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Private road supply in networks with heterogeneous users

Xinying Fu^a, Vincent A.C. van den Berg^{a,b,*}, Erik T. Verhoef^{a,b}

^a Vrije Universiteit Amsterdam, De Boelelaan 1105, 1081HV Amsterdam, the Netherlands

^b Tinbergen Institute, Gustav Mahlerplein 117, 1082 MS Amsterdam, the Netherlands

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ABSTRACT

We study different mixes of private and public supply of roads in a network with bottleneck congestion and heterogeneous users. There are two parallel links for one origin and destination pair and two groups of travellers, where the group with the higher value of time also has higher schedule delay values. Previous scholars argued that as users become more heterogeneous, they benefit more from product differentiation, making private supply of roads more efficient. However, we find that local monopoly power might also increase if there is a ‘separating equilibrium’, which is an equilibrium where at least one group only uses one private road due to the different combinations of toll and congestion of the two roads. The private road can thus increase its toll without worrying about the competition from the other road for this group: it has a local monopoly over them. This lowers the efficiency of private supply. The problem is especially severe with flat tolls, which are constant over the peak. With fine tolls – which vary continuously over the day – there tends to be a pooling equilibrium – where both types use both roads – and competition remains intense. Flat tolling is also worse for users than fine tolling, as it has higher generalized prices.

1. Introduction

Interest in traffic congestion management, including road pricing, is increasing with fast urbanization (Hensher and Puckett, 2007), and private supply of roads is considered by many scholars as a viable complement to the public supply of roads. Often mentioned considerations include shortages of public funds and higher operational efficiency of private sector firms. However, the possibility of excessive pricing under market power calls for caution. Winston (2013) argued that the under-performing US road system needs either massive improved public provision or an expansion of the role of the private sector. However, massive US public investment seems problematic at the moment.

The trade-off between public and private supply is complicated by user heterogeneity, where some people prefer a lower level of congestion at the cost of a higher toll, while others prefer a lower toll accepting a higher level of congestion. When preferences become more heterogeneous, on the one hand, product differentiation offered by different road providers can be expected to make travellers better off; on the other hand, different products become less close substitutes, and the increasing local monopoly market power of the operator may make travellers worse off. We study the impact of such trade-offs in various private, public and mixed regimes, using the bottleneck congestion model.

We develop a model that is as simple as possible, but not any simpler than that, to study the impacts of heterogeneity on the efficiency impacts of private road supply with parallel routes, with the objective to generate insights of a general nature; but not any

* Corresponding author at: Vrije Universiteit Amsterdam, De Boelelaan 1105, 1081HV Amsterdam, the Netherlands.

E-mail address: v.a.c.vanden.berg@vu.nl (V.A.C. van den Berg).

larger networks, which would make generalization more difficult. Yet, as Verhoef (2002) and Yang and Meng (2002) found for static networks, the insights obtained for such simple settings will often carry over to more complicated settings, where otherwise interpretation of results would be impossible. The problems studied seem highly relevant for contemporary transport policies. There is a growing interest in private supply of roads throughout the world. At the same time, only few would advocate a full privatization of roads, if only to limit undesirable concentration of market power. Competition with unpriced public roads will therefore be the rule rather than the exception. Once tolling has been introduced in a network it may be a more feasible option to consider public pricing instead of free supply of public infrastructure. Our paper gives insights that should help making this choice on a more informed basis.^{1,2}

The key assumption of the bottleneck model is that congestion cost comes from time wasted waiting in traffic jams and from schedule delays (i.e., the inconvenience of arriving at the destination earlier or later than desired). People in general differ in their values of time and schedule delay. As we will see this has important impacts. We consider two groups of commuters, and use ‘proportional heterogeneity’, as in Vickrey (1973) and Van den Berg and Verhoef (2011), where the values of time and schedule delay vary in a fixed proportion. This heterogeneity could stem from differences in income for otherwise identical preferences. The value of time is the ratio of the marginal utility of travel time to the marginal utility of income, and similarly for the values of schedule delay. A higher income decreases the marginal utility of income, thereby increasing all values by the same percentage. How people’s preferences trade-off travel time and schedule delay probably depends more on the type of trip (e.g. shopping vs doctors visit), family status, gender and job type.³

We examine two kinds of tolls: flat tolls that are constant over the peak, and fine tolls that vary continuously over the peak period so as to eliminate queueing. We find two types of equilibria: pooling and separating equilibria. In a pooling equilibrium, both types of users travel on both roads, as for both types the generalized price is the same on both roads. In a separating equilibrium, at least one type travels exclusively on one road.

As users become more heterogeneous, a separating equilibrium means that it becomes ever more unlikely that someone will switch to a different road if its type only travels on one road. As a result, for competing suppliers on parallel links, local monopoly power increases with user heterogeneity: these users will not want to switch to the other road, so in effect there is no competition for this type of user as long as the toll does not get too high. This local monopoly power lowers social welfare. On the other hand, with fine tolls, welfare and profit maximisation both result in a pooling equilibrium. Such a market structure means that both roads compete intensely for both types even if user heterogeneity increases. Accordingly, welfare increases with the degree of heterogeneity. The fine toll equilibrium often also gives a private supplier a higher profit than a flat toll equilibrium, as the time-varying part of the fine toll gives toll revenues without raising the generalized price of users by removing all queueing.

Our paper is related to three strands of literature. The first is the large literature on bottleneck congestion and user heterogeneity. Vickrey (1973) was the first, and studied proportional heterogeneity just as we do. Arnott et al. (1992) considered proportional heterogeneity and ‘ratio/flexibility’, heterogeneity, where the ratios of the value of time and values of schedule delay varies over users. With flat tolling, separation was found to be socially optimal if the degree of ‘ratio/flexibility’ heterogeneity is not too large compared to the degree of proportional heterogeneity. Separation is always optimal if the high-values type is also more inflexible.⁴ Yang and Meng (1998) developed socially optimal variable tolls for a network with heterogeneous users. Van den Berg and Verhoef (2011) assumed continuous heterogeneity both in values of time and schedule delay, and studied the welfare and distributional effects of fine tolls. Cantos-Sánchez et al. (2011) examined the viability of a new road into Madrid city centre with heterogeneous users. General heterogeneity has been studied by, for example, Newell (1987), Lindsey (2004), Wu and Huang (2015), Liu et al. (2015) and Takayama and Kuwahara (2017).

