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Intransitive Intertemporal Choice

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ABSTRACT

Multiattribute choice rules can be classified as being either alternative-based or attribute-based. Conventional accounts of intertemporal choice, hyperbolic and exponential discounting, assume alternative-based rules. One consequence of using these rules is that choices will be transitive, meaning that if *a* is preferred to *b*, and *b* is preferred to *c*, then *a* will be preferred to *c*. There have been many demonstrations of intransitivity in domains other than intertemporal choice, and in this paper we undertake to establish whether intransitive intertemporal choice can be explained by a stochastic specification of exponential discounting, or if we need to invoke an attribute-based choice process. In an experiment, we demonstrate that the pattern of intransitive responses is inconsistent with alternative-based choice. We argue that intransitive choices can best be explained by a version of Tversky's (1969) lexicographic-semiorder rule, in which choice is based on the amount of money when that amount exceeds a threshold, but on delay otherwise. Transitive choices, on the other hand, seem to be based on the rule that 'earlier is better' or else on a consistent rate of discount. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS intertemporal choice; hyperbolic discounting; exponential discounting; intransitivity

Multiattribute choice processes can be divided into two broad classes. In the first, decision makers make separate estimates of the value of each option before choosing between them. In the second, they compare options attribute by attribute, and choose the one which comes out best. Payne, Bettman and Johnson (1993) use *alternative-based* choice to refer to the first class, and *attribute-based* choice to refer to the second. To illustrate these for intertemporal choice, imagine that you have a choice between £100 in one month or £150 in three months. One way to choose is by calculating the 'present value' of both options and take the one with the highest value. This would be an alternative-based choice. Another strategy is to take the decision attribute by attribute: first decide if you are willing to wait an extra two months for any amount of money: if you are not, take the £100, but if you are, then take the £150. This would be an attribute-based choice, in which the attribute of delay is given priority over amount. The distinction between alternative- and attribute-based choices has been given other labels — Tversky (1969) distinguished between *independent* and *comparative* choice, Hsee *et al.* (1998) between *separate* and *joint* evaluation, and Russo and Dosher (1983) between *holistic* and *dimensional* choice strategies — but all draw on the same underlying definition.

Most accounts of intertemporal choice are alternative-based. This includes the conventional exponential discounting model, as well as recent models of non-constant or *hyperbolic* discounting (e.g. Ainslie, 1991; Loewenstein and Prelec, 1992). In these models, the subjective value of a delayed option is the utility or value it will have when received, weighted by a discount function which accounts for the effect of delay. As with all alternative-based processes, choices based on time discounting will be transitive. That is, if you prefer £100 in one month to £150 in three months, and £150 in three months to £200 in four months, then you will prefer £100 in one month to £200 in four months. Such transitivity is often cited as the central pillar of the theory of rational choice (e.g. Luce and Raiffa, 1957; Stigler, 1966; but see Anand, 1993). In this paper, we study whether intertemporal choices can be accounted for by an alternative-based choice model by examining patterns of intransitivity among these choices.

Our work was inspired by a classic paper by Amos Tversky (1969), who developed a reliable procedure for eliciting intransitive choice. His subjects made a series of pairwise choices between the following opportunities:

(A) 7/24 chance of $5.00
(B) 8/24 chance of $4.75
(C) 9/24 chance of $4.50
(D) 10/24 chance of $4.25
(E) 11/24 chance of $4.00

When subjects chose between successive pairs (e.g. between A and B), they typically chose the high-payoff/low-probability option (A). When choosing between the most distant pairs (A and E), however, they chose the low-payoff/high-probability option (E), thus displaying the following intransitive pattern of preference:

$$A \succ B, B \succ C, C \succ D, D \succ E, \text{but } E \succ A$$

where '$\succ$' means 'is chosen over'. Intransitive choice is a major empirical reason for proposing attribute-based models (Fishburn, 1982; Gigerenzer and Goldstein, 1996; Loomes and Sugden, 1982; Rubinstein, 1988). Tversky himself advanced two such models to account for his results. According to one of these, people choose based on a *lexicographic-semiorder* rule. First, they prioritize the attributes. Then, when making binary choices, if x exceeds y by more than a threshold amount on the first-priority attribute, x is chosen. Otherwise, the decision process moves to the second-priority attribute. Intransitive choice results when two or more sub-threshold differences add up to a supra-threshold difference, as they do when moving from A to E in Tversky's experiment.

