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Search intensity, wage dispersion and the minimum wage☆

Pieter A. Gautier[a,*], José L. Moraga-González[a,b,c]

[a] Department of Economics, Vrije Universiteit Amsterdam and Tinbergen Institute, Vrije Universiteit Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, Netherlands
[b] University of Groningen, CESifo, CEPR, Netherlands
[c] The Public-Private Sector Research Center at IESE, Barcelona, Spain

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ABSTRACT

We study a labor market where employers post wages and workers simultaneously choose the number of applications they send out. Firms offer the job to a worker at random; workers with multiple offers pick the best one. If the application costs are sufficiently low, workers contact multiple firms and there is wage dispersion in equilibrium. The number of applications workers send out is excessive from a welfare perspective due to a rent seeking externality. A mandatory minimum wage increases the mean and reduces the variance of the wage distribution. The net effect on welfare is ambiguous.

"The best guardian against abuses of labour is competition. Where competition is lacking, a statutory minimum wage is needed to prevent abuse."
– Chris Pissarides, Financial Times February 23, 2015.

1. Introduction

When it takes time to learn whether a job application is successful, it can be beneficial for a worker to simultaneously apply to multiple jobs. Similarly, for many goods and services, buyers contact multiple firms. Not only to get a good wage or price but also to avoid being rationed. Typically the number of wage offers or price quotes one receives is a random variable whose realization is in general smaller than the number of price quotes one asked for. In the labor market, rationing is particularly prevalent since a vacancy can typically be filled by one worker only. Both the academic job market and the Internet job search platforms (e.g. *monster.com*) have the following two features: (i) many posted vacancies give job descriptions and no exact wage information and (ii) workers observe many posted vacancies for a certain occupation within a given region and apply for various jobs at a time. Those features are consistent with the urn-ball meeting function which was first used by Butters (1977) and Pissarides (1979).[1]

This paper extends the consumer search model of Burdett and Judd (1983) to a job market setting with capacity-constrained firms where a vacancy can be filled by one worker only. In our model, each firm has a single vacancy and sets the wage to maximize profits. Workers decide whether to apply to one or multiple jobs (simultaneously). Once wages and the number of applications are decided upon, an individual firm selects one worker at random and offers the job to her and then the market closes. Workers with multiple offers take the one that gives them the highest payoff. The number of applications that a worker sends out and the wages that the firms set are endogenously and simultaneously determined in equilibrium. Search causes congestion effects which are not internalized by individual workers and firms. Therefore, the equilibrium tends to be inefficient.[2] The existence of wage dispersion increases

[1] For a discussion of more general many-on-one meeting technologies, see Cai et al. (2017).

[2] In labor search models, search intensity is often treated as a technology parameter that increases the match probability of a worker (e.g. Pissarides, 2000). For the aggregate number of matches it does not matter whether the number of workers or the search intensity is doubled. In our model, more search leads to more coordination problems between firms, as in Albrecht et al. (2004), and may thus lead to fewer matches; this may help explain the finding in Kuhn and Skuterud (2004) that Internet search does not reduce unemployment duration.

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* Corresponding author.
E-mail addresses: p.a.gautier@vu.nl (P.A. Gautier), j.l.moragagonzalez@vu.nl (J.L. Moraga-González).

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the individual incentives to search intensively even more. Although it is well known that because of rent seeking and congestion externalities, workers can send too many applications (from a social point of view) and that too much market power by firms leads to excessive entry, we focus in this paper on the interaction between wage setting and search intensity. When workers send out multiple applications, firm wages become dispersed and this rationalizes on its turn simultaneous search by workers.[3] In this context, we shed new light on the desirability of a binding minimum wage. A minimum wage has two effects on the wage distribution: (i) it increases the mean and (ii) it reduces the variance. The first effect increases while the second effect reduces the number of applications that a worker sends out.

Our model has the same two coordination frictions as the labor market model of Albrecht et al. (2006). First, because of rationing, if one worker searches harder he imposes a negative externality on the other workers. Second, when workers apply to two or more jobs, multiple firms can select the same candidate while only one of the firms will hire the worker. The efficient number of applications balances these forces by maximizing the number of matches minus total application costs. This optimal number is first increasing in the number of vacancies per unemployed worker (labor market tightness) and then decreasing. When there are few vacancies per worker, coordination frictions among workers are relatively severe compared to those among firms. An increase in the number of vacancies per worker then reduces the first coordination friction a lot while it increases the second one only moderately. As a result, the marginal social gains from sending out more applications increase. When there are relatively few workers per firm, the coordination problem among firms is large compared to the first coordination problem and the opposite result follows.

When search cost are sufficiently low, workers apply to multiple jobs in equilibrium. The incentive to contact multiple firms arises from the fear of a worker to be rationed and remain unemployed. This contrasts with Burdett and Judd (1983) and Acemoglu and Shimer (2000) where a monopsony equilibrium always exists. Since workers can be in contact with multiple firms, the Diamond (1971) result that a small search cost could lead to non existence of an equilibrium can be avoided.

