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Propagation of Aggregate Demand, Firm Specific and Policy Shocks in an Economy with Labour Flows within and between Firms of Different Sizes

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Abstract

This paper illustrates the propagation at the macro level of various types of shocks in a multifirm model representing a miniature economy which consists of firms of 4 different sizes. Our modelling exercise focuses on the interaction between internal and external labour flows. These flows are driven by the strategic behaviour of the firms with respect to fires, labour hoarding and internal versus external hires in case of opening of vacancies. The impulse response effects of our simulations show the mechanisms behind some findings of VAR-analyses, and of theoretical models of labour flows and the business cycle. Examples are the cleansing role of recessions and the larger spread of firm sizes due to technological change.

Keywords: labour market flows, hierarchical models of firms, interaction between internal and external labour markets, model simulations, impulse response effects, propagation of shocks.

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1. Introduction

A major question in business cycle analysis is how various types of shocks propagate through the economy. Traditional models of the business cycle consider two types of aggregate shocks: permanent or technology shocks, and cyclical or temporary shocks (Kydland and Prescott, 1982; Long and Plosser, 1993). When considering cyclical fluctuations at the labour market, the analysis nowadays takes account of labour market dynamics through an explicit description of gross labour flows. The major distinction here is between aggregate and reallocation shocks. Reallocation shocks originate from allocative disturbances which, in a general sense, are defined by Davis and Haltiwanger (1999) as events that alter the closeness of the match between the desired and the actual distributions of labour and capital inputs. These reallocation shocks include various types of idiosyncratic (firm specific) shocks but also e.g. skill-biased technical change (see e.g. Marimon and Zilibotti, 1999, Mortensen and Pissarides, 1999 and Acemoglu, 1999, for modelling the propagation of these shocks). Moreover, policy measures can lead to both aggregate and reallocation shocks, where, of course, the intention of these measures is to sterilise aggregate and allocation disturbances rather than to evoke them. The propagation of policy shocks may give an indication of the efficiency of various policy measures.

Most empirical studies in this vein use multivariate VAR-models or the common trends-cointegration approach in order to disentangle these various types of shocks and analyse their propagation through the economy (see e.g. Blanchard and Quah, 1989, Davis and Haltiwanger, 1999, Den Butter, Van Montfoort and Weitenberg, 2000). Small structural theories provide the identifying restrictions for these impulse-response analyses.

The present paper takes a complementary approach in analysing the propagation and cyclical effects of various shocks at the labour market by building, with some detail, a multifirm model, which describes a miniature economy consisting of firms of different sizes. At the basis of our modelling experiment is a general specification of a model of a firm, which describes strategic behaviour of the personnel management of
the firm with respect to fires, labour hoarding and internal versus external hires in case of opening of vacancies. In this way the single firm model endogenizes internal labour flows. The (optimal) size of the firm is determined by minimizing costs. Firms of different sizes are modelled by specifying differences in technology and cost structure. In our miniature economy four single firm models of different size classes are linked by using relative weights of the share of these respective size classes in the industrial sector of the (Dutch) economy. This makes our multifirm model reckon with the links between internal and external labour flows and the consequences of heterogeneity of both firms and workers, which is a novelty of our approach. Using the model we conduct impulse response simulations in order to see to what extent our model reproduces the stylised facts and findings of other studies, and to see what propagation mechanisms lay behind these impulse responses.

The contents of the paper is as follows. The next section introduces the specification and calibration of the single firm model and section 3 describes how the single firm models are linked to construct the multifirm model. A benchmark simulation is conducted as starting point for the impulse-response analyses of various shocks in sections 4 and 5. It includes price shocks, representing aggregate cyclical shocks (section 4), and 4 different types of reallocation shocks (section 5), which may be either interpreted as autonomous shocks or as (induced) policy shocks, namely (i) reduction of adjustment costs, (ii) shifts in wage formation, (iii) increases in human capital formation, and (iv) shifts in the degree of specialisation. The latter two types of shocks relate to skill-biased technical progress. Section 6 summarises the major conclusions from the simulation experiments.

2. Internal labour flows in a hierarchical model of the firm

The single firm model, which is at the hard of our multifirm simulation model, formalises the decisions of personnel managers with respect to the allocation of employees over the available jobs. It combines elements from the literature on internal labour markets, hierarchical models and equilibrium search theory in order to model the internal economics of firms. This section provides a broad outline of the model; for more details we refer to Van Gameren (2000) and to the short technical description in appendix 1.
The hierarchical model describes a firm with a number of hierarchical layers, where at the lowest layer, employees spend their time on production of output and at the other layers they divide their time between production and supervision. At the core of the model is a parabolic production function describing the benefits of co-operation. Individual productivity can benefit from co-operation with the other employees at the same hierarchical level but requires supervision; however, when a layer contains too many employees, it exceeds the span of control of the supervisor (in the next hierarchical layer) and the benefits of co-operation become negative. The time needed for supervision depends on the quality of the subordinates. Total production is equal to the sum of production of individual employees, including the benefits of co-operation (and division of work). In the basic version of the model we assume perfect competition and price taking firms so that the product price is constant and normalised to unity. Wage costs for production and supervision constitute an important part of total costs. Following the literature on internal labour markets wages are set according to fixed wage scales, which differ for each hierarchical layer. In calibrating the model we have selected the values for the parameters of the functions describing productivity and wages at the various hierarchical levels in such a way that they guarantee that the supply curve of the firm is upward sloping after a certain size limit. This is a necessary condition for the size of the firm to be finite as, due to the assumption of perfect competition, we do not have a downward sloping demand curve.

The key assumption of our hierarchical model is that the management of the firm decides for each employee whether he/she spends time on the production of output or on supervising subordinates (or on a combination of both). For each employee the optimal number of subordinates (which is none for the lowest hierarchical layer) is determined by maximisation of his contribution to the firm’s profits (production minus wage costs), weighting the costs and benefits of hiring additional subordinates. So, starting with the highest-ranked person in the firm, we conduct an iterative optimisation procedure which yields a long run equilibrium value of the optimal size and hierarchical structure of the firm given the parameters of the production functions and the wage scales. By construction, the optimal number of subordinates for each position is independent from the optimisation for other positions, which facilitates the optimisation procedure.

