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Tax Loss Carryovers in a Competitive Environment*

ANJA DE WAEGENAERE, *Tilburg University*

RICHARD SANSING, *Tilburg University, Dartmouth College*†

JACCO L. WIELHOUWER, *Vrije Universiteit Amsterdam*

ABSTRACT

The fact that incumbent firms can immediately deduct research and development (R&D) investments from taxable income is generally believed to give them a strategic advantage over new firms that cannot deduct the investment cost, but instead generate a net operating tax loss carryover. Using an analytical model, we show that this conventional wisdom need not hold in a competitive environment. We examine operating and investment decisions in a duopolistic industry in which an initial investment in R&D yields an immediate tax benefit for one firm, but creates a net operating loss carryover for the other firm. If both firms invest in R&D, the firm with the net operating loss carryover makes more aggressive capital investment decisions following successful R&D. This may deter the incumbent firm from investing in R&D despite the lower aftertax costs of this investment. Changing the tax loss carryover rules would thus not only affects start-up or loss firms, but would also affect the investment decisions of profitable firms in the same industry.

Keywords: net operating loss carryovers, R&D investments, capital expenditures

Reports de pertes fiscales dans un environnement concurrentiel

RÉSUMÉ

Le fait que les sociétés établies puissent déduire immédiatement de leur revenu imposable leurs investissements dans la recherche et le développement (R-D) leur confère, estime-t-on généralement, un avantage stratégique par rapport aux nouvelles sociétés qui ne peuvent déduire le coût de l'investissement mais obtiennent plutôt un report de perte fiscale nette d'exploitation. À l'aide d'un modèle analytique, les auteurs montrent que cette idée reçue ne tient pas nécessairement dans un environnement concurrentiel. Ils étudient les décisions d'exploitation et d'investissement en situation de duopole dans un secteur d'activité où un investissement initial en R-D produit un avantage fiscal immédiat pour une société, mais crée un report de perte nette d'exploitation pour l'autre société. Si les deux sociétés investissent en R-D, la société qui affiche le report de perte nette d'exploitation prend des décisions plus audacieuses en ce qui a trait aux dépenses d'investissement lorsque les activités de R-D sont fructueuses. Cette situation risque de dissuader la société établie d'investir dans les activités de R-D malgré le coût après impôt plus faible de cet investissement. La modification des règles relatives au report des pertes fiscales aurait donc une incidence non seulement sur les sociétés en démarrage ou les sociétés affichant des pertes, mais également sur les décisions d'investissement des sociétés rentables appartenant au même secteur d'activité.

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† Corresponding author.

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Mots clés : reports de perte nette d'exploitation, investissements dans les activités de R-D, dépenses d'investissement

1. Introduction

We study the effect of a net operating loss (NOL) carryover on operating and investment decisions in a competitive setting. NOL carryovers exist because of an asymmetry in the tax law in which gains are immediately taxed when they are realized, but losses do not necessarily provide immediate tax refunds. Auerbach (1986) and Auerbach and Poterba (1987) identify two effects of this asymmetry in the tax law. First, a firm that is able to get an immediate tax benefit from a tax-deductible investment has a stronger incentive to make that investment than does a firm for which the investment generates an NOL carryover. Second, a firm with an NOL carryover can only make use of this NOL carryover if it generates taxable income. Therefore, once a firm has an NOL carryover it has a stronger incentive to make an investment that generates taxable income than does a firm without an NOL carryover. This in turn would enable a firm with an NOL carryover to generate more aftertax profits from an investment opportunity than a firm without an NOL carryover.

The existing literature considers these issues by examining the investment decisions of a single firm. In contrast, in this study we examine how two firms competing in the same industry make research and development (R&D) and capital investment decisions when they are in different tax positions. One firm gets an immediate tax benefit from a tax-deductible R&D investment, the other creates an NOL carryover from its tax-deductible R&D investment. The conventional wisdom suggests that the firm receiving the immediate tax benefit is more likely to make the R&D investment than the firm that only creates an NOL carryover (Scholes et al. 2015). In contrast to what conventional wisdom suggests, we find that the strategic interaction between the two firms creates situations in which the firm receiving the immediate tax benefit is less likely to invest in R&D.

We study an industry that comprises two firms: an incumbent firm (the old firm) and a start-up firm (the new firm). The firms differ only in that the old firm is an existing firm that is profitable in a different business, whereas the new firm is in no other business. We consider a project that requires investment at two stages. In the first stage, each firm has the opportunity to make an investment in R&D. The old firm can deduct the investment from profits generated by its other business, yielding an immediate tax benefit. The new firm cannot deduct the investment from taxable income on other profits, so it creates an NOL carryover that it can only use to offset taxable income in the future. The R&D investment leads to a technological discovery at some date in the future, which we call the discovery date. At this point, the first stage ends and the second stage begins. At its discovery date, each firm can make a capital investment that allows it to manufacture products using the discovery. If the new firm makes the capital investment, it begins using its NOL carryover to offset the tax on profits at the start of the second stage. The tax asymmetry provides the old firm an *immediate tax deduction advantage* on date zero due to its lower aftertax investment cost of the initial R&D investment. However, it also provides the new firm an advantage on its discovery date because the capital investment allows the new firm to start using its NOL carryover, which implies that there are capital investments that have negative net present value (NPV) for the old firm but positive NPV for the new firm. We refer to this advantage as the new firm's *net operating loss advantage*. The greater willingness of the new firm to make capital investments can deter the old firm from making the initial tax-deductible R&D investment in the first place. On the other hand, the ability of the old firm to obtain an immediate tax benefit rather than to defer the tax benefit via an NOL carryover can make the initial investment more attractive to that firm. The net effect of these two considerations on the incentive to make the initial investment is ambiguous.

Our study has policy implications regarding the tax rules relating to net operating losses, such as making the tax loss carryforward period unlimited as was done under the Tax Cuts and Jobs Act (TCJA) in the United States. Most policy discussions involving net operating losses focus on the effects of tax losses on start-up firms. We find that these rules also affect the investment decisions of profitable firms operating in the same industry as loss firms. In particular, we find that the NOL carryover of a start-up firm can deter a profitable rival from making an investment. More generally, the apparent disadvantage from deferring the use of a tax benefit associated with a current expenditure can create a net advantage via its effects on a rival firm not subject to the same restriction.

The first stage of our model—the initial R&D investment—relates to studies on how taxes affect incentives to invest in internally developed intangible assets. Robinson and Sansing (2008) examine both the tax advantages and financial reporting disadvantages of investments in intangible assets. De Waegenaere et al. (2012) include both the tax deductibility of investments in internally developed intangible assets and the ability to shift income attributable to intangible assets to low-tax foreign jurisdictions. The tax-favored treatment of the initial investment in R&D also plays an important role in our study. However, we focus on how the asymmetric treatment of gains and losses affects competition.

Conventional wisdom suggests that profitable firms that are able to use tax losses immediately are more likely to invest in projects that generate tax losses early in the life of the project. Berger (1993) shows that R&D tax credits are effective in stimulating investments in R&D. Hall and Van Reenen (2000) and Bloom et al. (2002) also show that tax incentives stimulate R&D investments. Shevlin (1987) found that start-up firms are more likely to fund R&D investments via R&D limited partnerships, which enables the tax deductions to be used immediately as opposed to creating an NOL carryover. The usefulness of R&D limited partnerships was significantly reduced after the enactment of the Tax Reform Act of 1986. Auerbach and Poterba (1987) show that an NOL carryover can lower the incentive to invest in projects with rapid depreciation deductions, because a firm with an NOL carryover does not benefit from tax losses in the early years of the project. This certainly holds for R&D investments that are immediately expensed. Devereux et al. (1994) show that the asymmetric treatment of losses and gains increases the cost of capital of start-up firms.

The second stage of our model—the follow-up capital investment for which the start-up firm can use its NOL—relates to literature on the relation between capital investments and NOL carryovers. Once a firm has an NOL carryover, it has an additional incentive to invest in projects with positive expected taxable income, because the NOL carryover will shelter some or all of the income from the investment from tax. This can lead to higher capital expenditures (Cooper and Franks 1983; Wielhouwer et al. 2000) and earlier execution of real investment options (De Waegenaere et al. 2003). However, empirical evidence on the relation between tax loss carryforwards and investments is inconclusive. Edgerton (2010) finds that firms that pay no taxes generally make lower investments. In contrast, Dreßler and Overesch (2013) find that NOL carryforwards mitigate the negative effect of tax rates on investments.

Section 2 presents the model. In section 3, we characterize the equilibrium actions at each stage. We characterize the equilibrium outcomes in section 4. We present sensitivity and robustness analysis in section 5 and section 6 concludes.

2. Model

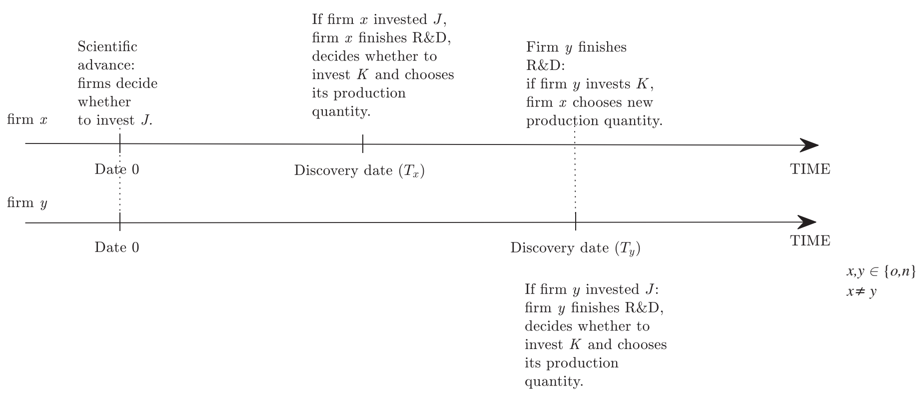
We consider two firms, which we denote "old" for the incumbent and "new" for the start-up. The firms are indexed $i \in \{o, n\}$. On date zero, the firms observe an exogenous scientific advance that allows them to develop a technology by investing J in R&D. For example, upon observing the availability of a new biofuel, the firms could decide whether to invest in R&D to develop an engine that is suitable for biofuels. Whether a firm invests in R&D is not observed by the competitor.

All profits are taxed at a constant statutory tax rate τ, $0 \leq \tau < 1$. The firms differ in that the old firm is an existing firm with annual profits in a different business that exceed J. The new firm is in no other business. This difference has implications for the tax treatment of the cost J that each firm incurs if it invests on date zero. The old firm has another profitable business that is sufficiently large that the R&D investment J can be fully deducted. Therefore, the aftertax cost of the investment on date zero to that firm equals $(1 - \tau)J$. Because the new firm has no other business, it creates an NOL that it is able to carry forward to future periods. For simplicity, we assume that the loss can be carried forward indefinitely into the future and used to offset all taxable income.[1] Because the focus of our study is on how the NOL carryover of one firm affects the interaction between the two firms, we deliberately suppress any nontax differences between the two firms. Accordingly, we assume that the pretax costs and benefits of the investments are the same for each firm.