The second strand is on private roads with bottleneck congestion and homogeneous users. Arnott et al. (1992) and de Palma and Lindsey (2002) demonstrated that private supply of roads generally enhances social welfare when congestion is severe. The efficiency is higher when both routes have fine tolls, as these eliminate queueing. Our study shows, in addition, that with heterogeneity, a fine toll is more likely than a flat toll to generate a pooling equilibrium and thereby promotes competition. With homogeneous preferences, de Palma and Lindsey (2002) also considered if a private supplier will choose to set a fine toll or a flat toll, whereas a public operator will always prefer to set a fine toll.

The final strand of literature is on static congestion and heterogeneous users. Edelson (1971) considered a monopolist private road with heterogeneous users, concluding that it may set a toll that is lower than socially optimal. Yang et al. (2002) considered a private monopolist operator in a network. Small and Yan (2001) and Verhoef and Small (2004) found that heterogeneity improves the performance of second-best and private regimes, as product differentiation better caters for the specific preferences of each group.

¹ In countries such as France and Spain, it is mandatory that there is an untolled public route parallel to tolled motorways. In the USA, on single motorways, it is common to see tolled pay-lanes and untolled free-lanes. So competition between (semi-) private and untolled roads is already uncommon.

² We investigate two objectives: pure welfare maximisation and pure profit maximisation. Firms may also care about consumers or the environment. In Public-Private Partnership (PPP) projects or with governmentally owned private firms the objective may be a mixture of welfare and profit. Finally, government or private employees maximise their own well-being instead of societal welfare or profit.

³ A surgeon or person in high finance often have strict work start times, while professors often do not. A shop-salesperson may have strict starting times, but a food deliverer or an (independent) plumber much less. Even in the centre peak, a large fraction of trips is for other activities such as trips shopping, visits and leisure – which all have flexible timings – or doctor and hospital appointments – which have very inflexible timings.

⁴ The terms proportional and ratio heterogeneity were later introduced by Van den Berg and Verhoef (2011) and Hall (2013) introduced the term flexibility heterogeneity to mean the same as ratio heterogeneity.

Competing private roads also generally offer different travel times and tolls, since differentiation raises profits (Luski, 1976; Calcott and Yao, 2005). Winston and Yan (2011) argued that full private supply of motorways can leave users better off than the current system with unpriced roads that offers no product differentiation. Yang and Huang (2004) and Holguín-Veras and Cetin (2009) studied multiple vehicle classes in a network.

The remainder of the paper is organized as follows: Section 2 briefly explains the model set-up. Section 3 derives, for both tolls, the optimal equilibrium for a welfare-maximising social planner and a profit-maximising monopolist in a network of two parallel links. For more complex ownership structures on this network, theoretical results are not clear-cut. Hence, Section 4 shows the simulation results for various ownership structures in the network: namely public, private and mixed supply of roads. Section 5 discusses the main limitations and concludes.

2. Model set-up

We assume there are two groups of travellers, denoted by $i = h, l$, where the h group has the higher values. The group size is N^i . Each group has an independent inverse demand function, $D^i(N^i)$, which is decreasing in N^i . Two parallel links, denoted by $j = 1, 2$, serve one origin and destination pair, whose bottleneck capacity is s_j .

The bottleneck model assumes that people dislike waiting in traffic jams, and dislike arriving either earlier or later than the desired arrival time, t^* . A person's congestion cost consists of two parts. The first part is the time costs associated with waiting at the bottleneck, which is the product of value of time, denoted by α , and time spent waiting, denoted by $T(t)$. The second part is the schedule delay costs. For people arriving early, the cost is the product of how early they arrive, measured by $t - t^*$, and the schedule delay value of arriving early, denoted by β . For people arriving late, the schedule delay cost is defined similarly, where γ denotes the schedule delay value of arriving late. In sum, the congestion cost for arriving at time t is:

$$C(t) = \alpha T(t) + \begin{cases} \beta(t^* - t) & \text{if } t \leq t^* \\ \gamma(t - t^*) & \text{if } t > t^* \end{cases}$$

For ease of analysis, we assume the two groups have proportional values of time and schedule delay, i.e. $\frac{\alpha^h}{\alpha^l} = \frac{\beta^h}{\beta^l} = \frac{\gamma^h}{\gamma^l}$ and $\alpha^h > \alpha^l$. This implies that people with a higher value of time also have a higher value of schedule delay. Given that all values have the marginal utility of income in the denominator, this pattern could result from income differences for otherwise identical individuals (Van den Berg and Verhoef, 2011). We will discuss the impact of relaxing these constraints later on.

3. Theoretical results

In this section, we study social welfare maximisation and monopolistic profit maximisation. The equilibrium characteristics of these two benchmark regimes carry over to more complex ownership structures, as shown later in the simulation section. We find that with flat tolls, social welfare maximisation for a social planner and profit maximisation for a monopolist result in a separating equilibrium. But with fine tolls, both settings result in a pooling equilibrium. With a pooling equilibrium, all types travel on both links. With a separating equilibrium, at least one type strictly prefers one road over the other. With full separation, each type solely uses its own road that it strictly prefers. With partial separation, one type solely uses one road, while the other uses both. We will not distinguish between the two forms of separated equilibrium: with both forms, one road has some captured users for whom it faces no competition from the other, and thus it has a 'local monopoly'. When the operator of this road raises its toll slightly, these users' only choice is between paying the higher toll or not travelling; the other road would remain unused by them and so given no competition for this type.

The next section will analyse flat tolls. The section thereafter will study fine tolls.

3.1. Flat toll

A flat toll is one that is constant over time. With proportional heterogeneity, when types of travellers use the same link, they travel jointly in time and cannot be distinguished by the time they enter. The equilibrium growth rates of the queueing time of group i are, $\frac{\beta^i}{\alpha^i}$ for early arrivals and $-\frac{\gamma^i}{\alpha^i}$ for late ones, and these ratios are constant with proportional heterogeneity. There is, therefore, no temporal separation between different types if they use the same link.

For social welfare and monopolistic profit maximisation, we show why the result is a separating equilibrium, where at least one group travel exclusively on one road only. We provide both mathematical proof and an intuitive explanation. Intuitively, we can show that starting with a pooling equilibrium, social welfare or total revenue can be further increased.