Until now we discussed the costs and benefits of optimal employment in the firm in the static sense of a long run equilibrium. However, a major feature of our model is it
describes labour market dynamics by also reckoning with the costs for the changing of the number of employees. Adjustment costs arise if the optimal number of employees differs from the actual number. We include three types of adjustment costs, namely firing costs, hiring costs and training costs. In the case of superfluous employees, the employees having the lowest individual qualities are fired which induces firing costs. If the firm has a shortage of employees it has to fill vacancies by searching for suitable employees. The firm searches, first, among the employees currently employed in other jobs at the firm. We assume the supervisor has insight in the qualities of the employees in the next lower rank in the hierarchy. If someone is available with sufficient capacity – where capacity improvement comes from ‘learning by doing’ (see later) – he/she is promoted, with a vacancy at the original job.

Second, if the required capacities are not available within the firm, the supervisor can decide to hire a new employee. This requires an external search procedure, which bears a higher cost level. External applicants are drawn from a random distribution; the firm has no influence on the arrival of candidates. If a candidate fails to meet the minimum requirements, a training procedure can be considered, at a certain cost. The result of the (external) application procedure may also be that the job remains vacant.

The profit maximisation of the entrepreneur, and his search for employees – the two mechanisms discussed above – are the first two steps in the operation of the simulation model (see table 1). The third step is that for each filled job, i.e. for each subordinate, we repeat the optimisation and search procedure. We assume the optimal number of subordinates to be independent from the decisions made at other ranks and other branches in the hierarchy. The number of repetitions of steps 1 and 2 is finite under the conditions of the model and makes it possible to provide a full description of the firm in the dynamic setting with respect to the number of employees, the hierarchical structure, total production, and the number of unfilled vacancies (step 4 in table 1). Notice that both the number of hierarchical ranks and the actual spans of control are endogenous in the model.

After steps 1 to 4, we have the hierarchical set-up of the firm at the onset of a period. All workers in the hierarchy remain at their jobs for (at least) one time period and produce output during this period. At the end of the period we simulate that a random number of employees decides to quit the firm. Here we can think of workers who find jobs elsewhere, or workers who have other reasons to leave the labour force. A fraction of the employees will retire; we impose a mandatory retirement age. Quits and retirements result in the opening of vacancies at the old positions. Furthermore,
the passing of time generates an increase in the experience of employees within the firm (‘learning by doing’), which is implemented as an increase in their personal measure of quality. This effect depends on the tenure in the current job and has a random component. Workers are fired if, after a training procedure, they still do not meet the requirements. Given the vacancies (and the unfilled vacancies remaining from the previous search process), we repeat the optimisation process (step 1) for this next period in order to determine whether it is optimal to search for employees to fill the vacancies, or whether it is best to close the vacancies altogether. The following steps in the modelling algorithm are again conducted successively, as described above.

Finally we have also included the possibility of horizontal flows in the modelling algorithm. Van Gameren and Lindeboom (1999) provide evidence on the frequency of horizontal mobility which also appears to play a role in promotion decisions: job rotation may enhance the capacity requirements for promotion. We implement horizontal mobility by creating a pool of employees who are fired because their branch of the firm was unprofitable. If there is a vacancy in another branch of the firm but at the same (or a lower) hierarchical level, then the employees in the pool are the first candidates to fill the vacancy. Admittedly, this is a rather restricted and simple way to model horizontal job mobility, but the maximisation procedure of the model covers only one period so that we are unable to model horizontal mobility as part of a multiperiod career plan which will eventually lead to a promotion.

Table 1. Set-up of the one-firm simulation model

<table>
<thead>
<tr>
<th>STEP 1</th>
<th>Determination of optimal number of subordinates (per supervisor, per time period)</th>
</tr>
</thead>
<tbody>
<tr>
<td>STEP 2</td>
<td>in the case of vacancies: search for employees</td>
</tr>
<tr>
<td></td>
<td>• promotion of insiders (causes vacancy chains)</td>
</tr>
<tr>
<td></td>
<td>• hire of outsiders (training might be necessary)</td>
</tr>
<tr>
<td></td>
<td>in the case of superfluous workers: dismiss the least qualified subordinates</td>
</tr>
<tr>
<td></td>
<td>(the result of this step might be that there remain unfilled vacancies)</td>
</tr>
<tr>
<td>STEP 3</td>
<td>Repeat steps 1 and 2 for each subordinate until reaching the rank where the</td>
</tr>
<tr>
<td></td>
<td>(optimal) number of subordinates equals zero.</td>
</tr>
<tr>
<td>STEP 4</td>
<td>Determine the number of employees, production (optimal, actual), hiring, firing</td>
</tr>
<tr>
<td></td>
<td>(flows, costs), organisational structure of the firm</td>
</tr>
<tr>
<td>STEP 5</td>
<td>• random quits will occur</td>
</tr>
<tr>
<td></td>
<td>• there will be an increase in the experience of the employees who stay (“learning</td>
</tr>
<tr>
<td></td>
<td>by doing”)</td>
</tr>
<tr>
<td></td>
<td>• repeat steps 1 to 4 for the following period</td>
</tr>
</tbody>
</table>
The parameters of the single firm model are calibrated upon scarcely available empirical evidence in this field. Our baseline calibration generates a benchmark representative firm where the flow characteristics (quits, fires, and internal and external worker flows) mimic the results found in a study by Hamermesh et al. (1996). That study presents estimates of the annual worker flows in the Netherlands in 1990, drawn from a stratified sample of about 1,000 firms with 10 or more employees. The values of the model parameters, which generate our benchmark firm, are based on case studies on the internal economics of firms by Baker et al. (1994) and Van Gameren et al. (1999). These studies use the personnel records of two large firms, and describe in the detail how the internal structure such as the span of control and the wage scales of the firm are organised. The calibration of search costs is based on a linear approximation of the quadratic adjustment cost function of Pfann & Verspagen (1989). Their results suggest that in the case of small adjustments, hiring costs seem to be somewhat higher than the firing costs, while for more expansionary firms hiring costs increase exponentially. We assume that external search is more expensive than internal search, which implies that the first option for a firm is to fill vacancies through internal moves.

3. The multifirm model

The one-firm model of the previous section is extended to a multi-firm model that is capable to describe labour flows between firms, in addition to the flows within the firm and the flows to and from non-employment. In doing so the model fits closer to the literature of the flow approach to the labour market and provides insight in the relationship between the dynamics of the internal and external labour markets.

