Chapter 3

Passive seismic studies at AdV

3.1 Objectives

The detection of gravitational waves (GW) from coalescing binary black holes [11, 14, 96-98] and a neutron star binary [17] has been the start of the era of GW science. The currently operating ground-based laser interferometers, the Advanced LIGO [99] and the AdV [35] GW detectors are sensitive in the frequency band 10 to 10000 Hz. In the future the sensitivity will be extended to frequencies as low as 1 Hz with third generation instruments such as Einstein Telescope [18]. The performance of all terrestrial interferometric GW detectors at frequencies below about 10 Hz is limited by seismic noise and by the direct coupling of mass density fluctuations to suspended detector elements. The effect of ground motion on the test masses can efficiently be suppressed by several orders of magnitude through advanced vibration isolation systems [100]. However, the seismically induced mass density fluctuations cannot be mechanically shielded and directly couple to the interferometer test masses. The density fluctuations create varying Newtonian forces that act on the test masses, hence the name Newtonian noise. Fig. 3.1 shows the AdV sensitivity curve along with the contribution from the seismic noise at the detector site and Newtonian noise caused due to it. At frequencies below 5 Hz, seismic, Newtonian and suspension thermal noise contribute most to the total detector noise. The maximum contribution at low frequencies is seismic in origin followed by Newtonian noise. Since the AdV detector’s observation frequency band starts at 10 Hz, no efforts are made to perform vibration isolation at low frequencies and hence the maximum contribution is observed from seismic noise as seen in Fig. 3.1. The Newtonian noise originates due to seismic motion of the subsurface and objects near the test masses cannot be shielded from.

Seismic motion recorded at the surface are mostly due to surface waves in the form of Rayleigh and Love waves. In order to subtract Newtonian noise, it is necessary to delineate and understand the sources of seismic noise near the detector. In this regard we restrict our

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1The contents of this chapter have been published as ‘Rayleigh wave phase velocity models for gravitational wave detectors using an array of nodal sensors’, Koley et al. 2018, First Break, 35(6):71-78 and ‘S-wave Velocity Model Estimation using Ambient Seismic Noise at Virgo, Italy’, Koley et al. 2017, SEG Technical Program Expanded Abstracts, DOI: 10.1190/segam2017-17681951.1
consideration to Newtonian noise from the seismic field near the site, assuming that noise generated by local equipment (e.g. pumps, air conditioning system) can be mitigated. Furthermore, we neglect the effect of Newtonian noise from atmospheric pressure variations, which has been discussed in [101]. The AdV GW detector is already equipped with a network of seismic sensors inside the detector buildings in order to identify local sources of noise and mitigate them. However due to the small array aperture of the existing sensor network, it is not possible to sample low-frequency seismic surface waves below 10 Hz. For this purpose, a passive seismic survey was carried out at the AdV GW detector in Italy, using a network of 5 Hz vertical component wireless geophones. An optimal seismic array with a maximum aperture of 3 km and a minimum of 6 m was designed to determine the direction of propagation and phase velocities of Rayleigh waves in the frequency band 0.4 to 8.0 Hz. Data were continuously recorded for a period of two weeks and analyzed offline. Direction of propagation of the seismic noise was estimated by using beamforming, while phase velocity estimation was performed with both beamforming and ESAC. Although the AdV’s detection band starts from 10 Hz, array analysis of low-frequency seismic noise is essential in order to estimate the phase velocities at low frequencies and subsequently compute a subsurface model that has better accuracy and resolution as compared to existing lithological models in the region. Previous Multi-channel Analysis of Surface Waves (MASW) [102] and micro-gravity gradient studies [103] at the AdV GW detector site revealed subsurface information up to depths of 35 m and 70 m respectively. This study improves upon the existing geological information of the region and estimates a horizontally stratified 1D subsurface model up to a depth of 800 m. The estimated phase velocities and the subsurface models are then further used to compute the subsurface quality factor model for the region. A simplistic subsurface quality factor model was previously estimated for the region by studying surface wave propagating to the detector site from a local wind park [104]. This study improves on the existing one-layer quality factor model with a detailed nine-layer model of the region.

Figure 3.1: Total noise curve of the AdV detector for an input laser power of 13 W (black curve) with the respective contribution from the relevant noise sources. The thick red and blue curves show the contribution from Newtonian and seismic noise which limit the detector sensitivity below 5 Hz.
3.2 Introduction

Array studies of ambient seismic noise have gained much importance in recent years for the purpose of classifying noise sources corresponding to different frequency bands. Stehly et al. in 2006 [105], Snieder et al. in 2009 [106], and Wapenaar et al. in 2010 [107] have demonstrated useful applications of using ambient noise recordings for surface wave tomography. Seismic motion generated by natural and artificial sources propagate through the subsurface both in the form of body and shear waves. However, the major contribution to the seismic noise field is in the form of Rayleigh and Love waves (Haubrich et al. 1963 [108]), especially at shallow depths. As stated in Chapter 2, a plane wave assumption is hence used for computing the phase velocity and direction of propagation of the seismic noise field as a function of frequency. Three approaches are of common use for analyzing signals: the frequency wavenumber (f-k) method (Lacoss et al. in 1969 [67]), the high resolution frequency wavenumber method (Capon in 1969 [109]; Asten and Henstridge in 1984 [68]) and the spatial auto-correlation technique (Ohori et al. in 2002 [71]; Asten et al. in 2004 [110]). In this chapter, we employ the f-k method for estimating the direction of propagation of different noise sources, and both f-k and ESAC for computing the phase velocities.

Seismic noise recorded at Virgo can be categorized into three different frequency bands,

- Secondary Oceanic Microseism: 0.2 – 1.0 Hz
- Road Bridge Noise: 1.5 – 4.0 Hz
- Local Noise sources: > 4.0 Hz

The AdV GW detector is located near Pisa, 30 km off the Western coast of Italy. Hence the secondary microseismic energy is mostly due to coastal reflection of ocean waves and it’s interaction with the incoming ocean waves. In our study, we focus on the secondary microseismic peak observed between 0.2 and 1.0 Hz, because at frequencies below 0.2 Hz the sensors suffer from inadequate sensitivity and the 1/f digital noise of the recorder becomes prominent. Moreover, this study aims to quantify only the fundamental mode of Rayleigh wave propagation. At frequencies greater than 1 Hz, noise originating from local road bridges is observed in the frequency band 1.5 to 4.0 Hz. A peak at 2.5 Hz is observed, dominant especially during working hours of a day. Studies by Acernese et al. in 2004 [111] presents a hypothesis that the noise peaked at 2.5 Hz is induced into the ground by oscillations due to local road bridges situated about 1.5 km away from the interferometer ends. In our analysis we try to verify such predictions, and match our observed noise propagation directions with possible sources on the field. At frequencies above 4.0 Hz sources of noise are mostly local and transient.

3.3 Seismic Array Design

Designing seismic arrays to estimate the phase velocities and direction of propagation of surface waves poses conflicting restrictions on the selection of inter-sensor spacing. Larger inter-sensor distance will increase the resolution of phase velocity estimates for long-period waves, while smaller inter-sensor distances prevents the high-frequency waves from getting
spatially aliased. Asten & Henstridge in 1984 [68] proposed that within a given frequency band, the maximum sensor separation $d_{\text{max}}$ should be at least greater than the maximum wavelength of interest $\lambda_{\text{max}}$ and the minimum sensor separation $d_{\text{min}}$ must be less than half the minimum wavelength $\lambda_{\text{min}}$. The second condition follows from the Nyquist criterion to avoid spatial aliasing at shorter wavelengths. However the measure proposed by Woods and Lintz in 1973 [75] and as described in Section 2.3.1 of using an array response function is the most widely used method. We use this approach for designing an array that is capable of resolving surface wave arrivals of frequencies as low as 0.4 Hz, and avoid aliasing up to a maximum frequency of 8.0 Hz.