If a firm invests J in R&D, the time to successful development of the technology is T_o for the old firm and T_n for the new firm, where T_o and T_n are independent and identically distributed random variables with T_o, $T_n \sim Exp(\theta)$ (see, for example, Dasgupta and Stiglitz 1980), so the expected time to discovery is $1/\theta$. After a firm successfully finishes its R&D, it can produce goods at marginal cost v per unit by making a fixed capital investment of K.[2] We assume that each firm observes when the rival starts production, which happens immediately following the capital investment K.[3] The first firm to invest K is a monopolist in the market. If the second firm also invests K, the firms engage in Cournot duopolistic competition from that point forward. Let the price p of the product be $p = d - Q$, where d is a demand parameter and $Q = q_o + q_n$ is total production by both the old and the new firm. The parameters d, v, K, J, and θ are common knowledge. The timeline is illustrated in Figure 1.

We define an equilibrium as a production decision, a capital investment decision, an R&D investment decision, and a belief regarding the rival's R&D investment strategy, capital investment strategy, and production quantities such that:

Figure 1 The timeline of events



Notes: If only firm $i \in \{o, n\}$ makes the R&D investment J on date 0, then $x = i$ and date T_y does not exist. If both firms invest J on date zero, $x \in \{o, n\}$ denotes the firm that finishes R&D first and $y \in \{o, n\}$, $y \in x$, denotes the firm that finishes R&D second. If both firms invest J on date 0 and make the capital investment K on their discovery date, firm x will be a monopolist from date T_x till date T_y; after date T_y, the firms engage in duopolistic competition. If only one firm invests both J on date 0 and K on its discovery date, that firm becomes a monopolist from the date it makes the discovery.

1. Countries may have limited loss carryforward periods or restrictions on the percentage of taxable income that the loss carryover can offset. Such restrictions do not qualitatively change our results.
2. For expositional convenience, we assume that the capital investment K does not decay over time and is not expensed or depreciated for tax purposes. Although depreciating K for tax purposes reduces the new firm's net operating loss advantage, it can still dominate the old firm's immediate tax deduction advantage.
3. Including a time delay between the capital investment K and the start of production does not qualitatively change our results.

- On each date after the date on which the firm has invested K, its production quantity maximizes its aftertax profits, given the rival's production quantity on that date.
- On each firm's discovery date T_i,
  - if the rival has already invested K, the firm chooses whether to invest K to maximize the present value of its future aftertax cash flows, taking its own future production quantity and its beliefs regarding the rival's future production quantity as given;
  - if the rival has not yet invested K, the firm chooses whether to make a capital investment K to maximize the present value of its future aftertax cash flows, taking its own future production quantity and its beliefs regarding the rival's future capital investment strategy and production quantity as given.
- On date zero, each firm chooses whether to invest J in R&D to maximize the present value of its future aftertax cash flows, given its own capital investment strategy and future production quantity and its beliefs regarding the rival's R&D investment strategy, capital investment strategy and production quantities.
- All beliefs are consistent with the firms' R&D investment strategies, capital investment strategies, and production quantities.

3. Equilibrium investment decisions

In this section, we investigate the effects of the new firm's NOL on the equilibrium investment decisions of the two firms. We first characterize the optimal production decisions and determine the value of the aftertax profits for each firm. In two subsequent subsections, we use these results to characterize the firms' equilibrium decisions whether to invest K on their discovery dates, that is, once they finish R&D, and whether to invest J in R&D on date zero.

Aftertax profits as of the discovery date

We now derive the present value of each firm's aftertax profits as of its discovery date, which depend on whether the rival also invests K on its discovery date, and, if so, on whether the firm is first or second to make the discovery. We characterize a firm's optimal production quantity in Lemma 3 in the Appendix.

We first determine the present value of aftertax profits to firm $i \in \{o, n\}$ if the rival does not invest K. If only firm i invests K, it earns pretax monopoly profits of $m = (d - v)^2/4$ per period in perpetuity; see Lemma 3 and its proof in the Appendix. The aftertax present value of these profits, ignoring the effects of the NOL carryover equals:

$$P(m) = \int_0^\infty (1-\tau)me^{-rt}dt = \frac{(1-\tau)m}{r}, \tag{1}$$

where $r > 0$ denotes the discount rate. Whereas the old firm expenses its investment J on date zero, the investment of J by the new firm creates an NOL that it uses after it invests K. Therefore, in addition to $P(m)$, the new firm earns the present value of the tax benefits from the NOL. Because the new firm can use its NOL at rate m per period from its discovery date until it is fully used, which occurs on date $t = T_n + J/m$, the present value of the tax benefits from the NOL at the date it invests K (i.e., the discovery date T_n) equals:

$$\int_0^{J/m} \tau m \mathrm{e}^{-rt}dt = \frac{\tau m}{r}\left(1-\mathrm{e}^{-rJ/m}\right) = \tau J \delta(m,J), \tag{2}$$

where

$$\delta(m,J) = \frac{m}{rJ}\left(1-\mathrm{e}^{-rJ/m}\right) \in (0,1). \tag{3}$$

The term $\delta(m, J)$ can be interpreted as a discount factor. When pretax income m becomes arbitrarily large or when the discount rate r becomes arbitrarily small, $\delta(m, J)$ converges to one. Thus the present value of tax benefits is close to τJ, the tax benefit that would be obtained if the initial investment J was deducted immediately. In contrast, if m is very small or r is very large, $\delta(m, J)$ converges to zero. In those cases, the present value of the tax benefits of the NOL is small.

Next, we determine the present value of aftertax profits to firm $i \in \{o, n\}$ if the rival also invests K, and firm i is first to finish R&D. Firm i then earns monopoly profits of m per period until the rival invests K, and earns duopoly profits of $c = (d - v)^2/9$ per period thereafter in perpetuity. See Lemma 3 and its proof in the Appendix for the derivation of the duopoly profits. The time that elapses until the rival finishes its R&D is an exponentially distributed random variable with mean $1/\theta$. We show in Lemma 2 in the Appendix that if firm $i \in \{o, n\}$ is first to finish R&D, the expected present value of aftertax profits, ignoring the effects of the NOL carryover, equals:

$$\tilde{P}(m,c) = \left(\frac{r}{\theta + r}\right)\frac{(1-\tau)m}{r} + \left(\frac{\theta}{\theta + r}\right)\frac{(1-\tau)c}{r}. \tag{4}$$

Because $m > c$, it follows from (4) that $\tilde{P}(m,c)$ is decreasing in θ. This occurs because the present value from investing K when a firm finishes its R&D first is higher when it expects to be a monopolist for a longer time, that is, when θ is lower.

In addition to $\tilde{P}(m,c)$, the new firm earns the present value of the tax benefits from its NOL. If it is first to finish R&D, it can use its NOL at rate m until the old firm finishes its R&D or the NOL is fully used, whichever occurs first. If the NOL is not fully used by the time the old firm finishes its R&D, the new firm uses its remaining NOL at rate c as of that date until it is fully used. We show in Lemma 2 in the Appendix that the expected present value of the tax benefits for the new firm as of its discovery date equals $\tau J\tilde{\delta}(m,c,J)$, where:

$$\begin{aligned}\tilde{\delta}(m,c,J) = &\left[\left(\frac{m}{rJ}\right)\cdot\left(\frac{r}{\theta + r}\right) + \left(\frac{c}{rJ}\right)\cdot\left(\frac{\theta}{\theta + r}\right)\right]\cdot\left(1 - \mathrm{e}^{-(r+\theta)\frac{J}{m}}\right) \\ &+ \left(\frac{c}{rJ}\right)\cdot\left(\frac{\theta}{\theta + r - rm/c}\right)\cdot\left(\mathrm{e}^{-(\theta + r)\frac{J}{m}} - \mathrm{e}^{-r\frac{J}{c}}\right) \in [0,1].\end{aligned} \tag{5}$$

The term $\tilde{\delta}(m,c,J)$ can be interpreted as the expected discount factor for the new firm's NOL when both firms invest K and the new firm finishes its R&D first.

Finally, we determine the present value of aftertax profits to firm $i \in \{o, n\}$ if the rival also invests K, and firm i is second to finish R&D. Firm i then earns duopoly profits of c per period into perpetuity. Therefore, the aftertax present value of these profits equals $P(c)$ for the old firm, and equals $P(c) + \tau J\delta(c, J)$ for the new firm, where $P(c)$ and $\delta(c, J)$ are given by (1) and (3) with m replaced by c.

In Table 1, we summarize the present value of aftertax profits from investing K on the discovery date.

We assume that the market is sufficiently lucrative that in the absence of a competitor, either firm would invest J on date zero and K on its discovery date. We show in Lemma 4 in the Appendix that this is satisfied if and only if:[4]

4. There are values of J above $J_{\max}$ for which the new firm will not invest in the absence of a competitor, whereas the old firm will invest because it gets an immediate tax benefit from investing J. We restrict our attention to projects that both firms could consider profitable in order to study the effects of the asymmetric tax treatment on competitive behavior.

TABLE 1
Present value of aftertax profits from investing K on the discovery date, conditional on being first or second to finish R&D and on the rival's strategy

	Present value for firm i if it invests K		
	Rival does not invest K	Rival will invest K and firm i is first to finish R&D	Rival invests K and firm i is second to finish R&D
$i = o$	$P(m)$	$\tilde{P}(m,c)$	$P(c)$
$i = n$	$P(m) + \tau J\delta(m, J)$	$\tilde{P}(m,c)+\tau J\tilde{\delta}(m,c,J)$	$P(c) + \tau J\delta(c, J)$

Notes: $P(m)$ ($P(c)$) is the after tax present value of future profits for the entire period as a monopolist (Cournot duopolist) excluding the effects of the NOL (see (1)). $\tilde{P}(m,c)$ is the after tax present value of future profits excluding the effects of the NOL when the firm finishes R&D first and the rival will also invest when it finishes R&D (see (4)). $\tau J\delta(m, J)$ ($\tau J\delta(c, J)$) is the present value of future tax savings from the NOL for the new firm when it is a monopolist (Cournot duopolist) for the entire period (see (3)). $\tau J\tilde{\delta}(m,c,J)$ is this present value when the new firm finishes R&D first and the old firm will also make the capital investment K when it finishes R&D (see (5)).

$$\left(\frac{\theta}{\theta+r}\right)\left[\frac{(1-\tau)m}{r}+\tau J\delta(m,J)-K\right]\geq J. \tag{6}$$

We refer to the highest value of J that satisfies (6) as $J_{\max}$. Because $J_{\max}$ depends on K, (6) puts a joint bound on J and K.

Equilibrium capital investment decisions on the discovery date

In this section, we use Table 1 to characterize the firms' equilibrium capital investment decisions on their respective discovery dates, that is, when they finish R&D.

If the rival did not invest J, condition (6) ensures that the firm will invest K on its discovery date. If both firms invest J on date zero, each firm's equilibrium decision whether to invest K on its discovery date depends on whether the rival also invests K, and if so, on whether the firm is first or second to make the discovery. To characterize the equilibrium decisions, we define three critical values of K in Table 2.[5]

Each firm is willing to invest K if, and only if, the investment has a non-negative NPV. Using Table 1, the maximum that the old firm is willing to invest is $K_o = P(c)$ when it is the second to finish R&D. The maximum that the new firm is willing to invest is $K_n = P(c) + \tau J\delta(c, J)$ when it is the second to finish R&D. Using (4), the maximum that the old firm is willing to invest is $K_f = \widetilde{P}(m,c)$ when it is the first to finish R&D, assuming that the new firm will invest K on its discovery date.