The model describes a collection of individual firms, in which we distinguish individual employees, like in the one-firm model. We consider the set of firms as being representative for a sector (or industry) of the economy. Within a sector we create firms that differ with regard to the size. Theoretically, we can include as many firms and as many employees in the model as we like. This would imply that we can describe a whole industry as an aggregate of all firms in that industry. Ultimately it would allow us to describe the whole economy, by including different industries as well. However, that would require an excessive amount of computer capacities and would also greatly obscure the working of the model. So we restricted ourselves to the modelling of one sector. For this sector, we build a small number of firms and
consider them representative for the whole sector. Our model is a partial model, in the sense that we assume, like in the one-firm model, completely elastic supply of labour, i.e. the supply of labour is always sufficiently large to meet the firms’ demand for labour without wage adjustments. The implication is that the growth in the number of employees can be (much) larger than the population growth which is not realistic for a model that mimics a whole (macro) economy. Therefore we label our multifirm model as a sectoral model, describing a miniature economy.

In order to keep the model relatively simple, we restrict the model to four size classes. In each class we specify and calibrate exactly one firm, and assume that this firm is representative for all firms of similar size. The firms in each of the four size classes are weighted by their relative contribution to the total employment in an industry. We choose to mimic an archetypical sector by using the firm size distribution of the total economy (of the Netherlands) as the basis for our multiplication weights. A further modelling problem is the treatment of flows between firms of the same size class. In fact, these flows are not modelled explicitly, unlike the flows between firms of different size classes. In the model, we cannot distinguish these flows between firms of the same size from the internal mobility, due to the choice to ‘blow up’ the firms according to their frequency in the sector under consideration. In modelling the firms of four different size classes we used our experience from a sensitivity analysis of the single firm model which provided information on how the size of the firm generated by the model depends on the values of the parameters. Before we come to the parameters, we discuss the parts where we have extended the one-firm model.

The most substantial extension of the one-firm model is the way in which the firm will search at the external labour market, when the internal search does not produce a suitable candidate. In the one-firm model, the only thing the firm can do is wait and see who applies for the job because arrivals at the external labour market are modelled as a completely random process. The randomised part still exists, but by combining several firms into one model, we can allow a firm to do an ‘informed search’ among the employees of the other firms in the sector. The fact that an employee is working at a certain hierarchical level provides information to other firms. We assume that a firm

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1 Flows between firms in the same size class can be studied if we construct firms of equal size. Heterogeneity can be introduced via different parameter sets (that lead to equal-sized firms), and we can analyse shocks that affect only one of the firms. Such simulations have not yet been conducted due to computer constraints.
can perfectly observe the qualities of the employees in other firms. To be more precise: we make a distinction between employees who can be hired directly from another firm, and employees who appear on the labour market because a firm fired them. We make the assumption that the first category (direct hires) will move to another firm only when they can make a 'promotion' to a higher rank. By assumption the second category may accept a job at any hierarchical rank. Qualities that are useful in one firm are not necessarily of the same value in other firms. The quality measure in our model consists of both general and firm-specific qualities. In case of a move to another firm we correct the quality measure for firm-specificity: the move results in a reduction of the quality measure, in comparison with its value if the employee would stay. In the one-firm model we provide training to external candidates that only narrowly fail to meet the job requirements. Now, in the multifirm model, we have to be more specific about training opportunities for candidates from other firms. We assumed that only candidates who are fired by the others are given the opportunity of training, and not candidates who are hired directly from another firm. The latter group will move only when they are better off in the new firm, and we suppose that they do not feel the need to move if this would imply that they have to do additional training. The former group, however, will be glad to receive training because it qualifies them for a new job so that they can minimise the duration of unemployment.

If neither the search within the own firm nor the search among employees of other firms results in a suitable candidate, then the last alternative that remains is to search among outsiders, who can be either non-employed or employed in another (not modelled) sector of the economy. This part is modelled like the external labour market in the one-firm model: as a random draw from probability distribution of qualities. The personnel department of the firm has no influence on the candidate. It has to wait and see the quality of the applicant, and has the choice between rejecting and accepting, if necessary after an additional training programme. Per period only one applicant arrives.

We summarise the search procedure by repeating the preference ordering of the personnel managers in the selection of employees for vacancies:

1. Move a candidate from another position within the firm. A candidate at the same hierarchical level who occupies a job which will be destroyed is preferred over a candidate whose job is not destroyed (hence whose departure causes a new vacancy with additional costs).
2. Do an ‘informed search’ among the employees of other firms in the same 
sector.

3. Hire someone who is not employed within the sector. If it turns out to be 
   impossible to find a suitable candidate, then the job remains 
vacant.

Whereas we assumed perfectly elastic demand and fixed prices in the single 
firm model, we allow for variability in the prices in the multifirm model. That is 
because we wanted to mimic the business cycle in the simulations as aggregate (or 
firm specific) shocks to the demand for products of the firm. These shocks are 
modelled via fluctuations in the price of output, and more specifically the cycle 
is simulated by means of a recursive sinus-function of output prices: 
\[ p_{t} = p_{t-1} + \delta f \sin(t) \]. The (potentially firm-specific) parameters \( \delta f \) govern the size of the business cycle. If \( \delta f = 0 \), 
cyclical behaviour does not occur at all (as in the one-firm model), and the higher the 
value of \( \delta f \) is, the stronger is the cyclical variation. When the \( \delta f \) parameters are equal 
for all firms \( f \), we have an aggregate demand shock that has the same direct effect on 
all the firms, but when these \( \delta f \) parameters differ per size class the respective firms are 
hit with different intensities by the cyclical shock. In the benchmark simulations we 
assume a constant time path for the wages and productivity so that the business cycle 
in the model concurs with fluctuations in the output prices but does not affect wages, 
(individual) productivity and the difference between wages and productivity.

Parameter Selection
Simulations with the one-firm model showed that the size of the firm is highly 
sensitive to the parameters associated with productivity and wages. We use those 
parameters as the main instrument for simulating firms of different sizes. Moreover 
we have to set values to the parameters which govern the flows between firms. In 
principle our calibration is based on the empirical results of Hamermesh et al. (1996) 
on flows between firms, although these results do not reveal how labour mobility 
differs between firms of different sizes. Yet Hassink (1996) uses these data for 
calculating mobility rates for two classes of firms: large firms, \emph{i.e.} firms with more 
than 100 employees, and small firms. His findings are shown in table 1. Obviously we 
would have liked to avail of more detailed empirical information on the various types 
of labour flows within and between firms; e.g. information on job-to-job mobility is 
sometimes used in the analysis of transitions in the labour market, but the link 
between internal and external flows is – as far as we know – never made. On the other 
hand, our model underestimates the total size of flows between firms because only
flows between firms in different size classes are visible in our simulation results. Therefore it would not make sense anyhow to calibrate the model in the present four-firm version on detailed flow data if they were available. The implication is that we use the data of table 2 only as kind if a guideline in the calibration and, in the presentation of the simulation results, we do not show the flows between firms separately, but instead include them in the more general category of external flows.