When workers contact multiple firms, as mentioned above, the equilibrium is characterized by wage dispersion.[4] Besides the fear from being rationed and remain unemployed, wage dispersion is an additional motivation to contact multiple firms, namely, to get a better offer. It turns out that in our setting, the number of firms that workers contact is *excessive* from a social welfare perspective. This result contrasts again with Acemoglu and Shimer (2000) where, like in Burdett and Judd (1983), there is no rationing. In those models, there is too little search in equilibrium because some workers free-ride on the search effort of others. The difference with our model arises from the different motives workers have to contact firms. In our setting: intensive search increases not only the payoff (lower price or higher wage depending on the context) but also the transaction probability. Individual workers do not internalize the congestion they cause on other workers and besides that, search is partly a rent seeking activity which does not have any social value.

The way the minimum wage affects workers incentives to send out applications hinges upon labor market tightness: when there are few firms per worker, an increase in the minimum wage leads to more applications. The intuition for this result is the following. When the wage distribution is skewed towards the competitive wage, an increase in the minimum wage reduces workers incentives to apply for additional jobs by compressing the wage distribution even more. Therefore, an increase in the minimum wage is desirable if the vacancy-to-unemployment ratio is relatively large because in that case the equilibrium number of applications is reduced and welfare rises. This mechanism is new to our knowledge. When there is already a lot of competition for workers, extra applications are not desirable. Introducing a minimum wage reduces the number of applications and this can increase employment by reducing coordination frictions.[5] By contrast, if the number of vacancies is small relative to the number of workers, the fact that the minimum wage increases the mean, gives workers incentives to send more applications in equilibrium, which worsens the situation.

Our model is related to the directed search/advertisement models of Butters (1977), Montgomery (1991), Burdett et al. (2003), Julien et al. (2000), Albrecht et al. (2006), Galenianos and Kircher (2009), Kircher (2009), and Wolthoff (2015) in the sense that workers know the location of firms. In directed search models, workers also observe the wages offered in the market before deciding where to send their applications. By contrast, here workers only learn about the wage of a firm after the contact has been made. This generates atomless wage distributions in equilibrium for the same reasons as in Burdett and Judd (1983). Whether search is random or directed depends on the context. Casual evidence shows that many advertisements do not provide detailed wage information. The focus of our paper is different from all of the papers mentioned above. We are mainly interested in the interaction between endogenous search intensity and the wage distribution and in the possible inefficiencies that are driven by this interaction. We also show that this interaction has important implications for the desirability of an institution like the minimum wage.

Our analysis is further related to the consumer search models of Burdett and Judd (1983), Janssen and Moraga-González (2004), Galeotti (2010) and Wildenbeest (2011). The fundamental difference between those models and ours is that we have rationing. In this sense it is similar to Lester (2011) but with endogenous search. Without capacity constraints workers tend to search less than what is collectively desirable; here workers anticipate the market frictions that arise from rationing and they contact an excessive number of firms.

The paper is organized as follows. Section 2 presents the main assumptions. Section 3 discusses efficiency. Section 4 derives the decentralized equilibrium. Section 5 shows how a minimum wage can be welfare improving. Finally, Section 6 concludes.

2. The labor market

We study a single period labor market with a large number of unemployed workers (u) and a large number of firms with exactly one vacancy (v). Let θ be the number of vacancies per unemployed worker; we refer to θ as the *vacancy-to-unemployment ratio*, or *labor-market tightness*. Firms, indexed by i, post wages. To allow for mixed strategies, the wage offer of a firm i is denoted $F_i(w)$. Wages must be higher than the minimum wage, denoted $\underline{w}$.[6] We normalize the value of the output produced by a firm-worker match to 1.

Workers, indexed by j, who get a job receive a gross utility equal to the wage w they are paid. In order to receive wage offers, workers must apply to vacancies. Let $k > 0$ be the (small) cost a worker must incur to complete and submit an application. A strategy for a worker j is the number of vacancies to apply for. Since workers do not observe the terms of trade ex ante, it is optimal for them to send their applications to the vacancies randomly (with equal probability). Thus, if worker j sends

[3] Because in equilibrium firms offer wages from a continuous wage offer distribution, our model is suitable for structural estimation. This is what we do in a companion paper, see Gautier et al. (2015).

[4] The nature of the wage dispersion is similar to Burdett and Judd (1983), Lang (1991) and Acemoglu and Shimer (2000) where some buyers/workers *observe* more than one price/wage while others just one. In our model, some workers *receive* one offer and others receive two or more. In Burdett and Mortensen (1998), by contrast, wage dispersion arises because a firm's wage offer may arrive either at an employed worker (who is choosier) or at an unemployed worker, while in Albrecht and Axell (1984) it arises because workers have different reservation wages.

[5] See also Flinn (2010) for a discussion of the minimum wage in equilibrium search models.