Following conventions, we use the following compound reference parameter $\delta^i = \frac{\beta^i \gamma^i}{\beta^i + \gamma^i}$. The congestion cost for group i on link j , as derived in Van den Berg and Verhoef (2011), is:

$$\begin{aligned} C_j^h &= \delta^h \frac{N_j^h + N_j^l}{s_j} \\ C_j^l &= \delta^l \frac{N_j^h + N_j^l}{s_j} \end{aligned} \quad (1)$$

More travellers on a link leads to higher congestion costs. Both types experience the same delays over time, but bear different congestion costs depending on the type-specific parameter δ^i . The link-group-specific toll is τ_j^i and the group-specific inverse demand functions is $D^i(N^h)$.

A social planner aims to maximise social welfare, which is the total consumer willingness to pay minus the congestion costs. When both types travel on both roads, i.e. $N_j^i > 0$, the social welfare is as follows:

$$SW = \int_0^{N_1^h+N_2^h} D^h(n)dn + \int_0^{N_1^l+N_2^l} D^l(n)dn - C_1^h N_1^h - C_1^l N_1^l - C_2^h N_2^h - C_2^l N_2^l \quad (2)$$

Proposition 1. *With a flat toll, to attain the highest social welfare, the equilibrium in traffic flows must be a (partially or fully) separated equilibrium.*

The detailed mathematical proof is in [Appendix A](#). For the candidate pooling optimum, the first order conditions imply that the traffic flow ratio of each group is equal to the link-capacity ratio, i.e. $\frac{N_1^h}{N_2^h} = \frac{N_1^l}{N_2^l} = \frac{s_1}{s_2}$. However, the second-order condition for a local maximum is then not satisfied, because the Hessian matrix is not negative definite.

Intuitively, starting from the above pooling equilibrium, which satisfies the first-order conditions, we can increase social welfare further in two steps.

In the first step, we move a h type user from link 2 to link 1 and move a l type from link 1 to link 2. In this way, both the total consumer benefit and the congestion cost on both links remain the same, so welfare is unchanged. Keep moving travellers in this way, until we arrive at a partial separating equilibrium.⁵ Although this separating equilibrium has the same congestion costs as the original pooling equilibrium, now the marginal social costs (MSC) differ: l type users have a lower MSC, and h type users have a bit higher MSC. So in the second step, we can increase welfare by increasing the number of l users and decreasing the number of h users. The candidate pooled optimum is akin to a saddle point but where welfare is first flat in division of user types over the routes. We first need to switch a mass of users between roads – while keeping welfare constant – before the derivatives of welfare to N^h and N^l become non-zero again.

So we have shown that a pooling equilibrium does not maximise social welfare, and thus it is not optimal for a social planner. The same logic applies to profit maximisation. Starting from a pooling equilibrium, the monopolistic profit can always be improved in two similar steps as in the social welfare maximisation problem. So a pooling equilibrium would also not be optimal for monopolistic profit maximisation.

That it is welfare maximising and profit maximising to have a separating equilibrium, often also holds for general heterogeneity. [Arnott et al. \(1992\)](#) consider proportional heterogeneity and ‘ratio heterogeneity’ between the value of time and values of schedule delay. With flat tolling, separation is still socially optimal, if the high-values type is also less flexible (i.e. $\alpha^h/\alpha^l < \beta^h/\beta^l$), or if it is not too flexible compared to the degree of proportional heterogeneity (i.e. $\beta^h/\beta^l \geq \alpha^h/\alpha^l > 2\sqrt{\beta^h/\beta^l} - 1$).

With continuous heterogeneity, it remains likely to be optimal to have separation with some user types solely using the one road and the others the other. This occurred with two dimensions of continuous heterogeneity in [Van den Berg and Verhoef \(2011\)](#) with public or private tolling with an untolled alternative. And similarly also in [Verhoef and Small \(2004\)](#) under static congestion and a heterogeneous value of time. However, especially in a competitive setting where both roads are tolled, a flat toll may lead to a higher profit than a fine toll, see also [de Palma and Lindsey \(2002\)](#) with homogeneous users.

3.2. Fine tolls

A fine toll varies continuously over time so as to eliminate queueing. Accordingly, the departure and arrival ratio on link j is s_j throughout the peak. The initial level of the toll (i.e. a flat component paid by all drivers on top of the time-varying component) can be set by the road operator. If both groups use the same link, h types travel at the centre of the peak, while l types travel at the tails under time-varying tolling. The toll rises at a rate β^i for early arrivals when type i travels, and falls at a rate γ^i for late arrivals. A type h driver would find it attractive to move towards t^* at any moment where type l drivers are in equilibrium. Reversely, a type l driver would move away from t^* when travelling with type h drivers. Hence, temporal separation will result spontaneously under time-varying tolling.

For social welfare and monopoly profit maximisation, we show that a pooling equilibrium is optimal with a fine toll. Our proof only assumes that the preferred arrival time and ratio β^i/γ^i are homogeneous, so the result also holds for more general heterogeneity. All queueing is eliminated, so users only experience tolls and schedule delay costs. The average congestion equal cost equals the average schedule delay cost. It follows from [Van den Berg and Verhoef \(2011\)](#)⁶:

⁵ All h types travel on link 1 when initially $N_2^h < N_1^l$, or all l types travel on link 2 when initially $N_2^h > N_1^l$.

⁶ Of each type, a fraction $\gamma^i/(\beta^i + \gamma^i)$ arrives early and the remainder late. As β^i/γ^i is fixed and users arrive a constant rate, the schedule delay for type h varies uniformly between 0 and $\delta^h N_j^h/s_j$, and for type l between $\delta^l N_j^h/s_j$ and $\delta^l N_j^h/s_j + \delta^l N_j^l/s_j$. Also making β^i/γ^i heterogeneous, means several possible temporal equilibria where each has comparable costs as here.

$$\begin{aligned}
C_j^h &= \delta_j^h \frac{N_j^h}{2s_j} \\
C_j^l &= \delta_j^l \frac{N_j^h}{s_j} + \delta_j^l \frac{N_j^l}{2s_j}
\end{aligned} \tag{3}$$

The factor $\frac{1}{2}$ stems from the fact that the travel delay cost is fully replaced by tolls: not a social cost, but a transfer. The term containing N_j^h for l types reflects the additional schedule delay costs that l types face because the h types occupy the central peak. Yet, as noted, they strictly prefer to use the shoulders, as the time-variant part of the toll in the centre peak is too high and steep for them.