Tversky (1969) pointed out that even alternative-based choices will sometimes be intransitive if the choice process has a *stochastic* or random component. Such a component can influence our preferences (sometimes we prefer x, sometimes y — it depends on our mood), or it can influence our choice behavior (sometimes we accidentally take y although we prefer x). There is a limit, however, to the number of intransitive choices that can be attributed to such stochastic factors. Tversky stated one of these limits as a choice axiom, *weak stochastic transitivity*:

$$\text{If } p(xy) \geqslant 0.5, \text{ and } p(yz) \geqslant 0.5 \text{ then } p(xz) \geqslant 0.5$$

where $p(xy)$ is the probability of choosing x over y, and so forth. In a study in which subjects repeatedly made the same set of choices, Tversky observed many violations of this axiom.

In this paper we undertake to determine whether intransitive intertemporal choice can be accounted for by a *stochastic specification* (Loomes and Sugden, 1998) of an alternative-based process, or if we must invoke an attribute-based process. Our research strategy is similar to Tversky's (1969). We first establish quantitative limits on the intransitive choices that can arise from alternative-based choices, and then test whether observed choices fall within these limits. There are two differences between our work and Tversky's. First, we are studying the domain of intertemporal choice rather than choice under risk. Second, because we study group rather than individual responses, subjects' responses are compared to a null hypothesis more suited to group data than weak stochastic transitivity.[1] This new null hypothesis is based on the ratio between the observed proportions of two classes of intransitive choice. In the next section, we describe these two classes, and how we can distinguish between them.

INTRANSITIVITY

We investigate the case in which subjects make binary choices between all possible pairs of four alternatives, which we designate as *a*, *b*, *c*, and *d*. Because there are 6 binary choices, there are 64 (2^6) possible choice patterns, all of which are depicted in Exhibit 1, 32 on each side of the exhibit. The middle column, labelled 'Preference order', indicates which alternative in each pair is preferred over the other. For Patterns 1 through 32, a ' + ' in the middle column indicates that the alternative on the left is chosen over the one on the right, while a ' − ' indicates that the alternative on the right was chosen; a ' + ' in the *ab* column means that this pattern involves a choice of *a* over *b*, and a ' − ' in this column means it involves a choice of *b* over *a*. For Patterns 33 through 64, this interpretation is reversed, and a ' + ' indicates a choice for the alternative on the right, while a ' − ' indicates a choice for the alternative on the left. The patterns on the right, therefore, are the mirror image of those on the left.

Some of the patterns, listed in the columns headed T, are transitive, and some, listed in the columns headed I_3 and I_4, are intransitive. For intransitive patterns, parentheses are placed around the alternatives which has a cyclical relationship — the one on the right of the parenthetical set is preferred to the one on the left. Some intransitivities, designated I_3, span three alternatives. To illustrate, Pattern 5 — *(abc)d* — contains a cyclical relationship between *a*, *b* and *c*: $a \succ b$, $b \succ c$, but $c \succ a$. Other intransitivities, designated I_4, span four alternatives. To illustrate, Pattern 7 — *(abcd)* — contains a cyclical relationship between *a*, *b*, *c* and *d*: $a \succ b$, $b \succ c$, $c \succ d$, but $d \succ a$. Appendix A gives a procedure for distinguishing between T, I_3 and I_4 choice patterns.

In the experiment reported below, the four options were as follows (monetary amounts are in Dutch Guilders):

A (7/1): Dfl 7, 1 week delay
B (8/2): Dfl 8, 2 week delay
C (9/4): Dfl 9, 4 week delay
D (10/7): Dfl 10, 7 week delay

Although we use upper-case letters corresponding to the lower-case letters of our discussion, it should be noted that each of the letters *a*, *b*, *c* and *d* can refer to any of the four alternatives, although the use of equivalent upper- and lower-case letters is not accidental (the modal choice pattern was ABCD, or Pattern 1). We use lower-case letters when speaking in general terms, and upper-case letters when referring to specific alternatives.

The four alternatives were chosen to permit a test of Tversky's (1969) lexicographic-semiorder rule. The alternatives have two dimensions, time and money. We expected that, for most subjects, money would be the first-priority attribute, and only when differences in money did not exceed a psychological

[1] We use a group-level test because it is virtually impossible to obtain a large number of independent responses to the same item from the same subject — if subjects remember their earlier choices, they will attempt to be consistent. Tversky (1969) was able to reduce this problem by representing probability on a spinner, so that subjects could not be sure that they had seen the same items before. There is no analogue to a spinner for representing time or amount.