[6] Varying $\underline{w}$ allows for the study of the effects of minimum wage policies.

ℓ applications, the probability that a randomly selected vacancy gets an application from the worker in question is ℓ/v. If a worker receives two or more wage offers, she chooses the vacancy that offers the highest wage. In the following, for convenience, we often treat search intensity as a continuous variable. This could be formalized by assuming that workers meet a random number of jobs (at some Poisson rate λ) and apply to all vacancies that they meet as in Kaas (2010).[7]

Workers and firms play a simultaneous moves game.[8] A firm chooses a wage offer taking as given the wages offered by the rest of the firms as well as the number of applications sent by each worker. A worker decides how many applications to send out, taking as given the wages offered by the firms as well as the number of applications sent by the other workers. Once applications have been received, each firm randomly picks a candidate and offers the job to her. If the candidate has no better offers, she accepts it while if she has better offer(s) she rejects it. We assume that firms can only consider one candidate.[9] As is standard in the literature, we restrict attention to symmetric Nash equilibria.

The labor market described above is similar to the nonsequential consumer search model of Burdett and Judd (1983) but with the important modification that each firm has just one vacancy to offer in the market. This assumption, which is quite common in labor markets, brings two important novelties: *first*, the incentive of a firm to overbid other firms wages is weakened; *second*, when determining her optimal search strategy, a worker must take into account that she may be rationed at some or at all vacancies she applied for. These two implications have a bearing on the nature of equilibria and their efficiency properties. We shall elaborate on these issues in the remainder of the paper.

3. Efficiency

We start by discussing the efficient number of applications. The planner's objective is to maximize aggregate output net of total application cost. Total output is simply equal to the number of matches times the value of a match. Since all workers are identical, we can focus on welfare per worker. Suppose every worker sends a job applications. Let $z(a,\theta)$ denote the probability that one of the a applications of a worker results in a job offer. Because a worker receives no job offer with probability $(1-z(a,\theta))^a$, the gross welfare per worker is:

$$W(a,\theta) = 1-(1-z(a,\theta))^a. \tag{1}$$

We now calculate $z(a,\theta)$. Let $\rho(a,\theta)$ be the ("urn ball") probability that a firm gets at least one applicant when each worker submits a applications:

$$\rho(a,\theta) = \lim_{u,v\to\infty,\frac{v}{u}=\theta}\left(1-\left(1-\frac{a}{v}\right)^u\right) = 1-e^{-\frac{a}{\theta}}.$$

Conditional on having sent applications to firms i and m, in a large labor market ($u, v\to\infty, \frac{v}{u}=\theta$), getting an offer from firm i is independent of getting an offer from firm m (see Albrecht et al. 2004). Then $z(a,\theta)$ is simply equal to the number of firms with applications divided by the total number of applications in the market:

$$z(a,\theta) = \lim_{u,v\to\infty,\frac{v}{u}=\theta}\frac{\left(1-\left(1-\frac{a}{v}\right)^u\right)v}{au} = \frac{\rho(a,\theta)\theta}{a}.$$

It is straightforward to show that $z(a,\theta)$ is strictly decreasing and convex in a, with $\lim_{a\to 0}z(a,\theta)=1$ and $\lim_{a\to\infty}z(a,\theta)=0$; obviously, the opposite properties hold with respect to θ.

Our first result characterizes the social optimum:

Proposition 1. *For any k and θ, there exists a unique socially optimal number of job applications $a^o(\theta,k)$. This number is independent of the minimum wage, decreasing in k and non-monotonic in the vacancy - to-unemployment ratio θ (first increasing and then decreasing).*

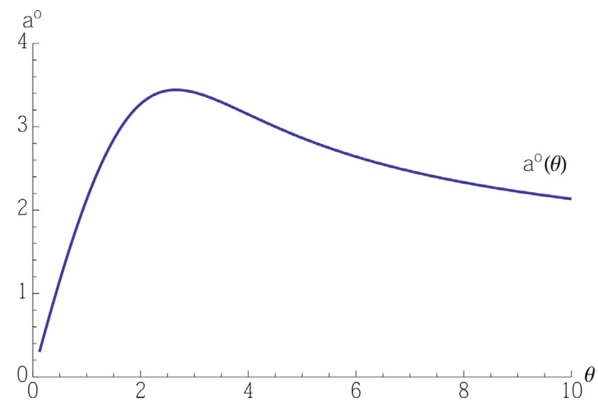

Fig. 1. The efficient number of job applications.

For a formal proof see the Appendix. The non-monotonicity of the efficient number of job applications with respect to market tightness can be seen in Fig. 1. While computing the optimal number of job applications we set k=0.01.