When both types travel on both roads (all $N_j^i > 0$), the social welfare is:

$$\begin{aligned}
SW &= \int_0^{N_1^h+N_2^h} D^h(n)dn + \int_0^{N_1^l+N_2^l} D^l(n)dn \\
&\quad - C_1^h N_1^h - C_1^l N_1^l - C_2^h N_2^h - C_2^l N_2^l
\end{aligned} \tag{4}$$

Proposition 2. *With a fine toll, to attain the highest social welfare, the equilibrium in traffic flows must be a pooling equilibrium.*

The detailed mathematical proof is in [Appendix B](#). The solution to the first order conditions implies a pooling equilibrium and the second order condition of a negative definite Hessian guarantees it is the local maximum. The resulting equilibrium initial toll is $\tau_j^i = 0$, the time-variant part of the toll already ensures that for both types the inverse demand equals the marginal social cost, as is also true in the standard bottleneck model with homogeneous users.

Similarly, we can prove that with fine tolling, the profit maximising equilibrium is also a pooling one with a group-specific toll $\tau^i = \frac{\partial D^i(N^i)}{\partial N^i} N^i$. This is the conventional mark-up of a monopoly (see also [de Palma and Lindsey \(2002\)](#)). The overall toll is the mark-up plus the time-variant marginal external cost. It is similar to static congestion ([Small and Verhoef, 2007](#)).

That with fine tolls a pooling equilibrium is optimal for both the social planner and the monopolist, does not depend on the assumption of proportional cost parameters. The logic can be applied to a general heterogeneity structure, and it stems from the notion that with eliminated queues, the best ordering of travellers is such that those with the highest schedule delay value travel closest to the preferred arrival time on both bottlenecks.

In sum, for two parallel links and welfare/monopoly profit maximisation, flat tolls and fine tolls generate different equilibria, i.e. separating versus pooling. We will show in the next section that those differences remain for more complex ownership structures on the network. As a result, flat tolls and fine tolls also have different implication for market structure and performance, which are interesting for both policy makers and practitioners. In addition, when different road operator interact with each other, sometimes Nash equilibrium does not exist and the results depend on the parameters used. So we will use simulations to examine more complex ownership structure on the network in the next section.

4. Simulation

To examine and compare more complex ownership structure on the network, such as public, private and mixed regimes, we use simulation. We consider the following seven regimes: free, public, monopoly, free-public, free-private, private-public and private-private. In the first three cases, one operator controls both links. The free regime, where both roads are free of tolling, is likely to be the status quo and a good starting point for comparison. The public regime aims to achieve the highest possible social welfare, and the efficiency of all other regimes can be evaluated against it. The monopoly regime aims to maximise joint profits, and can provide useful information for private supply of roads. However, in reality often only part of the network can be tolled, so it is important to also study the next two cases, where link 1 is left free of charge, but an operator on link 2 can charge a toll. We distinguish between a private and a public provider on link 2, because they have significantly different implications for travellers. Finally we analyse two competitive regimes, where the two roads are tolled by different operators, because competition is usually believed to increase efficiency. With competitive regimes, a private operator sets a toll on link 1, and either a public or private operator offers a competing service on link 2.

The parameters we use are: $\alpha^h = 10.59$, $\beta^h = 6.45$, $\gamma^h = 25.16$, and $\frac{\alpha^l}{\alpha^h} = 0.9$. This ensures that the ratios between the cost parameters are as in [Small \(1982\)](#), and the average value of time in the free regime is 10. The inverse demand function is linear, and is constructed to ensure that in the free regime, the price elasticity of demand is -0.4 for both groups, and the number of users is 1200 for h types and 1500 for l types. The capacity of link 1 is 4000 and that of link 2 is 8000.

In the simulation, we only consider link-specific tolls, τ_j , not link-group-specific tolls. This is because when we cannot tell the groups apart, there can be an incentive compatibility problem of a link-group-specific toll, where a member of one group can pretend to belong to the other group. A link-specific toll is also easier to implement in real life than a link-group-specific toll, due to costly verification for user types, even if this were technically possible.

4.1. Simulation: flat toll

4.1.1. Flat toll: base equilibrium

[Table 1](#) shows the equilibrium values of the key factors in all seven regimes. Similar to the theoretical results, we observe mostly separating equilibria, where at least one of the two links is only used by one type of travellers (see [Table 2](#)).

In the first three cases, at most one operator charges tolls on both roads. When both roads are free of charge, a pooling equilibrium

Table 1
Flat toll.

Pricing regime	Free	Public	Monopoly	Free-Pub	Free-Priv	Priv-Pub	Priv-Priv
τ_1	0	8.74	19.22	0	0	10.39	10.90
τ_2	0	8.37	18.97	3.53	11.03	9.32	12.71
N_1^h	4000	6805	4370	0	818	6165	0
N_2^h	8000	2737	2117	11,367	9000	3031	8520
N_1^l	5000	0	0	10,493	12,272	0	7189
N_2^l	10,000	11,444	7015	3512	0	10,958	2727
Π_1	0	59,446	83,967	0	0	64,046	78,388
Π_2	0	118,682	173,175	52,488	99,265	130,429	142,976
CS^h	173,321	109,583	50,650	155,512	116,025	101,794	87,367
CS^l	194,986	113,501	42,642	169,975	130,528	104,051	85,216
SW	368,307	401,213	350,434	377,975	345,818	400,321	393,947
ω	0	0.218	−0.118	0.064	−0.149	0.212	0.170

Flat toll (τ_j), traffic flow (N_j^i), profit on road j (Π_j), consumer surplus of type i (CS^i), social welfare (SW) and efficiency ($\omega = \frac{SW - SW^{Free}}{SW^{FirstBest} - SW^{Free}}$).

Table 2
Fine toll.