Exhibit 1. The 64 possible choice patterns that can arise from all possible binary choices between four alternatives, classified according to whether they are T, I_3 or I_4 patterns

	Pattern type			Preference order[a]						Pattern type			
	I_4	I_3	T	*ab*	*bc*	*cd*	*ac*	*bd*	*ad*	T	I_3	I_4	
1			*abcd*	+	+	+	+	+	+	*dcba*			33
2			*bacd*	−	+	+	+	+	+	*dcab*			34
3			*acbd*	+	−	+	+	+	+	*dbca*			35
4			*abdc*	+	+	−	+	+	+	*cdba*			36
5		(*abc*)*d*		+	+	+	−	+	+		*d*(*cba*)		37
6		*a*(*bcd*)		+	+	+	+	−	+		(*dcb*)*a*		38
7	(*abcd*)			+	+	+	+	+	−			(*dcba*)	39
8		(*cba*)*d*		−	−	+	+	+	+		*d*(*abc*)		40
9			*badc*	−	+	−	+	+	+	*cdab*			41
10			*bcad*	−	+	+	−	+	+	*dacb*			42
11	(*bacd*)			−	+	+	+	−	+			(*dcab*)	43
12		*b*(*acd*)		−	+	+	+	+	−		(*dca*)*b*		44
13		*a*(*dcb*)		+	−	−	+	+	+		(*bcd*)*a*		45
14			*cabd*	+	−	+	−	+	+	*dbac*			46
15			*acdb*	+	−	+	+	−	+	*bdca*			47
16	(*acbd*)			+	−	+	+	+	−			(*dbca*)	48
17	(*abdc*)			+	+	−	−	+	+			(*cdba*)	49
18			*adbc*	+	+	−	+	−	+	*cbda*			50
19		(*abd*)*c*		+	+	−	+	+	−		*c*(*dba*)		51
20	(*cadb*)			+	+	+	−	−	+			(*bdac*)	52
21	(*bcda*)			+	+	+	−	+	−			(*adcb*)	53
22	(*dabc*)			+	+	+	+	−	−			(*cbad*)	54
23	(*badc*)			−	−	−	+	+	+			(*cdab*)	55
24			*cbad*	−	−	+	−	+	+	*dabc*			56
25	(*acdb*)			−	−	+	+	−	+			(*bdca*)	57
26	(*cbda*)			−	−	+	+	+	−			(*adbc*)	58
27		*b*(*dca*)		−	+	−	−	+	+		(*acd*)*b*		59
28		(*dba*)*c*		−	+	−	+	−	+		*c*(*abd*)		60
29			*bdac*	−	+	−	+	+	−	*cadb*			61
30	(*bcad*)			−	+	+	−	−	+			(*dacb*)	62
31			*bcda*	−	+	+	−	+	−	*adcb*			63
32	(*dbac*)			−	+	+	+	−	−			(*cabd*)	64

[a]For choice patterns 1 through 32, a ' + ' in these columns mean that the left item was chosen over the right item. For the *ab* column, for instance, it means that *a* was chosen over *b*. For choice patterns 33 through 64, a ' + ' means that the right item was chosen over the left. For the *ab* column it means that *b* was chosen over *a*. A ' − ' in these columns has the opposite interpretation — e.g. *b* over *a* for items 1 through 32, and *a* over *b* for items 33 through 64.

threshold of significance would they turn to the time attribute. Because the difference in monetary payoffs between adjacent alternatives was quite small (Dfl 1, about the price of an expresso coffee), we expected that it would not exceed many thresholds, and that only for larger differences of Dfl 2 or Dfl 3 would the money dimension be salient. On the other hand, even the smallest differences in time (1 week) were likely to be above most thresholds. As a consequence, the specific patterns of intransitivity that we expected would involve preferences for the earlier of two adjacent alternatives (i.e. A ≻ B, B ≻ C, C ≻ D), but for the later of two non-adjacent alternatives (i.e. D ≻ A, D ≻ B, C ≻ A).

We next discuss the predictions made by two deterministic alternative-based models, those which propose that people discount the future exponentially or hyperbolically.