Table 2. Estimates of annual worker flows in the Netherlands, 1990
(in percentages of employment)

<table>
<thead>
<tr>
<th>Type &amp; direction</th>
<th>( L \geq 100 )</th>
<th>( L &lt; 100 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average number of employees</td>
<td>284</td>
<td>49</td>
</tr>
<tr>
<td>Hire to a newly-created job</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>Hire to an existing job</td>
<td>8.1</td>
<td>7.5</td>
</tr>
<tr>
<td>Outflow from an existing job</td>
<td>90</td>
<td>8.1</td>
</tr>
<tr>
<td>Outflow from a destroyed job</td>
<td>1.9</td>
<td>1.6</td>
</tr>
<tr>
<td>Internal mobility between existing jobs</td>
<td>2.9</td>
<td>11</td>
</tr>
<tr>
<td>Internal mobility from a destroyed to an existing job</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Internal mobility from an existing to a newly-created job</td>
<td>1.2</td>
<td>0.6</td>
</tr>
<tr>
<td>Internal mobility from a destroyed to a newly-created job</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Note: Based on tables 1 and 2 of Hassink (1996).

The mobility rates resulting from the benchmark simulations with the parameter values selected in the calibration procedure are presented in table 3. The table summarises the characteristics of the four firms generated by the model by the mobility rates for two size classes, large and small firms, to allow for comparison with table 2. These results are used as baseline projection in the impulse response analysis of the next sections. Because a number of inputs of the model are generated by stochastic processes we calculate 100 replications of each simulation and the simulation outcomes are averages over these 100 runs.

Table 3 shows that in the benchmark model the number of destroyed and newly-created jobs in small firms is small. This is due to the fact that in our model we have two types of fires which we label as ‘job destruction’: first, if a job/employee-pair is no longer profitable and the firm decides not to replace the employee, the job will be destroyed. If the employee is replaced, the job continues to exist. The second type of ‘job destruction’ occurs if an entire branch of the firm is destroyed. This happens if somewhere in the upper levels of the firm’s hierarchy a job becomes vacant, but a suitable applicant cannot be found. The lack of a suitable employee turns out to be the major source of job destruction. However, in small firms the number of levels is
small, and then seldom occurs that a complete branch is destroyed. The fact that we have little job destruction also reduces the possibilities for creation of new jobs in small firms. In larger firms we observe that the destruction of a job (or a branch) at one point in time is followed by the creation of new jobs in later time periods. In smaller firms there is less opportunity for this temporary destruction of jobs, and so job creation is lower as well.

Table 3. Simulation results of the benchmark model
(percentages of employment)

<table>
<thead>
<tr>
<th></th>
<th>Sector</th>
<th>large</th>
<th>Small</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm size</td>
<td>118</td>
<td>395</td>
<td>62.6</td>
</tr>
<tr>
<td>Ext. hire to a newly-created job</td>
<td>3.55</td>
<td>4.90</td>
<td>1.83</td>
</tr>
<tr>
<td>Ext. hire to an existing job</td>
<td>4.39</td>
<td>5.54</td>
<td>2.94</td>
</tr>
<tr>
<td>Outflow. from an existing job</td>
<td>7.53</td>
<td>9.78</td>
<td>4.70</td>
</tr>
<tr>
<td>Outflow from a destroyed job</td>
<td>2.31</td>
<td>4.14</td>
<td>0.00</td>
</tr>
<tr>
<td>Internal mobility to an existing job</td>
<td>2.67</td>
<td>4.44</td>
<td>0.42</td>
</tr>
<tr>
<td>Internal mobility to a newly-created job</td>
<td>0.64</td>
<td>1.14</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Note: Firm size is measured as the average number of employees.

4 Aggregate and size class specific demand shocks

This section presents a number of simulation experiments with aggregate and size class specific demand shocks representing various cyclical changes. First, we simulate a situation with larger swings of the business cycle, i.e. if there is more cyclical volatility than in the “normal” situation of the benchmark model. The second exercise leaves volatility unchanged, but we model a permanent negative shock by means of a sudden drop in the output prices. The permanent drop of the prices can be associated with a decrease in the demand but it may also be the result of technological changes. We model these two types of shocks as aggregate shocks which hit the entire industry with the same strength, and we also include simulations in which the shocks hit only small firms or large firms.

2 We do not allow for a whole size class to be destroyed. The destruction of one firm within a size class (i.e. a change in the weight factor) is ruled out, because we assume homogeneous firms within each size class.
Table 4. Shocks to the entire sector

<table>
<thead>
<tr>
<th>More volatility</th>
<th>Permanent shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector</td>
<td>Large</td>
</tr>
<tr>
<td>Size</td>
<td>+98</td>
</tr>
<tr>
<td>EH.N</td>
<td>-12</td>
</tr>
<tr>
<td>EH.E</td>
<td>-09</td>
</tr>
<tr>
<td>Out.E</td>
<td>-20</td>
</tr>
<tr>
<td>Out.D</td>
<td>-12</td>
</tr>
<tr>
<td>IM.E</td>
<td>-12</td>
</tr>
<tr>
<td>IM.N</td>
<td>-04</td>
</tr>
</tbody>
</table>

Note: All results are calculated as differences from the benchmark simulations in table 2. EH.N=Ext. hire to a newly-created job, EH.E=Ext. hire to an existing job, Out.E=Outflow from an existing job, Out.D=Outflow from a destroyed job, IM.E=Internal mobility to an existing job, IM.N=Internal mobility to a newly-created job.