The theoretical array response corresponding to five sensor geometries were tested and the sensor array that had the maximum resolution at the lowest frequency of 0.4 Hz and no spatial aliasing at the highest frequency 8.0 Hz was selected for deployment. For computing the theoretical array response, trial steering vectors were used in the slowness range $5 \times 10^{-4}$ to $10^{-2}$ s/m at an interval of $10^{-4}$ s/m and in the azimuth range 0° to 359.5° at an interval of 0.5°. The array response was then computed for a plane wave propagating through the array with directions in the range 0° to 315° at an interval of 45°. The phase velocity of the plane wave was decided based on prior surface wave studies in the region [104, 111]. Fig. 3.7(a) shows the theoretical phase velocities at nine discrete frequencies of 0.4, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, and 8.0 Hz that were used to mimic the propagation of the surface waves. The phase velocity is the highest at 0.4 Hz with a value of approximately 1400 m/s and then drops off sharply to 300 m/s at 2 Hz and then decreases gradually to about 120 m/s at 8 Hz. As a result the wavelength of the surface waves in the given frequency band approximately ranges from about 3000 m at 0.4 Hz to as short as 15 m at 8 Hz.

For designing the seismic array, the constraint on the total number of sensors that could be used was 70. The first sensor geometry (array $A$) that was studied was that of a regular square grid with a minimum spacing of 100 m and a maximum of 840 m with a sensor count of 64 as shown in Fig. 3.2(a). Figs. 3.2(c) and (d) show the computed theoretical array response values corresponding to two cases of plane wave propagation at 0.4 Hz and 2.0 Hz on a

Figure 3.2: (a) A regular square grid sensor array with minimum sensor spacing of 100 m and a total of 64 sensors. (b) Histogram of inter-sensor distances corresponding to all inter-sensor paths. (c) Theoretical array response for a plane wave propagating with a velocity of 1428 m/s along an azimuth of 0° at a frequency of 0.4 Hz showing the lack of resolution in the observed peak in the $(p - \phi)$ domain. (d) Same as (c), but corresponding to a velocity of 300 m/s and propagation azimuth of 0° at a frequency of 2.0 Hz with the arrow pointing to the actual slowness-azimuth of the incoming plane wave among all the aliased peaks.
polar graph with slowness $p$ increasing radially outwards and azimuth $\phi$ measured clockwise from the North (y-axis). Due to the limited aperture of the array, the plane wave propagating with a speed of 1428 m/s at 0.4 Hz along an azimuth of 0° is not well resolved with an azimuthal resolution of over 100° and phase velocity resolution of 800 m/s. Spatial aliasing is also observed at a frequency of 2.0 Hz corresponding to a phase velocity of 300 m/s due to the minimum sensor separation being 100 m. We draw two conclusions from the theoretical array response results of the uniformly spaced sensor array $A$. Firstly, in order to avoid spatial aliasing at high frequencies the distribution of sensors has to dense with a minimum spacing of less than one-third the minimum wavelength of interest. Secondly, the array aperture needs to be at least of the order of the maximum wavelength of interest. In order to satisfy both conditions with a uniform seismic array, a huge number of sensors would be necessary. Hence the need to use an irregular geometry with some sensors positioned closely and others at larger separation.

As a result, we test geometry $B$ as shown in Fig. 3.3(a). This array is again laid out as a square, but the inter-sensor spacing along each side of the square increases gradually as $d_0 \times r^n$ where $d_0$ is the minimum sensor separation of 4 m, $r = 2.5$ and $n$ takes integer values of $[0, 1, 2, ..., 7]$. Hence the sensors along each side of the square are spaced at distances of 4, 10, 25, ..., and 2414 m respectively from the first sensor. Array $B$ is characterized by a maximum array aperture of 2414 m and a minimum of 4 m, hence we have an improved resolution of 50° in azimuth and 600 m/s in phase velocity at the frequency of 0.4 Hz. No spatial aliasing is observed due to the closely spaced sensors near the origin of the array (Fig. 3.3(a)). However, the side-lobe amplitude in the computed theoretical array response are not damped, especially for the low frequencies and this can be seen in Fig. 3.3(c). The array also lacks azimuthal symmetry, with azimuthal resolution almost twice better when the plane wave impinges the array along its diagonals than when the plane wave propagates perpendicular to the orthogonal arms of the array.

In order to ensure azimuthal symmetry in the computed array response, array $C$ was designed in the form of concentric squares as shown in Fig. 3.4(a). A total of eight concentric squares

Figure 3.3: (a) Sensor array $B$ with minimum sensor spacing of 4 m and a maximum of 2414 m with a total of 64 sensors. (b) Histogram of inter-sensor distances corresponding to all inter-sensor paths. (c) Theoretical array response for a plane wave propagating with a velocity of 1428 m/s and propagation azimuth of 0° at a frequency of 0.4 Hz. Undamped side-lobe amplitudes can be observed. (d) Same as (c), but for a plane wave propagating with a velocity of 300 m/s and along an azimuth of 0° at a frequency of 2.0 Hz. No spatial aliasing is observed.
were designed with the side length of each square increasing as \( d_0 \times r^n \) with \( d_0 = 4 \) m, \( r = 2.2 \) and \( n \) takes integer values of \([0, 1, 2, \ldots, 7]\). The number of sensors per square was kept constant at eight. Hence a total of 65 sensors (64 + 1 at the center) were used. Although the array design is symmetric along the diagonals, but due to the same number of sensors being used for each square, the sensor density decreases as the inter-sensor spacing increases and is also shown in Fig. 3.4(b). This again leads to undamped side-lobe amplitudes and this can be observed in the computed theoretical array response shown in Figs. 3.4(c) and (d). The side-lobe energy especially at a frequency of 0.4 Hz is dominant and can cause erroneous estimation of propagation parameters especially in the case of multiple plane waves impinging on the array.

A solution to decrease the side lobe amplitudes in the computed array response is to increase the number of sensors in the outer squares such that a uniform sensor density is achieved. This is implemented in array \( D \) which is shown in Fig. 3.5(a). Array \( D \) is similar to \( C \) with the only difference being that the number of sensors per square increases as \( 4 \times n \) where \( n \) takes integer values of \([1, 2, 3, \ldots, 7]\). The outcome of the increased number of sensors is observed in Figs. 3.5(c) and (d) where the side-lobe magnitudes are damped and a good distribution of inter-sensor separation is observed for all azimuths which is shown in Fig. 3.5(b). This also leads to uniformity in the azimuthal resolution of the array. However, the number of sensors needed for this array type was 85 which was more than the constraint imposed on the sensor count. Hence, this sensor geometry could not implemented, despite the good results.

In order to mitigate all the pitfalls faced so far, we decided to test a circular array geometry. Geometry \( E \) shown in Fig. 3.6(a) is composed of eight circular rings of radii 6, 12, 24, 48, 96, 192, 768, and 1536 m respectively. The number of sensors from the innermost to the outermost ring varies as, 1(center), 3, 5, 7, 9, 11, 13, and 15. Alternate sensors in the penultimate ring are also shifted radially outwards from the central sensor by a distance of 100 m to achieve a more uniform distribution of inter-sensor distances as shown in Fig. 3.6(b). The array is characterized by a maximum aperture of 3000 m and hence can sample very frequency events of wavelengths in the same order as the maximum aperture. This
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Figure 3.5: (a) Sensor array $D$ with minimum sensor spacing of 4 m and a maximum of 1994 m with a total of 85 sensors. (b) Histogram of inter-sensor distances corresponding to all inter-sensor paths. (c) Theoretical array response for a plane wave propagating with a velocity of 1428 m/s and along an azimuth of 0° at a frequency of 0.4 Hz showing reduced side-lobe amplitudes and improved resolution. (d) Same as (c), but for a plane wave impinging the array corresponding to a velocity of 300 m/s and propagation azimuth of 0° at a frequency of 2.0 Hz. No spatial aliasing is observed.

is shown in Fig. 3.6(c). The array works well for high-frequency events because of the innermost ring of sensors which has a spacing of 6 m. Fig. 3.6(d) shows the computed theoretical array response at 2.0 Hz for a plane wave propagating along an azimuth of 0° and a slowness of 300 m/s. On comparing the theoretical array response between array $D$ and $E$, it is observed that the resolution of array $E$ is twice better than that of $D$, while ensuring no spatial aliasing up to 8.0 Hz. The circular layout of the array also ensures azimuthal symmetry in the array’s resolving power. Figs. 3.7(b) and (c) show the azimuthal and phase velocity resolution for array $E$ corresponding to nine different frequencies and eight different plane wave propagation directions between 0° and 315° at an interval of 45° respectively. The spread in the azimuthal resolution at the lowest frequency of interest was 10° corresponding
to the eight different plane wave propagation direction and it is almost constant at frequencies above 3.0 Hz. A similar trend is observed in the phase velocity resolution. Based on the theoretical array responses computed and the azimuthal symmetry of array $E$, it was finalized for deployment at the AdV detector site.