EXPONENTIAL AND HYPERBOLIC DISCOUNTING

Loomes and Sugden (1998) distinguish between a *core theory*, which in their work is expected utility theory, and a *stochastic specification* of that theory, which modifies the core theory by incorporating a stochastic component. For intertemporal choice, the alternative-based core theories are *exponential* and *hyperbolic* discounting, both of which describe the present value of future outcomes as their value when they occur, discounted by a function of delay. The mainstream model in economic analysis is exponential discounting:

$$V_0 = V_t\left(\frac{1}{1+r}\right)^t \tag{1}$$

where V_0 is the present value of an option (at time = 0), t is the amount by which it will be delayed, V_t is the value it will have when it is received, and r is the discount rate (equivalent to a personal rate of interest). If r is positive, then options will be valued less the more they are delayed.

Hyperbolic discounting was proposed in the context of psychology (e.g. Ainslie, 1975; Herrnstein, 1961; Mazur, 1987), but is now finding increasing application in economics (e.g. Laibson, 1997, 1998; O'Donaghue and Rabin, 1997). The most commonly cited specification of this model is

$$V_0 = \left(\frac{V_t}{1+kt}\right) \tag{2}$$

where k is a hyperbolic discount parameter. As with the exponential parameter r, a positive k means that people would prefer to get good things sooner rather than later.

Non-stochastic specifications of both exponential and hyperbolic discounting hold that we always choose the option with the highest present value. As shown in Appendix B, assuming that there is no indifference between pairs, only 7 of the 64 possible patterns of choice between the alternatives in our experiment are compatible with exponential or hyperbolic discounting: ABCD, BACD, BCAD, BCDA, CBDA, CDBA, DCBA (from Exhibit 1, these are patterns 1, 2, 10, 31, 50, 36, 33). We refer to these as T* patterns.

Alternative-based choice models can also incorporate idiosyncratic decision weights. For instance, imagine someone who would normally prefer the alternatives in the order DCBA. His birthday is next week, however, and so he will get more utility out of having money then. This could change his preference order to ADCB. In his classic study of dynamic inconsistency, Strotz (1956) incorporated both factors: 'The relative weight which a person may assign to the satisfaction of a future act of consumption (the manner of discounting) may depend on either or both of two things: (1) the *time distance* of the future date from the present moment [corresponding to r or k], or (2) the *calendar date* of the future act of consumption (p. 167)'. The calendar date refers to the moment of experience and preferences for consumption at special times can lead to choice patterns not predicted by time discounting. If we introduce the possibility of idiosyncratic choice into the core theory, then time discounting models are consistent with all *transitive* choice patterns.[2]

Intransitivity can arise if choice has a stochastic component. We have already alluded to two ways of incorporating such a component (discussed in Hey, 1995). First, the randomness can be a form of *choice error*, in which decision makers know what they want but accidentally choose the wrong thing (Harless and Camerer, 1994); second, it can be due to preferences that are themselves subject to

[2] In fact, time discounting models are consistent with a 'few' such patterns. If the majority of observed patterns were transitive and yet not predictable by time discounting theories then we would have to search for other theories.

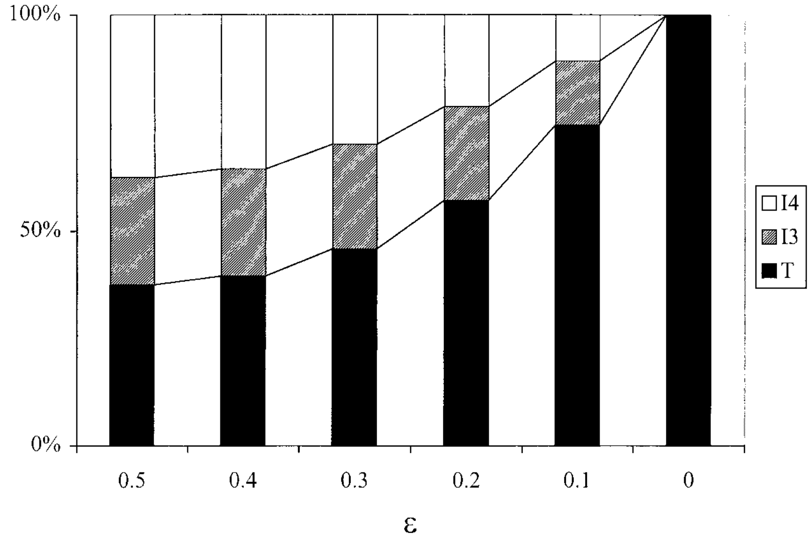

Exhibit 2. Graphical representation of the expected proportion of T, I_3 and I_4 choice patterns as a function of ε in the choice error model

variability (e.g. Becker, DeGroot and Marschak, 1963; Thurstone, 1927a,b). We consider these two models in turn.