The simulation results in table 4 show that a small negative shock (of about 1%) to the output prices of all the firms hardly affects the size of firms, nor does it affect mobility rates. Hires to and departures from existing jobs increase somewhat. Larger effects occur when there is a slightly higher volatility in the business cycle. A volatility of 3.3% instead of 3% in the baseline leads to a large increase in the size of small firms, while large firms are hardly affected. Note, however, that job creation and destruction in large firms declines, and that external mobility replaces internal moves to existing jobs. In small firms both inflow and outflow rates decrease, while the internal mobility rates remain the same, although the increase in the number of employees potentially creates more room for internal mobility. An interesting implication of our simulation experiment is that small firms seem to benefit from the ‘cleansing effect of recessions’ as higher volatility leads to more employment, and, as a consequence, to more output. Caballero & Hammour (1994) investigate this kind of response of industries to cyclical variations in demand in a theoretical model and conclude that the structure of adjustment costs has a determinant role in these outcomes. In this respect it is of interest to note that further experiments with higher volatility in our model even reveal an optimal volatility of the business cycle. Firm size begins to decrease if volatility increases to values above 4%.
Table 5. Size-specific demand shocks

<table>
<thead>
<tr>
<th></th>
<th>shock to large firms</th>
<th>Shock to small firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>More volatility</td>
<td>Permanent shock</td>
</tr>
<tr>
<td></td>
<td>Sector</td>
<td>Large</td>
</tr>
<tr>
<td>Size</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>EH.N</td>
<td>-0.2</td>
<td>-0.4</td>
</tr>
<tr>
<td>EH.E</td>
<td>+0.3</td>
<td>+0.5</td>
</tr>
<tr>
<td>Out.E</td>
<td>+0.1</td>
<td>+0.1</td>
</tr>
<tr>
<td>Out.D</td>
<td>-0.3</td>
<td>-0.6</td>
</tr>
<tr>
<td>IM.E</td>
<td>-0.2</td>
<td>-0.4</td>
</tr>
<tr>
<td>IM.N</td>
<td>-0.1</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

Note: see table 4.

In our experiments with size specific shocks we see (table 5) that shocks that hit small firms hardly affect large firms. The same holds for shocks to large firms: they have hardly an effect on the size of small firms. There is some propagation with respect to the flows. Shocks to small firms reduce job creation and destruction rates in large firms, but mobility to and from existing jobs increases. We see that existing vacancies are taken by external candidates more often than via internal mobility. When a shock of the same size hits larger firms only, we find effects with the same sign but larger in size. The increase in size of the small firms occurs only if small firms are hit directly.

A shock to large firms has a negligible effect on the small firms. If the permanent shock hits only the small firms, we see an effect on the flow rates of large firms, but we see no effect on the flows of the small firms. Again, further analysis indicates that there is an optimal volatility of the business cycle.

5 (Policy induced) reallocation shocks

This section shows the outcomes of our simulation experiments with four different types of shocks which can be associated with policy measures.

Differences in adjustment costs

Policies aimed to reduce adjustment costs are expected to increase employee mobility. Higher mobility is one of the features of a more flexible labour market. A lack of flexibility is often considered as a drawback of the Dutch (and even more of the European) labour market, in comparison with, e.g. the United States (Calmfors & Drifill, 1988). Against this background we conducted a number of experiments in order to simulate the effects of various adjustment costs on the mobility of employees and on the size of firms.
As noted in section 3 the model basically contains three types of adjustment costs: costs of firing an employee and two types of search costs – namely for those already employed in the firm, and for those who are not employed or are employed in one of the other firms in the modelled sector. Firing costs can be changed by policy measures; e.g., policies that reduce the protection of employees will make it easier to fire people who are not performing well or who became superfluous because of a downturn in the economic activities. Therefore a reduction of the firing costs can be regarded as the implementation in the model of a reduction of protection. It seems more difficult to reduce search (or hiring) costs by means of policy measures. In order to reduce these type of costs, it is important for firms that it becomes easier to locate correct information on qualities of (potential) applicants. For the internal search costs, this depends on the information structure within the firm – government policy will have (at most) a secondary effect. The role of the government is larger in reducing external search costs; e.g., through a better information system of the labour offices. Accurate information on the available employees and the jobs will facilitate good matches between jobs and employees.

### Table 6. Reduction of adjustment costs

<table>
<thead>
<tr>
<th></th>
<th>Firing costs</th>
<th>External search costs</th>
<th>Internal search costs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sector</td>
<td>Large</td>
<td>Small</td>
</tr>
<tr>
<td>size</td>
<td>-2</td>
<td>-10</td>
<td>0</td>
</tr>
<tr>
<td>EH,N</td>
<td>-0.3</td>
<td>-0.6</td>
<td>+0.1</td>
</tr>
<tr>
<td>E.H,E</td>
<td>+0.4</td>
<td>+0.6</td>
<td>+0.3</td>
</tr>
<tr>
<td>Out.E</td>
<td>+0.2</td>
<td>+0.1</td>
<td>+0.3</td>
</tr>
<tr>
<td>Out.D</td>
<td>0.5</td>
<td>0.9</td>
<td>0</td>
</tr>
<tr>
<td>IM.E</td>
<td>-0.3</td>
<td>-0.6</td>
<td>+0.1</td>
</tr>
<tr>
<td>IM.N</td>
<td>-0.2</td>
<td>-0.3</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: see table 4.

In table 6 we show the results of simulations in which we reduce the adjustment costs (one at a time) by 5%. Compared with the benchmark simulations, we see that the firing costs and the costs of search for employees within the firm have only small effects on economic activity. Reduction of firing costs leads to a decrease in the employment in large firms. Note that lower firing costs also lead to a decrease in job creation and destruction. On the other hand, external hiring increases while internal mobility drops. Hence it appears that more flexibility, implemented as a reduction of firing costs, proves not to be favourable to employment and output according to our simulations. Large firms even reduce in size because there is less reason for these
firms for labour hoarding. External hiring is a good alternative, since it is always possible to fire employees if they become superfluous. This relative unimportance and somewhat counterintuitive effect of a reduction of firing costs shows up in other models as well; see e.g. Booth (1997) for a discussion.

As we would expect, we find that internal mobility in large firms increases if it becomes relatively cheaper to search internally. In small firms we see no effect on internal mobility; as it remains unprofitable to make moves within small firms. Note that lower internal search costs result in an increase in job destruction and in the creation of new jobs. These results suggest that a good information system concerning the qualities of employees is important for firms.

The operation of the external labour market is highly important to the growth of a firm. A reduction of the costs of search among non-employed people leads to a large increase of firm sizes. Mobility rates decrease but the most prominent effect is a substantial increase in size of all firms if information on external candidates becomes cheaper. A transparent labour market, in which it is relatively easy to find employees with the required qualities is very important, even without additional investment in employee qualities. Part of this result is driven by the assumptions of perfectly elastic demand for the production, as well as a supply of labour which is always sufficiently large to fill all vacancies.

