {}^{12}\text{C}$ ($l=0,3$) partial wave amplitudes and doorway-state matrix elements for different ways of treating the Δ - N interaction.

		$l=0$		$l=3$	
		$\text{Im}\langle D_0 H D_0 \rangle$ (MeV)	a^0	$\text{Im}\langle D_0 H D_0 \rangle$ (MeV)	a^0
$\Sigma + \Gamma$	$r_0=0$	-127	-0.039 + $i0.686$	-32.6	-0.007 + $i0.373$
	$r_0=2.5$ fm	-125	-0.039 + $i0.694$	-30.3	-0.004 + $i0.381$
$\frac{4}{3}\Sigma$	$r_0=0$	-128	-0.039 + $i0.685$	-32.7	-0.007 + $i0.373$
	$r_0=2.5$ fm	-129	-0.039 + $i0.680$	-31.7	-0.005 + $i0.374$
4Γ	$r_0=0$	-124	-0.039 + $i0.688$	-31.5	-0.007 + $i0.373$
	$r_0=2.5$ fm	-114	-0.039 + $i0.730$	-27.6	-0.003 + $i0.395$
					a
					-0.001 + $i0.348$
					-0.011 + $i0.362$
					-0.001 + $i0.348$
					-0.009 + $i0.359$
					+0.001 + $i0.346$
					-0.027 + $i0.371$

as the relation between vertex corrections and self-energies is concerned. Similar results are obtained for finite range interactions and we present those corresponding to $r_0=2.5$ fm, i.e., the extreme case where the range is as large as the rms of ${}^{12}\text{C}$.

The value for the coupling constant is

$$t_0(r_0=2.5 \text{ fm}) = (830 - i5800) \text{ MeV fm}^3. \quad (46)$$

As the doorway state expectation values in Table I show, finite range effects modify severely for $l=0$ the ratio between vertex corrections and self-energies. The 15 MeV difference in the doorway state expectation values corresponding to self energies and vertex corrections only constitute a 45% change in the expectation value of the interaction. In $l_\pi=3$, the 4 MeV difference corresponds to a 15% change. Owing to strong rescattering in $l=0$, this drastic finite range effect modifies the amplitude by not more than 10%. In the more important $l_\pi=3$ partial wave, self-energy and vertex correction parametrizations lead to partial wave amplitudes which differ by about 6%. Trivially, the effects of vertex corrections in the realistic case (first line of Table I) as compared to those of a renormalized self-energy (second line in Table I) become practically identical. Figure 3 shows that the angular distributions corresponding to the three cases of Table I do not display any significant difference due to different treatments of the Δ - N interaction.

We have evaluated the three-dimensional doorway state expectation value for the interaction [(44a) and (44b)] and found that for $r_0 \leq 1.5$ fm the main effect of the finite range is a renormalization of the corresponding zero range result by

FIG. 3. $\pi^+ - {}^{12}\text{C}$ elastic cross section at 162 MeV. Dashed curve: Δ self-energy only; dashed-dotted curve: vertex corrections only; solid curve: both, renormalized in each case according to Table I. Data from Ref. (13).

$$\left. \begin{array}{l} \Sigma_{r_0}(\vec{k}', \vec{k}) \\ \Gamma_{r_0}(\vec{k}', \vec{k}) \end{array} \right\} \approx \left\{ \begin{array}{l} \Sigma_{r_0=0}(\vec{k}', \vec{k}) \\ \Gamma_{r_0=0}(\vec{k}', \vec{k}) \end{array} \right\} \left[\frac{1}{1 + \left[\frac{r_0}{2b} \right]^2} \right]^3 \times \exp \left[- \frac{k^2 r_0^2 / 8}{1 + \left[\frac{r_0}{2b} \right]^2} \right] \quad (47)$$

(b is the harmonic oscillator parameter). This result is exact for ${}^4\text{He}$.

Note that the renormalization is both energy and A dependent. The renormalization factor $e^{-k^2 r_0^2 / 8}$ would be obtained in our approximate evaluation of the doorway state expectation values (24) by assuming initial and final momenta $-q/2$ and $q/2$, respectively, for the two nucleons. In this Breit frame the Δ - N relative momentum is

$$k_{\Delta N} = \frac{1}{2}k \quad (48)$$

and this exponential factor is the corresponding form factor of the Δ - N interaction. Fermi motion effects apparently become important for $(r_0/2b)^2 \approx 1$.

To summarize, we stress that an s -wave Δ - N interaction can be accounted for phenomenologically by a Δ -nucleus potential which has the shape of the nuclear density. This does not imply that vertex corrections are negligible as compared to self energies. Their effect, however, as well as nonlocality

effects in the Δ - N interaction, can be taken into account by renormalizing a local self-energy. For the interpretation of such effective self energies in terms of Δ - N coupling constants like those in Eqs. (45) and (46), inclusion of the vertex corrections is required. This in turn requires a model for the spin-isospin dependence of the Δ - N interaction.

We now discuss the influence of higher partial waves in the Δ - N interaction on π -nucleus scattering. Phenomenologically, it has been found that a spin orbit interaction is required for the interpretation of elastic angular distributions; this indicates the importance of p -wave Δ - N interactions. In the following, we evaluate the self-energies corresponding to p -wave interactions and assume that the vertex corrections mainly renormalize the strength-parameters as is the case in our approximate evaluation Eq. (25b), and as has been shown to be more generally valid for the s -wave interaction. In contrast to the s wave, higher partial wave ΔN interactions change the typical $\vec{k} \cdot \vec{k}'$ dependence of the impulse approximation. The angular dependence in general is described by the rank K tensor [cf. Eqs. (25a) and (25b)]

$$X_0(K, K, L', L) = \frac{4\pi}{3} k^2 \{ [Y_1(\hat{\vec{k}}') \otimes Y_1(\hat{\vec{k}})]_K \otimes [Y_L(\hat{\vec{k}}') \otimes Y_L(\hat{\vec{k}})]_K \}_0, \quad (49)$$

where we have evaluated the Δ - N relative momentum in the Breit frame (cf. Eq. 48). After recoupling we find

$$X_0(K, K, L', L) = \frac{k^2}{4\pi} (-1)^{L+1+K} [(2L+1)(2L'+1)(2K+1)]^{1/2} \times \sum_{\lambda} \langle L0, 10 | \lambda 0 \rangle \langle L'0, 10 | \lambda 0 \rangle \begin{Bmatrix} L & L' & K \\ 1 & 1 & \lambda \end{Bmatrix} P_{\lambda}(\hat{\vec{k}}' \cdot \hat{\vec{k}}). \quad (50)$$

For s -wave interaction, $L, L' = 0$, we trivially recover the angular dependence of the impulse approximation $P_1(\hat{\vec{k}} \cdot \hat{\vec{k}}')$. P -wave partial waves $L = L' = 1$, give rise to "Kisslinger" type interactions ($K=0$), spin-orbit interactions ($K=1$), and second rank tensor interactions ($K=2$). However, as Eq. (50) shows, there are only two independent kinds of angular dependences, $P_0(\hat{\vec{k}} \cdot \hat{\vec{k}}')$ and $P_2(\hat{\vec{k}} \cdot \hat{\vec{k}}')$. In other words, an arbitrary p -wave interaction can be described as a superposition of a Kisslinger and a spin orbit potential. We therefore have parameterized the spreading potential in the form

$$W(R) = W_0 \rho(R) / \rho(0) + 2 \vec{L}_{\Delta} \cdot \vec{\Sigma}_{\Delta} V_{LS}(R) + V_{LK} \vec{\nabla} \cdot \rho(r) \vec{\nabla} / (\mu \rho(0)), \quad (51)$$

$$V_{LS}(R) = V_{LS}^0 \mu R^2 \exp(-\mu R^2)$$

and performed a Δ -h calculation for π - ${}^{12}\text{C}$ scattering.