To determine the optimal investment strategies, we rank the three critical values. First, because $m > c$, investing is more attractive to the old firm if it is first to finish R&D than if it is second to finish R&D, and so it holds that $K_o < K_f$. Second, because the new firm still has the NOL that it can use to reduce the tax burden of the investment revenues, investing K if the rival is already in the market is more attractive to the new firm than to the old firm, that is, $K_o < K_n$. Finally, K_f can be smaller or greater than K_n. The difference between them is:

5. There is a fourth critical value, $K_{fn} = \tilde{P}(m,c)+\tau J\tilde{\delta}(m,c,J)$, below which the new firm will make the capital investment if (1) it finishes R&D first and (2) the old firm will invest K on its discovery date. However, the old firm will only invest when it is second to finish R&D if $K \leq K_0 \leq K_{fn}$. Therefore, this fourth critical value has no effect on the investment behavior of the firms.

TABLE 2
Critical values of K

Maximum value of K	For which
$K_o = \frac{(1-\tau)c}{r} = P(c)$	Old firm invests K if it is second to finish R&D and the new firm has already invested K on its discovery date
$K_n = P(c) + \tau J\delta(c, J)$	New firm invests K if it is second to finish R&D and the old firm has already invested K on its discovery date
$K_f = \left(\frac{r}{\theta+r}\right)\frac{(1-\tau)m}{r} + \left(\frac{\theta}{\theta+r}\right)\frac{(1-\tau)c}{r} = \widetilde{P}(m,c)$	Old firm invests K if (i) it is first to finish R&D and (ii) the new firm will invest K on its discovery date

Notes: K is the capital investment that a firm needs to make to start production once it finishes R&D. The critical value mentioned in column 1 is the maximum value of K for which a firm wants to invest K in the scenario mentioned in column 2.

$$K_f - K_n = \frac{(m-c)(1-\tau)}{\theta+r} - \tau J\delta(c,J). \tag{7}$$

Recall that K_f is the critical value below which the old firm invests if it is first to finish R&D, assuming that the new firm will invest K when it finishes its R&D. The first term on the right hand side of (7) is the difference between the expected present value from investing K to the old firm and to the new firm in the scenario in which the old firm finishes R&D first and both firms invest K on their respective discovery dates. The second term is the present value of the new firm's tax benefits from the NOL in that scenario. The new firm's tax benefit from the NOL ($\tau J\delta(c, J)$) is increasing in the tax rate τ and in the initial investment J. If both J and τ are sufficiently high, the new firm's tax benefit is so high that the value it derives from investing K when it is second to finish R&D exceeds the value that the old firm derives from investing K when it is first to finish research, that is, $K_n > K_f$. If the tax rate τ is too low, there is no value of J for which $K_n > K_f$. We summarize the possible rankings of K_o, K_n, and K_f in the following lemma.

Lemma 1. *The three critical values K_o, K_n, and K_f are ranked as follows*:

$$\begin{aligned} &K_o < K_f < K_n, \quad \text{if } \tau > \tau^* \text{ and } J > J^*, \\ &K_o < K_n \le K_f, \quad \text{otherwise}, \end{aligned} \tag{8}$$

where

$$\tau^* = \frac{r(m-c)}{rm+\theta c}, \tag{9}$$

$$J^* = \frac{c}{r}\log\left[\frac{c\tau(\theta+r)}{c(\theta\tau+r)-(1-\tau)rm}\right]. \tag{10}$$

We now use the rankings in (8) to characterize the optimal investment decisions on the firms' discovery dates. To do so, we identify four regions of values of K, denoted as regions A–D. The regions are defined in Table 3.

Because the new firm has the NOL that it can use to shield revenues from taxes once it starts producing, the NPV from making the capital investment is higher for the new firm that for the old firm. This affects the firm's equilibrium investment decisions in regions B and C, but not in regions A and D. In region A, K is sufficiently low that investing K has positive NPV even if the rival has invested K and even without the NOL (because $K \le K_o$). Therefore each firm invests

TABLE 3
Definitions of regions A–D

	Region			
	A	B	C	D
If $\tau > \tau^*$ and $J > J^*$:	$K \leq K_o$	$K_o < K \leq K_f$	$K_f < K \leq K_n$	$K > K_n$
Otherwise:	$K \leq K_o$	$K_o < K \leq K_n$	–	$K > K_n$

Notes: The critical values K_o, K_f, and K_n are as defined in Table 2. If the tax rate τ satisfies $\tau > \tau^*$ and the R&D investment J satisfies $J > J^*$, it holds that $K_n > K_f$ (see (8)), and the values of the capital investment K between K_o and K_n are divided between regions B and C. Otherwise, it holds that $K_n \leq K_f$. In this case, region C is empty and all values of K between K_o and K_n are in region B.

K even if it is second to finish R&D. In region D, K is sufficiently high that investing if the rival has already invested K has negative NPV even with the NOL (because $K > K_n$), so each firm invests only if it is first to finish R&D.

In contrast, for intermediate values of K (regions B and C), the new firm's NOL carryover implies that some projects that have negative NPV to the old firm have positive NPV for the new firm. In region B, $K \leq K_f$ ensures that the capital investment has positive NPV for either firm if it finishes R&D first. However, investing if the rival finishes R&D first only has positive NPV for the new firm because $K > K_o$. Hence, the new firm invests even if it finishes R&D second (because $K < K_n$), whereas the old firm only invests if it finishes R&D first. In region C, $K \leq K_n$ ensures that the new firm will invest K on its discovery date even if the old firm has already invested K. Anticipating this behavior, the old firm will not invest K on its discovery date because $K > K_f$. Hence, in region C the new firm's credible threat to invest if it is second to finish R&D deters the old firm from investing even if it finishes R&D first.

We summarize the firms' investment strategies on their respective discovery dates in each of the four regions in Proposition 1.

PROPOSITION 1. *If both firms invest J on date zero, the optimal investment decisions on their discovery dates are as displayed in Table 4.*

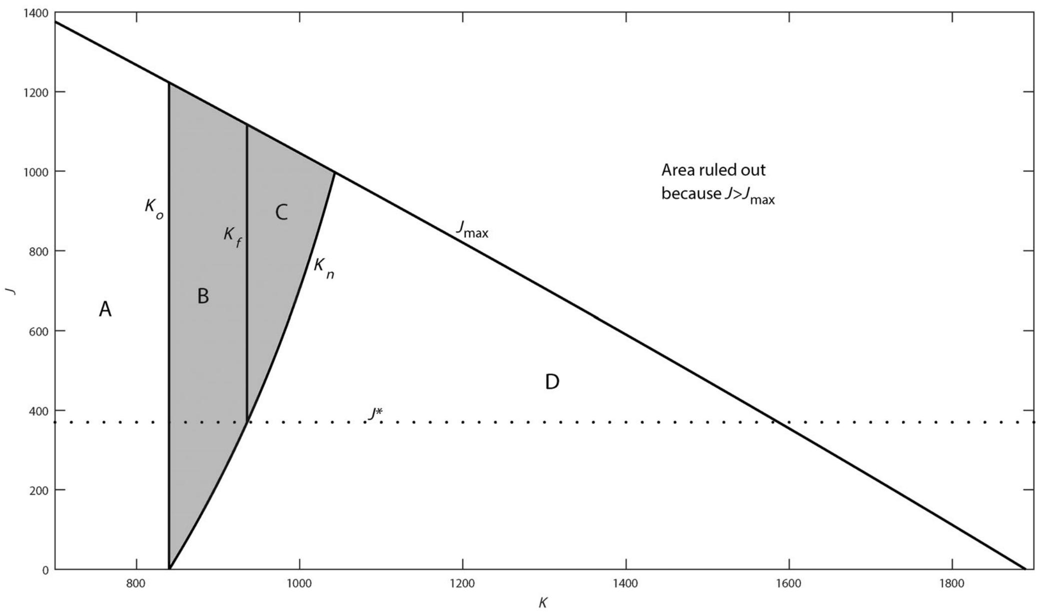
The firms' equilibrium behavior in regions B and C is consistent with the idea that a firm with an NOL carryover will be more aggressive than a firm without an NOL carryover when making an

TABLE 4
Decision whether to invest K on the discovery date if both firms invest J on date zero

Region	Investment decision by new firm	Investment decision by old firm
Region A	always invest	always invest
Region B	**always invest**	**invest only when first**
Region C	**always invest**	**never invest**
Region D	invest only when first	invest only when first

Notes: always invest means that the firm makes the capital investment K regardless of whether it is first or second to finish R&D; *invest only when first* means that the firm invests K only if it finishes R&D first. The bold entries represent cases in which the firms make different investment decisions when in similar circumstances.

Figure 2 Regions that determine the equilibrium investment decisions at the discovery dates



Notes: The regions A–D from Table 3 that determine the equilibrium investment decisions on the firms' discovery dates, as a function of the capital investment *K* and the R&D investment *J*. The parameter values are $\tau = 30\%$, $r = 10\,\%$, $c = 120$, $m = 270$, and $\theta = 1$. The areas in which the new firm invests more aggressively on its discovery date if both invest *J* on date zero are shaded.

investment that generates taxable income (e.g., Auerbach 1986; Cooper and Franks 1983; Wielhouwer et al. 2000).

We illustrate the four regions A–D (defined in Table 3) in Figure 2. The parameter values are $\tau = 30\%$, $r = 10\%$, $c = 120$, $m = 270$, and $\theta = 1$. Combinations of *K* and *J* in the upper right part of the graph are ruled out because $J > J_{\max}$, and so investing in R&D would not be attractive to the new firm even in the absence of competition. Because $J \le J_{\max}$ and because the boundary K_n is a function of *J*, the regions depend jointly on *K* and *J*. For all values of *J*, the vertical line that represents the value of K_o separates regions A and B. The separations between regions B, C and D depend on whether $K_n < K_f$ or $K_n \ge K_f$, which in turn depends on the values of τ and *J*. Because $\tau > \tau^* \approx 0.102$, there exists a J^* such that $K_n \le K_f$ for $J \le J^*$ and $K_n > K_f$ for $J > J^*$ (see (8)). The dotted line represents the value of $J^* \approx 370$. For $J \le J^*$ the sloping line that represents the value of K_n separates regions B and D and region C does not exist (see the second row in Table 3). For $J > J^*$, the vertical line that represents K_f separates regions B and C, and K_n separates regions C and D (see the first row in Table 3).

Intuitively, one might expect that an increase in *K* makes it less likely that a firm makes the investment. However, this is not the case for the old firm's capital investment decision when it is first to finish R&D and both $\tau > \tau^*$ and $J > J^*$. In that case, region C is not empty (see Table 3) and Table 4 shows that the old firm does not invest in region C but it does invest in region D, even though *K* is higher in region D.

Corollary 1. *If $\tau > \tau^*$ and $J > J^*$, the old firm's decision to invest K on its discovery date when it is second to finish R&D is not monotonic in K: it invests if $K \le K_f$ and if $K > K_n$, but does not invest if $K \in (K_f, K_n]$.*

The nonmonotonicity in the old firm's investment decision occurs because an increase in K affects both firms' incentives to invest. As can be seen from Table 4, when K increases from $K \in (K_f, K_n]$ (region C) to $K > K_n$ (region D), the new firm's strategy changes from investing even if it is second to finish R&D (in region C), to only investing if it is first to finish R&D (in region D). The fact that the new firm does not invest K if it is second to finish R&D in region D makes investing K attractive to the old firm when it is first to finish R&D.