**Shocks in wage formation**

In the benchmark simulation we assume that wages do not increase faster than productivity and do not react to changes in output prices. In this section we study the effect of this policy of relatively moderate wage adjustments (a prominent feature of the so called Dutch “polder-model”), by comparing the benchmark simulations with the results of simulations where the wages show more variability over time. We create two different time paths: one in which the wages show a constant growth pattern, and one in which wages depend on the (lagged) business cycle. In the first simulation experiment, there is a constant, autonomous increase in real wages as compared with productivity so that the labour income ratio increases in the long run. Wages in the second alternative depend on lagged output price movements which represent changes in the business cycle. It reflects a (not explicitly modelled) wage bargaining process that takes one period, and where the outcome is to adjust the wages with the same percentage as the change in output prices. The time lag also represents uncertainties.
over future productivity; the wages are set at the beginning of each period, while production occurs during the that period.

### Table 7. Wage push

<table>
<thead>
<tr>
<th>Size</th>
<th>Constant wage growth</th>
<th>Wages dependent on business cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sector</td>
<td>large</td>
</tr>
<tr>
<td>EH.N</td>
<td>-0.4</td>
<td>-0.7</td>
</tr>
<tr>
<td>EH.E</td>
<td>+0.4</td>
<td>+0.6</td>
</tr>
<tr>
<td>Out.E</td>
<td>+0.1</td>
<td>0</td>
</tr>
<tr>
<td>Out.D</td>
<td>-0.4</td>
<td>-0.7</td>
</tr>
<tr>
<td>IM.E</td>
<td>-0.3</td>
<td>-0.6</td>
</tr>
<tr>
<td>IM.N</td>
<td>-0.1</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

Note: see table 4.

Table 7 indicates that wage growth has a negative effect on the economic activities of large firms. Small firms are hardly affected. The results suggest that the moderate wage development in the benchmark simulation is relatively unimportant for the size of economic activities. A constant (limited) wage growth has only small effects on the employment figures. Job creation and destruction rates drop, while external mobility to existing jobs increases and replaces internal mobility. Wages that fluctuate over time, by following the business cycle with a delay of one period, have an almost negligible effect on the size of firms. In large firms it causes a slight decrease in job creation and destruction and a shift from internal to external mobility. Small firms are remarkably unaffected by both kinds of wage variation. They manage to adapt the qualities of their existing work force to compensate the wage changes.

The simulation results suggest that wage policy does not have much influence on labour market dynamics. A bargaining strategy resulting in a wage development which closely follows the prices reaches the same level of employment as a strategy where wage growth follows long-term productivity increases. Note that we do not model excessive wage patterns; the simulated time paths of the wages all remain within a range similar to the fluctuations in prices and productivity. If we redo the simulations with larger wage growth (either constant or dependent on the prices), we do find negative employment effects, which is in accordance with the notion that a substantial part of the success of the Dutch “polder model” can be ascribed to a policy of wage moderation.
Shocks in human capital formation

The impulse-response analysis of this subsection studies the effects of education policies designed to raise the average education level of the population. The average level can be raised in three ways: first, through additional investments in basic (elementary) schooling; this investment will increase the minimum schooling level in the country. Second, by investing more in higher and academic education; this policy will expand the upper level of the education pyramid. The third option is to combine the two policies by spreading the additional investments throughout the education system. As in the model implementation the latter is a linear combination of the other two options; we do not discuss it separately.

In our model the effects of education policies are driven by the parameters of the function that describes the qualities of external applicants. The education policies of our simulations are directed towards people who are non-participants, and we assume that schooling will enhance their capacities.3 It is implemented in the model via a shift in the probability density function from which we draw the quality of an external applicant. Policy 1 (elementary education) increases the lower bound of the distribution without changing the upper bound. Investments in higher education (policy 2) work the opposite way: an increase of the upper bound and no change in the lower bound.

Table 8 Investments in education

<table>
<thead>
<tr>
<th>Sector</th>
<th>Elementary education</th>
<th>Higher education</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>large</td>
<td>small</td>
</tr>
<tr>
<td>EH.N</td>
<td>-1.7</td>
<td>-2.8</td>
</tr>
<tr>
<td>EH.E</td>
<td>+1.4</td>
<td>+2.3</td>
</tr>
<tr>
<td>Out.E</td>
<td>-0.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>Out.D</td>
<td>-1.6</td>
<td>-2.8</td>
</tr>
<tr>
<td>IM.E</td>
<td>-2.2</td>
<td>-2.1</td>
</tr>
<tr>
<td>IM.N</td>
<td>-0.5</td>
<td>-0.9</td>
</tr>
</tbody>
</table>

Note: see table 4.

In table 8 we show the results of simulations with increases of 20% in the lower bound (left panel) or the upper bound (right panel) of the quality distribution from which external applicants are drawn. Our simulations show that investments in higher

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3 Due to the construction of the model we must assume that people who are not employed in the modelled sector also benefit from additional investments in education. The applicant is randomly drawn from the probability density function and we do not know if he is unemployed or has a job elsewhere outside the industry.
education have the highest return for employment. If we invest only in elementary education - which increases the minimum quality available - it does not lead to a change in the average size of firms. Large firms even decrease in size, while small firms show a small increase. The mechanism behind it is that, because of the enhanced quality of workers at elementary levels, more mismatches occur between the employees and the jobs on the lowest hierarchical ranks of large firms. The higher qualifications of the employees translate into higher wages. However, the increase in skills creates a situation whereby they become too expensive to occupy the job in the lower ranks of the hierarchy; as the productivity increase is too small as compared with the wage increase. The implication is that fewer employees can actually find a job. A prominent effect of investment in elementary education is the large drop in job destruction. Once a job is occupied, it is almost always profitable to maintain that job because qualifications will almost never fall below the required level. On the other hand, investments in higher education lead to an increase in firm sizes, but also to more job destruction. Investment in higher education generally leads to greater mobility and climbing of the career ladder - see also the internal mobility rates - thus leading to a boost in economic activity. Education investments which are directed to the (already) higher educated serve to increase employment, while investments in lower education hardly have an effect. Den Butter and Gorter (1999) come to a similar results in simulation experiments in a flow model of the labour market with two different types of jobs: here lowering the costs associated with jobs which require skilled labour is more effective in enhancing employment than lowering the costs associated with all kind of jobs.