In Fig. 4 we show the results of the partial-wave doorway-state expectation values for pure central and Kisslinger potentials, respectively. The large l behavior is the same for Kisslinger and spin-orbit interactions as can be seen easily from a partial-

FIG. 4. Doorway-state expectation values as a function of pion angular momentum, for 162 MeV π^+ - ^{12}C . Solid curve: central; dashed curve: spin-orbit; dotted curve: Kisslinger potential (arbitrary units).

wave projection of $\tilde{R}^{\text{el}}(q) X_0(K, K, L', L)$ [cf. Eq. (22)]. The doorway state matrix elements of the p -wave induced interactions are enhanced by l_π as compared to those of the s -wave interaction (see Ref. 2). As compared to the spin-orbit interaction, the Kisslinger potential does not lead to a change of sign in the central doorway state matrix elements. One can see from Fig. 4 that, qualitatively, either of the two p -wave potentials can be approximated by a superposition of the other one and a modified central piece. This is confirmed by explicit calculation of the angular distributions.

As for the finite range effects, details of the angular distribution are obviously dependent on the details of the parametrization of the spreading potential (51), and for a given angular distribution the extra strength parameter V_{2K} allows us to improve the fit to the data. Such an example is shown in Fig. 5(a) for π - ^{12}C scattering at 162 MeV. The comparison of solid with dashed-dotted curves displays the effect of shifting the strength between central and Kisslinger potentials, and the comparison with the dashed curve displays the effect of shifting strength between spin-orbit and Kisslinger potentials.

At this particular energy, inclusion of the Kisslinger potential improves the fit to the data. The relevance of this improvement is rather doubtful in view of the impressive disagreement of the same parametrizations at 226 MeV [Fig. 5(b)]. We could not find any weakly energy dependent parametrization which would improve significantly the results of Ref. 2 corresponding to $V_{LK}=0$ over the range of pion energies $80 \leq T_\pi \leq 260$ MeV.

FIG. 5. π^+ - ^{12}C elastic cross section at 162 MeV (a) and 226 MeV (b). Solid curve: $W_0=(-48-i66)$ MeV, $V_{LS}^0=(-6.8-i2.2)$ MeV, $V_{LK}=0$. Dashed curve: $W_0=(-48-i66)$ MeV, $V_{LS}^0=(-18-i6)$ MeV, $V_{LK}=(-7.2-i2.4)$ MeV. Dashed-dotted curve: $W_0=(15-i45)$ MeV, $V_{LS}^0=(-6.8-i2.2)$ MeV, $V_{LK}=(9+i3)$ MeV [cf. Eq. (51)].

The parametrization (51) actually allows us to account for s - d and p - f components of the Δ - N interaction as well. According to (50), an s - d tensor force leads back to the impulse approximation $\vec{k} \cdot \vec{k}'$ dependence, and therefore the doorway state l dependence of an s -wave Δ - N interaction; in this case, however, we expect an increase in the strength parameter with increasing pion energy as a result of the necessary threshold factors in such a tensor force. In the same way a p - f tensor force is equivalent to a pure p -wave interaction with, again, an increasing strength parameter. Only for an $L=L'=2$ interaction does a new angular dependence appear in (50) corresponding to $\lambda=3$. With the possible exception of pion rescattering we do not

see any mechanism which in the considered pion energy range corresponding to Δ - N relative momenta of up to 200 MeV/c would produce considerable Δ - N interactions in relative d or higher angular momentum states. In this sense (51) turns out to be a rather general form for a phenomenological treatment of the Δ -nucleus interaction

We finally reemphasize that the form (51) has only been obtained in the effective zero-range approximation. Unlike the case of the pure ΔN s -wave interaction we have so far not been able to test it against an exact numerical calculation.

IV. THE Δ - N INTERACTION IN π -NUCLEUS REACTIONS

In this section we discuss the role of the Δ - N interaction in π -nucleus reactions. We use our approximate evaluation of the two-body nuclear matrix element to calculate cross sections for pion induced particle-hole excitations and nucleon knockout.

As we have seen in the last section, the effect of the Δ - N interaction in elastic scattering can be accounted for completely by a Δ -nucleus optical potential, i.e., a one body operator acting on the Δ . This obviously is not possible anymore in π -nucleus reactions. In addition to processes in which the Δ undergoes a collision with the residual nucleus, before it decays into π and the final nuclear state, there are processes in which the nucleus gets excited by a Δ - N collision. These two classes of processes, the pion induced (M_2, M_3) and Δ -induced processes (M_0, M_1), are shown diagrammatically in Fig. 6. Obviously, processes corresponding to M_0 and M_1 cannot be accounted for by a one body operator; i.e., an effective Δ -nucleus potential describing all the four processes of Fig. 6 has to depend on the quantum numbers of the excited state.

In the modified DWIA calculations for excitation of discrete states in ^{12}C (Ref. 3) and quasielastic scattering⁴ done so far, the genuine two-body pro-

with

$$f_{\lambda M} = (-1)^{\lambda} h(\lambda) (k' \otimes k)_{\lambda M} D_0^{-1} R_{l_h}(Q) \langle 1 + \vec{t}_\pi \cdot \vec{t}_N \rangle + \frac{V_a}{4\pi} D_1^{-1} D_2^{-1} \tilde{R}_{l_h}(Q) \frac{(2S'+1)(2T+1)}{64} \\ \times \sum_{i, \Lambda} (-1)^i \bar{\chi}_i(S', S, K, \lambda, \Lambda) \langle \bar{\eta}_i(T, 0) - \bar{\eta}_i(T, 1) \vec{t}_\pi \cdot \vec{t}_N \rangle X_\lambda(\Lambda, K, L', L). \quad (53)$$

This result can be straightforwardly obtained from Eq. (10) by replacing the particle wave function by a constant in configuration space ($l_p=0$) in Eq. (20) and replacing the pion momentum transfer q by the momentum transfer Q to the residual nucleus,

FIG. 6. Basic four processes of Fig. 1 in the particle-hole representation. M_0, M_1 : “inelastic” vertex corrections ($\Delta h \rightarrow \Delta p-2h$); M_3 : “elastic” vertex correction ($\Delta h \rightarrow \Delta h$).

cesses of the type M_0 and M_1 have been left out completely. At the end of the present section we shall be in a position to estimate the corrections to these earlier calculations.