Figure 3.7: (a) Theoretical phase velocity as a function of frequency used for computing the azimuthal and phase velocity resolution of different sensor geometries in the frequency band 0.4 to 8.0 Hz. Estimated (b) azimuthal resolution and (c) phase velocity resolution for array $E$ as a function of phase velocity corresponding to eight different plane wave propagation azimuths and it shows a near-uniform response for the different plane wave propagation directions.

### 3.4 Array Deployment

Figure 3.8: (a) The blue circle shows the location of the seismic array on a map of Italy. The red circle points to the Côte d’Azur buoy which is situated approximately 200 km off the coast of Pisa in the Ligurian sea. (b) Latitude and longitude of the sensor locations marked along with the two orthogonal arms of the AdV detector. (c) Sensor layout same as (b), but shown on a map of the AdV detector [16].

The array of 70 wireless seismic sensors equipped with 5 Hz sensitive geophones was deployed at the AdV GW detector for a period of two weeks between August 13 and 28,
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2016. The blue circle in Fig. 3.8(a) shows the location of the array on a map of Italy. The array is situated approximately 30 km off the coast of Pisa and spans the two orthogonal arms of the AdV detector in the form of several concentric circular rings. Fig. 3.8(b) shows the array layout with the latitude and longitude of the sensor locations marked along the y and x-axis respectively. Fig. 3.8(c) shows the sensor layout on the field with the blue balloons showing the sensor locations. Sensor locations on the three outer rings were measured with the GPS receiver within the nodes and had a horizontal location accuracy of within 5 m. However, the inner ring of sensors which were within a 100 m radius of the central sensor were positioned with an accuracy of a meter using a measuring tape and a compass. Since the azimuth and radial distance of the inner ring of sensors were known, usage of a measuring tape and a compass was deemed more suitable for accurate deployment. Moreover, the first two sensor rings are situated at radial distances of 6 m and 12 m respectively and the use of GPS receiver within the sensors to note their location could lead to inaccuracy.

3.5 Passive Seismic Sources

Seismic noise recorded at Virgo can be categorized into three different frequency bands, namely the oceanic microseism (0.4 to 0.8 Hz), road bridge noise (1.5 to 4.0 Hz), and local sources (4 to 8 Hz).

3.5.1 Oceanic microseism

The oceanic microseism observed in the Mediterranean is characterized by a double peak in the frequency band 0.2 to 1.0 Hz. Fig. 3.9(a) shows daily averaged power spectral density (PSD) corresponding to one of the sensors. A peak in the frequency band 0.6 to 0.8 Hz is observed on all days irrespective of the swell in the Mediterranean Sea. On days when the significant wave height\(^2\) in the Mediterranean sea goes above one meter a secondary peak in the frequency band 0.3 to 0.5 Hz can be observed. During the 16 days of measurement, we classified the microseismic energy into three types of events, namely A, B and C. Fig. 3.9(b) shows the estimated PSD for one of the stations. Type A event observed during 13th and 15th August, is characterized by weak microseismic energy and a peak frequency between 0.6 and 0.8 Hz. Consequently, the significant wave height in the Mediterranean sea obtained at the Côte d’Azur Buoy (43.38° N, 7.83° E, depth of anchoring 2300 m) was below 0.5 m and as shown in Fig. 3.9(b). The peak frequency of the microseism shifts to frequencies between 0.3 and 0.4 Hz for event B recorded during 16th to 19th of August. This shift in frequency can be attributed to an increased wave height of about 1 m recorded during the same period. Event C observed between 21st and 22nd of August sees a moderate storm pass through the Mediterranean and features a significant wave height of about 1.5 m. As a consequence, the peak frequency of the secondary microseism shifts down to lower frequencies along with an additional peak in the frequency range 0.2 to 0.3 Hz. Overall a good correlation between the swell in the Mediterranean and the microseismic energy was observed. However, since the distance between the buoy and the seismic array was approximately 200 km, a slight delay between the changes in the wave height and changes in the microseismic energy was noted.

\(^2\)Significant wave height is the mean of the one-third highest wave heights measured during a given period.
Figure 3.9: (a) Estimated Power Spectral Density averaged over every day of measurement from 13 to 28 August, 2016 for station 144 showing a double peak in the secondary microseismic band. (b) Spectrogram of the seismic acceleration in the frequency range 0.2 to 1.0 Hz for days between 13th and 28th of August 2016 corresponding to sensor 144. Significant wave height (in meters) measured during the same period at the Côte d’Azur buoy shows good correlation with the secondary microseismic energy.

3.5.2 Road bridge noise

Seismic noise originating from the interaction of the sea waves with the coast diminishes gradually beyond 1 Hz and between 1.0 and 1.5 Hz a trough in the estimated PSD is observed (Fig. 3.9(a)). The magnitude of seismic noise starts to increase again from 1.5 Hz and a peak in its amplitude spectrum is observed at 2.5 Hz. The frequency band of the observed noise is approximately 1.5 to 4.0 Hz. Fig. 3.10(a) shows a spectrogram of the seismic data measured by sensor 144 for all days of measurement with a temporal resolution of 10 minutes. There are two observations from the estimated spectrogram of the seismic data. Firstly, the magnitude of the seismic noise is half an order of magnitude less on weekends (14th and 15th of August, 2016), which is expected due to less traffic on the nearby roads and bridges surrounding the detector site. Secondly, on weekdays a diurnal variation in the seismic noise amplitude is observed. The magnitude of seismic noise measured during the daytime is approximately one and a half orders of magnitude more than that measured during the night. Both these observations justify the fact that the noise source is mainly due to traffic and human activity on nearby roads and bridges. However, in Fig. 3.10(a) we show the spectrogram of the measured seismic ground motion pertaining to only one sensor and the characteristic of the observed noise might be due to local noise sources in action. As a result, we compute the average PSD of the measured seismic noise from all the sensors for every hour of data in the frequency band 1.5 to 4.0 Hz. This is shown in Fig. 3.10(b) where all the green dots show the hourly average of the measured PSDs for all the sensors and the black curve shows the average variation of PSD for the entire detector site. The spread of the observed seismic noise PSD is an order of magnitude over all the sensors and shows the characteristic daily variation that was also observed in the spectrogram in Fig. 3.10(a). A slight dip in the seis-
mic noise PSD is observed around noon everyday, which could be attributed to the slightly reduced traffic and human activities during lunch hours. Fig. 3.10(c) shows a similar plot as in Fig. 3.10(b), except that it is for days between 19th and 23rd of August, 2016. The seismic noise PSD on weekends is half an order of magnitude less as compared to the noise level on weekdays.

Seismic noise measured at each of the sensor locations is a superposition of waves originating from both far and nearby sources. While during the day time it is difficult to find any effect of far away noise sources due to traffic on nearby roads or due to other local human activities, during the night when the local noise sources have a diminished contribution, it is easier to quantify and validate the contribution of any persistent far away noise sources. From previous studies in the region [111] it was hypothesized that five road bridges present within distances of 1 to 3 km away from the end buildings of the AdV detector contribute to the seismic noise measured in the frequency band 1.5 to 4.0 Hz. For this reason, seismic sensors were specifically positioned at the base of these five road bridges, the location of which are shown in Figs. 3.11(a) and (b). During the same time, seismic sensors were also installed near the three End Buildings of the detector. The objective of the exercise was to find any correlation between the ground motion measured at the road bridges and that at the detector site. Bridges A1, A2, B and D which are located about 1.5 km away from the interferometer ends, had the maximum contribution to the noise observed at the site. The bridges induce oscillations in the ground in the frequency bands 1.5 to 4.0 Hz and 5.5 to 8.0 Hz. However, only the low-frequency component of the noise propagates to the detector site, while the high frequency is attenuated much earlier [112]. Figs. 3.12(a) and (b) show the PSD of the recorded ground motion beneath bridge A1 and that recorded at the central building during the night when

Figure 3.10: (a) Estimated spectrogram of the seismic noise measured by sensor 144 with a temporal resolution of 10 minutes showing the decreased seismic activity during night and on weekends. (b) Green dots showing the hourly averaged PSDs of the seismic noise from all the sensors in the frequency band 1.5 to 4.0 Hz and the black curve shows the mean PSD variation at the detector site. (c) Same as (b) but for days between 19th and 23rd of August, 2016 showing a reduced seismic noise level on weekends.
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Figure 3.11: (a) Location of the seismic sensors beneath the road bridges and at the arms-ends of the AdV detector in cartesian coordinates along with the respective sensor numbers. (b) Locations of five road bridges on a map of the region [16]. (c) Example of local noise sources X and Y in the frequency band 4.0 to 8.0 Hz impinging on the center of the seismic array.