	Free	Public	Monopoly	Free-Pub	Free-Priv	Priv-Pub	Priv-Priv
τ_1	0	0	15.92	0	0	2.83	3.55
τ_2	0	0	15.92	−3.99	9.71	1.32	7.10
n_1^h	4000	4077	2507	0	0	3897	3494
n_2^h	8000	8155	5014	12,919	10,080	7794	6988
n_1^l	5000	4986	2766	7368	12,600	3862	6209
n_2^l	10,000	9972	5532	8721	0	10,332	6274
Π_1	0	48,525	100,425	0	0	57,724	89,624
Π_2	0	97,049	200,850	54,308	130,448	120,869	146,526
CS^h	173,321	180,092	68,075	200,885	122,295	164,528	132,230
CS^l	194,986	193,890	59,665	224,304	137,582	174,588	135,039
SW	368,307	519,555	429,015	479,497	390,325	517,708	503,419
ω	0	1	0.401	0.735	0.146	0.988	0.893

Flat part of the toll (τ_j), traffic flow (N_j^i), profit on road j (Π_j), consumer surplus (CS^i), social welfare (SW) and the efficiency measure ($\omega = \frac{SW - SW^{Free}}{SW^{FirstBest} - SW^{Free}}$).

results both in the temporal sense as between the roads, since both groups require the same growth rate of the queue to be in equilibrium, and hence have no group-specific preferences of one used arrival moment over the other. There is therefore actually a continuum of equilibria at the group-route level, as long as the ratio of aggregate traffic to capacity is the same on both roads.

When a social planner charges link-specific tolls on both roads, social welfare increases by 10% compared to the free regime. Due to the higher price and the lower congestion on link 1, the h types travel on both links and the l types only travel on link 2. Consumer surplus decreases, but the increase in toll revenue more than compensates it. When a private operator sets tolls to maximise the joint toll revenue of both roads, the tolls are more than twice those in the public regime. A sharp drop in consumer surplus leads to social welfare that is lower than in the free regime.

In the next two cases, link 1 is free but an operator can charge a toll on link 2. In the free-public regime, the increase in social welfare compared to the free regime is moderate, because only one third of the total road capacity is tolled. The consumer surplus is the second highest among all regimes. On the contrary, in the free-private regime, a private provider on link 2 decreases social welfare compared to the free regime. It is even worse than the monopoly regime, because the monopolist of both roads wants to reduce congestion on link 1 in order to raise total revenue, while the private operator on link 2 only cares about profit on link 2 and causes too much congestion on link 1. This result was also found in a static single-type setting (Verhoef et al., 1996).

The last two cases show the effect of competition between two operators. We assume Nash behaviour between operators. When a public operator on link 2 competes with a private operator on link 1, the efficiency and consumer surplus are nearly as high as in the public regime. When two private operators compete, we can achieve 17% of the welfare gain we get from moving from the free regime to the first best regime (public pricing with fine tolls). The private-private regime has even higher efficiency than the free-public regime, because the higher tolls greatly reduce congestion. On the other hand, the consumer surplus in the private-private regime is much less than in the free-public regime.

Fig. 1. Flat toll: heterogeneity and efficiency. Efficiency (ω), degree of heterogeneity ($\frac{\alpha^h}{\alpha^l}$).

4.1.2. Flat toll: heterogeneity and efficiency

What happens to the welfare effects of the regimes when the two groups become more different in terms of value of time and schedule delay value; in other words, as the degree of heterogeneity (defined as $\frac{\alpha^h}{\alpha^l}$) increases?⁷ Fig. 1 shows the result in terms of the relative efficiency.

The free regime has an efficiency of zero. The competing private-private regime does not have a Nash equilibrium for some parameters, due to a discontinuity in the pay-off function that arises when a marginal change in the toll leads to a switch between a fully separated equilibrium (each group uses one bottleneck exclusively) and a partially separated equilibrium (one groups uses both bottlenecks, one group uses only one). So these two regimes are omitted (Appendix A).⁸

Fig. 1 shows that for the remaining five regimes, the efficiency generally increases with the degree of heterogeneity. Similar as in Small and Yan (2001) with static congestion, when the two groups' values of time and schedule delay become more different, product differentiation is more appreciated by both groups. In other words, h types prefer the link with a higher toll and lower congestion level, while l types prefer the link with a lower toll and higher congestion level. This increases the efficiency of free-private regime. However, we also find that the efficiency of private-public regime decreases over a certain range with the degree of heterogeneity, due to increasing local monopoly power. More specifically, as the groups become more different, the optimal toll on link 2 has to be very high to correct for the congestion externality of the h group. This leaves the l group travelling on link 1, and subject to the monopoly power of the private supplier.

In addition, the efficiency does not change monotonically with the degree of heterogeneity, because there can be regime changes. When $\frac{\alpha^h}{\alpha^l} = 1$, both types are the same, so a pooling equilibrium results for all regimes. But as $\frac{\alpha^h}{\alpha^l}$ increases, separating equilibrium starts to emerge, hence the kinks. For the monopoly case, a regime change happens again at $\frac{\alpha^h}{\alpha^l} = 1.83$, where the most profitable equilibrium changes from partial separating to full separating (see Appendix A). As a result, efficiency drops at that point because full separating equilibrium means more local monopoly power. Similar is true for public and private-public regimes. The free-public and free-private regime do not show efficiency decrease, because the free road offers relatively strong competition.

4.1.3. Flat toll: heterogeneity and distributional effects

Apart from social welfare concerns, policy makers and practitioners are also interested in the distributional effects of pricing, not in the least place because it has a strong impact on the social and political feasibility. A project is likely to meet resistance from travellers if they are made worse off. The percentage change in generalized price for both types are summarized in Figs. 2 and 3 respectively.

First of all, for both types all pricing regimes with a flat toll lead to a higher price. So all projects are likely to meet resistance from travellers unless the allocation of toll revenues convinces them otherwise. Secondly, as the degree of heterogeneity increases ($\frac{\alpha^h}{\alpha^l} > 3$ in the both figures), each link becomes more specialized in serving one group of travellers: h types travel on link 2 and l types travel on link 1. That is why the price for h types is the same for the public, free-public and private-public regimes, and that for l types is the same for the free-public and free-private regimes. As a result, the percentage price change for h types increases with the degree of heterogeneity in the free-private regime, because the private operator on link 2 has increasingly more market power over h types. Similarly, the percentage price change for l types increases most significantly with monopoly and private-public regimes. And finally,

⁷ To make the results comparable when changing cost parameters, we also change the intercept of the inverse demand function, so as to keep the slope and the number of travellers at the free regime unchanged.