In the following we shall focus our discussion on the characteristics and possible experimental signals of these Δ -induced processes. In this context, the isospin dependence of the Δ - N interaction which leads to modifications of quasifree isospin ratios is particularly important.

We have already presented our general result for the particle-hole transitions in the presence of the Δ - N interaction. Starting from the expression (10) we have also calculated the amplitude for nucleon knockout ($\pi, \pi'N$). The result for the unpolarized cross section corresponding to one nucleon knockout from a closed L - S shell is

$$\sigma \sim 2 \frac{2l_h+1}{4\pi} \sum_{\lambda M} |f_{\lambda M}|^2 \quad (52)$$

$$\vec{Q} = \vec{k} - \vec{k}' - \vec{p}. \quad (54)$$

The relevant form factors appearing in (53) are

$$R_{l_h}(Q) = \int_0^\infty r^2 dr j_{l_h}(Qr) y_{l_h}(r), \quad (55a)$$

$$\tilde{R}_h(Q) = \int_0^\infty r^2 dr j_{l_h}(Qr) y_{l_h}(r) \rho(r). \quad (55b)$$

We obviously have neglected the nucleon final state interaction. The isospin brackets stand for matrix elements of the appropriate initial and final pion and nucleon states.

As for the particle-hole excitations (20), Eq. (53) displays how the Δ - N interaction modifies the 3-3 characteristics of the quasifree knockout process. The different form of the isospin dependence in first and second order terms shows that in the presence of the Δ - N interaction, the cross sections for the $\pi^\pm p \rightarrow \pi^\pm p$ and $\pi^+ n \rightarrow \pi^0 p$ processes will not follow in general the quasifree isospin ratios. Furthermore, in the second order process, isospin dependence and angular dependence are not factorized anymore; i.e., $\pi^+ p$, $\pi^- p$, or charge exchange processes have in

general different angular distributions due to the Δ - N interaction. Similarly, spin-flip ($\lambda=1$) and non-spin-flip contributions ($\lambda=0$) are modified differently.

Trivially, the presence of the Δ - N interaction leads to more complicated angular dependences as described by X_λ in comparison to the $(k' \otimes k)_\lambda$ dependence of the impulse approximation. Finally, for later applications, we emphasize the smoother dependence of the nuclear form factor $\tilde{R}_h(Q)$ on the momentum transfer to the residual nucleus as compared to the first order term $R_h(Q)$.

We now discuss in detail the effects of the Δ - N interaction for the two classes of π -nucleus reactions. We first rewrite the sum over the four different processes [the i sum in Eqs. (20) and (53)] in the following way by using Eqs. (14), (16), and (17),

$$\begin{aligned} \omega_{ph}(S', S, K, \lambda, \Lambda, T, T_f) &= \sum_{i=0}^3 (-1)^i \tilde{\chi}_i(S', S, K, \lambda, \Lambda) \bar{\eta}_i(T, T_f) \\ &= \bar{\sigma} (-1)^{T_f} [1 + (-1)^\lambda] (1 - \frac{4}{3} \delta_{T,1}) + \frac{4}{3} \delta_{T,1} (-1)^{T_f} \{ \bar{\chi}_0 [1 + (-1)^{T_f}] - \bar{\delta} \}, \end{aligned} \quad (56a)$$

$$\begin{aligned} \omega_{qr}(S', S, K, \lambda, \Lambda, T) &= \sum_{i=0}^3 (-1)^i \tilde{\chi}_i(S', S, K, \lambda, \Lambda) \langle \bar{\eta}_i(T, 0) - \bar{\eta}_i(T, 1) \vec{t}_\pi \cdot \vec{t}_N \rangle \\ &= \langle 1 + \vec{t}_\pi \cdot \vec{t}_N \rangle \{ \frac{4}{3} \bar{\chi}_0 - \bar{\delta} \} \delta_{T,1} + \bar{\sigma} [1 + (-1)^\lambda] (1 - \frac{4}{3} \delta_{T,1}) \\ &\quad + \frac{8}{9} \langle 1 - 2 \vec{t}_\pi \cdot \vec{t}_N \rangle \bar{\chi}_0 \delta_{T,1}. \end{aligned} \quad (56b)$$

We have omitted the corresponding arguments in the functions $\tilde{\chi}_i$, $\bar{\sigma}$, and $\bar{\delta}$ in Eq. (56).

Equation (56) makes explicit the dependence of the second order term on the isospin of both the Δ - N interaction and the final-state.

As a general result we find from (56) that the second order term has the same isospin dependence on the final state if only a $T=2$ Δ - N interaction is present. In the case of the particle hole excitation, the dependence on the isospin of the final nuclear state is completely contained in the phase $(-1)^{T_f}$ as in the impulse approximation [cf. Eq. (20)].

Similarly, the second order knockout amplitude becomes proportional to the projector on the isospin $\frac{3}{2}$ π - N state ($\langle 1 + \vec{t}_\pi \cdot \vec{t}_N \rangle$).

This implies that in quasifree scattering, the cross sections for the processes $\pi^\pm p \rightarrow \pi^\pm p$ and $\pi^+ n \rightarrow \pi^0 p$ still exhibit the free 3-3 isospin ratios. Analogously, the cross section ratio of particle-hole excitations which differ only in their isospin is unchanged from its value 4 ($\sigma_{T=0}/\sigma_{T=1}$) if only $T=2$

Δ - N interactions are present.

This result is far more general than our derivation indicates. Allowing for only $\Delta N T=2$ states in the four processes of Fig. 1, the initial and final pion must be in an isospin $\frac{3}{2}$ state with either of the two nucleons in the initial or final state, and consequently the amplitude must contain the isospin $\frac{3}{2}$ projector. Thus, these isospin ratios allow us, in principle, to separate the $T=1$ component of the Δ - N interaction from the $T=2$ component. This is very important for our understanding of the Δ - N interaction since, as mentioned above, the two important inelastic processes, pion rescattering and true absorption feed preferentially either one of the two isospin channels.

Furthermore, we note from (56) that for $T=2$ the amplitudes are proportional to $\bar{\delta} [1 + (-1)^\lambda]$; i.e., medium corrections for a $\Delta N T=2$ interaction cancel for a spin-flip induced transition. Thus for particle hole excitations which are dominated by coupling to the magnetization current like the 1^+

$T=0,1$ excitations at 12.7 and 15.1 MeV in ^{12}C , there are no $T=2$ Δ - N medium corrections to lowest order. Similarly the spin-flip contribution in quasifree scattering is not modified in contrast to the non-spin-flip piece.

We note that magnetization current dominated $\Delta T=1$ transitions are very selectively modified by the various components of the Δ - N interaction. For this class of transition we have

$$\omega_{\text{ph}} \sim \delta_{T,1} \bar{\delta}(S', S, K, 1, \Lambda)$$

but

$$\bar{\delta} \sim (1 - \delta_{S,2} \delta_{S',2}) ;$$

i.e., a nonzero effect is obtained only if the Δ - N isospin is 1 and if the Δ - N spin is 1 in either the initial and/or final channel. Thus, the dominant absorption modes $^5S_2(\Delta N) \rightarrow ^1D_2(NN)$ and $^5P_3(\Delta N) \rightarrow ^3F_3(NN)$ give rise to medium corrections which cancel to lowest order for these special nuclear transitions.