Figure 3.12: Spectrogram of the ground motion measured (a) beneath bridge A1, (b) the central building of the AdV detector, (c) beneath bridge B, (d) at the North End of AdV detector, (e) beneath bridge D, and the (e) the West End Building. The arrow in the figures point to the structures in the spectrogram where a strong imprint of the ground motion measured beneath the road bridges was also observed at the detector site.

Seismic noise from local sources was minimal. A normalized cross-correlation magnitude of 0.25 is observed between the two. Similar examples of spectrograms of the ground motion recorded beneath bridge B and the North End of the AdV detector are shown in Figs. 3.12(c) and (d). The normalized cross-correlation magnitude of the ground motion beneath bridge B and the North End was approximately 0.2. The normalized cross-correlation magnitude was the lowest between measurements at bridge D and the West End building as shown in Figs. 3.12(d) and (e). The weak correlation between the two measurements was because the bridge
is located 2.5 km from the site which is farther than bridge A1 and B. Moreover, from the spectrogram in Fig. 3.12(e) we observe that the ground motion measurements at the West End building was much noisier than at the other End Buildings. This could be attributed to wind-noise impacting the measurement. The sensor was not buried properly in the ground and a significant part of the sensor enclosure was vulnerable to strong gusts of wind.

### 3.5.3 Local noise sources

In the frequency band 4 to 8 Hz, noise sources are mostly local and transient, and attenuate over small propagation distances of a few hundred meters. Hence, a dense network of 25 sensors in the center of the array (Fig. 3.11(c)) was used for phase velocity estimation. For example Fig. 3.13(a) shows a noise gather recorded by the sensors in the center of the array and originating from source X shown in Fig. 3.11(c). Other miscellaneous noise sources in this frequency range include ground motion induced by shaking of local structures during periods of high wind speed. Fig. 3.13(c) shows the PSD of the ground motion recorded by a sensor near one of the electric towers and compares it with the wind speed measured by the Virgo anemometer during the time (Fig. 3.13(d)). A significant peak in the frequency domain was observed when the wind speed is in excess of 10 km/hr.

![Figure 3.13](image)

**Figure 3.13:** (a) A noise gather comprising data from sensors in the central part of the array in the frequency band 4.0 to 8.0 Hz due to the noise source X as marked in Fig. 3.11(c). (b) Locations of sensor 149 and the electrical power-grid tower near it which induces local ground motion due to shaking during high wind speed. (c) PSD of the ground motion recorded near the same tower shown in (b). (d) Wind speed recorded by the AdV anemometer during the same period.

### 3.6 Beamforming Results

The first passive seismic data analysis method implemented on the ambient seismic noise measured at the AdV detector site was beamforming. The methodology of beamforming was explained in Section 2.3.1. The first step in beamforming is computing the data covariance matrix at a desired frequency $f$. For the oceanic microseism, beampower is computed for
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every hour of data and the data covariance matrix $R_{xx}(f)$ given by Eq. (2.15) is evaluated by further subdividing every hour of data into 12 consecutive windows with a 50% overlap between consecutive windows. Hence for a network of 70 sensors, the data matrix $X(f)$ is of the form $70 \times 12$ and $R_{xx}(f) = X(f)X^*(f)$ is a $70 \times 70$ matrix. Since, the estimated beampower is also a function of slowness $p$ and azimuth $\phi$, the next step in computing the beampower is the selection of the number of trial steering vectors to be used. For performing beamforming in the frequency band $0.4$ to $1.5$ Hz, trial steering vectors were computed in the slowness range $2.5 \times 10^{-4}$ to $2 \times 10^{-3}$ s/m at an interval of $2 \times 10^{-5}$ s/m and in the azimuth range $0^\circ$ to $-359.5^\circ$ at an interval of $0.5^\circ$, which equals a total of 60,564 steering vectors. Next, we represent the output beampower in two types of plots. First, we fix the time window and the frequency band, and express the beampower only as a function of $p$ and $\phi$. Fig. 3.14 is an example of such a representation. The beampower is plotted as a surface in the $p - \phi$ domain for a day’s measurement in six different frequency bands. Secondly, we express the beampower as a function of azimuth of propagation $\phi$ and time for a given frequency band. The beampower is initially stacked for all values of slowness and expressed only as a function of azimuth for a given time window and frequency. The process is then repeated for several time windows and the beampower is plotted as a surface, where the azimuth is measured anti-clockwise from the x-axis and the time is plotted radially outwards. This plot gives us an idea about the change in the estimated propagation direction as a function of time. Fig. 3.15 shows such a plot for six days of measurement. The radial axis in all the plots in Fig. 3.15 represents time in hours and azimuth is measured anti-clockwise from the x-axis. Propagation velocities in the secondary microseismic band range between 2 to 0.8 km/s, as can be seen in Fig. 3.14. Propagation azimuth for the oceanic microseism was observed to be close to 180° and can be attributed to the waves from the Mediterranean Sea hitting the

Figure 3.14: Beamforming output at frequencies centered at 0.45, 0.55, 0.65, 0.75, 0.85, and 0.95 Hz with a bandwidth of 0.1 Hz. At each of the frequencies the normalized beampower peak is observed at an azimuth of 180° corresponding to the Mediterranean Sea as a source.
3.6 Beamforming Results

**Figure 3.15:** Beamforming output showing normalized beampower as a function of time and azimuth for days between 16th and 21st of August, 2016 at \( f = 0.45 \) Hz. The radial axis represents time (1 to 24 hours) and the azimuth is measured anticlockwise from East (x-axis). The arrow points to the direction of the Mediterranean Sea.

**Figure 3.16:** A histogram of the oceanic noise propagation direction measured anticlockwise from the East (x-axis) at three frequencies of 0.45, 0.65, and 0.85 Hz.

cost at Pisa. A histogram plot (Fig. 3.16) of the direction of propagation of the oceanic noise shows a slight shift from \( 154^\circ \) at 0.45 Hz to \( 180^\circ \) at 0.85 Hz.

Beamforming was also used for estimating the direction of propagation of road bridge noise in the frequency band 1.5 to 4.0 Hz. Beamforming was performed on smaller time segments because of the presence of multiple sources of noise in this frequency band. The data matrix \( X(f) \) is computed for every 300 s of data at a time. Each stretch of 300 s is further subdivided into 5 segments of 60 s each, and hence the data matrix takes the form of a \( 70 \times 5 \) matrix. Steering vectors for this frequency band were computed in the slowness range \( 1.0 \times 10^{-3} \) to \( 10^{-2} \) s/m at an interval of \( 2 \times 10^{-5} \) s/m and in the azimuth range \( 0^\circ \) to \( 359.5^\circ \) at an interval of 0.5\(^\circ\). Fig. 3.17 shows the beampower as a function of azimuth and time for six days of measurement centered at a frequency of 2.5 Hz and a bandwidth of 0.1 Hz. The arrows in the figure show the directions along which the road bridges are located. Noise was dominantly seen to originate from the bridges at azimuth of \( 350^\circ \) and
Figure 3.17: Beamforming output showing normalized beampower as a function of time and azimuth for days between 16th and 21st of August, 2016 at $f = 2.5$ Hz. On the radial axis is time (1 to 24 hours) and the azimuth is measured anticlockwise from East (x-axis). The arrow points to the location of the road bridges near the Virgo site.