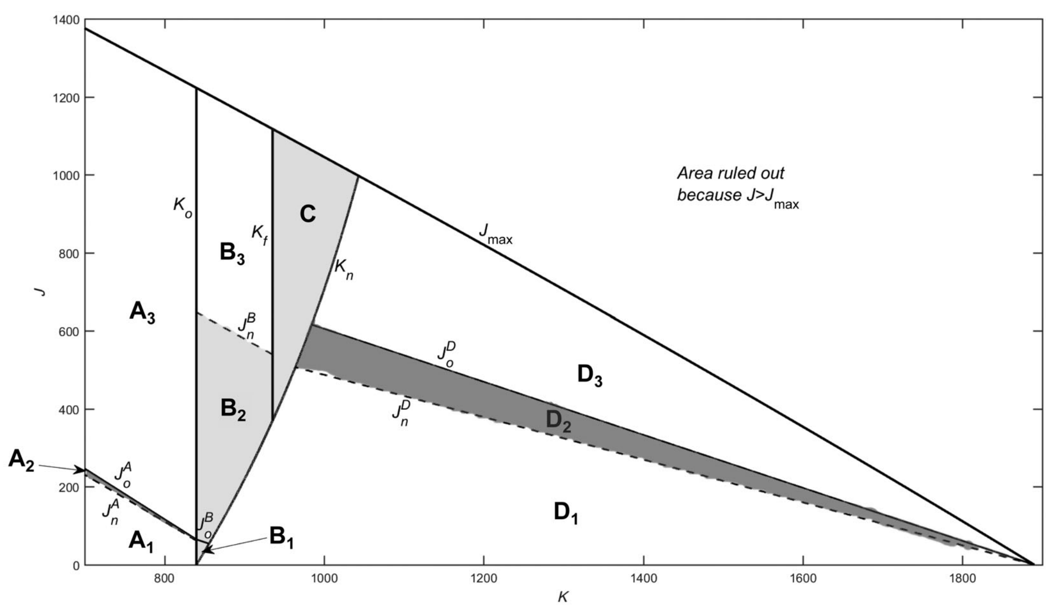
Equilibrium R&D investment decisions on date zero

In this section we use the firms' equilibrium capital investment strategies on their respective discovery dates, as characterized in Proposition 1, to determine their equilibrium R&D investment decisions on date zero.

We first consider region C. Table 4 shows that in this region, the new firm's aggressive investment strategy on its discovery date deters the old firm from investing K even if it makes the discovery first. This in turn implies that if the new firm invests J on date zero, the old firm should not invest J on date zero as it will never make the capital investment K. Hence, only the new firm invests J on date zero in region C.

In contrast, in regions A, B, and D in Figure 3, each firm will invest K if it finishes R&D first (see Table 4). Therefore, for each firm there exists a critical value of J below which the firm is willing to invest J even if the rival does so too (see Lemma 6 in the Appendix). Let J_o be the maximum value of J for which the old firm wants to invest J on date zero if the rival also invests on date zero, and let J_n be this maximum value for the new firm. If J exceeds both J_o and J_n (J high), both firms only want to invest if the rival does not. If J is lower than both J_o and J_n

Figure 3 Regions that determine the equilibrium investment decisions on date zero



Notes: The regions A–D from Proposition 2 that determine the equilibrium investment decisions on date zero, as a function of the capital investment K and the R&D investment J. The parameter values are τ = 30%, r = 10%, c = 120, m = 270, and θ = 1. The areas where tax asymmetry leads to asymmetric investment behavior are shaded. In the dark shaded regions (A_2 and D_2) the old firm's immediate tax deduction advantage deters the new firm from investing J on date zero. In the light shaded regions, the new firm's net operating loss advantage deters the old firm from investing J on date zero (in regions B_2 and C), or implies that the new firm invests more aggressively on its discovery date when they both invest J (in region B_1).

TABLE 5
J conditions and optimal strategies in regions A, B, and D

	J values	Strategy
J high	$J_n, J_o < J < J_{\max}$	Each firm only invests if rival does not
J intermediate	$J_o < J \le J_n$	Only new firm invests
	or $J_n < J \le J_o$	Only old firm invests
J low	$J \le J_n, J_o$	Both firms invest

Notes: This table presents the optimal investment strategies (column 3) dependent on the level of the R&D investment J. The regions where J is high, intermediate, or low are defined by the ranking of the critical values J_o, J_n, and $J_{\max}$. J_o (J_n) is the maximum value of J for which the old (new) firm wants to invest J when the rival also invests J. $J_{\max}$ is the maximum value of J for which each firm would invest J in the absence of competition.

(J low), both firms will invest on date zero. For values between these critical values (J intermediate), one firm is willing to invest if its rival also invests, whereas the other firm only wants to invest if the rival does not. As a result, the firm that is always willing to invest does so, thereby deterring its rival from investing J on date zero. We summarize the general pattern of the investment decisions in regions A, B, and D in Table 5.

The critical values J_o and J_n that determine the equilibrium behavior in regions A, B, and D differ across the three regions because the firms use different capital investment strategies on their respective discovery dates in each region (see Proposition 1), which affects the NPV of investing J. Therefore, henceforth we denote the critical values in region $k \in \{\text{A, B, D}\}$ by J_o^k and J_n^k.

Table 5 shows that for intermediate values of J in regions A, B and D, two cases are possible: only the old firm invests if $J_o^k > J_n^k$; only the new firm invests if $J_o^k < J_n^k$. Hence, to fully characterize the equilibrium investment decisions of the two firms in regions A, B, and D, we need to rank the critical values J_o^k and J_n^k in each region. In regions A and D, it holds that $J_n^k \le J_o^k$. This occurs because: (i) the two firms have the same investment strategies on their respective discovery dates (see Table 4); and (ii) the present value of the tax deductions is higher for a firm that can deduct the loss immediately. Combined this implies that the aftertax value from investing J on date zero is higher for the old firm than for the new firm, so $J_n^k \le J_o^k$ for $k \in \{A, D\}$. Hence, only the old firm invests for intermediate J in regions A and D. In contrast, in region B the new firm always invests K on its discovery date whereas the old firm only invests K if it finishes R&D first (see Table 4). If the old firm's benefit from immediately deducting the investment J on date zero dominates the benefit to the new firm of its more aggressive investment strategy on its discovery date, the present value from investing J is higher for the old firm than for the new firm. Thus, as was the case in regions A and D, $J_o^B > J_n^B$ and so only the old firm invests for intermediate J. However, if the new firm's benefit from its more aggressive investment strategy dominates the old firm's immediate tax deduction advantage, then $J_n^B > J_o^B$ and only the new firm invests for intermediate J. We present the condition for $J_n^B > J_o^B$ in Lemma 7 in the Appendix.

The following proposition summarizes the equilibrium investment decisions on date zero in each of the four regions A–D.

Proposition 2. *The firms' equilibrium investment decisions on date zero are as follows*:

(i) In regions $k \in \{A, D\}$, there exist critical values $J_n^k \le J_o^k$ such that
if $J \le J_n^k$, both firms invest J;
if $J_n^k < J \le J_o^k$, only the old firm invests J;

if $J_o^k < J \leq J_{\max}$, there are two pure strategy equilibria; one in which only the old firm invests J and one in which only the new firm invests J.

(ii) In region B, there exist critical values J_n^B and J_o^B and two cases are possible:

If $J_n^B \leq J_o^B$ the equilibrium decisions are as in (i) with k = B.

If $J_n^B > J_o^B$, the equilibrium investment decisions on date zero are as follows:

if $J \leq J_o^B$, both firms invest J;

if $J_o^B < J \leq J_n^B$, only the new firm invests J;

if $J_n^B < J \leq J_{\max}$, there are two pure strategy equilibria; one in which only the old firm invests J and one in which only the new firm invests J.

(iii) In region C, only the new firm invests J.

Conventional wisdom suggests that a firm that can obtain an immediate tax benefit from an investment will invest more aggressively than a firm whose tax benefit would be deferred. Proposition 2 shows that this conventional wisdom does not hold in region C and for intermediate J in region B when $J_o^B < J_n^B$. In these two regions, the new firm's more aggressive investment strategy on its discovery date deters the old firm from investing J on date zero.

The different regions in Proposition 2 are illustrated in Figure 3, for the parameter values from Figure 2, that is, $\tau = 30\%$, $r = 10\%$, $c = 120$, $m = 270$, and $\theta = 1$.

Figure 3 displays the critical values K_o, K_f, and K_n that determine the boundaries of regions A, B, C, and D. Combinations of K and J in the upper right part of the graph are ruled out because $J > J_{\max}$. In regions $k \in \{A, B, D\}$, the downward sloping solid line represents the critical value J_o^k below which the old firm invests J on date zero if the new firm also invests J. The downward sloping dashed line represents the critical value J_n^k below which the new firm invests J on date zero if the old firm also invests J. The subregions for low, intermediate, and high J are indicated with subindices 1, 2, and 3, respectively. The areas in between the dashed and the solid downward sloping lines are the intermediate values of J. In the case illustrated in Figure 3, it holds that $J_o^B < J_n^B$, so that the new firm keeps the old firm out of the market in region B_2. If instead we let $\theta = 0.01$ and keep all other parameters the same, it holds that $J_n^B < J_o^B$. In this case the old firm keeps the new firm out of the market in region B_2.

4. Equilibrium outcomes

In this section we combine the firms' decisions whether to invest J on date zero (from Proposition 2) and their decisions to invest K on their respective discovery dates (from Proposition 1) to derive the possible equilibrium outcomes. We focus our discussion on how the asymmetric tax treatment of the initial investment J affects the equilibrium outcome. We distinguish between cases where the effect is in line with the conventional wisdom that suggests that the firm with the immediate tax deduction advantage on date zero (the old firm) has a strategic advantage, and cases where the conventional wisdom does not hold.

Because we have deliberately suppressed nontax differences between the two firms, equilibrium outcomes are symmetric when the tax rate is zero. Indeed, when $\tau = 0$ it follows from Proposition 1 and Proposition 2 that the firms would follow the same investment strategies on date zero and the same investment strategies on their respective discovery dates, so neither firm would have a strategic advantage over the other.[6] Whenever $\tau > 0$, however, the different tax treatment of the initial investment cost J implies that the present value from an investment is different for

6. If $\tau = 0$, it follows from Tables 2 and 3 that regions B and C are empty. Moreover, it follows from Lemma 5 in the Appendix that in regions $k \in \{A, D\}$, it holds that $V_n^k = V_o^k$ if $\tau = 0$, and so $J_o^k = J_n^k$. Combined with Propositions 1 and 2, this implies that the two firms have the same investment strategies. Hence, they are in strategically symmetric positions.

the two firms. This in turn implies that the firms may make different investment decisions and asymmetric equilibrium outcomes can arise.

We define the following equilibrium outcomes for firm $i \in \{o, n\}$ as follows:

- *Uncontested Monopolist* (UM(i)): only firm i invests J on date zero and thus becomes a monopolist on the date it makes the discovery.
- *Contested Monopolist* (CM(i)): both firms invest J on date zero but only firm i invests K, and so firm i becomes a monopolist on the date it makes the discovery.
- *Contested Duopolist* (CD): both firms invest J on date zero and invest K on their discovery dates, and so eventually the firms engage in duopolistic competition when both have finished R&D.