The medium modification in dominantly longitudinal p-h transitions $\lambda=0$ [Eq. (56a)] can be related to those occurring in elastic scattering. For $\lambda=0$, we have $\bar{\chi}_0 = \bar{\chi}_2$ and $\bar{\chi}_1 = \bar{\chi}_3$ [Eq. (17b) and (13)], and therefore

$$\omega_{\text{ph}} = \bar{\chi}_0(\bar{\eta}_0 + \bar{\eta}_2) - 2\bar{\chi}_1\bar{\eta}_1 \quad (\lambda=0). \quad (57)$$

with

$$\begin{aligned} \tilde{M} = & g_{81}^8 [(2j_p + 1)(2j_h + 1)]^{1/2} \langle j_h \frac{1}{2}, j_p - \frac{1}{2} | \mathcal{J}0 \rangle h(T_f) \Theta_{T_f, -T_z} \\ & \times \left[D_0^{-1}(\vec{k}', \vec{k}) R_{\mathcal{J}}^{\text{tr}}(q) \vec{k}' \cdot \vec{k} + \frac{V_a}{4\pi} \tilde{R}_{\mathcal{J}}^{\text{tr}}(q) D_1^{-1}(\vec{k}', \vec{k}) D_2^{-1}(\vec{k}', \vec{k}) \right. \\ & \left. \times (-1)^{T_f} \frac{(2S' + 1)(2T + 1)}{64} \sqrt{2} \omega_{\text{ph}}(S', S, K, 0, K, T, T_f) X_0(K, K, L', L) \right]. \end{aligned} \quad (59b)$$

The dependence on the final particle hole state is, apart from the normalization, contained on the one hand in radial form factors, R and $\tilde{R}$. On the other hand, the second order form factor is multiplied with matrix elements of the Δ - N interaction which depend only via the isospin on the final nuclear excited state. Thus, for longitudinal excitations, there are only two independent superpositions of matrix elements in the Δ - N interaction; i.e., the medium modifications for $T=0$ longitudinal excitations on the one hand, and $T=1$ longitudinal excitations on the other hand, are each described by one common function characterizing the Δ - N interaction. This implies, in particular, that the same matrix elements of the Δ - N interaction determine the medium

Since

$$\bar{\eta}_0 = \bar{\eta}_2 \quad \text{unless } T_f = 1, T = 1, \quad (58a)$$

we find

$$\omega_{\text{ph}} = 2(\bar{\chi}_0\bar{\eta}_0 - \bar{\chi}_1\bar{\eta}_1) \quad (58b)$$

which implies that the correction from the Δ induced process M_0 in Fig. 6 becomes identical to the self-energy correction M_2 , and, analogously, M_1 and M_3 are equal. Thus, in this perturbative treatment, the Δ -induced processes M_0, M_1 double the effect of the medium corrections to the Δ -h propagator M_2, M_3 . In elastic scattering this pairwise identity is trivial since M_0 and M_2 represent identical processes, and therefore addition of M_0 and M_2 would be double counting. In inelastic scattering we have in our approximation numerically identical corrections from different processes. Formally this doubling effect is completely analogous to initial and final state distortion in inelastic scattering as compared to the one sided distortion only in elastic scattering.

For longitudinal excitations the unpolarized cross section corresponding to (20) can be written as

$$\sum_M |M_{\text{ph}}^{\lambda=0}|^2 = |\tilde{M}|^2 \quad (59a)$$

corrections in elastic scattering and isoscalar longitudinal excitations. We finally note from (58a) that differences in the medium modifications related to the Δ - N interaction between isoscalar and isovector longitudinal excitations are due exclusively to the $T=1$ part of the Δ - N interaction.

So far we have discussed the general structure and selection rules of Δ - N induced medium modifications. In this last part we discuss the order of magnitude of these effects for specific reactions. For these estimates we need a model for the Δ - N interaction. We select the Δ - N s -wave with the quantum numbers $T=1, S=2$ and assume for the following a zero range force. On the one hand this interaction is simple enough to lead to transparent results, and on

the other hand it accounts for a large part for the pion absorption induced Δ - N interaction.

For these estimates we have to go beyond a simple perturbative treatment of the Δ - N interaction, in particular we want all the Δ propagators to be dressed by the Δ -nucleus interaction. To do this in an approximate way we use our elastic scattering results. The equivalence between vertex corrections and self-energy insertions to high accuracy in the elastic differential cross section apparently implies that the dressing of the Δ -h Green's function can always be described as an effect of a Δ -nucleus potential. We therefore assume that in inelastic scattering as well, the iterated effect of processes M_2 and M_3

(Fig. 6) is simply given by a distorted Δ propagator. In the following we therefore consider the three processes shown in Fig. 7.

This separation of "elastic" Δ -nucleus scatterings summed up into a distorted propagator and inelastic Δ - N transitions treated to lowest order is in the spirit of a standard distorted wave procedure.

Formally, this simply amounts to interpreting the Δ Green's functions D_i^{-1} in (20) and (53) as distorted ones, $\tilde{D}_i^{-1}$, and in turn restricting the i summation to the values 0,1.

This gives, for the particle-hole matrix element (20) with our simple Δ - N interaction,

$$\begin{aligned} \tilde{M}_{ph} = & g \frac{16}{81} \sqrt{4\pi} (-1)^{M+l_h+T_f+T_z} \Theta_{T_f, -T_z} h(T_f) [(2l_p+1)(2l_h+1)(2j_p+1)(2j_h+1)]^{1/2} \\ & \times \sum_{\lambda\lambda'} i^{\lambda'} \langle l_p 0, l_h 0 | \Lambda' 0 \rangle \sqrt{2\lambda+1} (-1)^{\lambda} h(\lambda) \\ & \times \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \lambda \\ l_p & l_h & \Lambda' \\ j_p & j_h & J \end{pmatrix} [Y_{\Lambda'}(\hat{q}) \otimes (k' \otimes k)_{\lambda}]_{J, -M} \cdot \tilde{m}_{ph}(\vec{k}', \vec{k}). \end{aligned} \quad (60a)$$

$$\tilde{m}_{ph}(\vec{k}', \vec{k}) = \tilde{D}_0^{-1}(\vec{k}', \vec{k}) R_{\Lambda'}^T(q) + \frac{V_\alpha}{(4\pi)^2} \tilde{D}_1^{-1}(\vec{k}', \vec{k}) \tilde{D}_2^{-1}(\vec{k}', \vec{k}) \tilde{R}_{\Lambda'}^T(q) \frac{5}{16} (-1)^{T_f} \quad (60b)$$

and similarly for the knockout amplitude $f_{\lambda M}$ [Eq. (53)],

$$\begin{aligned} \tilde{f}_{\lambda M} = & (-1)^{\lambda} h(\lambda) (k' \otimes k)_{\lambda M} \left[\tilde{D}_0^{-1}(\vec{k}', \vec{k}) R_{\Lambda'}(Q) \langle 1 + \vec{t}_\pi \cdot \vec{t}_N \rangle \right. \\ & \left. + \frac{V_\alpha}{(4\pi)^2} \tilde{D}_1^{-1}(\vec{k}', \vec{k}) \tilde{D}_2^{-1}(\vec{k}', \vec{k}) \tilde{R}_{\Lambda'}(Q) \frac{5}{16} \langle 1 - \vec{t}_\pi \cdot \vec{t}_N \rangle \right]. \end{aligned} \quad (61)$$