Figure 3.18: Rayleigh wave phase velocity dispersion curve estimated using beamforming corresponding to the (a) oceanic microseism in the frequency band 0.4 to 1.0 Hz and (b) road bridge noise in the frequency band 1.5 to 4.0 Hz.

30°. Road Bridge C near the North End of the interferometer was also observed to contribute to the measured noise on a few days. Besides an estimate of the direction of the noise sources and the estimated Rayleigh wave phase velocities obtained from beamforming are shown in Figs. 3.18(a) and (b) corresponding to the oceanic microseism and the road bridge noise respectively. Noise originating from the road bridges in the frequency band 1.5 to 4.0 Hz propagates with velocities in the range 450 to 150 m/s. This is consistent with previous measurements at the site [104]. Due to the circular geometry of the seismic array it was
possible to estimate the phase velocities at higher frequencies by using ESAC. We discuss the results from the ESAC method in the next section.

3.7 ESAC Results

The ESAC method as described in Section 2.3.2 was used to estimate the Rayleigh wave phase velocity in the frequency band 1.5 to 8.0 Hz at intervals of 0.2 Hz. Since the inner six sensor rings with a maximum radial distance of about 100 m from the central sensor were positioned along the circumference of circles, they could be used for performing ESAC.

Figure 3.19: Spatially averaged real values of the normalized cross-correlation shown with the green dots expressed as function of the radius of the sensor rings and the best fitting zero-th order Bessel function shown with the black curve at frequencies of 1.5, 2.5, 3.5, 4.5, 5.5, and 7.5 Hz. The dotted red and blue curve show the Bessel functions corresponding to a ±10% error in the estimated phase velocity.

Cross-correlations were initially computed for every 10 minutes of data and then stacked for every hour of data. This process was then repeated for a day’s noise records and a dispersion curve was estimated for every day of ambient noise data. Fig. 3.19 shows the best fitted zero-th order Bessel function \( J_0(r, f) \) corresponding to the observed cross-correlations as a function of the radial distance \( r \) from the central sensor and at discrete frequencies of 1.5, 2.5, 3.5, 4.5, 5.5, and 7.5 Hz. The green dots in the figures show the observed cross-correlations and the black curve shows the best fitting \( J_0(r, f) \). We also show the Bessel functions corresponding to 110% and 90% of the estimated phase velocity. These are indicated with the blue and the red dotted curves respectively. Because we only use the inner six sensor rings with maximum radial distance of 96 m, it was only possible to estimate the phase velocity reliably upto a frequency of 2.0 Hz. Fig. 3.20 shows the dispersion curves obtained from beamforming and from the ESAC method with the black dashed and the red curve respectively. A mismatch of ±40 m/s is observed between the two in the frequency band 3 to 5 Hz.
Figure 3.20: Rayleigh wave phase velocity dispersion curve obtained using ESAC (red curve) and beamforming (black dotted curve).

and 20 m/s at frequencies above 5 Hz.

3.8 Estimation of a 1D S-wave velocity model at AdV

The Rayleigh wave phase velocity dispersion curve is inverted to obtain a 1D S-wave velocity model for the region. The parameter space for the inversion comprises the P-wave velocity, S-wave velocity, density and depth of each layer. Given a subsurface model, it is possible to compute a dispersion curve corresponding to the model. The forward problem of computing a theoretical dispersion curve from a subsurface model is accomplished by using the Thomson-Haskell propagator matrix method [44] as described in Section 1.2.5. In order to obtain the best fitting subsurface model, an inversion is carried out with the neighborhood algorithm as proposed by Sambridge in 1999 [86]. The solution to such an inverse problem is non-unique, and ill-posed with respect to the subsurface parameters. Hence, a priori information about the subsurface must be incorporated while solving the inverse problem.

Virgo is located on the southern basin of river Arno, which is characterized by marine and continental deposits over Mesozoic bedrock, formed mainly during the Middle Miocene period (Patacca et al. in 1990 [113]). A set of North-West striking normal faults also span the basin. Hence as one goes from East to West a dipping carbonate bedrock formation that stretches from 700 m deep in the Eastern end, to 2500 m deep or more to the West end is present (Cantini et al. in 2001 [114]). Shallow subsurface geology up to 70 m is well studied through boreholes, and gravimetric studies at the site (Stefanelli et al. in 2008 [103]). According to these studies the upper 70 m of subsurface was formed mainly due to glacial activity and eustatic changes during Pleistocene period. The top layer of the subsurface at AdV site is composed of mud and clay of density 1500 kg/m³ and extends up to a depth of 25 m. This is followed by a thin layer of sand, and conglomerates of density 1700 kg/m³ and 2100 kg/m³ respectively. The last layer as evident from gravimetric studies is of organic clay and mud with a density of 1800 kg/m³. Multichannel Analysis of Surface Waves (MASW) with a linear 1D array have also been carried out at the AdV site in 2013. Based on the results of these studies we further subdivide the top 25 m of the subsurface into layers.
3.8 Estimation of a 1D S-wave velocity model at AdV

<table>
<thead>
<tr>
<th>Layer no.</th>
<th>$V_p$ (m/s)</th>
<th>$V_s$ (m/s)</th>
<th>Depth range (m)</th>
<th>density (kg/m$^3$)</th>
<th>Formation type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150 − 250</td>
<td>75 − 140</td>
<td>0 − 5</td>
<td>1700</td>
<td>Mud and Clay</td>
</tr>
<tr>
<td>2</td>
<td>200 − 350</td>
<td>100 − 160</td>
<td>5 − 10</td>
<td>1800</td>
<td>Sand</td>
</tr>
<tr>
<td>3</td>
<td>200 − 350</td>
<td>120 − 180</td>
<td>10 − 15</td>
<td>1800</td>
<td>Sand and Clay</td>
</tr>
<tr>
<td>4</td>
<td>350 − 500</td>
<td>150 − 250</td>
<td>15 − 25</td>
<td>1900</td>
<td>Sand</td>
</tr>
<tr>
<td>5</td>
<td>400 − 700</td>
<td>150 − 300</td>
<td>25 − 40</td>
<td>1900</td>
<td>Gravel</td>
</tr>
<tr>
<td>6</td>
<td>600 − 1200</td>
<td>150 − 400</td>
<td>40 − 80</td>
<td>1900−2100</td>
<td>Organic Clay</td>
</tr>
<tr>
<td>7</td>
<td>1200 − 2000</td>
<td>300 − 800</td>
<td>100 − 250</td>
<td>2100−2500</td>
<td>Pliocene</td>
</tr>
<tr>
<td>8</td>
<td>1500 − 3000</td>
<td>500 − 1500</td>
<td>600 − 800</td>
<td>2300−2600</td>
<td>Carbonate</td>
</tr>
<tr>
<td>9</td>
<td>2000 − 4000</td>
<td>1000 − 3500</td>
<td>800 − ∞</td>
<td>2600−2700</td>
<td>Half-space</td>
</tr>
</tbody>
</table>

Table 3.1: Parameter search space used for obtaining a 1D subsurface model from the dispersion measurements with the AdV array.

Figure 3.21: (a) S-wave velocity models explored for a nine-layer model, with the red line showing the model having minimum misfit. (b) Theoretical dispersion curves corresponding to all explored models in (a) and the observed dispersion curve shown in black. (c) $V_s$ model from this study and previous MASW measurements for top the 50 m.

The parameter space, over which a search is performed for estimating the best P-wave, S-wave, density and depth of each layer of the subsurface, is shown in Table 3.1. Since the values of density were known a priori from micro-gravity gradient studies, they were fixed
for the top 70 m of the subsurface. The inversion is also carried out corresponding to a nine-
layer model and a total of 60,000 models were explored. Figs. 3.21(a) and (b) show all the
S-wave velocity models explored and their corresponding theoretical dispersion curves, with
the relative misfit values of each model shown by using the colorbar. The velocity model
shown by the red color in Fig. 3.21(a) corresponds to the best fit model, and the estimated
dispersion curve matches well with the observed dispersion curve (black) and is shown in
Fig. 3.21(b). Fig. 3.21(c) shows a comparison of the velocity model obtained from this study
with the estimated model values from previous MASW studies. It is worth noting that in the
top 30 m of the subsurface, the geology varies significantly among the three interferometer
ends and the center of our array.