The finding of decreasing employment due to investments in lower education may be counterintuitive, but a similar reaction is found in models on skill-biased technological shocks. In those models (see e.g. Marimon & Zilibotti, 1999) the effects of shocks that change the distribution of required skills depend on the prevailing system of unemployment benefits. If unemployment benefits are generous, then employment decreases if the mismatch between capacities of applicants and the job requirements increases. In a generous system employees have the opportunity to continue searching for a better match. In a system with limited unemployment benefits it may be necessary to make a match even if a better match is possible. Unemployment benefits are not modelled in our specification, but we assume that employees do not adjust their wages downward. Investments in higher education also result in higher wages, but the balance between the increase of productivity and wages is positive. This leads to a larger total number of jobs, mainly at the lower hierarchical
grades. Crowding-out (see e.g. Van den Berg et al., 1998) can occur, but is not likely because the increased qualities (due to the additional education) make it such that higher educated people are too expensive to occupy a low-productive job (cf. the investments in the lower educated). The main mechanism that drives these results in the model is, however, that increased productivity in the higher ranks of the hierarchy is used to supervise more employees at the lower ranks.

**Shifts in the degree of specialisation**

Our last simulation experiments consider the role of the internal structure of firms in the size of labour mobility. In the benchmark model the hierarchical structure (and the “flatness” of the organisation) of the firms is determined by the parameters of the function which describes the ‘benefits of co-operation’ and which reaches a maximum at a given number of employees. Technological innovations, e.g. in the field of ITC, may lead to changes in the hierarchical structure of the firm. In the model these changes can be implemented by changing the parameters of the ‘benefits of co-operation’ function. In our simulation experiments we keep the average size of the firms as given. It means that we try to select the new parameters in such a way that the total number of employees remains equal, and that changes occur only in the number of levels and the number of employees per level.

The width of the hierarchical levels refers to the internal practise of specialised task sets (small levels) or of more general (multiple) tasks and abilities (wide levels). The latter, where every employee can perform more tasks within the firms, can still contain a high amount of firm-specific capacities. Firm-specific qualities result in a loss of productivity whenever an employee moves to another firm. This can be modelled by the quality indicator of the employees, and by the additional requirements employees from other firms should fulfil before being hired.
Table 9. Internal structure

<table>
<thead>
<tr>
<th></th>
<th>Wider Levels</th>
<th>Smaller Levels</th>
<th>More firm-specificity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sector</td>
<td>large</td>
<td>Small</td>
</tr>
<tr>
<td>Size</td>
<td>+7</td>
<td>+184</td>
<td>−28</td>
</tr>
<tr>
<td>EH.N</td>
<td>0</td>
<td>−0.8</td>
<td>−0.3</td>
</tr>
<tr>
<td>FH.F</td>
<td>+0.9</td>
<td>+0.6</td>
<td>−0.8</td>
</tr>
<tr>
<td>Out.E</td>
<td>+0.9</td>
<td>0</td>
<td>−1.0</td>
</tr>
<tr>
<td>Out.D</td>
<td>−0.2</td>
<td>−1.5</td>
<td>0</td>
</tr>
<tr>
<td>IM.E</td>
<td>+0.2</td>
<td>−0.8</td>
<td>−0.2</td>
</tr>
<tr>
<td>IM.N</td>
<td>−0.6</td>
<td>−1.1</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: see table 4.

A larger optimal ‘span of control’ makes large firms larger and small firms smaller (see table 9). Hence, the spread between firm sizes becomes wider by these technological changes. This larger span of control could be used to make the organisation flatter, i.e. to reduce the number of hierarchical levels while having more employees on each remaining level. However, our simulations indicate that in large firms it is profitable to leave the number of hierarchical levels unchanged. But each level has more employees, and by implication, the total size of the firm increases. Conversely, in smaller firms we find that widening the span of control leads to flattening of the organisation and even to a reduction in firm sizes.

A smaller size of the span of control (middle panel of table 9) is expected to lead to more hierarchical levels. If a firm wants to maintain its production level it must create more levels and hire more employees. At these new levels it offers a lower salary (because these levels consist of jobs with lower productivity) and attracts employees with lower qualities. The simulations show that in large firms the creation of new levels does not occur. Due to the smaller levels farther up in the hierarchy, the total firm size reduces. Note that almost all mobility rates drop. Internal mobility to new jobs is almost reduced to zero. In small firms we hardly see an effect, but simulation experiments not reported here indicate that, when we use a slightly higher value of the benefits of co-operation than in table 9, larger firms will in fact increase their number of hierarchical levels.

The third section of table 9 shows that. According to our model simulations, the amount of firm-specific capital is rather unimportant to the size of firms and for the mobility rates. We would expected to see a change from external to internal mobility, but this is not found in the simulations. It appears that these effects occur only if the
loss of productivity is much higher than in our simulations. Conversely, these results underline the importance of learning by doing for the employability (and ability to move to other jobs) for employees.

6 Conclusions

This paper simulates impulse response effects of aggregate demand, firm specific and policy induced reallocation shocks, using a multifirm model which mimics an ‘archetypical’ sector (industry) of the economy. The focus is on the interaction between flows on the internal and external labour markets. For computational reasons we restrict the model to an interaction of four firms. We identify the firms based on size categories, and we use multiplication factors in order to mimic an ‘archetypical’ sector. Therefore we label it a model of a miniature economy. A limitation is that, in doing so, flows between firms of the same size cannot be studied in the model.

The first set of simulations considers the impulse response effects of aggregate and firm (size) specific demand shocks, which are implemented as product price changes. A major and intriguing finding is that, when the price fluctuations remain within certain bounds and uncertainty does not become too large, the resulting process of ‘cyclical cleansing’ has an overall positive effect. Larger firms appear to be relatively less sensitive to cyclical shocks than small firms. The propagation of firm specific shocks from small firms to large firms, and vice versa, appears to be small.

Our second set of model experiments gives impulse response effects of various reallocation shocks associated with policy measures. They lead to the following conclusions.

1. A reduction of the costs of external search are very beneficial for firms, much more so than changes in firing costs and the costs of internal mobility.
2. An autonomous wage increase, or fluctuations of wages along with the lagged business cycle, appears to have a small negative effect on the size of the firms, as compared to the baseline model where wage formation is in line with long run productivity increases. Furthermore, we find that job creation and destruction rates drop and internal mobility is replaced by external hiring.
3. No favourable effects are found when we, by a policy of schooling, enhance the quality of the lower educated applicants. That is because higher-qualified applicants demand higher wages as well, thus making them more expensive and less profitable for filling jobs at the lower ranks of the hierarchy. It is better for the
economy (as modelled here) to invest in higher education, which leads to an increase in employment across the entire range. In that case higher job destruction rates are more than offset by the increase in job creation and increased mobility within the firm.