We now evaluate approximately the second order expression in Eqs. (60) and (61). We first observe that the combination

$$\frac{V_\alpha}{(4\pi)^2} \frac{5}{16} \rho(r)$$

is just the configuration space expression for $\Sigma + \Gamma$ in Eq. (24) for this particular interaction and can therefore be identified with the spreading potential

$$\frac{V_\alpha}{(4\pi)^2} \frac{5}{16} \rho(r) = W_{sp} \rho(r) / \rho(0). \quad (62)$$

We rewrite the second order terms

$$\frac{5}{16} \frac{V_\alpha}{(4\pi)^2} \tilde{D}_1^{-1} \tilde{D}_2^{-1} \tilde{R}(\kappa) = \alpha(\kappa) W_{sp} \tilde{D}_1^{-1} \tilde{D}_2^{-1} R(\kappa) \quad (63a)$$

with the reduction α of the central strength of the spreading potential

$$\alpha(\kappa) = \frac{\tilde{R}(\kappa)}{\rho(0) R(\kappa)}. \quad (63b)$$

Equation (63) applies to both (60) and (61) with $\kappa = q$ or Q , respectively, and the $R, \tilde{R}$ being the appropriate form factors.

In this way we determine the effective strength of the spreading interaction depending on the specific process via the form factors R . This procedure gives a sufficiently reliable estimate of the medium corrections even in the presence of pion distortion as shown in Ref. 3.

Similarly, we define the reduction of the central strength of the spreading potential in the energy denominator of the second order term by

$$\beta(\kappa) = \frac{\tilde{\tilde{R}}(\kappa)}{\rho(0) \tilde{R}(\kappa)}, \quad (64)$$

where $\tilde{\tilde{R}}$ differs from $\tilde{R}$ by an extra power of ρ in the integrand [Eqs. (21b) and (55b)]. With this choice one accounts for the fact that the second order process necessarily occurs in a region of higher nuclear densities, and therefore the reduction of the

central strength becomes smaller [$\beta(0) \geq \alpha(0)$].

Assuming a Breit-Wigner form of the free Δ propagator, we find, for the amplitude $\tilde{m}_{ph}$ in eq. (60),

$$\tilde{m}_{ph}(\vec{k}', \vec{k}) = R_{\Lambda}^T(q) \{ [E - R + i\Gamma(E)/2 - k^2/2M^* - \alpha(q)W_{sp}]^{-1} + (-1)^{T_f} \alpha(q)W_{sp} [E - R + i\Gamma(E)/2 - k^2/2M^* - \beta(q)W_{sp}]^{-2} \}. \quad (65)$$

At resonance ($E - R - k^2/2M^* = 0$) and for a purely absorptive spreading potential we obtain, for the order of magnitude of the ratio of second order to impulse approximation,

$$\tilde{\rho}_{ph} = \frac{\tilde{m}^2}{\tilde{m}_{IA}} = (-1)^{T_f+1} \alpha |W_{sp}| \frac{\Gamma/2 + \alpha |W_{sp}|}{(\Gamma/2 + \beta |W_{sp}|)^2} \approx (-1)^{T_f+1} \alpha \frac{|W_{sp}|}{\Gamma/2 + \alpha |W_{sp}|}. \quad (66)$$

$\mathcal{J}mW_{sp}$ is of the order of the half width and α about 0.3 for a 1^+ and 2^+ excitation and 0.45 for a 0^+ excitation (Ref. 3).

The resulting value

$$\tilde{\rho}_{ph} \approx 0.2(-1)^{T_f+1} \quad (67)$$

is typical for the medium corrections at resonance.

This order of magnitude is primarily given by the strength of the spreading potential. The assignment of the $T=1, S=2$ ΔN quantum numbers, however, is crucial for the discussion of differences in the medium corrections for different types of nuclear excitations. To illustrate this relation we calculate the ratio assuming a spin-isospin independent interaction. We find

$$\langle \tilde{\rho}_{ph} \rangle_{ST} \approx -\alpha \frac{|W_{sp}| (-1)^{\lambda+T_f}}{\Gamma/2 + \alpha |W_{sp}|} [\delta_{\lambda,0} \delta_{T_f,0} + \frac{1}{3} (\delta_{\lambda,0} \delta_{T_f,1} + \delta_{\lambda,1} \delta_{T_f,0} - \delta_{\lambda,1} \delta_{T_f,1})]. \quad (68)$$

The comparison of (68) with (67) shows that the medium modifications (68) of the isoscalar longitudinal excitation are identical provided the different interactions give rise, in elastic scattering, to the same spreading potential. In this case the Δ -induced processes enhance the effect of the spreading potential in the transition operator. This is in agreement with our general results. For the choice $S=2, T=1$ there is no dependence on λ , i.e., longitudinal and magnetization current dominated transitions are affected in exactly the same way. This follows from the $S=2$ assumption which requires the pion angular momentum to couple with the spin of either of the two nucleons to $\frac{3}{2}$. In turn, the λ dependence in (68) comes from the $S=1$ piece of the interaction.

Obviously, a quantity which is very sensitive to the presence of the absorptive $T=1, S=2$ Δ - N interaction is the ratio of cross sections corresponding to excitations which differ only in their isospin. Note from (66) that in contrast to isoscalar excitations, the Δ - N induced transitions tend to cancel the effect of the spreading potential for isovector transi-

tions. Thus, as a general rule we expect, at resonance, isovector transitions to be enhanced and isoscalar transitions to be suppressed by the processes $\tilde{M}_0$ and $\tilde{M}_1$ in Fig. 7.

In Fig. 8 we show the ratio of cross sections for $\Delta T=0,1$ transitions with otherwise identical structure. For this ratio we only need to specify the parameters $\alpha(0)$ and $\beta(0)$ which we have evaluated cor-

FIG. 7. Diagrammatic representation of the processes estimated in the text. The dashed double lines represent dressed Δ propagators, obtained by iterating the Δ self energy. M_{IA} : impulse approximation; $\tilde{M}_0, \tilde{M}_1$: inelastic vertex corrections.

FIG. 8. Ratio of 1^+ ($T=0$) to 1^+ ($T=1$) excitations in ^{12}C as a function of pion energy. Solid curve: $W_{sp} = (-48 - i66)$ MeV in Eq. (65). Dashed curve: W_{sp} in the numerator of Eq. (65) replaced by $(15 - i45)$ MeV.

responding to a 1^+ excitation in ^{12}C :

$$\alpha(0) \approx 0.35, \quad \beta(0) \approx 0.55. \quad (69)$$

It is clear from our estimates that at resonance, the quasifree ratio of 4 is typically decreased by a value of 2 and not strongly dependent on the chosen value for the real part of the interaction. Thus this decrease in the ratio follows directly from the observed absorptive strength of the spreading potential and its interpretation in terms of true absorption ($T=1$ dominance). This latter assumption is corroborated by the correctly predicted order of magnitude of the experimentally observed true absorption cross section.