3.9 Near surface quality factor estimation

Computing the near surface shear wave and body wave velocities from the observed Rayleigh
wave dispersion is a well established method and has been widely explored by geophysicists,
for example Xia et al. in 1997 [115] and Park et al. in 1999 [52]. However, computing the
near surface quality factor from the observed Rayleigh wave attenuation and dispersion is not
straightforward under the assumption that the near surface quality factor might be a function
of frequency as reported by Jeng et al. in 1999 [116]. In this section we aim to compute
the near surface quality factor depth model based on the assumption that in the frequency
band 1 to 10 Hz, the quality factor model is independent of frequency [117, 118]. Based on
studies by Anderson et al. in 1965 [119], the near surface quality factor can be obtained from
the near surface P-wave, S-wave, density model and the Rayleigh wave phase velocity and
frequency dependent Rayleigh wave attenuation coefficients. Since we already know the near
surface velocity models at the AdV site along with the observed Rayleigh wave dispersion,
the next step is to compute the frequency dependent Rayleigh wave attenuation coefficients.
Next we discuss the method used to compute the Rayleigh wave attenuation coefficient.

3.9.1 Rayleigh wave attenuation model

The propagation velocity of surface waves is a function of frequency as different wavelengths
penetrate to different depths in the subsurface. This observed dispersion is also responsible
for different attenuation levels for different frequencies or equivalently different depths of
penetration. In general, the attenuation of surface waves as measured on the surface can be
either due to geometrical spreading or due to attenuation properties of the medium. While
the energy decay of surface waves in the form of geometrical spreading is independent of
frequency and only a function of the distance the wave has propagated from the source, the
medium attenuation of surface waves is in general frequency dependent. Attenuation of sur-
face waves can be of two types: Apparent attenuation and Intrinsic attenuation. Apparent
attenuation $\alpha_{app}$ is caused due to phenomena like scattering, leakage, and events like reflection
and transmission due to inhomogeneities in the propagation medium. On the contrary,
intrinsic attenuation $\alpha_I$ is attributed to conversion of the propagating seismic energy into heat
and is dependent on the medium properties. In general when we talk about Rayleigh wave
attenuation we refer to the intrinsic attenuation only and in this Section we are only interested
3.9 Near surface quality factor estimation

in computing the intrinsic attenuation of Rayleigh waves. The attenuation model for Rayleigh waves with amplitude $A(r_0, f)$ at the source point $r_0$ following the work of Kudo and Shima, 1970 [120] is given by

$$A(r, f) = \frac{A(r_0, f)}{\sqrt{|r - r_0|}} e^{-(\alpha_I(f) + \alpha_{app}(f))(|r - r_0|)},$$

(3.1)

where $r_0$, $r$ are the position vectors of the source and the receiver respectively. Now there are two challenges in using this attenuation model with ambient noise data. Firstly, since we are interested only in estimating the intrinsic attenuation coefficients, the surface wave events to be used for the analysis must not be scattered events. Secondly, the exact location of the ambient noise source has to be determined in order to correct for the geometrical spreading of the surface waves. Hence, after careful examination of two weeks of ambient noise data, seven surface wave events in the frequency band 1 to 8 Hz were selected for computing the frequency dependent attenuation coefficients. Also, care must be taken that in the frequency domain the events recorded at different sensor locations must have a consistent amplitude response or else the estimated attenuation coefficients will only yield correct values corresponding to frequencies where peaks in the amplitude spectrum are observed. Fig. 3.22(a) shows the location of such a source near the center of the array along with the 25 sensors in the center of the array where were used for the analysis. Some sensors had missing data during the event and hence they were not shown in Fig. 3.22(a). The seismograms recorded by the 25 sensors with maximum source-receiver offset of 690 m is shown in Fig. 3.22(b). In order to compare the amplitude of the seismograms in frequency domain, we averaged the PSD of the observed seismograms in bins of 0.4 Hz at an interval of 0.2 Hz. Fig. 3.22(c) shows the average PSD corresponding to all the 25 sensors with a distinct color for each sensor. As stated earlier, the amplitude response of the seismogram in Fig. 3.22(c) at each of the sensors show a consistent trend, implying that spectral-energy content of the event is maintained throughout the propagation medium. We observe that the attenuation of the low-frequency surface waves (2.0 to 4.0 Hz) is an order of magnitude less as compared to the higher frequencies (4.0 to 8.0 Hz). However, it must be noted that the amplitude spectra shown in Fig. 3.22(c) have not yet been corrected for the geometrical spreading.

3.9.2 Estimating the attenuation coefficient

The amplitude spectrum of the seismic noise measured at each of the sensor locations is a function of frequency, where the number of frequency bins depend on the length of the input seismic signal. Since, the attenuation coefficients are estimated at discrete frequency intervals, the first step is to represent the Fourier amplitude of the seismic noise appropriately at these desired frequencies. The Fourier amplitude at each frequency is computed as the average of the Fourier amplitude of the seismic noise centered at the particular frequency and with a desired bandwidth. In this application, we use frequency intervals of 0.2 Hz and a bandwidth of 0.4 Hz. The bandwidth of 0.4 Hz is used in order to make successive frequency windows 50% overlapping. Fig. 3.23 shows the average spectral amplitude as a function of source-receiver offset at frequencies of 3.0, 4.0, 6.0, and 8.0 Hz corresponding to the source-receiver geometry shown in Fig. 3.22(a). After the average amplitude spectra has been estimated centered at the desired frequencies, we correct the amplitude spectrum for
Figure 3.22: (a) Blue dots show the sensor locations and the red dot show the estimated location of the source. (b) Seismic noise gather (time vs offset) for the source-receiver layout shown in (a). (c) Amplitude spectral density of the ground motion measured by each of the sensors corresponding to the source-receiver layout shown in (a).

Figure 3.23: Average spectral amplitude as a function of source-receiver offset corresponding to the source-receiver geometry shown in Fig. 3.22(a) computed at frequencies centered at 3.0, 4.0, 6.0, and 8.0 Hz with a bandwidth of 0.4 Hz.

to the source-receiver geometry shown in Fig. 3.22(a) computed at frequencies centered at 3.0, 4.0, 6.0, and 8.0 Hz with a bandwidth of 0.4 Hz.

geometrical spreading. After correcting for the geometric spreading we can rewrite Eq. (3.1) as

\[
\ln(A(r, f) \times |r - r_0|) = \ln(A_0(r_0, f)) - \alpha_R(f)(|r - r_0|)
\]

where \(\alpha_R\) is the intrinsic attenuation coefficient for Rayleigh wave propagation. It must be noted that in Eq. (3.2) the apparent attenuation coefficient has been neglected assuming that the events considered for estimating the attenuation coefficient are free of scattering. Now, Eq. (3.2) takes the intercept-form of a straight line \(y = -mx + c\) where the slope \(m\) of the line is the estimated Rayleigh wave attenuation coefficient. Fig. 3.24(a) shows the plots of \(\ln(A(r, f) \times |r - r_0|)\) as a function of \(|r - r_0|\) corresponding to the source-receiver location shown in Fig. 3.22(a) at frequencies 3.0, 4.0, 6.0, and 8.0 Hz. The black curve in Fig. 3.24(a)
shows the best fitting straight line where the slope of each line gives an estimate of \( \alpha_R \) at each of the frequencies. Fig. 3.24(b) shows the estimated Rayleigh wave attenuation coefficient in the frequency band 1.0 to 8.0 Hz at intervals of every 0.2 Hz corresponding to seven sets of source-receiver combination (only one of the combinations have been shown in Fig. 3.22(a)).

**Figure 3.24:** (a) Geometrical spreading corrected spectral amplitude as a function of source-receiver offset at frequencies 3.0, 4.0, 6.0, and 8.0 Hz. The black line corresponds to the best fitting straight line of the form \( y = -mx + c \) at each frequency. (b) Estimated Rayleigh wave intrinsic attenuation coefficient as a function of frequency with the error bar showing the standard deviation of the estimation over all the seven source-receiver combinations.