We first consider the cases where both firms invest J on date zero. This occurs for J low in regions A, B, and D. As in Figure 3, we label these regions A_1, B_1, and D_1 in Table 6. The equilibrium outcomes follow immediately from the firm's capital investment decisions as summarized in Table 4. If K is sufficiently low (region A_1), both firms invest on their respective discovery dates yielding a Contested Duopoly. If K is sufficiently high (region D_1), only the first firm to discover invests K, and so each firm has a 50% chance of becoming a Contested Monopolist. However, if both firms invest J in region B, but K takes intermediate values (region B_1), the new firm invests K even if it finishes R&D second, whereas the old firm only invests K if it finishes R&D first. Therefore, if the new firm discovers first it becomes a Contested Monopolist whereas if the old firm discovers first the two firms become Contested Duopolists. We summarize the equilibrium outcomes when both firms invest J in Table 6. The case in which the new firm becomes a Contested Monopolist is highlighted in bold.

Next, we consider the cases in which there is a unique equilibrium in which one firm invests J on date zero and the other firm does not, resulting in one of the firms being an Uncontested Monopolist. This occurs in region C as well as for intermediate values of J in regions A, B, and D. As in Figure 3, we label these regions A_2, B_2, and D_2 in Table 7. We summarize the equilibrium outcomes in Table 7, highlighting the results that are contrary to the conventional wisdom in bold.

When K is either low or high (regions A_2 and D_2), the conventional wisdom holds because the two firms have the same investment strategies on their respective discovery dates and the old firm's immediate tax deduction advantage makes the investment of J more attractive to the old firm than to the new firm. For intermediate values of K (regions B_2 and C), the conventional

TABLE 6
Equilibrium outcomes when both firms invest J in R&D (regions A_1, B_1, and D_1)

Region	Capital investment K	Equilibrium	Asymmetry effect
A_1	Both firms invest K	CD	No asymmetry
B_1	Old finishes first: both invest K	CD	No asymmetry
	New finishes first: only new invests K	**CM(n)**	**Lack of a loss carryover leads to less aggressive investment by the old firm**
D_1	Only first to finish invests K	CM(o) or CM(n)	No asymmetry

Notes: CD means Contested Duopoly; CM(i) means Contested Monopoly in which firm $i \in \{o, n\}$ finishes R&D first and becomes a monopolist as of that date. In regions B and D, each of the two possible equilibrium outcomes in the third column occurs with probability 50%. The entry in bold is an equilibrium outcome where the lack of an NOL for the old firm implies that it invests less aggressively than the new firm.

TABLE 7
Asymmetric equilibrium outcomes in regions A_2, B_2, C, and D_2

Region	R&D investment	Equilibrium	Asymmetry effect
A_2	Only the old firm invests	UM(o)	Conventional wisdom: immediate tax deduction gives competitive advantage to the old firm
B_2	If $J_o^B > J_n^B$: Only old invests	UM(o)	Conventional wisdom
	If $J_o^B < J_n^B$: Only new invests	**UM(n)**	**Loss carryover leads to more aggressive capital investment by the new firm, deterring R&D investment by the old firm**
C	Only new invests	**UM(n)**	**Loss carryover leads to more aggressive capital investment by the new firm, deterring R&D investment by the old firm**
D_2	Only the old firm invests	UM(o)	Conventional wisdom

Notes: UM(i) means Uncontested Monopoly in which firm $i \in \{o, n\}$ has the competitive advantage and only firm i makes the R&D investment J on date zero. The entries in bold are equilibrium outcomes where the new firm's NOL implies that it invests more aggressively than the old firm. J_o^B (J_n^B) is the maximum value of J for which the old (new) firm wants to invest J when the rival also invests J.

wisdom holds in region B_2 when $J_o^B \geq J_n^B$. However, in region C and in region B_2 when $J_n^B > J_o^B$, the conventional wisdom does not hold because the new firm's more aggressive investment behavior on its discovery date deters the old firm from investing on date zero.

Finally, when J is high in regions A, B, and D (regions A_3, B_3, and D_3 in Figure 3), there are two pure strategy equilibria in which one firm invests J on date zero and the other does not, and so one of the firms becomes an Uncontested Monopolist. The difference in the tax treatment of the initial investment J does not cause a difference in investment behavior in these regions because J is so high that neither firm is willing to invest J if the rival invests J.

5. Sensitivity and robustness analysis

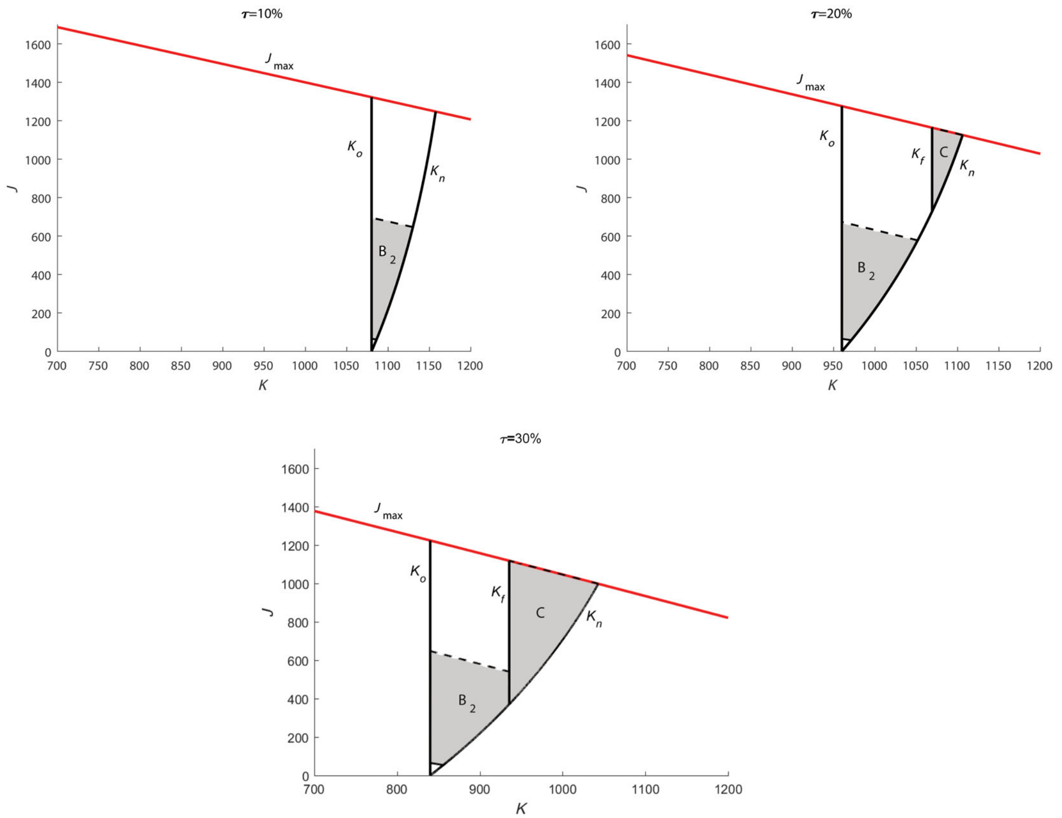
Sensitivity with respect to the tax rate and expected length of the R&D stage

In this section we illustrate the effects of two important parameters, the tax rate (τ) and the expected length of the R&D process ($1/\theta$), on the regions in which, contrary to conventional wisdom, the new firm becomes an Uncontested Monopolist. We show that changes in these parameters have ambiguous effects on the likelihood that this equilibrium outcome occurs.

Recall that the new firm's NOL advantage implies that it becomes an Uncontested Monopolist in region C and in region B_2 if $J_n^B > J_o^B$. Figure 4 shows regions B_2 and C for three values of the tax rate and Figure 5 shows these two regions for three values of θ. All other parameter values are as in Figures 2 and 3. In each case, it holds that $J_n^B > J_o^B$ so that the new firm becomes an Uncontested Monopolist both in region B_2 and in region C.

In Figure 4, a change in τ implies that some (K, J) values will enter B_2 or C and some (K, J) values will drop out of these two regions in response to the change. In Figure 5, the set of (K, J) values in the combined regions B_2 and C expands when θ increases. However, if we take the same parameters as in Figure 5 and we let θ increase from 0.04 to 0.06, then J_o^B increases and as a result some (K, J) values drop out of region B_2. Hence, whether a change in the tax rate or in the expected length of the R&D stage makes it more or less likely for the new firm to become an Uncontested Monopolist is ambiguous and depends on the probability distribution of (K, J) values.

Figure 4 Sensitivity with respect to the tax rate



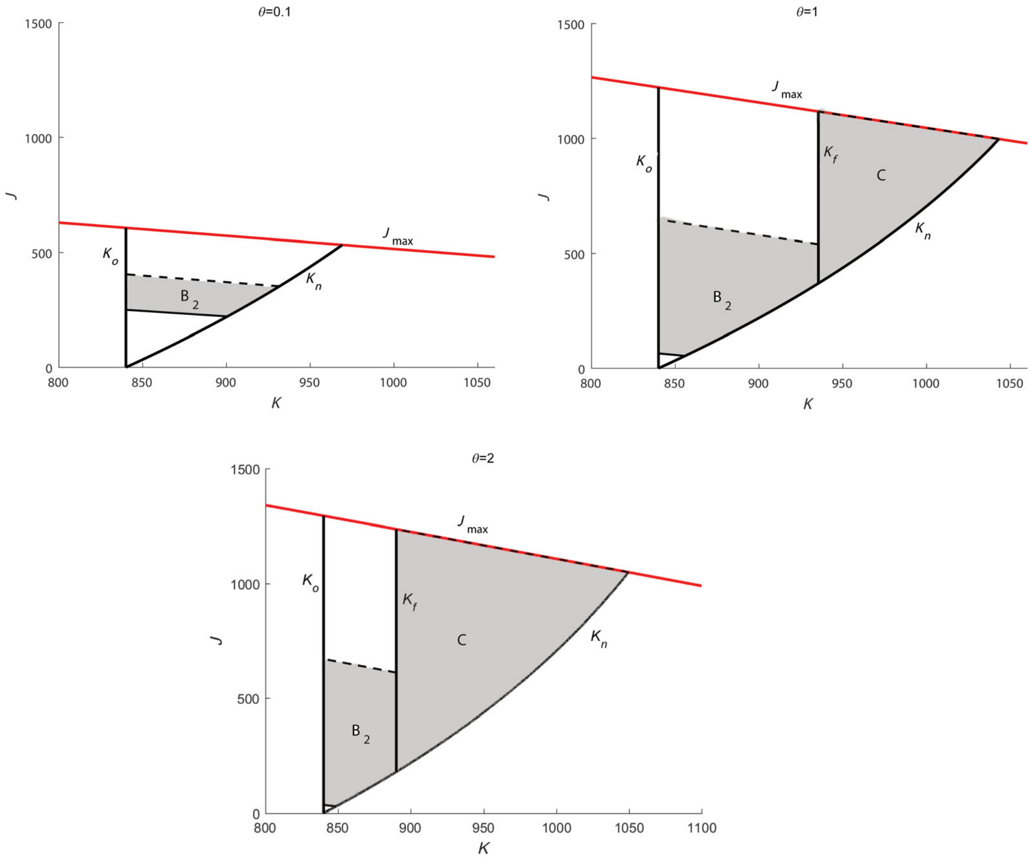
Notes: Regions B_2 and C (shaded areas) as a function of the capital investment K and the R&D investment J for $r = 10\%$, $c = 120$, $m = 270$, and $\theta = 1$, and three values of the tax rate τ. In regions B_2 and C, only the new firm makes the R&D investment J.