Away from the resonance and in particular at

FIG. 9. Lowest order contributions of the Δ - N interaction to a single particle transition (e.g., charge exchange in ^{13}C). In the numerical estimates, the self-energy (M_2) has been iterated by using dressed Δ propagators in all four diagrams.

lower energies our results depend sensitively on the assumed real part of the Δ - N interaction. As pointed out above, there is no Δ - N channel which, in an obvious way, would be singled out in generating the real part of the Δ - N interaction; this makes our results away from resonance model dependent as is shown explicitly for the two models (66) and (68).

This enhancement of the "impulse-approximation" $\Delta T=1$ cross section at resonance due to the Δ -induced transitions is obviously of relevance for any charge exchange reaction. In view of the widespread interest in the particular $^{13}\text{C}(\pi^+, \pi^0)^{13}\text{N}_{g.s.}$ reaction we have applied our formalism to this case. Starting from our general two-body nuclear matrix element (10) we have derived the following expression for the single particle transition ($j_p l_p \rightarrow j'_p l'_p$) of a valence nucleon outside a closed shell core:

$$M_{s.p.} = g \frac{8}{81} \sqrt{2\pi} [(2l_p + 1)(2l'_p + 1)(2j_p + 1)(2j'_p + 1)]^{1/2} (-1)^{j_p - m_p + l_p + M} \\ \times \sum_{\lambda, \Lambda'} i^{\Lambda'} \langle l'_p 0, l_p 0 | \Lambda' 0 \rangle \sqrt{2\lambda + 1} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \lambda \\ l'_p & l_p & \Lambda' \\ j'_p & j_p & \mathcal{J} \end{bmatrix} \langle j'_p m'_p, j_p m_p | \mathcal{J} M \rangle m_{s.p.}(\vec{k}', \vec{k}), \quad (70a)$$

$$m_{s.p.}(\vec{k}', \vec{k}) = (-1)^{\lambda h}(\lambda) R_{l'_p l_p}^{\Lambda'}(q) D_0^{-1}(\vec{k}', \vec{k}) [Y_{\Lambda'}(\hat{q}) \otimes (k' \otimes k)_{\lambda}]_{\mathcal{J}, -M} \langle 1 + \vec{t}_{\pi} \cdot \vec{t}_N \rangle \\ + \frac{V_{\alpha}}{4\pi} \tilde{R}_{l'_p l_p}^{\Lambda'}(q) D_1^{-1}(\vec{k}', \vec{k}) D_2^{-1}(\vec{k}', \vec{k}) \frac{(2S' + 1)(2T + 1)}{64} \\ \times \sum_{i, \Lambda} (-1)^i \bar{\chi}_i(S', S, K, \lambda, \Lambda) \langle \bar{\eta}_i(T, 0) - \bar{\eta}_i(T, 1) \vec{t}_{\pi} \cdot \vec{t}_N \rangle [Y_{\Lambda'}(\hat{q}) \otimes X_{\lambda}(\Lambda, K, L', L)]_{\mathcal{J}, -M} \quad (70b)$$

with

$$\left. \begin{matrix} R_{l'_p l_p}^{\Lambda'}(q) \\ \tilde{R}_{l'_p l_p}^{\Lambda'}(q) \end{matrix} \right\} = \int_0^\infty r^2 dr y_{l'_p}(r) y_{l_p}(r) j_{\Lambda'}(qr) \left\{ \begin{matrix} 1 \\ \rho(r) \end{matrix} \right\}. \quad (70c)$$

The four terms of the i sum in Eq. (70b) correspond to the processes shown diagrammatically in Fig. 9. In the same spirit as before we sum up the “elastic” part of the Δ nucleus interaction into dressed Δ propagators. Here only M_2 describes such an elastic interaction; i.e., it involves only 1Δ , $0p$ - $0h$ states while the other processes always couple to 1Δ , $1p$ - $1h$ states. This leads to a modified impulse approximation where the Δ self energy only appears and the three remaining processes now define the second order correction. Instead of giving the by now obvious general expression for this summed amplitude we immediately specialize to our standard case of a Δ - N interaction in the $S=2$, $T=1$ s -wave channel only:

$$\begin{aligned} m_{s.p.} = & (-1)^\lambda h(\lambda) [Y_{\Lambda'}(\hat{\mathbf{q}}) \otimes (k' \otimes k)_\lambda]_{\mathcal{F}, -M} R_{l'_p l_p}^{\Lambda'}(q) \\ & \times \{ [E - R + i\Gamma(E)/2 - k^2/2M^* - \frac{3}{4}\alpha(q)W_{sp}]^{-1} \langle 1 + \vec{t}_\pi \cdot \vec{t}_N \rangle \\ & + \frac{5}{4}\alpha(q)W_{sp} [E - R + i\Gamma(E)/2 - k^2/2M^* - \frac{3}{4}\beta(q)W_{sp}]^{-2} \langle 1 - \frac{3}{5}\vec{t}_\pi \cdot \vec{t}_N \rangle \}. \end{aligned} \quad (71)$$

$\alpha(q)$ and $\beta(q)$ are generally defined in Eqs. (63b) and (64). For the special case of elastic scattering or charge exchange on the valence neutron in ^{13}C we have for the non-spin-flip part of the amplitude ($\lambda=0$) $\Lambda'=0$, and we find

$$\alpha(0)=0.47, \quad \beta(0)=0.78. \quad (72)$$

With the same simplifying assumption as in (66) we find, for the ratio of second to first order terms at resonance,

$$\tilde{\rho}_{s.p.} = -\frac{5}{4}\alpha(0) \frac{|W_{sp}|}{\Gamma/2 + \frac{3}{4}\alpha(0)|W_{sp}|} \frac{\langle 1 - \frac{3}{5}\vec{t}_\pi \cdot \vec{t}_N \rangle}{\langle 1 + \vec{t}_\pi \cdot \vec{t}_N \rangle} \approx -0.43 \frac{\langle 1 - \frac{3}{5}\vec{t}_\pi \cdot \vec{t}_N \rangle}{\langle 1 + \vec{t}_\pi \cdot \vec{t}_N \rangle} = \begin{cases} -0.20 & (\pi^- \rightarrow \pi^-), \\ +0.26 & (\pi^+ \rightarrow \pi^0), \\ -1.1 & (\pi^+ \rightarrow \pi^+). \end{cases} \quad (73)$$

As for isovector transitions in closed-shell nuclei, we find for the charge exchange an enhancement of the impulse approximation amplitude by typically 20 to 30 %. Note that this model implies an almost complete cancellation of the impulse approximation due to Δ -induced transitions for π^+ elastic scattering on the valence neutron at resonance.