### 3.9.3 Quality factor forward problem

Following the work of Anderson *et al.* in 1965 [119], the frequency dependent Rayleigh wave attenuation coefficient \( \alpha_R(f) \) is related to the P-wave quality factor \( Q_P \) and the S-wave quality factor \( Q_S \) as

\[
\alpha_R(f) = \frac{\pi f}{V^2_R(f)} \times \left[ \sum_{i=1}^{N} P_i(f)Q_P^{-1} + \sum_{i=1}^{N} S_i(f)Q_S^{-1} \right],
\]

where \( V_R(f) \) represents the Rayleigh wave phase velocity, \( P_i = V_{P_i} \frac{\partial V_R}{\partial V_{P_i}}, \) \( S_i = V_{S_i} \frac{\partial V_R}{\partial V_{S_i}}, \) \( V_{P_i} \) is the P-wave velocity of the \( i^{th} \) layer and \( V_{S_i} \) is the S-wave velocity of the \( i^{th} \) layer. Hence, for all frequencies of interest \([f_1, f_2, ..., f_n]\) and for a \( N \)-layer subsurface model, Eq.
can be expressed in matrix form as

\[
\begin{bmatrix}
\alpha_R(f_1) \\
\alpha_R(f_2) \\
\vdots \\
\alpha_R(f_n)
\end{bmatrix}
= 
\begin{bmatrix}
V_{P1} \frac{\partial V_R(f_1)}{\partial V_P} & \cdots & V_{PN} \frac{\partial V_R(f_1)}{\partial V_P} \\
V_{P1} \frac{\partial V_R(f_2)}{\partial V_P} & \cdots & V_{PN} \frac{\partial V_R(f_2)}{\partial V_P} \\
\vdots & \ddots & \vdots \\
V_{P1} \frac{\partial V_R(f_n)}{\partial V_P} & \cdots & V_{PN} \frac{\partial V_R(f_n)}{\partial V_P}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial V_R(f_1)}{\partial Q_P} \\
\frac{\partial V_R(f_2)}{\partial Q_P} \\
\vdots \\
\frac{\partial V_R(f_n)}{\partial Q_P}
\end{bmatrix}
= 
\begin{bmatrix}
Q_{P1}^{-1} \\
Q_{P2}^{-1} \\
\vdots \\
Q_{PN}^{-1}
\end{bmatrix}
\tag{3.4}
\]

The forward problem stated in Eq. (3.4) is linear and of the form \( Ax = B \) where \( x = [Q_{P1}, ..., Q_{PN}, Q_{S1}, ..., Q_{SN}]^T \), \( B = [\alpha_R(f_1), \alpha_R(f_2), ..., \alpha_R(f_n)] \) and \( A \) is the kernel matrix comprising the partial derivatives of the Rayleigh wave phase velocity with respect to the P-wave and S-wave velocities of each layer of the medium. The values of \( V_{P1}, V_{S1}, V_R(f) \) and \( \alpha_R(f) \) in Eq. (3.4) are known except for the values of the partial derivatives \( \frac{\partial V_R(f)}{\partial Q_P} \) and \( \frac{\partial V_R(f)}{\partial Q_S} \). Following Haskell, 1953 [44] we know that the Rayleigh wave phase velocity for any \( N \)-layer subsurface model is a solution to the implicit equation \( F(f_j, c_j, V_p, V_s, \rho, h) = 0 \) as stated in Eq. (2.21). We estimate the partial derivatives of the Rayleigh waves with respect to the layer parameters following the methodology stated by Cercato in 2007 [121] in the frequency band 1.0 to 8.0 Hz. Figs. 3.25(a) and (b) show the partial derivatives of the Rayleigh wave phase velocity with respect to the P-wave and the S-wave velocity respectively for the nine-layer model that we derived for the AdV detector site in Section 3.8. The values of the partial derivatives are an order of magnitude more sensitive to the S-wave velocities of the subsurface layers than to the P-wave velocities.

### 3.9.4 Quality factor inversion

The quality factor depth model is derived by solving the linear system of equations in Eq. (3.4). From Figs. 3.25(a) and (b) we observe that \( V_R(f) \) is more sensitive to changes in \( V_S \) than to \( V_P \). Hence, the observed Rayleigh wave attenuation \( \alpha_R(f) \) is less sensitive to changes in \( Q_P \) as compared to \( Q_S \). Xia et al. in 2002 [122] performed a quantitative study of the dependence of \( Q_P \) and \( Q_S \) to the \( V_P \) and \( V_S \) values for a six-layer model. They state that both \( Q_P \) and \( Q_S \) can be estimated reliably only in situations when the ratio \( \frac{V_S}{V_P} > 0.45 \). In cases when the ratio is less than 0.45 only \( Q_S \) can be estimated reliably. Hence, for our case we impose two constraints while solving the linear system of equations in Eq. (3.4). We estimate \( (Q_P^{-1}, Q_S^{-1}) \) from the linear system of the type \( Ax = B \) under the constraint \( x > 0 \) and \( Q_P = 2Q_S \) [52]. Based on previous studies in the region [104] we also set the upper and lower bounds on the search space of \( Q_P \) and \( Q_S \). The upper and lower bounds on \( Q_P \) are given as

\[
Q_{PLB} = [40, 40, 40, 40, 20, 30, 50, 100, 100]
\]

\[
Q_{UB} = [80, 100, 100, 100, 100, 120, 140, 200, 200].
\]

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3.9 Near surface quality factor estimation

Figure 3.25: (a) $\frac{\partial V(f)}{\partial V_{Pi}}$ and (b) $\frac{\partial V(f)}{\partial V_{Si}}$ as a function of frequency for all the nine layers of the subsurface model of the AdV site. The partial derivatives $\frac{\partial V_R(f)}{\partial V_{Pi}}$ are greater than $\frac{\partial V_R(f)}{\partial V_{Si}}$ as expected.

Similarly the bounds on $Q_S$ are set as

$$Q_{SLB} = [20, 20, 20, 20, 25, 30, 50, 50]$$

$$Q_{SUB} = [40, 45, 45, 45, 50, 60, 60, 100, 100].$$

Solving Eq. (3.4) under the set of constraints mentioned above is no more a simple linear problem but rather an optimization problem.

We make use of Genetic algorithm [123] which is an evolutionary technique for obtaining global solution to the optimization problem stated earlier. The algorithm starts off with an initial population and computes the misfit between the estimated and the observed $\alpha_R(f)$. Based on the misfit of the models explored in the initial population, new children are generated in the succeeding generation. A cross-over fraction is defined at this stage which controls the probability of cross-over between initial models to generate newer models in the next generation. A larger cross-over fraction forces the solution in the next generation to be close to the initial population, and a lower cross-over fraction ensures diversity in the children of the next generation. For our case we run the algorithm to a maximum of 50 generations with a maximum of 20 children per generation and the cross-over fraction is fixed to 0.2 for the first 10 generations and fixed at 0.8 for the next 40 generations. These values of the cross-over fraction ensure solution diversity in the initial stages and a value of 0.8 later streamlines the solution. Fig. 3.26(a) shows the $Q_P$ and $Q_S$ models explored with the algorithm with the red curve showing the best fitting model under the given set of bounds on the $Q_P$, $Q_S$ values stated in Eqs. (3.6) and (3.7). The evolution of the solution misfit as a function of the population count is shown in Fig. 3.27(b). The misfit stays approximately constant after five generations even with a small cross-over fraction value of 0.2 which was set till the tenth generation. Fig. 3.27(a) shows the observed $\alpha_R(f)$ as the blue curve and the final best fit model as the red curve. The inverted best fitting S-wave quality factor model in Fig. 3.26(b)
Figure 3.26: (a) Blue curves show the $Q_P$, $Q_S$ models explored and the red curve the best fit model. (b) Best fitting quality factor model as a function of depth.