Robustness to model assumptions

In this section we discuss two model assumptions. First, in our main model we assume no limitations on the use of the tax loss carryforward. In some jurisdictions the NOL can only be carried forward for a limited period of time or used to offset part of the taxable income (e.g., a maximum of 80 percent under US tax law following the enactment of the Tax Cuts and Jobs Act of 2017). These limitations lower the value of the NOL carryover and therefore reduce the advantage of the new firm when it makes the follow-up capital investment. Therefore, the regions in which the new firm's aggressive investment behavior on its discovery date deters the old firm from investing J on date zero would tend to shrink.

Second, in our model there is a time lag between when the first and the second firm make the discovery. This occurs because the discovery times are independent and exponentially distributed with parameter θ. Consider instead the case where at the time the first firm discovers the technology, the information completely spills over to the second firm (see Reinganum 1985). In that case, both firms need to decide whether they want to invest K at the time the first firm makes the discovery. The value of investing K if the rival does so too would be K_o for the old firm and K_n for the new firm, with $K_o < K_n$. Therefore, the new firm would deter the old firm from investing K for all $K_o < K \leq K_n$, which in turn would deter the old firm from investing J on date

Figure 5 Sensitivity with respect to the expected length of the R&D stage



Notes: Regions B_2 and C (shaded areas) as a function of the capital investment K and the R&D investment J for τ = 30%, r = 10%, c = 120, m = 270, and three values of the expected speed of discovery θ. In regions B_2 and C, only the new firm makes the R&D investment J.

zero. Hence, all region B would then become part of region C. This occurs because in the combined regions B and C, the new firm is willing to invest K if the old firm does so too, but the old firm is not willing to invest K if the new firm does so too. So in case of information spillover on the date the first firm makes the discovery, the region in which the new firm's tax advantage deters the old firm from investing K in R&D increases.[7]

6. Conclusion

To the extent that start-up firms cannot get an immediate tax benefit from R&D expenditures, they are thought to be at a competitive disadvantage relative to profitable older firms. We find that this conventional wisdom need not hold in our two-stage investment setting.

A start-up firm's loss carryover affects competition with a firm without a loss carryover. The firm without the loss carryover is less likely to make a capital investment at the end of its

7. The same occurs when we let θ go to infinity. However, when θ goes to infinity, two things happen simultaneously: almost immediate discovery on date zero by both firms (but not at the same time); and almost no time lag between the first and second to discover.

R&D stage than is the start-up firm, because the start-up firm can only obtain the tax benefits from its NOL carryover by making the capital investment. In contrast, the older firm received the full tax benefit from its R&D investment when it was made. This implies that the start-up firm invests more aggressively when it finishes its R&D, which in turn may deter the older firm from making an initial investment in R&D. Therefore, even though the aftertax cost of an initial R&D investment is higher for the start-up firm than it is for an older profitable firm, there are settings in which a start-up firm would make an initial R&D investment that the older firm would reject.

Our results indicate that there are situations in which being able to deduct R&D immediately is not a competitive advantage. The advantage of the start-up firm when making the capital investment may partly explain the inconclusive evidence regarding the relation between tax loss carryforwards and corporate investment (Edgerton 2010; Dreßler and Overesch 2013). This in turn suggests that it is important to control for the competitive environment when analyzing the effects of tax asymmetries on investment behavior.

The competitive effects should also be considered in policy decisions regarding the asymmetric treatment of losses. Making the tax loss carryforward period unlimited, as was done under the Tax Cuts and Jobs Act (TCJA) in the United States, makes tax-deductible investments more attractive. Annual limitations on the use of net operating loss carryovers, as also done under the TCJA, does the opposite. Our analysis shows that policy decisions that increase or decrease the attractiveness of investments for loss-making firms may have the opposite effect on profitable firms.

We have deliberately suppressed any nontax differences between the two firms so as to focus on the effects of the NOL carryover on firm investment and operating decisions. To the extent that start-up firms and established firms have different pretax costs and benefits, our predicted differences in the behaviors of the two types of firms could be either exacerbated or attenuated.

Appendix

Lemmas and proofs

Lemmas

In Lemma 2 we derive expressions for the present value of aftertax profit to firm i from investing K on the date R&D finishes, when both firms invest K on the date they finish R&D and firm i is first to finish R&D. We derive these values as a function of the pretax profit that firm i will earn when it is a monopolist, which we denote x, and the pretax profit that it will earn when the two firms compete, which we denote y. In the proof of Lemma 3 we will use these expressions to determine the firms' optimal production decisions and the corresponding optimal pretax profits x and y.

LEMMA 2. *Let the pretax profit per period in case of monopoly be x, and let the pretax profit per period in case of duopoly be y. Then, the expected present value of aftertax profits to the old firm from investing K on the date R&D finishes if it is first to finish R&D, and the new firm will invest too, equals*

$$\tilde{P}(x,y) = \left(\frac{r}{\theta + r}\right)\frac{(1-\tau)x}{r} + \left(\frac{\theta}{\theta + r}\right)\frac{(1-\tau)y}{r}. \tag{11}$$

The expected present value of aftertax profits to the new firm from investing K on the date R&D finishes if it is first to finish R&D, and the old firm will invest too, equals

$$\tilde{P}(x,y) + \tau J\tilde{\delta}(x,y,J),$$

where

$$\tilde{\delta}(x,y,J) = \left[\left(\frac{x}{rJ}\right)\cdot\left(\frac{r}{\theta+r}\right) + \left(\frac{y}{rJ}\right)\left(\frac{\theta}{\theta+r}\right)\right]\cdot\left(1-\mathrm{e}^{-(r+\theta)\frac{J}{x}}\right)$$
$$+\left(\frac{y}{rJ}\right)\cdot\left(\frac{\theta}{\theta+r-rx/y}\right)\cdot\left(\mathrm{e}^{-(\theta+r)\frac{J}{x}} - \mathrm{e}^{-r\frac{J}{y}}\right) \in [0,1]. \tag{12}$$

Proof. First consider the case where the old firm finishes first, that is, $i = o$. Let T be the remaining time until the new firm invests K. The present value of aftertax profits for the old firm on date T_o if pretax profit equals x for $t < T_o + T$ and y for $t \geq T_o + T$ equals

$$h(x,y,T) = \int_0^T (1-\tau)x\mathrm{e}^{-rt}dt + P(y)\cdot\mathrm{e}^{-rT}$$
$$= \frac{(1-\tau)x}{r}\left(1-\mathrm{e}^{-rT}\right) + \frac{(1-\tau)y}{r}\mathrm{e}^{-rT}. \tag{13}$$

The remaining time T is an exponentially distributed random variable with a density function of $f_T(t) = \theta e^{-\theta t}$. Therefore, the expected payoff as of the date the old firm finishes its R&D, when it finishes first, is

$$\tilde{P}(x,y) = E[h(x,y,T)]$$
$$= \int_0^\infty \left(\frac{(1-\tau)x}{r}(1-\mathrm{e}^{-rt}) + \frac{(1-\tau)y}{r}\mathrm{e}^{-rt}\right)\theta\mathrm{e}^{-\theta t}dt$$
$$= \left(\frac{r}{\theta+r}\right)\frac{(1-\tau)x}{r} + \left(\frac{\theta}{\theta+r}\right)\frac{(1-\tau)y}{r}.$$

Now, we consider the the present value of the tax benefits for the new firm if it finishes its R&D first. We let T be the remaining time until the old firm invests K, and we let $d(x, y, J, T)$ be the present value on date T_n of the tax benefits of the new firm if pretax profit equals x until the old from enters and y thereafter. The present value depends on whether the NOL is fully used up by the time the old firm invests K, which depends on T. If $T < J/x$, the NOL is used at rate x until date $T_n + T$. The remaining NOL of $J - xT$ on date $T_n + T$ is used at rate y until it is fully used. Hence, it follows from (3) with $m = x$ and $L = J - xT$ that

$$d(x,y,J,T) = \int_0^T \tau x\mathrm{e}^{-rt}dt + \tau J\delta(y, J-xT)\cdot\mathrm{e}^{-rT}$$
$$= \frac{\tau x(1-\mathrm{e}^{-rT})}{r} + \left\{\frac{\tau c\left(1-\mathrm{e}^{-r(J-xT)/y}\right)}{r}\right\}\mathrm{e}^{-rT}. \tag{14}$$

If $T \geq J/x$, the NOL is used at rate x as of date T_n until it is fully used, and, hence,

$$d(x,y,J,T)=\int_0^{J/x}\tau x\mathrm{e}^{-rt}dt=\frac{x\left(1-\mathrm{e}^{-rJ/x}\right)}{r}. \tag{15}$$

The remaining time T until the old firm invests K is an exponentially distributed random variable with a density function of $f_T(t) = \theta e^{-\theta t}$. Therefore, the expected payoff as of the date the first firm finishes its R&D, when it finishes first, is

$$\begin{aligned}\tau J\tilde{\delta}(x,y,J)&=E[d(x,y,J,T)]\\&=\int_0^{J/x}\left[\frac{\tau x(1-\mathrm{e}^{-rt})}{r}+\left\{\frac{\tau y\left(1-\mathrm{e}^{-r(J-xt)/y}\right)}{r}\right\}\mathrm{e}^{-rt}\right]\theta\mathrm{e}^{-\theta t}dt\\&+\int_{J/x}^{\infty}\frac{x\left(1-\mathrm{e}^{-rJ/x}\right)}{r}\theta\mathrm{e}^{-\theta t}dt.\end{aligned}$$

Solving the integral and dividing by τJ yields (12). ■

LEMMA 3. *The new firm's NOL does not affect the optimal production quantities of either firm. For both firms, equilibrium pretax profit per period equals $c = (d - v)^2/9$ when the two firms are duopolists, and equals $m = (d - v)^2/4$ when the firm is a monopolist.*

PROOF. We first consider production decisions as of the date the firm invests K if the rival firm does not invest. If only firm i invests, it chooses a production quantity q_i so as to maximize

$$P(x(q_i)),\quad \text{if } i=o, \tag{16}$$

$$P(x(q_i))+\tau J\delta(x(q_i),J),\quad \text{if } i=n \tag{17}$$

where

$$P(x)=\frac{(1-\tau)x}{r}, \tag{18}$$

$$\delta(x,J)=\frac{\tau x}{rJ}\left(1-\mathrm{e}^{-rJ/x}\right), \tag{19}$$

and $x(q_i) = (p - v)q_i = (d - q_i - v)q_i$ is pretax profit as a function of production quantity q_i. It holds that

$$\frac{\partial}{\partial q_i}P(x(q_i))=\frac{\partial}{\partial x}P(x)\frac{\partial}{\partial q_i}x(q_i) \tag{20}$$

$$\frac{\partial}{\partial q_i}\{P(x(q_i)+\tau J\delta(x(q_i),J)\}=\frac{\partial}{\partial x}\{P(x)+\tau J\delta(x,J)\}\frac{\partial}{\partial q_i}x(q_i). \tag{21}$$

It follows from (18) that $\frac{\partial}{\partial x}P(x)>0$ for all x. Moreover, it follows from (19) that $\frac{\partial}{\partial x}\{\tau J\delta(x,J)\}=\frac{\tau}{r}\left(1-(1+a/x)\mathrm{e}^{-(a/x)}\right)$ with $a=rJ$. Using the Taylor expansion $\mathrm{e}^{(a/x)}=\sum_{n=0}^{\infty}(a/x)^n>1+a/x$ yields $\frac{\partial}{\partial x}\{\tau J\delta(x,J)\}>0$ for all x. Because $\frac{\partial}{\partial x}P(x)>0$ and $\frac{\partial}{\partial x}\tau J\delta(x,J)>0$, it follows from (20) and (21) that the optimal production quantity satisfies $\frac{\partial}{\partial q_i}x(q_i)=0$, that is, it is the quantity that maximizes pretax profit $x(q_i)$. Therefore, $q_i^*=\text{argmax}_q x(q_i)=\frac{d-v}{2}$ and $x(q_i^*)=m$.

