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published in

Behavioral and Brain Sciences
2021

DOI (link to publisher)

[10.1017/S0140525X21000984](https://doi.org/10.1017/S0140525X21000984)

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citation for published version (APA)

Dutilh Novaes, C., & Dos Santos, C. F. (2021). Numerosities are not ersatz numbers. *Behavioral and Brain Sciences*, 44, Article e198. <https://doi.org/10.1017/S0140525X21000984>

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Numerosities are not ersatz numbers

Published online by Cambridge University Press: **15 December 2021**

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Abstract

In describing numerosity as “a kind of ersatz number,” Clarke and Beck fail to consider a familiar and compelling definition of numerosity, which conceptualizes numerosity as the cognitive counterpart of the mathematical concept of cardinality; numerosity is the magnitude, whereas number is a scale through which numerosity/cardinality is measured. We argue that these distinctions should be considered.

Type

Open Peer Commentary

Information

Behavioral and Brain Sciences, Volume 44, 2021, e198


DOI: <https://doi.org/10.1017/S0140525X21000984>



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The difference between cardinality and number is often overlooked by those who defend the numerical character of the so-called number sense. Clarke and Beck are no exception. This can be gathered from their description of numerosity as “a kind of ersatz number” 

(sect. 6, para. 3). Although they correctly point out that approximate number system (ANS) researchers rarely state explicitly what they mean by “numerosity,” they miss those who do. This term was introduced by the psychophysicist Stevens (1939/2006) to disambiguate the term “number,” ambiguously used to designate a scale for the measurement of cardinality and the very magnitude this scale measures. Stevens reserved the word “number” for the first use, and introduced “numerosity” for the second. Consistent with Stevens's definition, in the contemporary literature one finds “numerosity” defined as a synonym for cardinality (e.g., Butterworth, 2005, p. 3; Nieder, 2016, p. 366; Piazza & Izard, 2009, p. 261).

Stevens's distinction between number and numerosity is not arbitrary. In mathematics, “cardinality” and “number” refer to different concepts. Cardinality refers to set size, whereas numbers provide a way of determining and expressing set size; it is a trivial observation that the cardinality of a set can be determined and expressed without even mentioning numbers. Indeed, suppose that we want to know whether the cardinality of the set of people in a room is equal to the cardinality of the set of chairs in that room. We need not count people and chairs; we can just ask people to sit down, one person per chair. If no person remains standing and no chair remains empty, we conclude immediately that both sets have the same size. If someone remains standing, then the set of people is larger than the set of chairs; if chairs remain empty, then the opposite is the case. No number is involved in this procedure; it can be carried out recruiting only the notion of one-to-one correspondence or equinumerosity, which, despite its name, is defined without invoking numbers (Enderton, 1977, p. 129). In principle, we can use any set as a “yardstick” to evaluate and express cardinalities. For example, let P be the set $\{a, b\}$; we can say that the cardinality of the set of authors of this commentary is equal to P 's cardinality; that the cardinality of the set of planets in our solar system is larger than P 's cardinality; and so on. In sum, cardinality is the magnitude, whereas number is a scale for the measurement of this magnitude. In principle, there can be scales other than numbers, such as the one based on P .

C&B compare perception of “number” with perception of other magnitudes, such as distance. They claim that, if we acknowledge that the visual system represents distance, we should also acknowledge that the ANS represents number. But this is where the confusion between cardinality and number misleads them. Although the visual system represents distance, it does not represent meters or feet (scales for the measurement of distance); by the same token, we may acknowledge that the ANS represents *numerosities*, but this does not mean that it represents number. Scales are not the sorts of things that can be perceived. At most, it may be that subjects use numbers as a mental scale to evaluate numerosity, but they certainly do not *perceive* numbers as such (which arise from applying a specific scale to magnitudes).



We, thus, have one way of resisting C&B's attack on the argument from imprecision. Whatever it is that the ANS uses to measure numerosity, it is imprecise. Therefore, it is unlikely that the ANS uses numbers, because by definition numbers constitute an exact scale for the measurement of cardinality (notice that there may be inexact scales; the one based on P above is one example; an approximate accumulator is another).

But is it correct to say that the ANS *represents* numerosities imprecisely? As C&B notice, there is a crucial difference between (what they call) number (and we call numerosity) and other magnitudes such as distance. Although the latter are “first-order,” or intrinsic, properties of the environment, “numerical quantities are higher order in that they can only be assigned relative to a sortal – a criterion for individuating the entities being counted” (sect. 5.3, para. 4). C&B are to be praised for highlighting this distinction, which is often overlooked in the literature. But it has a consequence they do not seem to consider: In contrast to distances, there are no numerosities “out there” to be perceived, because numerosities emerge only after an agent has adopted a given sortal. We can say that distances are imprecisely represented by the visual system because there seem to be exact distances in the external environment, regardless of some agent noticing them. But we cannot say the same about numerosities, because there are no numerosities out there, inexact or otherwise, to be represented: they emerge relative to a given sortal. As C&B acknowledge, “numbers [numerosities] ... enter into contents via property attribution, not as objects of perception” (sect. 2.2.3, para. 1). But attribution and representation seem to have different directions of fit: Attribution goes from subject to stimulus, whereas representation goes from stimulus to subject. Ultimately, the notion of representation that they adopt seems in tension with their own higher-order understanding of numerical quantities.

Finally, if numerosity is understood as pertaining to cardinality, it is also incorrect to say that subjects *attribute* numerosities to stimuli, because cardinality is an exact property by definition. The property the ANS assigns to stimuli, however, is vague. Stevens introduced the term “numerousness” to designate this property: “a property or attribute which we are able to discriminate when we regard a collection of objects” without counting (Stevens, 1939/2006, p. 1). This term has (regrettably) fallen into disuse, but we think that accounts of the so-called number sense would still benefit from a clear distinction between number (a scale), numerosity (a precise property), and numerousness (a vague property attributed in perception) (dos Santos, 2021).

Financial support

C. F. dos Santos is funded by FAPEMA (Foundation for Research and Scientific and Technological Development of Maranhão) under Grant Number BD-08292/17. C. Dutilh



Novaes is funded by the European Research Council (ERC), with Horizon 2020 Grant 771074 – SEA.

Conflict of interest

None.

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