Our final topic in this discussion of various π -nucleus reactions is the $(\pi, \pi N)$ reaction. If again we simplify as much as possible Eq. (61) we arrive at the following expression for the ratio of second to first order terms:

$$\tilde{\rho}_{qf} \approx -\alpha \frac{|W_{sp}|}{\Gamma/2 + \alpha|W_{sp}|} \frac{\langle 1 - \vec{t}_\pi \cdot \vec{t}_N \rangle}{\langle 1 + \vec{t}_\pi \cdot \vec{t}_N \rangle} = \begin{cases} -0.08 & (\pi^+ p \rightarrow \pi^+ p), \\ 0.23 & (\pi^+ n \rightarrow \pi^0 p), \\ -0.70 & (\pi^+ n \rightarrow \pi^+ n). \end{cases} \quad (74)$$

Note the order of magnitude difference in the second order corrections to the $\pi^+ p \rightarrow \pi^+ p$ and $\pi^- p \rightarrow \pi^- p$ reactions, respectively. As can be read off from Eq. (74), this difference arises from the quasifree suppression by a factor of 3 of the $\pi^- p$ as compared to the $\pi^+ p$ impulse approximation amplitudes and the enhancement by a factor of 3 of the

corresponding second order terms. Effectively, this means that the $\pi^+ p \rightarrow \pi^+ p$ reaction is basically a quasifree reaction, while in the $\pi^- p \rightarrow \pi^- p$ reaction one and two body pieces are of equal importance.

Because of the large energy transfers involved in the knockout reactions, Eq. (74) should be taken only as an order of magnitude estimate. We have

evaluated numerically the amplitude (61). The most important kinematical correction is to take properly into account the energy transfer to the knocked-out nucleon. The effective pion energy which enters the Δ propagator is up to the hole energy equal to the incident energy in D_1 but equal to the outgoing pion energy in D_2 . This induces a non-negligible phase between first and second order terms. We have found the second order corrections to be most important for incident energies above the resonance; in this case, the second order term is enhanced since the energy transfer ΔE to the nucleon can be chosen such that the Δ is put on its "mass shell" (i.e., $E-R-\Delta E=0$). In Fig. 10 we show the ratio of cross sections for proton knockout from the p shell in ^{16}O induced by π^+ and π^- for two different kinematics ($\theta_\pi=130^\circ$, $\theta_p=-17.5^\circ$, $\phi_p=0^\circ$, solid curve, and $\theta_\pi=60^\circ$, $\theta_p=-50^\circ$, $\phi_p=0^\circ$, dashed curve) as a function of the outgoing pion energy. At the quasi-elastic peaks indicated by the arrows, we find almost no change for $\theta_\pi=130^\circ$ and a large enhancement of the ratio for $\theta_\pi=60^\circ$. In the latter case, the final Δ propagates close to its mass shell, while for $\theta_\pi=130^\circ$, the large second energy denominator suppresses strongly the Δ induced knockout. Furthermore, for $\theta_\pi=60^\circ$ we observe a drastic decrease in the ratio with decreasing outgoing pion energy. For energies below 100 MeV, the second order term becomes actually comparable to or even larger than the first order one.

With increasing momentum transfer to the residual nucleus, the second order term decreases less rapidly than the impulse approximation due to the possibility of sharing the momentum transfer between two nucleons. Thus, close to the minimum of the

FIG. 10. Ratio of 240 MeV π^+ to π^- proton knockout cross section for ^{16}O (p -shell knockout), as a function of the energy of the scattered pion. Solid curve: $\theta_\pi=130^\circ$, $\theta_p=-17.5^\circ$; dashed curve: $\theta_\pi=60^\circ$, $\theta_p=-50^\circ$ (coplanar geometry). The arrows indicate the quasielastic point.

elementary π - N cross section and at large momentum transfers, the π^-p knockout reaction is dominantly a two-nucleon process.

Before closing this section, we briefly summarize the implications for our earlier calculations of π -nucleus reactions within the DWIA framework.^{3,4} Here, the transition operator has simply been modified by the spreading potential and all other contributions to the Δ self energy. For low-lying isoscalar excitations in ^{12}C , we have seen that the Δ -induced processes double the effect of the spreading potential: According to both Ref. 3 and the estimate in Eq. (67), this leads to a 30–40% decrease of the cross section in the resonance region. Such changes will improve the agreement between theory and experiment for the 0^+ and 3^- excitations, but make it deteriorate for the 2^+ state. For isovector transitions on the contrary, we expect a near cancellation between self-energy and vertex corrections, with a corresponding increase in cross section by a similar amount. Qualitatively, this may help to explain the dramatic failure of all earlier calculations with regard to the isospin ratio of the 1^+ excitation in ^{12}C .¹⁴ Detailed numerical studies are now in progress.¹⁵ For quasielastic scattering,⁴ we expect only minor modifications of the inclusive reaction. ($\pi^+, \pi^{+'}$) is dominated to $\approx 90\%$ by π^+-p knockout, in the resonance region, where the corrections coming from Δ induced processes are only of the order of $\approx 8\%$ [Eq. (74)]. A change in the absolute normalization of $\leq 15\%$ would not affect any of the conclusions drawn in Ref. 4. The situation is of course completely different for the quasifree charge exchange reaction ($\pi^+, \pi^{0'}$), where we would predict an increase in cross section over a DWIA calculation of up to 40% [Eq. (74)]. It is interesting that very recently, a marked enhancement of the inclusive single charge exchange in ^{16}O at 160 MeV relative to the ($\pi^+, \pi^{+'}$) reaction has indeed been reported by Ashery *et al.*¹⁶ Whether quantitative calculations of this reaction in the presence of the Δ -induced processes are technically feasible remains to be seen.

V. CONCLUSIONS

In this paper we have investigated consequences of the Δ - N interaction in π -nucleus scattering. The aim of this investigation has been twofold. On the one side, we have attempted to relate the phenomenologically determined spreading potential to the interaction parameters of an assumed effective two body Δ - N interaction. On the other side, we have shown how such a two-body interaction allows us to relate the medium modifications in various π -nucleus reactions to each other and to the

medium modifications occurring in elastic scattering. By introduction of the effective Δ - N interaction as one of the building blocks rather than the Δ spreading potential, the Δ -h formalism provides, in principle, a closed framework into which the various phenomena of π -nucleus scattering can be incorporated.

With regard to elastic scattering, we have found as the main result that here all the medium corrections can effectively be described by a Δ -nucleus optical potential. This result does not imply that Δ -h vertex corrections are negligible as compared to Δ self energies. We have found, however, in both analytical calculations and numerical evaluations of the medium corrections that vertex corrections simply renormalize the strength of the spreading potential. Similarly, finite range effects in the Δ - N interaction and the resulting nonlocalities of the Δ optical potential can be accounted for by a renormalization of the strength of a local potential.