We now determine the optimal production quantities of the two firms as of the date the second firm finishes its R&D, if they both invest K. Then, aftertax profit is given by (16) and (17) with $x(q_i)$ replaced by $x(q_i| q_j) = (p - v)q_i = (d - q_i - q_j - v)q_i$, the pretax profit of firm i if it produces q_i units and the rival produces q_j units. Because $\frac{\partial}{\partial x}P(x)>0$ and $\frac{\partial}{\partial x}\tau J\delta(x,J)>0$, it follows from (20) and (21) that the optimal production quantities satisfy $q_i^*=\text{argmax}_{q_i} x\left(q_i|q_j^*\right)$, for $i,j\in\{n, o\}$. This yields $q_o^*=q_n^*=\frac{d-v}{3}$ and $x\left(q_i^*|q_j^*\right)=c$.

Still to be determined is the optimal production quantity q_i of the first firm to finish R&D as of the date it finishes its R&D until the second firm finishes its R&D. Whichever firm invests first, it earns pretax profit of c as of the date the second firm invests. Therefore, if firm i finishes its R&D first, production quantity q_i for the time until the second firm enters is chosen so as to maximize

$$\tilde{P}(x(q_i),c), \text{if } i=o,$$
$$\tilde{P}(x(q_i),c)+\tau J\tilde{\delta}(x(q_i),c,J), \text{if } i=n,$$

where $x(q_i)$ is pretax profit of firm i until the second firm enters, as a function of production quantity q_i, and the functions $\tilde{P}(\cdot,c)$ and $\tilde{\delta}(\cdot,c,J)$ are as defined in (11) and (12), respectively, with $y=x\left(q_i^*|q_j^*\right)=c$. It follows from (11) that $\tilde{P}(x,c)$ is increasing in x. To show that $\tau J\tilde{\delta}(x,c,J)$ is increasing in x, recall that $\tau J\tilde{\delta}(x,c,J)=E[d(x,c,J,T)]$, where T is the time that elapses until the old firm enters, and $d(x, c, J, T)$ is given by (14) for $x\le J/T$ and by (15) for $x>J/T$. For all values of T, it holds that $\partial d(x, c, J, T)/\partial x>0$. This in turn implies that $\tau J\tilde{\delta}(x,c,J)$ is increasing in pretax profit x earned until the second firm enters. Because $\frac{\partial}{\partial x}\tilde{P}(x,c)>0$ and $\frac{\partial}{\partial x}\tau J\tilde{\delta}(x,c,J)>0$, the optimal production quantity satisfies $\frac{\partial}{\partial q_i}x(q_i)=0$. Therefore, $q_i^*=\text{argmax}_q x(q_i)=\frac{d-v}{2}$ and $x(q_i^*)=m$. ■

LEMMA 4. *Let $J_{\max}$ be the maximum value of J for which* (6) *is satisfied. Then*

(i) In the absence of a competitor, the new firm invests J on date zero if and only if $J\le J_{\max}$. The old firm invests J on date zero if $J\le J_{\max}$.

(ii) In the absence of a competitor, each firm invests K on its discovery date if $J\le J_{\max}$.

PROOF. (i) We first consider the decision to invest J on date zero. Let M_i, $i\in\{o, n\}$ denote the expected present value to firm i when only firm i invests J on date zero. If only firm $i\in\{o, n\}$ invests J on date zero, that firm invests K and becomes a monopolist on date T_i when its R&D stage ends. Using Table 1 yields that the date-zero present value of investing J if the rival firm does not invest J is $\mathrm{e}^{-rT_i}(P(m)-K)-(1-\tau)J$ for the old firm, and $\mathrm{e}^{-rT_i}(P(m)+\tau J\delta(m,J)-K)$ for the new firm, where T_i is a random variable with density function $f_{T_i}(t)=\theta\mathrm{e}^{-\theta t}$. Hence,

$$M_o=\int_0^\infty \mathrm{e}^{-rt}(P(m)-K)\theta\mathrm{e}^{-\theta t}dt-(1-\tau)J$$
$$=\left(\frac{\theta}{\theta+r}\right)[P(m)-K]-(1-\tau)J, \tag{22}$$

and

$$M_n = \int_0^\infty e^{-rt}[P(m) + \tau J\delta(m,J) - K]\theta e^{-\theta t} dt$$
$$= \left(\frac{\theta}{\theta + r}\right)[P(m) + \tau J\delta(m,J) - K] - J. \tag{23}$$

Condition (6) is satisfied if $M_n \geq 0$. Because M_n is decreasing in J, $J_{\max}$ is the value of J at which $M_n = 0$ and $M_n \geq 0$ if and only if $J \leq J_{\max}$. Hence, in the absence of a competitor, the new firm invests J on date zero if and only if $J \leq J_{\max}$. Because M_o is also decreasing in J and $M_o \geq M_n$, it holds that $M_o \geq 0$ if $J \leq J_{\max}$. Hence, in the absence of a competitor the old firm invests J on date zero if $J \leq J_{\max}$.

(ii) We now show that if $0 \leq J \leq J_{\max}$, each firm invests K on its discovery date in the absence of a competitor. In the absence of a competitor, the net present value on discovery date from investing K is $P(m) - K$ for the old firm and $P(m) + \tau J\delta(m, J) - K$ for the new firm. We have shown that $J \leq J_{\max}$ implies $M_o \geq 0$. Therefore, when $0 \leq J \leq J_{\max}$ it follows from (22) that $P(m) - K \geq 0$, and hence also $P(m) + \tau J\delta(m, J) - K \geq 0$. ■

Proof of Lemma 1

First, $m > c$ implies that $K_o < K_f$. Next, $K_n > K_o$ follows from the fact that (see (2) with m replaced by c)

$$\tau J\delta(c,J) = \frac{\tau c}{r}\left(1 - e^{-rJ/c}\right) > 0. \tag{24}$$

Finally, to rank K_n and K_f, note that $K_n > K_f$ iff $h(J) < 0$, where the function $h : \mathbb{R} \to \mathbb{R}$ is given by

$$h(J) := \frac{(m-c)(1-\tau)}{\theta + r} - \tau J\delta(c,J).$$

It follows from (24) that h is continuous and decreasing in J, with $h(0) > 0$, and

$$\lim_{J\to\infty} h(J) = \frac{(m-c)(1-\tau)}{\theta + r} - \frac{\tau c}{r} = \frac{m-c}{\theta + r} - \frac{\tau}{r}\left(\frac{rm + \theta c}{\theta + r}\right).$$

Hence, two cases are possible:

- If $\tau \leq r(m - c)/(rm + \theta c)$, then $\lim_{J\to\infty} h(J) \geq 0$. Because h is decreasing in J, this implies that $h(J) \geq 0$ for all J.
- If $\tau > r(m - c)/(rm + \theta c)$, then $\lim_{J\to\infty} h(J) < 0$. Because h is decreasing in J and $h(0) > 0$, there exists a $J^* > 0$ such that $h(J^*) = 0$, $h(J) > 0$ for $J < J^*$, and $h(J) < 0$ for $J > J^*$.

Together, this implies that $h(J) < 0$ iff $\tau > \tau^*$ and $J > J^*$, and so we conclude that

$$K_f < K_n, \quad \text{if } \tau > \frac{r(m-c)}{rm + \theta c} \text{ and } J > J^*,$$
$$K_n \leq K_f, \quad \text{otherwise.} \tag{25}$$

Solving $h(J^*) = 0$ yields the value of J^* in (10). ■

Proof of Proposition 2

We have shown in Lemma 4 that for both firms, investing J on date zero if the rival does not invest is optimal because $J \le J_{\max}$. Now let V_i^k, $i \in \{o, n\}$ be the payoff to firm i when both firms invest J on date zero. Investing, if the rival firm does too, is optimal for firm $i \in \{o, n\}$ if and only if $V_i^k \ge 0$. In Lemma 5 we derive expressions for V_o^k and V_n^k.

LEMMA 5. *The date-zero present value of investing J if the rival firm also invests J is*

$$V_o^k = \left(\frac{\theta}{2\theta + r}\right)\Pi_{o,1}^k + \left(\frac{\theta}{2\theta + r}\right)\left(\frac{\theta}{\theta + r}\right)\Pi_{o,2}^k - (1-\tau)J, \tag{26}$$

for the old firm, and

$$V_n^k = \left(\frac{\theta}{2\theta + r}\right)\Pi_{n,1}^k + \left(\frac{\theta}{2\theta + r}\right)\left(\frac{\theta}{\theta + r}\right)\Pi_{n,2}^k - J, \tag{27}$$

for the new firm, where the values of $\Pi_{o,1}^k$, $\Pi_{o,2}^k$, $\Pi_{n,1}^k$, *and* $\Pi_{n,2}^k$ *depend on the region, as displayed in Table 8.*

PROOF. We first determine the value of each firm as of the date it finishes R&D, distinguishing the case where the firm is first to finish R&D, and the case where it is second to finish R&D. For firm $i \in \{o, n\}$, we determine

- $\Pi_{i,1}^k$: the value of firm i as of the date that it finishes R&D, conditional on being first to finish R&D.
- $\Pi_{i,2}^k$: the value of firm i as of the date that it finishes its R&D stage, conditional on being second to finish R&D.

The values of $\Pi_{i,1}^k$ and $\Pi_{i,2}^k$ depend on the two firms' investment decisions on the date they finish their R&D stages. Combining Tables 1 and 4 yields the expressions in Table 8.

TABLE 8
Firm value when it finishes R&D

	$\Pi_{o,1}^k$	$\Pi_{n,1}^k$	$\Pi_{o,2}^k$	$\Pi_{n,2}^k$
$k = \text{A}$	$\tilde{P}(m,c) - K$	$\tilde{P}(m,c) - K + \tau J\tilde{\delta}(m,c,J)$	$P(c) - K$	$P(c) - K + \tau J\delta(c, J)$
$k = \text{B}$	$\tilde{P}(m,c) - K$	$P(m) - K + \tau J\delta(m, J)$	0	$P(c) - K + \tau J\delta(c, J)$
$k = \text{C}$	0	$P(m) - K + \tau J\delta(m, J)$	0	$P(m) - K + \tau J\delta(m, J)$
$k = \text{D}$	$P(m) - K$	$P(m) - K + \tau J\delta(m, J)$	0	0

Notes: The value of firm $i \in \{o, n\}$ on the date it finishes R&D, conditional on being first to finish R&D ($\Pi_{i,1}^k$) and conditional on being second to finish R&D ($\Pi_{i,2}^k$). $P(m)$ ($P(c)$) is the after tax present value of future profits for the entire period as a monopolist (Cournot duopolist) excluding the effects of the NOL (see (1)). $\tilde{P}(m,c)$ is the after tax present value of future profits excluding the effects of the NOL when the firm finishes R&D first and the rival will also invest when it finishes R&D (see (4)). $\tau J\delta(m, J)$ ($\tau J\delta(c, J)$) is the present value of future tax savings from the NOL for the new firm when it is a monopolist (Cournot duopolist) for the entire period (see (3)). $\tau J\tilde{\delta}(m,c,J)$ is this present value when the new firm finishes R&D first and the old firm will also make the capital investment K when it finishes R&D (see (5)).