This non-monotonicity of the efficient number of job applications with respect to market tightness is related to the two types of coordination frictions that exist in this market. First, since workers send job applications randomly, the applications of a worker exert a negative externality on the applications of other workers. These congestion effects between the workers weaken as the firm-worker ratio goes up. Second, a firm may choose the same candidate as some other firm(s), in which case only one of the firms hires the worker and the others remain idle. These coordination problems between the firms strengthen as the firm-worker ratio goes up. When the firm-worker ratio is initially low each firm receives many applications and therefore the first coordination friction is relatively large compared to the second one because the probability that multiple firms consider the same candidate is low while the probability that multiple workers apply to the same vacancy is high. In this case, an increase in θ reduces the congestion effects between workers more than when θ is already large. In the latter case, applications are spread across many vacancies and therefore the congestion effects are practically absent but the firms coordination problem is severe. An increase in θ then aggravates the firm coordination problem further and this reduces the marginal social gains from applications. The socially optimal number of job applications balances the two coordination problems mentioned above to maximize the number of matches net of job applications costs.

4. Labor market equilibrium

We now turn to the private incentives in the labor market. We focus on situations where workers send more than one application so there is

[7] Loosely speaking, allowing a to be continuous is approximately similar to have a subset of workers observing more vacancies than the rest. For example, if a takes on value 2.3 it is as if 30% of the workers observes and applies to 2 vacancies and 70% observes and applies to 3 vacancies.

[8] Modelling the interaction between firms and workers as a simultaneous moves game captures a situation where a typical worker knows the locations of the firms with vacancies but only learns the wage of a firm after she has applied there and her application results in a job offer. Alternatively, in the directed search literature, workers observe the posted wages before applying. See Burdett and Wright (2003), Albrecht et al. (2006) and Galenianos and Kircher (2009).

[9] This recruitment technology describes a situation where frictions are large so that a firm that fails to hire its first candidate closes its vacancy (Albrecht et al., 2004). The other extreme is to assume that screening workers is costless so that a firm that fails to hire its first candidate can always go back to the next candidate and so on till the very last worker (complete recall) as in Kircher (2009), Gautier and Holzner (2016) and Elliot (2014). In the working paper version of our paper (see Gautier and Moraga-González, 2004), we also discussed the case of complete recall, which gave qualitatively similar results.

potential competition for workers. In such a situation, it is known that wage equilibria are in mixed strategies.[10]

We start by examining wage setting behavior by firms, given workers strategies. Suppose workers apply for $a \geq 2$ vacancies. A typical firm may receive one or more applications. If a firm receives more than one application, the firm picks one of the candidates at random. The (randomly chosen) applicant has sent a total of a applications, so she can potentially receive 1, 2, 3,…, up to a total of $a-1$ other wage offers. The firm does not compete for the worker in question with probability $(1-z(a,\theta))^{a-1}$, competes with just one other firm with probability $(a-1)z(a,\theta)(1-z(a,\theta))^{a-2}$, and so on. Thus the expected payoff to a firm i offering a wage w given that the other firms offer a wage from the wage offer distribution $F(\cdot)$ is:

$$\pi^i(w;F(\cdot)) = \rho(a,\theta)\left[\sum_{j=0}^{a-1}\binom{a-1}{j} z(a,\theta)^j(1-z(a,\theta))^{a-1-j}F(w)^j\right](1-w). \tag{2}$$

We can use the binomial theorem to simplify (2) to:

$$\pi^i(w;F(\cdot)) = \rho(a,\theta)[1-z(a,\theta)+z(a,\theta)F(w)]^{a-1}(1-w).$$

Because of the standard reasons (see Burdett and Judd, 1983), the wage offer distribution cannot contain mass points. Since in a mixed strategy equilibrium, all firms must be indifferent between strategies, profits of any firm must be equal to the profits of a firm that offers the lower bound of the support of $F(\cdot)$; this lower bound must be equal to the minimum wage because otherwise a firm offering such a wage would gain by decreasing it:

$$\pi^i(\underline{w};F(\cdot)) = \rho(a,\theta)(1-z(a,\theta))^{a-1}(1-\underline{w}). \tag{3}$$

Solving $\pi^i(w;F(\cdot)) = \pi^i(\underline{w};F(\cdot))$ yields the equilibrium wage distribution:

$$F(w;a,\theta,\underline{w}) = \frac{1-z(a,\theta)}{z(a,\theta)}\left[\left(\frac{1-\underline{w}}{1-w}\right)^{\frac{1}{a-1}} - 1\right]. \tag{4}$$

The upper bound of the support of the wage distribution can be found by setting $F(\overline{w};a,\theta,\underline{w}) = 1$ and solving for $\overline{w}$, which yields:

$$\overline{w} = 1-(1-\underline{w})(1-z(a,\theta))^{a-1}. \tag{5}$$

The maximum wage offered in the market is increasing in $\underline{w}$ and decreasing in a and θ.