Choice error can be modelled by assuming that while there is a definite preference order, there is a fixed probability that each choice will be mistaken and the less-preferred alternative will be chosen. Consider a decision maker who prefers *a* to *b*, but who has a constant probability ε ($0 \leqslant \varepsilon \leqslant 0.5$) of taking the dispreferred alternative *b*. The probability that the preferred alternative *a* will be chosen will then be $p(ab) = 1 - \varepsilon$.[3] We can readily calculate how many intransitivities of each kind (I_3 and I_4) will occur for a given value of ε, by first calculating the probability of every choice pattern, and then summing up the probabilities of each pattern in each category. The probability of Pattern 1, *abcd*, for instance, is given by:

$$P[abcd] = p(ab) \times p(bc) \times p(cd) \times p(ac) \times p(bd) \times p(ad)$$

Exhibit 2 depicts the proportion of I_4 and I_3 patterns for representative values of ε.

The implications of *preference variability* were first described by Thurstone (1927a,b) who proposed that *evaluations* (and not choices) are subject to random variation. Consequently, the subjective value of an option is better described as a distribution of values rather than a single point. When making a binary choice, the decision maker draws one value from the distribution for each option, and then chooses the one with the highest value. The degree of overlap between distributions, and the corresponding probability of choosing one item over another is a function of their separation and their shape.

A traditional Thurstone analysis of choice assumes that the value distributions for each option are normal and have equal variance (see Coombs, Dawes and Tversky, 1970, for a review). Although this assumption is restrictive, it also predicts more intransitive choice than any other assumption. The distance between adjacent distributions is given in standard deviation units and designated δ

[3] Harless and Camerer also allow the decision maker to be indifferent between *a* and *b*, in which case $p(ab) = 0.5$, although to simplify the present analysis we will assume that there is no indifference. In practice, complete indifference is indistinguishable from ε=0.5.

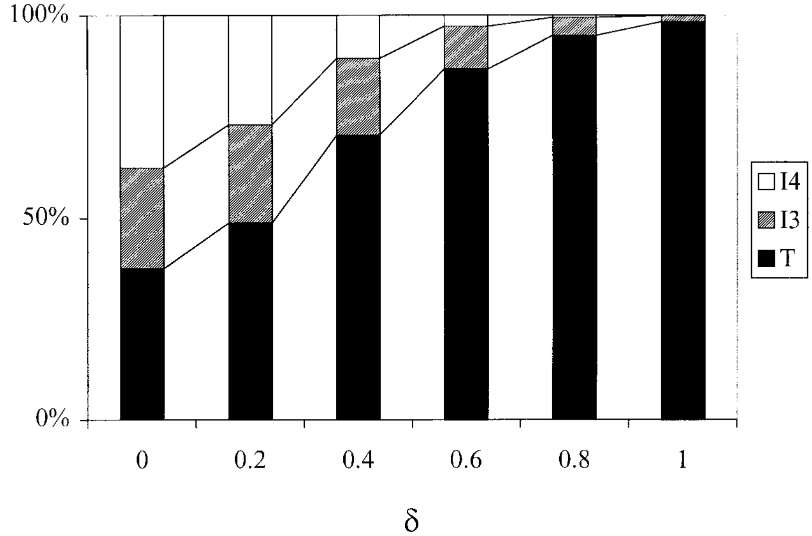

Exhibit 3. Graphical representation of the expected proportion of T, I_3 and I_4 choice patterns as a function of δ in the preference variability (Thurstone) model

(analogous to the statistic d' used in detection theory, e.g. Macmillan and Creelman, 1991). Consider two alternatives, *a* and *b*, where the mean of the *a* distribution is greater than that of the *b* distribution by δ standard deviation units. In a binary choice the probability of choosing *a* over *b* is given by $F(\delta/\sqrt{2})$ or the area under the normal curve in the interval bounded by $-\infty$ and $\delta/\sqrt{2}$.[4] If we know (or can make some assumptions about) the distances between alternatives, we can predict the probability of all the patterns depicted in Exhibit 1.[5] Exhibit 3 shows the predicted proportion of T, I_3 and I_4 choices for different values of δ.