Figure 3.27: (a) Computed and the measured Rayleigh wave attenuation coefficient model. (b) Evolving misfit values between the measured and the computed Rayleigh wave attenuation vs population count.

shows a smooth increase from 30 to 45 for the first four layers and then shows a sudden decrease. This decrease is associated with the fifth and the sixth layer of the subsurface model. The loosely packed gravel layer overlaying the organic clay deposits are responsible for this increased attenuation. Also, from Figs. 3.25(a) and (b) we observe that the Rayleigh wave dispersion changes are most sensitive to velocity changes in layer five and six corresponding to the frequency band of 1.5 to 2.5 Hz. Since a steep rise in the $\alpha_R(f)$ value is observed for the same frequency band (Fig. 3.24(b)) of 1.5 to 2.5 Hz, implying stronger attenuation, the quality factor estimates for these two layers are less than for the top four layers. The jump in the quality factor values for layers seven and eight to higher values which implies weaker
attenuation are again due to the smaller and almost constant \( \alpha_R(f) \) in the frequency band 1.0 to 1.5 Hz.

### 3.10 Newtonian Noise Estimate

The horizontally stratified P-wave, S-wave, density and quality factor model derived from the ambient noise measurements serve as the starting point for computing the Newtonian noise contribution to the AdV detector’s sensitivity. The current Newtonian noise estimate for the AdV design sensitivity curve is based on a solution to the elastic wave equation for the case of a homogeneous half space \([66, 124]\). Caveats to using a simple homogeneous half space model are due to a constant P-wave solution throughout the medium with no reflected or transmitted phases. In the context of Rayleigh waves which is the only type of surface waves observed in a homogeneous half space, the ground motion attenuation as a function of frequency for a layered subsurface model reveals a complex relationship with depth unlike the case of a homogeneous half-space. Hence, we choose to use a complete solution to the elastic wave equation for computing the displacement of the subsurface near the test masses. We use the Elastodynamic Toolbox \([125]\) for computing the ground motion as a function of frequency. The azimuthal distribution of sources near the detector is based on the beamforming results. Although the density of sources as a function of azimuth surrounding a test mass

![Figure 3.28: Schematic of the simulated setup, with the test mass (red) at the center above the receiver grid (blue), and 180 excitation points (black) where the soil is excited horizontally and vertically at 3.0 Hz. The size of the excitation points scales to the relative scaling factor of the source strength and varies for each frequency.](image)

is obtained from beamforming, the polarization of the source and the horizontal to vertical
down-force at the source point are stochastic variables. The sources for each frequency bin are also positioned at radial distances between seven and eight times the dominant Rayleigh wavelength. Fig. 3.28 shows the source distribution with black dots around the test mass which is shown in blue. The strength of each source is re-scaled such that the surface PSD of the simulated and the observed ground motion match at all frequencies.

To compute the Newtonian noise associated with the specific test mass, we need to be able to compute the density fluctuations near the test mass. For this we use the equation derived in [21] that relates the Newtonian noise directly to the computed displacement field of each subsurface element instead of density fluctuations near the test mass. The total gravity fluctuation \( \delta_{a_{NN,gen}}(t) \) near the test mass can then be computed as the integral

\[
\delta_{a_{NN,gen}}(t) = G \int_V \left( \rho(x)u(x, t) \right) \nabla \left( \frac{x'}{|x'|^3} \right) dV,
\]

where \( u(x, t) \) represents the seismic displacement at a point with position vector \( x \) at time \( t \), \( x' \) is the distance vector between the infinitesimal volume element in the subsurface and the test mass and \( \rho(x) \) is the mass density. In order to compute the integral in Eq. (3.7) we need to define the volume surrounding the test mass that must be considered. The radius of this volume which we refer to as the integration radius is again a function of frequency since different frequency waves suffer a different level of attenuation in the subsurface. Technically the integration radius used at a given frequency is half the dominant Rayleigh wavelength \( \lambda_R \) at that frequency [21]. For the nine-layer model we found that a minimum integration radius equal to the dominant Rayleigh wavelength was necessary to obtain stable estimates of Newtonian noise. For a detailed analysis on the selection of parameters necessary for computing Newtonian noise at the AdV site, the reader is encouraged to read PhD thesis of M. Bader, 2020 [126].

Fig. 3.29 shows the Newtonian noise estimate from our array and compares it with the estimate obtained using seismic ground motion reported by Acernese et al. in 2014 [127]. For low frequencies both Newtonian noise estimates are in reasonable agreement, while for higher frequencies, the Newtonian noise estimated from previous study exceeds the noise estimate from our study by about one order of magnitude. The reason for the deviation is due to the different seismic models used in the two approaches: The constant P-wave model neglects the attenuation of seismic waves with depth. In reality, more than 80% of the surface wave amplitudes are attenuated after \( \lambda_R \). This means that in the constant P-wave model subsurface wave amplitudes are overestimated, which leads to an overestimate of Newtonian noise. At low frequencies, where the wavelength of surface waves are longer and only attenuate at significant distances from the test mass, the mismatch between the two estimates is less pronounced. Furthermore, we compare the Newtonian noise under the same mathematical treatment for a horizontally layered Virgo-like geology with two different surface PSDs (blue and green curve in figure 3.29). The Newtonian noise based on the seismic spectrum measured with the sensor array in 2016 is less than noise expected with the conservative, high seismic noise measured in 2014 [127].

The two comparisons show that we expect the seismic Newtonian noise at the Virgo site to be lower than what is currently assumed in the Virgo design sensitivity curve. We conclude that this is due to a lower seismic noise than previously expected at the site and also due to an overestimate of seismic amplitudes in the previously used seismic wave propagation models.
3.11 Summary

Near surface imaging by using Multi-Channel Analysis of Surface Waves (MASW) has been widely used in the geophysical community since the late 1990’s. Surface waves in such surveys are typically generated with a hammer or a weight dropper. There are also instances when the surface waves generated from a controlled source active seismic survey were used for shallow subsurface imaging, usually to perform near surface static correction of the data. These studies convey information about the top 20 to 30 m of the subsurface due to the high-frequency content of surface wave signals generated by such excitation. For imaging deeper subsurface, surface wave studies rely on natural sources of noise like the microseismic energy at frequencies lower than 1.0 Hz.

For our study, we relied entirely on the ambient seismic noise in the frequency band 0.4 to 8.0 Hz for measuring the Rayleigh wave dispersion. At frequencies below 1.0 Hz we measured the Rayleigh wave dispersion from the secondary microseismic peak. In the frequency band 1.5 to 4.0 Hz noise originating from nearby road bridges served as the source of surface waves. Besides measuring the Rayleigh wave dispersion by using beamforming and ESAC, we estimated the directions of the propagating noise as a function of time and frequency by using beamforming. While the Rayleigh wave dispersion could be used for estimating a suitable S-wave velocity model for the region, the direction of noise propagation gave us

Figure 3.29: Newtonian noise at AdV site from a full solution of the wave equation in a layered geology, evaluated at the central building and for the characteristic site PSD $[127]$. 

![Newtonian noise at AdV site](image-url)
information about the azimuthal distribution of noise sources surrounding the detector which was later used for simulating the ambient seismic noise field near the detector. In the end the main objective of the passive seismic campaign was to be able to make a more accurate computation of the Newtonian noise contribution to the current AdV sensitivity. Given that we measured seismic noise up to the lowest frequency of 0.4 Hz, we could reliably invert for a subsurface velocity model up to a depth of 800 m. As an extension to the entire exercise of processing ambient seismic noise, we also estimated the near surface quality factor model by making use of the observed Rayleigh wave attenuation in the frequency band 1.0 to 8.0 Hz. Since the observed Rayleigh wave dispersion was less sensitive to changes in P-wave velocity model of the region, obtaining reliable estimate of the P-wave quality factor is difficult. Hence we posed bounds on the expected quality factor values for each layer and assumed that P-wave quality factor was twice that of the S-wave quality factor. The inverted subsurface parameters were then further used to simulate the ambient seismic noise field near the detector. This simulated ground motion was subsequently used to estimate the Newtonian noise as a function of frequency at the AdV detector site. We observed that the previous models overestimate Newtonian noise at the detector, as it assumes a homogeneous half-space for solving the elastic wave equation unlike this study where we use a horizontally layered subsurface model. Moreover, previous estimates of Newtonian noise at the AdV site does not consider contribution of body waves due to reflection and transmission unlike this study where we used the full solution of the elastic wave equation.