Eqs. (4) and (5) characterize wage setting behavior by firms given worker strategies. A close look at the equilibrium wage distribution reveals that an increase in θ results in a rightward shift of the distribution. Thus, wages become more competitive as market tightness increases. An increase in the minimum wage $\underline{w}$ has a similar influence on the wage distribution. These remarks follow from three observations: (i) the upper bound of the distribution of wages is less than 1, and increasing in a and θ; (ii) $F(\cdot)$ is monotonically decreasing in $z(a,\theta)$ and in $\underline{w}$; and (iii) $z(a,\theta)$ is monotonically increasing in θ. These results are summarized next:

Lemma 1. *For any value of θ and $a \geq 2$, there exists a unique equilibrium wage distribution given by (4) with support $[\underline{w}, 1-(1-\underline{w})(1-z(a,\theta))^{a-1}]$. Further, an increase in the minimum wage $\underline{w}$, or in market tightness θ, results in higher (in the sense of first-order stochastic dominance) equilibrium wages.*

One of the reasons why workers find it attractive to apply for several jobs is that this increases the chance to get an offer. The other motive is that applying for various jobs increases the probability of getting several offers, i.e., a better paid job. Therefore, it is important to clarify how wage dispersion depends on the parameters of the model. A result that will be useful later is that wage dispersion is non-monotonic in market tightness. As in Burdett and Judd (1983), when $\theta \to 0$, the probability that two or more firms compete for the same worker is arbitrarily low so firms have no incentives to offer wages above the minimum wage. As θ increases, firms expect to face competition for a worker with positive probability and this leads to significant wage dispersion. When $\theta \to \infty$ every worker receives more than one wage offer almost surely so firms must pay workers their marginal productivity.

To complete the characterization of an equilibrium with wage dispersion and multiple applications, we need to study workers' optimal strategies. A worker maximizes utility taking firm wage setting and other workers' strategies as given. The (gross) expected utility of a worker sending a_ℓ applications when the rest of the workers sends a applications is:

$$u^j(a_\ell;a) = \int_{\underline{w}}^{\overline{w}}\left(\sum_{j=1}^{a_\ell}\binom{a_\ell}{j} z(a,\theta)^j(1-z(a,\theta))^{a_\ell-j}\cdot j\cdot F(w)^{j-1}\right)wf(w)dw. \tag{6}$$

This expression tells us that a worker who sends a_ℓ applications receives a single offer with probability $a_\ell z(a,\theta)(1-z(a,\theta))^{a_\ell-1}$ and in this case the wage the worker expects to get is the mean wage; the worker gets two offers with probability $\binom{a_\ell}{2}z(a,\theta)^2(1-z(a,\theta))^{a_\ell-2}$ and in this case the wage the worker expects to get is the maximum of a random sample of size 2; and so on and so forth.[11] Using the binomial theorem, Eq. (6) can be rewritten as

$$\begin{aligned} u^j(a_\ell;a) &= a_\ell z(a,\theta)\int_{\underline{w}}^{\overline{w}}[1-z(a,\theta)(1-F(w))]^{a_\ell-1}wf(w)dw \\ &= \overline{w} - \underline{w}(1-z(a,\theta))^{a_\ell} - \int_{\underline{w}}^{\overline{w}}[1-z(a,\theta)(1-F(w))]^{a_\ell}dw. \end{aligned} \tag{7}$$

where the last equality follows from integration by parts.

We now use the wage distribution (4) to obtain an expression for:

$$[1-z(a,\theta)(1-F(w))]^{a_\ell} = (1-z(a,\theta))^{a_\ell}\left(\frac{1-\underline{w}}{1-w}\right)^{\frac{a_\ell}{a-1}}.$$

Using this equality, we rewrite the utility of workers in (7) as:

$$\begin{aligned} u^j(a_\ell;a) &= \overline{w} - \underline{w}(1-z(a,\theta))^{a_\ell} - \int_{\underline{w}}^{\overline{w}}(1-z(a,\theta))^{a_\ell}\left(\frac{1-\underline{w}}{1-w}\right)^{\frac{a_\ell}{a-1}}dw \\ &= \overline{w} - \underline{w}(1-z(a,\theta))^{a_\ell} - (1-z(a,\theta))^{a_\ell}\cdot \\ &\quad \frac{(a-1)(1-\underline{w})\left[1-\left(\frac{1-\underline{w}}{1-\overline{w}}\right)^{\frac{a_\ell+1-a}{a-1}}\right]}{a-1+a_\ell}. \end{aligned}$$

Using the expression for the upper bound of the wage distribution in (5) and simplifying gives:

$$u^j(a_\ell;a) = 1-(1-\underline{w})(1-z)^{a-1} - \underline{w}(1-z)^{a_\ell} - \frac{(a-1)(1-\underline{w})(1-z)^{a_\ell}(1-(1-z)^{a-1-a_\ell})}{a-1-a_\ell},$$

[10] In the working paper version of this paper we also study single-wage equilibria. Such equilibria arise when all workers send just one job application. In that case, there is no competition for workers and firms set a wage equal to the minimum wage. This type of equilibrium can be sustained when the cost of sending job applications is relatively high. A disproportionately high application cost for all workers together with the absence of wage dispersion are not very realistic features so we have skipped single-wage equilibria in this version of the paper.