⁸ If a Nash equilibrium exists, its efficiency is bounded above by the private-public case.

Fig. 2. Flat toll: heterogeneity and percentage price change (H types). Percentage change in generalized price ($\frac{100(P^H - P_{free}^H)}{P_{free}^H}$), degree of heterogeneity ($\frac{\alpha^H}{\alpha^L}$).

Fig. 3. Flat toll: heterogeneity and percentage price change (L types). Percentage change in generalized price ($\frac{100(P^L - P_{free}^L)}{P_{free}^L}$), degree of heterogeneity ($\frac{\alpha^H}{\alpha^L}$).

the percentage change in generalized prices are mostly decreasing for h types, but increasing for l types. This is because the prices in free regime (P_{free}^i) increases for h types but decreases for l types when the degree of heterogeneity increases. We observe a kink for h types in the free-public regime, because the increase in price in the free-public regime is smaller compared with that in the free regime between regime changes for h types.

In sum, for the parameters used in the simulation for flat tolls, monopoly performs consistently worse than the free regime, while the free-private regime performs better than the free regime when the degree of heterogeneity is sufficiently high, due to the benefit of product differentiation. Private-public is better than free-regime, because of competition between the two links. However, the competition effect can be undermined by local monopoly power when the two types become so different that they travel on separate links. If the most efficient public regime is not available, the free-public regime is the best in terms of social welfare when heterogeneity is large and private-public regime is the best when heterogeneity is small so that the private operator has less market power. Finally, the generalized price for both types are lowest with the free regime.

Fig. 4. Fine toll: heterogeneity and efficiency. Efficiency (ω), degree of heterogeneity ($\frac{\alpha^H}{\alpha^L}$).

4.2. Simulation: fine toll

4.2.1. Fine toll: base equilibrium

We now turn to fine time-varying tolling on both roads. Table 1 shows the equilibrium values of the key variables. In line with the theoretical discussion, we observe mostly pooling equilibria in a spatial sense, where both groups travel on both links. The free-public and free-private regimes have separating equilibrium, because link 1 is free and does not have a fine toll component.

As a benchmark, the free regime is the same as with the flat toll. If both roads are tolled by a social planner, the increase in social welfare is 41%. This is because first queueing is eliminated, and second the users with higher schedule delay value now travel closer to the desired arrival time. The consumer surplus is higher with a fine toll. If both roads are tolled by a monopolist, the social welfare is still higher than in the free regime, but consumer surplus naturally drops due to the private monopolist's pricing behaviour.

The next two regimes concern cases where link 1 is free. When a public supplier on link 2 hands out a fixed time-independent subsidy to attract travellers similar to in Braid (1996), the consumer surpluses of both groups are higher, and the public supplier earns a positive return, so it is a Pareto improvement compared to the free regime. This differs from in the static model. When a private supplier charges a toll on link 2, the increase in social welfare is still positive but much smaller.

Both regimes with competition again perform quite well. The private-public and private-private regimes have a pooling equilibrium. Similar to the case of flat tolls, the social welfare of the private-private regime is higher than that of free-public regime, because congestion is greatly reduced due to having less travellers.

With the monopoly and the free-private setting, the private operator sees a higher profit with a fine toll than with a flat toll. In the public-private regime, the public operator will ensure a fine tolling outcome. Yet, at least for these parameter values, in the competitive private-private regime, both operators see the higher profit with flat tolling, as it gives a local monopoly. So then regulation, contracting or subsidies may be needed to ensure that a fine-toll equilibrium will result. As de Palma and Lindsey (2002) also find, it depends on parameter combinations if a private operator prefers flat or fine tolling. For instance, if demand would be more price sensitive, this would reduce local monopoly power without affecting the toll revenue of a fine part of the toll, making fine tolling relatively more profitable.

With (mixed) private regimes, consumers tend to be better off with fine tolling than with flat tolling. This is partly due to the absence of local monopoly power with fine tolling, but also due to the efficiency gain from fine tolling, in that eliminates queueing and reduces schedule delay costs for the high values users. It is not the case that fine tolling allows consumers to adapt their departure time choice to prevent paying the private mark-up. The mark-up is constant over the day, while the time variant-part of the toll equals the time-variant marginal external cost.

4.2.2. Fine toll: heterogeneity and efficiency

As the degree of heterogeneity increases, the ranking of the regimes in terms of social welfare remains the same, as is shown by Fig. 4.

The free and public regimes have an efficiency of zero and one respectively, and the competing private-private regime does not have a Nash equilibrium for some parameters for the same reason as for flat tolls (see Appendix A), so these are omitted.⁹

For the remaining four regimes, the efficiency of a fine tolling increases with the degree of heterogeneity, because product differentiation is appreciated more by a more diverse population of travellers. The slopes appear somewhat flatter for private-public

⁹ If a Nash equilibrium exists, its efficiency is bounded above by the private-public case.

Fig. 5. Fine toll: heterogeneity and percentage price change (H types). Percentage change in generalized price ($\frac{100(P^H - P_{free}^H)}{P_{free}^H}$), degree of heterogeneity ($\frac{\alpha^H}{\alpha^L}$).

Fig. 6. Fine toll: heterogeneity and percentage price change (L types). Percentage change in generalized price ($\frac{100(P^L - P_{free}^L)}{P_{free}^L}$), degree of heterogeneity ($\frac{\alpha^H}{\alpha^L}$).

and monopoly regimes and steeper for free-public and free-private regimes. This is because the first two regimes have fine toll on both roads and the degree of heterogeneity matters less. Unlike with flat tolls, both types travel on both links with a fine toll in the private-public regime, so there is enough competition between the two links to prevent the efficiency from dropping. The free-private regime is less efficient than monopoly, because the monopoly operator charges a much more efficient time-varying toll on link 1 (that remains untolled in the free-private regime). We still observe a kink for the free-private regime, because the equilibrium changes from pooling when the two groups are the same to separating when the degree of heterogeneity increases, and the local monopoly power of link 2 increases. Overall, it seems that the relative efficiency from different pricing regimes is rather robust, and independent of the degree of heterogeneity. This has to do with the fact that time-varying pricing always eliminate all delays on the priced roads. The remaining inefficiency due to mark-up pricing depends on demand elasticities, is therefore largely independent on the degree of heterogeneity.