It is only in virtue of this equivalence between Δ -h vertex corrections and Δ self-energies that the Δ -h formalism can form the basis of an economical phenomenology of pion-nucleus elastic scattering. If, in particular, one, furthermore, restricts Δ - N relative orbital angular momenta L, L' to be less than 2 in either the initial or final channel (therefore allowing for up to p interactions) we have shown that the medium corrections in elastic scattering can be parametrized by three complex strength parameters multiplying the central, spin-orbit, and Kisslinger potentials. We have illustrated how the additional strength parameter associated with the Kisslinger Δ -nucleus potential can be used to improve considerably the agreement between theoretically calculated and experimentally observed angular distributions. In particular, the strong discrepancy between theory and experiment in the backward π - ^{12}C cross sections at 160 MeV can be removed. However, we have not been able to find sets of parameters which would vary smoothly with pion energy. Although we do not see any other possibility of accounting for the experimentally large backward cross sections, the supposedly strong energy variation in the Δ - N strength parameters definitely requires confirmation from analyses of π -nucleus reactions.

In our discussion of the role of the Δ - N interaction in π -nucleus reactions we have not performed a similarly detailed evaluation of the medium corrections for specific reactions. We rather have focused on relations between medium corrections corresponding to various reactions and have given the order of magnitude of the expected effects.

In contrast to elastic scattering, the effect of the Δ - N interaction cannot be parametrized, in general, into a few optical potential strength parameters.

This follows from the fact that in inelastic scattering, the Δ - N interaction not only modifies the Δ propagation through the nuclear medium but in addition plays an active role in inducing nuclear transitions. Roughly speaking, in these additional processes, the nuclear transition occurs through a hard Δ - N collision rather than through the decay of the Δ -h state into pion and excited nuclear states. Depending on the quantum numbers of the excited state certain components of the Δ - N interaction may get enhanced, other components suppressed. Only in isoscalar longitudinal excitations are the medium corrections determined by the same superposition of the various components of the Δ - N interaction which also generates the Δ spreading potential in elastic scattering. In isovector transitions or spin-flip transitions (coupling to the magnetization current) the effects of the Δ - N interaction can differ significantly from those in elastic scattering or longitudinal isoscalar excitations. Of particular interest in this context are cross section ratios corresponding to nuclear transitions which differ only in isospin ($\rho_{\Delta T} = \sigma_{T=0}/\sigma_{T=1}$) or corresponding to the various charge states in the exclusive knockout reaction,

$$\rho_{\text{qf}} = \sigma(\pi^+, \pi^+ p) / \sigma(\pi^-, \pi^- p) .$$

In both cases, deviations from the quasifree values of 4 and 9, respectively, occur exclusively via the $T=1$ component of the Δ - N interaction.

Although, in general, this interplay between the quantum numbers of the nuclear excited states and the channel quantum numbers of the Δ - N interaction makes any detailed prediction dependent on the specific model for the Δ - N interaction, the model dependence is strongly reduced for inelastic reactions very close to resonance. Here first and second order terms interfere only via the absorptive part of the Δ - N interaction. This absorptive part, however, is known to be dominantly generated through coupling to the pion absorption channel. The value of this $T=1$ absorptive component can therefore be inferred from the imaginary part of the Δ spreading potential. In this sense, medium corrections to inelastic scattering at resonance are implicitly related to the true pion absorption cross sections which have been predicted within the Δ -h formalism.

Thus, our order of magnitude estimate of the medium corrections at resonances involves only known strength parameters. We have found for $\Delta J = 1, 2$ transitions that $\Delta T = 1$ amplitudes are enhanced over the first order values by typically 20% and that the $\Delta T = 0$ amplitudes are suppressed by the same amount. Thus, intermediate coupling to the pion absorption channel diminishes the large quasifree differences between $\Delta T = 0$ and $\Delta T = 1$ excitations.

In the knockout reaction at quasifree kinematics, the Δ - N interaction has the opposite effect. It suppresses even more the already small π^-p cross sections leading to ratios $\sigma(\pi^+, \pi^+p)/\sigma(\pi^-, \pi^-p)$ which are up to a factor of 2 larger than the quasifree value of 9.

The strongest experimental evidence for medium corrections induced by the Δ - N interaction comes from the measured ratio of the knockout cross sections corresponding to different charge states.^{17,18} Relative suppressions of the π^-p cross section of up to a factor of 2 at the quasifree peak and enhancement of the π^-p channel in the tails of the quasifree peak have been observed in qualitative agreement with our estimate.

Similarly, the measured ratio of 1^+ excitation cross sections in ^{12}C (Ref. 14) indicate the presence of medium corrections to the impulse approximation which are comparable in size to those obtained in our calculations.

Only in such ratios can the experiment provide directly a quantitative measure of the importance of second order processes, although, in principle, any cross section for nuclear transitions of low multipolarity should contain, at resonance, according to our estimates, a 30–40% contribution from second order processes. To demonstrate the presence of these corrections requires, in general, a rather involved evaluation of the first order term which has to include all the medium corrections present in elastic scattering. Such calculations have been performed for very few cases only. The low lying isoscalar longitudinal excitations in ^{12}C do not show uniformly significant deviations between experiment and first order calculations. Inclusion of second order processes will decrease the first order values and thereby remove existing discrepancies in the 0^+ and 3^- cross sections. They, however, will produce discrepancies between theory and experiment in the 2^+ excitation. With the exception of the 1^+ excitations, discussed above, in all other measured and calculated excitations in ^{12}C , nuclear structure uncertainties or large uncertainties in the pion data prohibit any definite conclusion.

The only other carefully investigated transition which is of relevance here is the exclusive charge exchange reaction in ^{13}C . Forward charge exchange scattering on ^{13}C corresponds to a $\Delta\mathcal{J}=0$ transition which makes this reaction particularly sensitive to medium modifications. As in every $\Delta T=1$ transition, second order processes enhance, at resonance, the cross section. Our estimated 25% increase of the first order amplitude, calculated within the Δ - h formalism,¹⁹ will remove a large fraction of the existing discrepancy.

To summarize, there is strong experimental evidence for violations of quasifree ratios at resonance which in sign and size are compatible with medium modifications induced by the absorptive $T=1$ component of the Δ - N interaction. Cross-sections for $\Delta T=1$ excitations in ^{12}C and ^{13}C exceed the predictions of the first order Δ - h calculations by an amount which appears to be compatible with Δ - N interaction induced corrections. Experimental $\Delta T=0$ cross sections corresponding to the 0^+ , 3^- , and 1^+ excitations in ^{12}C are smaller than the calculated first order cross sections. Again, second order corrections will remove at least part of the existing discrepancy. Only for the $\Delta T=0$ 2^+ excitation in ^{12}C , does there seem to be no evidence for any second order correction. First order Δ - h calculation and experiment agree almost completely near the first maximum.³ Inclusion of second order correction will lead, at resonance, to a discrepancy between theory and experiment of about 30%.

These experimental indications require further confirmations. In particular it still has to be established experimentally that the discussed violations of quasifree isospin ratios or the enhancement of $\Delta T=1$ excitations are a general phenomenon valid for reactions on any $T=0$ nucleus, as our theoretical description implies. On the theoretical side, technically complicated detailed calculations are unfortunately unavoidable if the already existing experimental material is to be used to specify the Δ - N interaction beyond the average strength parameters obtained already from the analysis of elastic scattering.

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