[11] In the empirical job search literature an important distinction is made between the offer distribution, F and the distribution of accepted wage offers which is in our case very similar to the bracketed term in (6): $G(a,\theta,w) = \sum_{j=0}^{a-1}\binom{a-1}{j}z(\theta,a)^j(1-z(\theta,a))^{a-1-j}\cdot F(w)^j$. This is basically a combination of the wage offer distribution $F(w)$ and the probability that the worker accepts the offer.

where, to shorten the expression, we have suppressed the dependency of z on a and θ.

A worker j should continue to send applications till the marginal gains from applying equal the marginal cost k. Taking the first order derivative of $u^j(a_\ell; a)$, the equilibrium condition is:[12]

$$\frac{(a-1)(1-\underline{w})\left((1-z)^{a-1}-(1-z)^{a_\ell}\right)}{\left(a-a_\ell-1\right)^2} - \frac{(a-1-a_\ell \underline{w})(1-z)^{a_\ell}\log[1-z]}{a-a_\ell-1} - k = 0.$$

After invoking symmetry, the first order condition for utility maximization is:

$$\Psi(a;\theta,\underline{w}) = k, \tag{8}$$

where

$$\Psi(a;\theta,\underline{w}) = (1-z)^{a-1}\left\{(a-1)(1-\underline{w})z + (a(1-\underline{w})-1)(1-z)\log[1-z]\right\}. \tag{9}$$

Numerical analysis of the marginal gains from applying reveals that, for $a \geq 2$, $\Psi(a;\theta,\underline{w})$ is positive and continuously decreasing in a, converging to zero as a approaches infinity. These conditions guarantee the existence and uniqueness of symmetric equilibrium.

Proposition 2. *Let $0 < k \leq \Psi(2,\theta,\underline{w})$. Then an equilibrium exists where workers send $a^*(\geq 2)$ applications and firms randomly choose wages from the set $[\underline{w}, 1-(1-\underline{w})(1-z(a^*,\theta))^{a^*-1}]$ according to the wage distribution*

$$F(w;a^*,\theta,\underline{w}) = \frac{1-z(a^*,\theta)}{z(a^*,\theta)}\left[\left(\frac{1-\underline{w}}{1-w}\right)^{\frac{1}{a^*-1}} - 1\right];$$

the number a^* solves $\Psi(a^*;\theta,\underline{w}) - k = 0$. The equilibrium unemployment rate is $(1-z(a^*,\theta))^{a^*}$.

This result characterizes labor market equilibria with multiple applications and wage dispersion. We note that the number of job applications workers send in equilibrium increases as k falls and becomes arbitrarily large as the cost of applications converges to zero. This contrasts with the work of Burdett and Judd (1983) and Acemoglu and Shimer (2000) where workers sample a maximum of *two* wages in equilibrium.

The equilibrium number of job applications is represented in Fig. 2. While computing a^* we set k=0.01 and $\underline{w} = 0$. Note that a^* first increases and then decreases in the vacancy-to-unemployment ratio θ. On the one hand, a higher θ gives workers incentives to apply for more vacancies because each application has a higher chance of resulting in an offer. On the other hand, a higher θ tends to decrease wage dispersion and this reduces the incentives workers have to send multiple applications. As Fig. 2 shows, the first effect dominates when market tightness is low while the second effect dominates otherwise.

5. Inefficiency of market equilibrium and minimum wage policies

A comparison of the workers equilibrium condition with the efficiency condition yields the following result:

Proposition 3. *Suppose k, θ and $\underline{w}$ are such that workers send multiple applications in equilibrium. Then the equilibrium number of applications is excessive from the point of view of social welfare maximization.*

This result, illustrated in Fig. 3, shows that workers send too many applications in equilibrium. The graph plots the equilibrium number of applications $a^*(\theta)$ and the efficient amount $a^o(\theta)$ (for k=0.01 and $\underline{w} = 0$). As mentioned above the social planner cares about the number

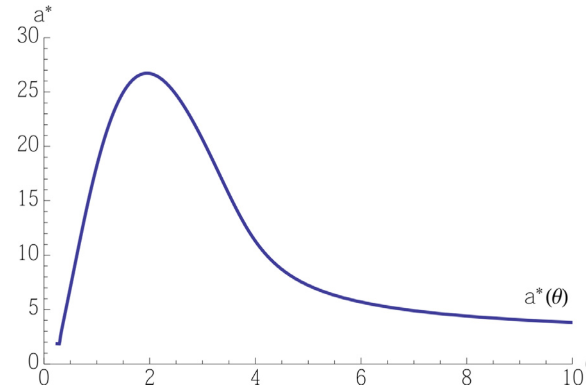

Fig. 2. The equilibrium number of job applications.