4.2.3. Fine toll: heterogeneity and distributional effects

The distributional effects for the different fine tolling regimes are summarized in Figs. 5 and 6.

Compared with flat tolls, the generalized prices with fine tolls are lower for all regimes, because fine tolls are more efficient. More

travellers will therefore use the roads. This is in accordance with [Van den Berg \(2012\)](#), who found for homogeneous users that a continuous toll is much better for users than a flat toll; and that for a toll with discrete steps in it, the toll is better for users the more steps it has.

Similar to with flat tolls, with a fine toll as the degree of heterogeneity increases the percentage price change in general decreases for *h* types and increases for *l* types. However, the percentage price change in the free-private regime increases for *h* types, because *h* types travel on link 2 only and are subject to the local monopoly power of the operator on link 2. Similarly, the percentage price changes increases most steeply in the monopoly regime for *l* types, because in other regimes with tolling on link 1, the operator is either public or faces competition from link 2. The free-public regime has in general lower prices for both types than the public regime, because link 2 offers a subsidy. However, there are kinks in the free-public regime for *h* types, which is caused again by a regime change. For example, at first when both types are the same, they travel on both roads. Then as the subsidy increases with the degree of heterogeneity, *h* types only travel on link 1 and *l* types on both roads. Finally as the subsidy increases further, all *l* types move to link 2 and *h* types travel on both links instead. In addition, the free-public regime generates a lower price than the free regime for both types, and public and private-public regime generate a lower price for *h* types.

In sum, for the parameters used in the simulation for fine tolls, all regimes perform better in terms of social welfare than the free regime. The free-public regime also offers a higher consumer surplus for both types due to the flat subsidy it entails. Unlike in the flat toll case, the private-public regime remains more efficient than the free-public regime, even as the degree of heterogeneity increases, because the pooling equilibrium ensures relatively intense competition between the two links.

We have done robustness checks with respect to several supply and demand parameters: total capacity, capacity share of link 1 and elasticity of demand. The main conclusions just reported remain robust.

5. Conclusion

Our study shows that as travellers become more different in values of time and of schedule delay, product differentiation offered by operators on parallel roads is more appreciated by both groups, but each operator might also have more local monopoly power. With flat tolls, separating equilibria are more likely to arise and competition softens, while with a fine toll, pooling equilibria are more commonly observed so that the competitive force remains relatively strong.

Under flat tolls, all our regimes raise the generalized price of both types compared to the no-toll regime. A constant toll does not remove the queuing, so travel costs and externalities remain high and this also implies a high toll. With a larger degree of heterogeneity, this is less so for the users with high values and more so for low values users. Time-variant tolling leads to lower increases in generalized price, and for the high values often *lowers* prices. So no matter the competition regime, time-variant tolling seems preferable over a flat toll scheme that does not remove queuing. A flat toll regime has a lower welfare gain, hurts road users more and is more susceptible to a local monopoly. It is important for policy-makers to take heterogeneity into account in evaluating the efficiency of various road pricing policies. If a flat toll is used, special care should be taken to make sure the reduced competition within the network does not become too harmful for social welfare. As fine tolling offers the efficiency gain from removing queuing, profit(s) are often higher than with flat tolling. If this is not the case, measures may be needed to ensure time-variant tolling: this could for example be direct regulation, clauses in the agreement to set-up a private road or lane, or subsidies.

We see two most interesting and important extensions to our study. Firstly, we consider a discrete setting with only two types of users. This results in the non-existence of Nash equilibrium in some cases and separating equilibria. When the heterogeneity follows a continuous distribution, the non-existence problem should be solved. However, the basic intuition still holds, with flat tolls each user type typically strictly prefers one link over the other, while with fine tolls all types use both roads. As a result, the implications for local monopoly power and competition would remain. Secondly, we only look at proportional heterogeneity, and for example not also at 'ratio heterogeneity' – which is between the value of time and the values of schedule delay – or at heterogeneity in the preferred arrival time. For fine tolls, this is not restricting, as a pooling equilibrium is socially optimal for more general heterogeneity. For flat tolls, it is known that a separating equilibrium remains socially optimal if the degree of ratio heterogeneity is not too large compared to the degree of proportional heterogeneity. But what about profit maximisation and mixed regimes under flat tolling?

Also interesting would be to consider larger networks with many parallel competing routes, and connecting roads that serve as complements to each other. See, for example, [Verhoef and Small \(2004\)](#) for this type of analysis for static congestion and homogeneous users. It would also be important to consider private roads that use step tolls, which vary in steps over time. Public step tolls under heterogeneity have been extensively studied,¹⁰ but not private ones. Finally, it would be interesting to also study the long-run decision of investing in new roads and expanding existing ones (e.g., [Yang and Meng, 2002](#); [Verhoef, 2008](#); [Wu et al., 2011](#); [Lu and Meng, 2017](#)).

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¹⁰ See, for example, [Xiao et al. \(2011\)](#), [Börjesson and Kristoffersson \(2014\)](#), [Van den Berg \(2014\)](#), [Chen et al. \(2015\)](#) and [Li et al. \(2017\)](#).

Beijing Jiaotong University (economics), Beijing Jiaotong University (engineering) and Tongji University (Shanghai) for their suggestions. Any remaining errors are ours.