4. Experiments with technology induced changes in the span of control indicate that a kind of skill biased technical change which makes the span of control larger, has a large positive effect on employment in large firms, but reduces employment in small firms. Hence these simulations concord with stylised facts that the new technologies lead to a larger spread in firm sizes. Small firms perform better with a smaller span of control. It allows employees to focus on specialised task sets and the firms to become profitable by filling the niches in the economy left open by large firms.

References


Gameren, E. van (2000), *The Internal Economics of Firms: an Investigation into the Labour Mobility within Firms*, Tinbergen Institute Research Series, nr. 2 18.

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Gameren, E. van, J.G. Treble, S. Bridges and T. Barmby (1999), The internal economics of the firm: Further evidence from personnel data, mimeo


Appendix Technical description of single firm model

The profit function in period \( t \) of each supervisor (occupied at hierarchical level \( i \)) is specified as

\[
\text{prf}_i = \sum_{j=0}^{n_r} \left( p_i a(n_j) q_{j,i+1,t} - w_{j,i+1,t} - \text{fp}_{j,i+1,t} - AC(n_j, n^*_r) \right),
\]

and is maximised with respect to \( n_t \). Here, \( n^*_r \) is the currently available number of subordinates.

The constituent parts are the following functions:

Production \( q_{j,i,t} = Y_{q,t} {\gamma_{q,1}}^t \gamma_{q,t} \) with \( \gamma_{q,1} = 150 \) and \( \gamma_{q,t} = 0.85 \).

Benefits of Cooperation \( a(n) = an^2 + bn + c \) with \( a = -0.05625 \) and \( b = 0.45 \) (derived from the parameters \( \max = 1.9 \) and \( n_{\max} = 4 \)).

Supervision Costs \( \text{fp}_{j,i,t} = Y_{\text{fp},1} (1/c_j,t) \) with \( \gamma_{\text{fp,1}} = 37.5 \) and \( \gamma_{q,t} = 1 \).

Wage \( w_{j,i,t} = Y_{w,1} (\gamma_{w,2})^t \gamma_{w,t} \) with \( \gamma_{w,1} = 175 \) and \( \gamma_{w,2} = 0.75 \).

The variables \( \text{tenF}_{jt}, \text{tenL}_{jt} \) and \( \text{Age}_{jt} \) indicate the tenure of employee \( j \) with the firm, the time he is employed in his current job and his age. \( At \) allows for autonomous wage increases, \( A_{t+1} = A_t + \gamma_{w,t} \). In the baseline simulations \( \gamma_{w,0} = 0 \); there is no autonomous wage increase.

The function \( p_t \) defines the price of the output as it will be received by the firm. It is specified as an exogenous function, \( p_t = 1 + \delta_t \), but in the baseline simulations we keep price constant: \( \delta = 0 \).
The relative quality measure \( c_{ijt} \) is defined as
\[
    c_{ijt} = \frac{\text{qual}_{ijt}}{E(q_{Ait})}
\]
if the job is occupied by subordinate \( j \), and
\[
    c_{ijt} = 1
\]
if the job is vacant,
where \( \text{qual}_{ijt} \) is the actual quality of subordinate \( j \), and \( E(q_{Ait}) \) the expected quality of an external applicant.

Dynamics are incorporated in the model through the adjustment costs,
\[
    AC(n_l, n_{l-1}) = -\frac{\text{fr}_{ijt} (n_{l-1}^* - n_l^*)}{L(n_l - n_{l-1}^*)} + \frac{\text{si}_{ijt} \min(n_l - n_{l-1}^*, n_{l-1}^*)}{L(n_l - n_{l-1}^*)} + \frac{\text{se}_{ijt} \max(n_l - n_{l-1}^*, 0)}{L(n_l - n_{l-1}^*)},
\]
with \( n_{l-1}^* \) the number of potential internal candidates for the job (defined as the number of employees on the next lower level), and \( I(\cdot) \) the indicator function. The three different search costs are:
- firing costs \( \text{fr}_{ijt} = \gamma_{fr} = 25 \)
- internal search costs \( \text{si}_{ijt} = \gamma_{si} = 33.33 \)
- external search costs \( \text{se}_{ijt} = \gamma_{se} = 50 \)

External candidates are drawn from a uniform probability density function with upper and lower bounds that vary per level:
\[
    q_{Ait} \sim \text{UNIF}(lb_{it}, ub_{it}),
\]
\[
    lb_{it} = y_{lb,i}(\gamma_{prob,i}^t),
\]
\[
    ub_{it} = y_{ub,i}(\gamma_{prob,i}^t),
\]
with \( y_{lb,i} = 75 \) and \( y_{ub,i} = 225 \). Hence, expected quality is equal to \( E(q_{Ait}) = y_{prob,i}^t(\gamma_{prob,i}^t)^2 \), with \( y_{prob,i} = (y_{lb,i} + y_{ub,i})/2 = 300 \).

The ‘baseline’ minimum requirements are given by
\[
    q_{mi,t} = y_{mq,i}(\gamma_{mq,i}^t),
\]
with \( y_{mq,i} = 250 \) and \( y_{mq,i} = 0.75 \).

For an internal applicant the minimum requirements are
\[
    q_{mi,t} = (1 + \gamma_{RQIE}^t) q_{mi,t},
\]
with \( \gamma_{RQIE} = 0.10 \), while for an external applicant the minimum requirements equal
\[
    q_{mi,t} = (1 - \gamma_{RQTR}^t) q_{mi,t},
\]
with \( \gamma_{RQTR} = 0.20 \).

Furthermore, we have a number of functions that specify the relations between successive periods. We have a random quit probability, which is actually defined as the probability that an employee remains in the firm \( (1 - \gamma_{QUIT}^t) \), with \( \gamma_{QUIT} = 0.02 \). There is a retirement age: an employee who reaches the age of \( y_{RETR} = 60 \) is retired. For each worker who does not quit, we introduce an accumulation of quality,
\[
    \text{qual}_{i,t+1} = \text{qual}_{i,t} + (1 + 2(\gamma_{enj})^t U),
\]
where \( U \) is the random factor drawn from a uniform distribution \( U \sim \text{UNIF}(0, \gamma_{GRWE}) \) with \( \gamma_{GRWE} = 0.20 \).