Next, we discount these values back to date zero, recognizing that the date that a firm finishes its R&D stage is the realization of a random variable, the distribution of which depends on whether it was the first or second to finish its R&D. When a firm finishes its R&D first, the date upon which it finishes is $T_1 = \min\{T_n, T_o\}$ (the first-order statistic of T_n and T_o), which has a density function of $f_1(t) = 2\theta e^{-2\theta t}$. Moreover, because T_n and T_o are independent and exponentially distributed, $\Pi^k_{i,1}$ is independent of T_1. Hence, the date zero expected present value of firm i conditional on being first to finish R&D equals

$$
\begin{aligned}
E\left[e^{-rT_1}\cdot\Pi^k_{i,1}\right] &= E\left[e^{-rT_1}\right]\cdot\Pi^k_{i,1} \\
&= \left(\int_0^\infty 2\theta e^{-2\theta t}\cdot e^{-rt}dt\right)\cdot\Pi^k_{i,1} \\
&= \left(\frac{2\theta}{2\theta+r}\right)\cdot\Pi^k_{i,1}. \qquad (28)
\end{aligned}
$$

When a firm finishes its R&D second, the date upon which it finishes is $T_2 = \max\{T_n, T_o\}$ (the second-order statistic of T_n and T_o), which has a density function of $f_2(t) = 2\theta e^{-\theta t}(1 - e^{-\theta t})$. Because $\Pi^k_{i,2}$ is independent of T_n and T_o, the date zero present value conditional on being second to finish R&D is

$$
\begin{aligned}
E\left[e^{-rT_2}\cdot\Pi^k_{i,2}\right] &= E\left[e^{-rT_2}\right]\cdot\Pi^k_{i,2} \\
&= \left(\int_0^\infty 2\theta e^{-\theta t}\left(1-e^{-\theta t}\right)\cdot e^{-rt}dt\right)\cdot\Pi^k_{i,2} \\
&= \left(\frac{2\theta}{2\theta+r}\right)\left(\frac{\theta}{\theta+r}\right)\cdot\Pi^k_{i,2}. \qquad (29)
\end{aligned}
$$

Because each firm has a 50% probability of being first to finish R&D, and because the new firm can only deduct the loss J after it invests K, it holds that

$$V^k_o = \frac{1}{2}E\left[e^{-rT_1}\cdot\Pi^k_{o,1}\right] + \frac{1}{2}E\left[e^{-rT_2}\cdot\Pi^k_{o,2}\right] - (1-\tau)J,$$

$$V^k_n = \frac{1}{2}E\left[e^{-rT_1}\cdot\Pi^k_{n,1}\right] + \frac{1}{2}E\left[e^{-rT_2}\cdot\Pi^k_{n,2}\right] - J.$$

Combined with (28) and (29), this yields the expressions in (26) and (27). ■

LEMMA 6. *In regions $k \in \{A, B, D\}$, V^k_o and V^k_n are decreasing in J*, and *there exist critical values J^k_n and J^k_o such that*

$$V^k_o \geq 0 \text{ if and only if } J \leq J^k_o, \qquad (30)$$

$$V^k_n \geq 0 \text{ if and only if } J \leq J^k_n. \qquad (31)$$

PROOF. Because $V^k_o \geq 0$ and $V^k_n \geq 0$ at $J = 0$, and because $\lim_{J\to\infty} V^k_o < 0$ and $\lim_{J\to\infty} V^k_n < 0$, it is sufficient to show that V_o^k and V_n^k are monotonically decreasing in J. Because $\Pi_{o,1}^k$ and $\Pi_{o,2}^k$ are independent of J, it holds that $\frac{\partial}{\partial J}V_o^k = -(1-\tau)$ in each region, and so V_o^k is decreasing in J. ■

To show that V_n^k is decreasing in J, it is sufficient to show that $[1-\tau\Delta_k(J)]J$ is increasing in J, where $\Delta_k(J)$ is given by

$$\Delta^k(J) = \begin{cases} \left(\frac{\theta}{2\theta+r}\right)\tilde{\delta}(m,c,J) + \left(\frac{\theta}{2\theta+r}\right)\left(\frac{\theta}{\theta+r}\right)\delta(c,J), & \text{in region } k=\text{A}, \\ \left(\frac{\theta}{2\theta+r}\right)\delta(m,J) + \left(\frac{\theta}{2\theta+r}\right)\left(\frac{\theta}{\theta+r}\right)\delta(c,J), & \text{in region } k=\text{B}, \\ \left(\frac{\theta}{\theta+r}\right)\delta(m,J), & \text{in region } k=\text{C}, \\ \left(\frac{\theta}{2\theta+r}\right)\delta(m,J), & \text{in region } k=\text{D}. \end{cases} \tag{32}$$

$\Delta^k(J)$ is the date zero expected discount factor for the speed at which the new firm can use its NOL. For regions B, C, and D, this follows from taking the derivative of $[1-\tau\Delta^k(J)]J$ with respect to J, using the fact that $\frac{\partial}{\partial J}\{\tau J\delta(x,J)\} = \tau e^{-rJ/x} < 1$ for $x \in \{m, c\}$. For region A, we let $D(J, T_n, T_o)$ be the date zero present value of the tax benefits from the NOL, as a function of T_n and T_o. Then,

$$\frac{\partial}{\partial J}\{J-\tau\Delta^k(J)J\} = \frac{\partial}{\partial J}\mathbb{E}[J-D(J,T_n,T_o)] = \mathbb{E}\left[\frac{\partial}{\partial J}\{J-D(J,T_n,T_o)\}\right],$$

and so it suffices to show that $\frac{\partial}{\partial J}\{D(J,T_n,T_o)\} < 1$ for all realizations of T_o and T_n. Two cases are possible:

- $T_o < T_n$: the new firm finishes its R&D second. It uses its NOL of J at rate c as of date T_n until it is fully used. It follows from (3) with $x = c$ and $L = J$ that the present value on date T_n equals $\frac{\tau c\left(1-e^{-rJ/c}\right)}{r}$. Hence,

$$D(J,T_n,T_o) = \left[\frac{\tau c\left(1-e^{-rJ/c}\right)}{r}\right]e^{-rT_n}. \tag{33}$$

  Taking the derivative of $D(J, T_n, T_o)$ with respect to J shows that $\frac{\partial}{\partial J}\{D(J,T_n,T_o)\} < 1$ for all J.
- $T_n \le T_o$: the new firm finishes first. Let $T = T_o - T_n$. Then, it follows from (14) and (15) that

$$D(J,T_n,T_o) = \left[\frac{m\left(1-e^{-rJ/m}\right)}{r}\right]e^{-rT_n}, \text{if } J \le mT,$$
$$= \left[\frac{\tau m(1-e^{-rT})}{r} + \left\{\frac{\tau c\left(1-e^{-r(J-mT)/c}\right)}{r}\right\}e^{-rT}\right]e^{-rT_n}, \text{ if } J > mT$$

  $D(J, T_n, T_o)$ is continuous at $J = mT$. Moreover, taking the derivative of $D(J, T_n, T_o)$ with respect to J shows that $\frac{\partial}{\partial J}\{D(J,T_n,T_o)\} < 1$ for all J. ■

To characterize the equilibrium investment decisions of the two firms, it remains to determine whether $J_n^k \le J_o^k$ or $J_n^k > J_o^k$.

LEMMA 7. *The critical values are ranked as follows*:

(i) In regions $k \in \{A, D\}$, it holds that $J_n^k \le J_o^k$.

(ii) In region B, it holds that $J_n^B > J_o^B$ if and only if

$$\left(\frac{\theta}{2\theta+r}\right)\left(\frac{\theta}{\theta+r}\right)[P(m)-K]-\tau\left[1-\Delta^B\left(J_o^B\right)\right]J_o^B>0, \tag{34}$$

with $\Delta^B(J)$ as defined in (32) for region B, and

$$J_o^B=\left(\frac{\theta}{2\theta+r}\right)\left[\tilde{P}(m,c)-K\right]/(1-\tau). \tag{35}$$

PROOF. (i) In regions $k \in \{A, D\}$, the value from investing J if the rival does so too (V_n^k and V_o^k) differs only due to a different present value of tax deductions from the initial loss J. The old firm can deduct the loss J immediately, whereas the new firm can only use it once it has made the discovery. Using Lemma 5 yields that in regions A and D, it holds that

$$V_n^k-V_o^k=-\tau\left[1-\Delta^k(J)\right]J, \tag{36}$$

where $\Delta^k(J)$ is defined in (32). Because $\Delta^k(J) < 1$, it holds that $V_n^k-V_o^k<0$; due to the higher present value of the aftertax cost of the investment J, investing if the rival does so too is less attractive to the new firm than to the old firm. Because V_n^k and V_o^k are both decreasing in J (Lemma 6) this implies that in regions A and D the critical value of J below which the new firm wants to invest, if the old firm does too, is lower than the critical value of J, below which the old firm wants to invest if the new firm does too, that is,

$$J_n^A \le J_o^A \text{ and } J_n^D \le J_o^D. \tag{37}$$

(ii) Using (26), (27), and Table 8 yields that in region B

$$V_n^B-V_o^B=\left(\frac{\theta}{2\theta+r}\right)\left(\frac{\theta}{\theta+r}\right)[P(m)-K]-\tau\left[1-\Delta^B(J)\right]J, \tag{38}$$

with $\Delta^B(J)$ as defined in (32) for region B. Lemma 6 shows that V_n^B is decreasing in J and $V_o^B=0$ at $J=J_o^B$. Therefore, it holds that $J_n^B>J_o^B$ if and only if $V_n^B>0$ at $J=J_o^B$, or, equivalently, $V_n^B-V_o^B>0$ at $J=J_o^B$. Hence, it follows from (38) that $J_n^B>J_o^B$ if and only if (34) is satisfied. It remains to determine J_o^B, the critical value of J below which the old firm wants to invest if the new firm does too. In region B, solving $V_o^B=0$ using (26) and Table 8 shows that J_o^B is given by (35). ■

PROOF OF PROPOSITION 2. For regions A, B, and D, using Lemma 6 yields

- If $J \le \min\{J_n^k, J_o^k\}$, then $V_o^k \ge 0$ and $V_n^k \ge 0$. Hence, both firms are willing to invest J in R&D on date zero even if the rival does too, so they both invest.

- If $J_n^k < J_o^k$ and $J_n^k < J \le J_o^k$, then $V_n^k < 0 \le V_o^k$. The old firm wants to invest if the rival does too, but the new firm does not want to invest if the old firm invests too. Hence, only the old firm invests.
- If $J_n^k > J_o^k$ and $J_o^k < J \le J_n^k$, then $V_o^k < 0 \le V_n^k$. The new firm wants to invest if the rival does too, but the old firm does not want to invest if the new firm invests too. Hence, only the new firm invests.
- If $J > \max\{J_n^k, J_o^k\}$, then $V_o^k < 0$ and $V_n^k < 0$. Therefore, each firm prefers to invest J if its rival does not invest J, but prefers not to invest if its rival does invest. Accordingly, there are two pure strategy equilibria: one in which the old firm invests J on date zero and the new firm does not; and one in which the new firm invests J on date zero and the old firm does not.

Combined with the rankings in Lemma 7, this yields the equilibrium strategies in Proposition 2(i) and (ii). The equilibrium behavior in region C follows immediately from (6) and the fact that the old firm never invests K in this region (Proposition 1). ■

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