Appendix A. Proof of Proposition I

As mentioned in Section 3.1, the public operator maximises social welfare defined as:

$$SW = \int_0^{N_1^h + N_2^h} D^h(n)dn + \int_0^{N_1^l + N_2^l} D^l(n)dn - C_1^h N_1^h - C_1^l N_1^l - C_2^h N_2^h - C_2^l N_2^l \quad (5)$$

The first-order conditions for traffic flows in a local maximum is:

$$\begin{aligned} \frac{\partial SW}{\partial N_1^h} &= D^h(N_1^h + N_2^h) - \frac{\delta^h}{s_1}(N_1^h + N_1^l) - \frac{\delta^h}{s_1}N_1^h - \frac{\delta^l}{s_1}N_1^l = 0 \\ \frac{\partial SW}{\partial N_1^l} &= D^l(N_1^l + N_2^l) - \frac{\delta^l}{s_1}(N_1^h + N_1^l) - \frac{\delta^h}{s_1}N_1^h - \frac{\delta^l}{s_1}N_1^l = 0 \\ \frac{\partial SW}{\partial N_2^h} &= D^h(N_1^h + N_2^h) - \frac{\delta^h}{s_2}(N_2^h + N_2^l) - \frac{\delta^h}{s_2}N_2^h - \frac{\delta^l}{s_2}N_2^l = 0 \\ \frac{\partial SW}{\partial N_2^l} &= D^l(N_1^l + N_2^l) - \frac{\delta^l}{s_2}(N_2^h + N_2^l) - \frac{\delta^h}{s_2}N_2^h - \frac{\delta^l}{s_2}N_2^l = 0 \end{aligned} \quad (6)$$

Solving the first order conditions results in that the traffic flow ratio of each group is equal to the link-capacity ratio, i.e. $\frac{N_1^h}{N_2^h} = \frac{N_1^l}{N_2^l} = \frac{s_1}{s_2}$. However, the second order condition for a local maximum – i.e. the Hessian matrix being negative definite – is not satisfied. A detailed mathematical proof is available upon request. We sketch the proof here. A matrix is negative definite if its k-th order leading principal minor is negative when k is odd, and positive when k is even. For any positive value of the capacity on road 1, either the third-order leading principle minor is positive or the fourth-order one is negative, so the Hessian matrix is not negative definite. As a result, the pooling equilibrium which satisfies the first order conditions is not a local maximum. Therefore, a pooling equilibrium cannot produce the highest social welfare.

The same logic applies to profit maximisation. First-order condition for an interior maximum produces a pooling equilibrium, but the second-order condition for a maximum is not then satisfied.

Appendix B. Fine toll

As shown in Section 3.2, the social welfare is the following:

$$SW = \int_0^{N_1^h + N_2^h} D^h(n)dn + \int_0^{N_1^l + N_2^l} D^l(n)dn - C_1^h N_1^h - C_1^l N_1^l - C_2^h N_2^h - C_2^l N_2^l \quad (7)$$

To maximise social welfare, the first-order conditions for traffic flows in a local maximum is as follows:

$$\begin{aligned} \frac{\partial SW}{\partial N_1^h} &= D^h(N_1^h + N_2^h) - \frac{\delta^h}{s_1}N_1^h - \frac{\delta^l}{s_1}N_1^l = 0 \\ \frac{\partial SW}{\partial N_1^l} &= D^l(N_1^l + N_2^l) - \frac{\delta^l}{s_1}N_1^h - \frac{\delta^l}{s_1}N_1^l = 0 \\ \frac{\partial SW}{\partial N_2^h} &= D^h(N_1^h + N_2^h) - \frac{\delta^h}{s_2}N_2^h - \frac{\delta^l}{s_2}N_2^l = 0 \\ \frac{\partial SW}{\partial N_2^l} &= D^l(N_1^l + N_2^l) - \frac{\delta^l}{s_2}N_2^h - \frac{\delta^l}{s_2}N_2^l = 0 \end{aligned} \quad (8)$$

We can solve the first order conditions and arrive at a pooling equilibrium. In addition, the Hessian matrix is negative definite, so this pooling equilibrium is indeed the local maximum. After solving the equilibrium in traffic flows, it follows naturally that the equilibrium flat part of the toll is $\tau_j^i = 0$.

Similarly, we can prove that with fine tolling, the profit maximising equilibrium is also pooling with a group-specific toll: $\tau^i = \frac{\partial D^i(N^i)}{\partial N^i} N^i$.

Appendix C. Discontinuity

There are two types of separating equilibria: a fully separating equilibrium, where one group travels on one link and the other group on the other link; and a semi-separating equilibrium, where one group travels on one link, but the other group on both links. The optimal toll is different for these two equilibria. As a result, we have observed discontinuity in best response functions.

For a numerical example, consider a private supplier on link 1, who chooses τ_1 to maximise its profit, given τ_2 is around 8.229. If he charges a high toll, $\tau_1 = 14.546$, a full separating equilibrium results: h types will travel on link 1 and l types on link 2. If he charges a

Table 3
Private supplier on link 1.

Type	τ_1	τ_2	N_1^h	N_2^h	N_1^l	N_2^l	Π_1
Separating	14.546	8.228	8087	0	0	9254	117,628
Semi-Separating	10.837	8.228	9143	0	1711	8116	117,623
Separating	14.546	8.230	8087	0	0	9253	117,628
Semi-Separating	10.837	8.230	9143	0	1712	8115	117,634

Fig. 7. Discontinuity in optimal toll.

lower toll, $\tau_1 = 10.837$, a semi-separating equilibrium results: h types will travel on link 1 but l types on both links. The optimal toll for the full separating equilibrium is different from the one for the semi-separating equilibrium, as shown in Table 3. So when τ_2 changes from 8.228 to 8.230, the optimal τ_1 jumps from 10.837 to 14.546.

Fig. 7 shows how profit on link 1 changes with τ_1 , given three different levels of τ_2 . The top curve is for $\tau_2 = 0$, the middle for $\tau_2 = 8.228$, and the lowest one for $\tau_2 = 15$. When τ_2 is small, i.e. $\tau_2 = 0$, it's better to have full separating equilibrium, so the highest profit for operator 1 is achieved on the left part of the green top curve. For the middle curve, with an intermediate $\tau_2 = 0 = 8.228$, the full separating and semi-separating equilibria are equally profitable. But when τ_2 is large, i.e. $\tau_2 = 15$, it's better to have semi-separating equilibrium, and the highest profit is achieved on the right part of the blue dots. The discontinuity arises when τ_2 is some value in between, i.e. $\tau_2 = 8.228$, both left or right parts of the orange dots can generate the highest profit and a small change in τ_2 will discontinuously change the optimal value of τ_1 . As a result, the best response function in toll is not continuous in the opponent's toll, and sometimes a Nash equilibrium does not exist. It is likely that this feature disappears if the more realistic case of a continuous distribution were considered.

Appendix D. Supplementary material

Supplementary data associated with this article can be found, in the online version, at <https://doi.org/10.1016/j.tra.2018.09.025>.

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