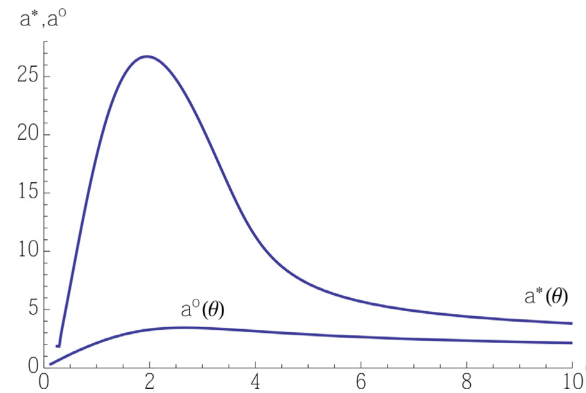

Fig. 3. Private and social incentives to send job applications.

of matches and thus internalizes the congestions caused by the job applications; by contrast, workers only care about their individual utility, which results in an excessive number of job applications.

Given the inefficiency in the decentralized market economy, we now ask whether there is a role for the minimum wage to improve or restore efficiency? We show that for high θ, a positive minimum wage is desirable but it can never restore efficiency because as long as there is wage dispersion there is inefficient rent-seeking behavior.

Proposition 4. *Assume that k, θ and $\underline{w}$ are such that workers send multiple job applications in equilibrium. Then, there exists a $\tilde{\theta}$ such that a^* is increasing (decreasing) in $\underline{w}$ for all $\theta < (>)\tilde{\theta}$. As a result, an increase in $\underline{w}$ reduces (increases) welfare for $\theta < (>)\tilde{\theta}$.*

For fixed a, an increase in the minimum wage does not affect the probability that a worker is hired but it does affect the wage a worker expects to get. When θ is low, the wage distribution is close to the degenerate distribution at the minimum wage; an increase in the minimum wage shifts the wage distribution upwards, which results in larger marginal gains from sending applications. In this case, a raise in the minimum wage is welfare reducing.

By contrast, when θ is large the wage distribution is close to the degenerate distribution at the competitive wage and, thus, an increase in the minimum wage reduces the marginal gains from applications by making the wage distribution even more compressed. In this case, a raise in the minimum wage is socially desirable.[13] Proposition 4 is illustrated in Fig. 4 where we have computed the equilibrium number of

[12] The second order condition is very difficult to verify analytically. We have nevertheless checked numerically that it holds.

[13] This mechanism is quite different from others currently emphasized in the literature: a minimum wage increase raises the likelihood a job is accepted (e.g. Burdett and Mortensen, 1998), increases the search intensity or drives out less productive firms (e.g. Van den Berg, 2003). For evidence on minimum wage increases compressing the wage distribution see Teulings (2003).

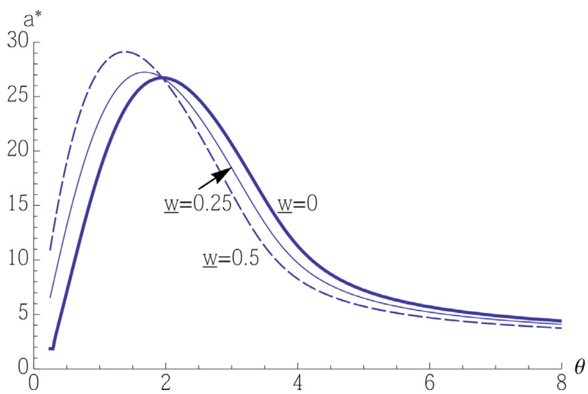

Fig. 4. Equilibrium number of applications, market tightness and minimum wage.

applications as a function of market tightness for different values of the minimum wage and k=0.01.

A minimum wage can by itself not restore efficiency because search is partly a rent-seeking activity. For a given mean wage, a higher variance of the wage distribution increases the incentives to apply for jobs because the worker's pay-off depends not on the average wage offer but on the expected highest offer. In our working paper we show that a mandatory wage as is offered in collective wage arrangements could restore efficiency. However, when workers and firms are heterogeneous, such policies may lead to a sub-optimal allocation (see Cai et al. 2015).

6. Final remarks

We have presented a simple model of the labor market where workers decide upon the number of jobs they apply for and firms simultaneously post a wage. If firms are capacity constrained, the fear of being rationed and remain unemployed gives workers incentives to send out multiple applications. However, sending out multiple applications can by itself generate more wage dispersion, which in turn increases the desire to apply for multiple vacancies and stimulates rent-seeking behavior. As a result, the number of applications in the market is socially excessive.

We started our paper with a quote from Chris Pissarides where he argued that a minimum wage can be desirable if there is not enough competition for workers. Here, we show that in sectors where the vacancy-to-unemployment ratio is high, a minimum wage can also be desirable. This is because in the absence of a minimum wage, workers spend too many resources to increase competition for them.

Appendix A

Proof of Proposition 1. Rewrite gross welfare as

$$W(x,\theta)=1-\left(1-\frac{1-e^{-x}}{x}\right)^{x\theta},$$

where $x=a/\theta$. Taking the derivative of W with respect to x we obtain the social marginal gains of an increase in x. This yields $\partial W(x,\theta)/\partial x=g(x)^{\theta}$, where

$$g(x)=-\left(\log\left[1-\frac{1-e^{-x}}{x}\right]+\frac{1-e^{-x}-xe^{-x}}{x\left(1-\frac{1-e^{-x}}{x}\right)}\right)\left(1-\frac{1-e^{-x}}{x}\right)^{x}.$$

Rewrite the social costs as $C(x,\theta)=kx\theta$. Taking the derivative of C with respect to x we obtain the social marginal costs of an increase in x. This gives $\partial C(x,\theta)/dx=k\theta$. If welfare is maximized, x must be such that $g(x)=k\theta^{1/\theta}$.

The following properties can be easily seen by simply plotting $g(x)$. The function $g(x)$ is continuous, with $\lim_{x\to 0}g(x)=\infty, \lim_{x\to\infty}g(x)=0$; moreover, there exists a unique $\tilde{x}$ for which $g(\tilde{x})=0$. As a result, $g(x)>0$ and monotonically decreases in x for all $x\le\tilde{x}$. This implies that for any k and θ, there exists a unique x, denoted x^o, that equalizes the marginal social gains to the marginal social costs.

The second order condition for welfare maximization requires that

$$\partial^2 W(x,\theta)/\partial x^2=\theta g(x)^{\theta-1}g\prime(x)<0,$$

which obviously holds for $x=x^o$ because $g\prime(x^o)<0$. Moreover, x^o is a global maximum because the properties of $g(x)$ guarantee that for any $x>x^o$ the marginal social gains are lower than the marginal social costs. For fixed θ, the social optimum obtains as $a^o=\theta x^o$.

To prove, finally, that the socially optimal number of applications first increases in θ and then decreases, we observe that

$$\frac{\partial a^o}{\partial\theta}=x^o+\theta\frac{\partial x^o}{\partial\theta},$$

where, using the implicit function theorem,

$$\frac{\partial x^o}{\partial\theta}=-\frac{k\theta^{\frac{1}{\theta}-2}(1-\log[\theta])}{g\prime(x^o)}.$$

Because $g\prime(x^o)<0$, the sign of $\partial x^o/\partial\theta$ is equal to the sign of $\theta^{\frac{1}{\theta}-2}(1-\log[\theta])$. In a neighborhood of $\theta=0$, this expression is positive and goes to 0. Therefore we conclude that $\lim_{\theta\to 0}\partial a^o/\partial\theta>0$ so a^o first increases in θ. When θ is larger, $\partial x^o/\partial\theta$ turns negative. Despite this, it is difficult to show analytically that a^o decreases for sufficiently large θ. Numerically solving for the optimum reveals that this is the case (see graph in the main text of the paper). □

Proof of Proposition 3. Assuming a is continuous, the efficient number of applications maximizes $W(a,\theta)-ak$, where $W(a,\theta)$ is given by (1). The first order condition is

$$-(1-z(a,\theta))^{a}\log\left[1-z(a,\theta)\right]+(1-z(a,\theta))^{a-1}\left(1-z(a,\theta)-\frac{az(a,\theta)}{\theta}\right)=k. \quad (10)$$

There is an excessive number of applications in market equilibrium when the LHS of (10) is smaller than the LHS of (8). For this, after simplification, it must be the case that:

$$(a-1)(1-\underline{w})z + a(1-\underline{w})(1-z)\log[1-z] - \left(1 - z - \frac{az(a,\theta)}{\theta}\right) > 0. \tag{11}$$

Because the expression $1 - z - az/\theta$ is negative, the LHS of (11) is larger than:

$$(1-\underline{w})\left\{(a-1)z + a(1-z)\log[1-z] - \left(1 - z - \frac{az(a,\theta)}{\theta}\right)\right\} > 0.$$

The last inequality can be seen by plotting the expression in curly brackets as a function of a and θ. □

Proof of Proposition 4. We examine how the marginal gains from applications change when the minimum wage goes up. This is given by the following derivative:

$$\frac{d\Psi(a,\theta,\underline{w})}{d\underline{w}} = -(1-z(a,\theta))^{a-1}[(a-1)z(a,\theta) + a(1-z(a,\theta))\log[1-z(a,\theta)]] \tag{12}$$

The sign of this derivative depends on the sign of:

$$(a-1)z(a,\theta) + a(1-z(a,\theta))\log[1-z(a,\theta)].$$

For a fixed a, this expression converges to zero as θ approaches zero and is negative and decreasing in θ in a neighborhood of $\theta = 0$. This implies that for a sufficiently small θ the number of applications workers send in equilibrium is increasing in the minimum wage. Since workers apply for too many jobs in equilibrium, an increase in the minimum wage results in a higher welfare. The expression becomes positive for large θ. Then for large θ, the equilibrium number of applications is decreasing in the minimum wage. In this case a minimum wage raise is desirable because it aligns social and private incentives. □

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