EXPERIMENT

Hypotheses

Because not everyone has the same utility function, aggregating responses from different people can conceal true patterns or create spurious ones (Hey, 1995). Kirby (1997), for example, shows that an aggregate of exponential discount functions can lead to a collective hyperbolic function. Perhaps more often, individual errors can be washed out or diluted. Since we are concerned with intransitivity, consider an aggregate version of Tversky's axiom of weak stochastic transitivity. Imagine two groups of subjects, half of whom choose (*abcd*) and the other half choose (*dcba*). The responses of every subject would be intransitive, yet if the results were combined there would be no way to distinguish between these subjects and another group of whom half chose *abcd* and the other half *dcba*, two T* patterns.

To test hypotheses based on group data we need an index of group intransitivity that is independent of individual differences. One index has already been introduced — the proportion of intransitive

[4] Let the standard deviation of the x and y distributions be 1, then the standard deviation of the differences is $\sqrt{2}$, and the mean is δ. In binary choice, the decision maker randomly chooses a value for this distribution. If the value exceeds 0, then they will choose x, otherwise they will choose y. The probability that it will exceed 0 is $F(\delta\sqrt{2})$.

[5] An Excel spreadsheet is available from the authors which calculates the probability of all 64 choice patterns for any specified distributions of preferences, and distances between distributions.

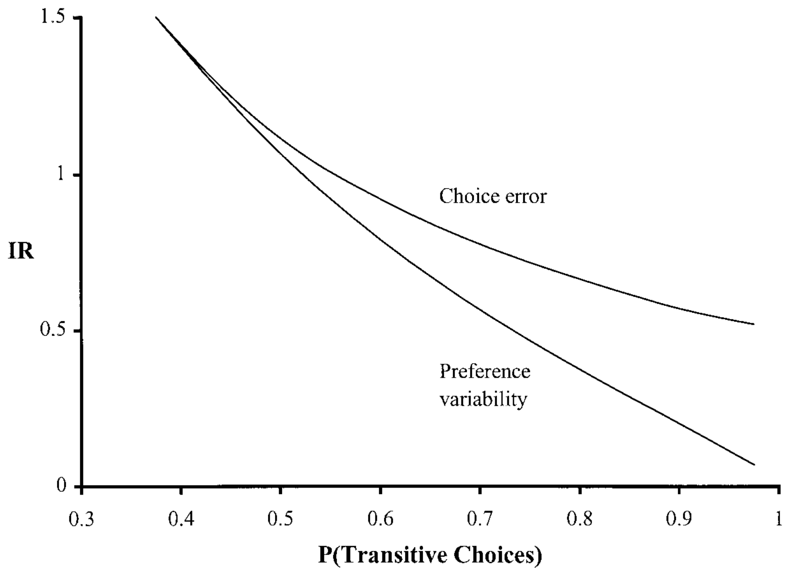

Exhibit 4. The *IR* ratio as a function of proportion of transitive responses for both the choice error and preference variability models

responses. A shortcoming of this index is that it is insensitive. If subjects are indifferent between all alternatives (if $\varepsilon = 0.05$ or $\delta = 0$), then the expected proportion of intransitive responses is 65%, and only under extreme circumstances will this value be exceeded. A better statistic is the ratio of I_4 patterns to I_3 patterns, which we call the *intransitivity ratio* (*IR*). As can be seen from Exhibits 2 and 3, when the ability to discriminate between items is low — i.e. ε is high (Exhibit 2) or δ is low (Exhibit 3) — there will be more I_4 than I_3 patterns ($IR > 1$). As the ability to discriminate increases, *IR* decreases. Exhibit 4 shows *IR* as a function of the number of transitive choice patterns for both stochastic specifications. The maximum value of *IR*, which occurs when $\varepsilon = 0.5$ or $\delta = 0$, is 1.5.

In the experiment reported, we consider two hypotheses. First, we test whether *IR* is within the theoretically permissible range:

H_1: $IR \leqslant 1.5$

If this hypothesis is not rejected, then we have no basis for counting that a stochastic specification of the conventional discounting models can describe intertemporal choice. If, on the other hand, H_1 is rejected, we will investigate whether our results can be explained by application of the lexicographic-semiorder rule.

The results of applying the lexicographic-semiorder rule depend on whether the *first-priority* attitude is *delay*-to-reward or *amount*-of-reward, and on whether there is a significant difference between alternatives on this attribute. As already explained, we anticipated that, for most subjects, the highest-priority attribute would be amount. If so, then subjects would be most likely to prefer the longer-delay alternative when the difference in amount was at its maximum (i.e. $D \succ A$), they would be next most likely to choose the longer-delay alternative when the difference is intermediate (i.e. $C \succ A$, and $D \succ B$), and they would be least likely to choose the longer-delay option when the difference is small (i.e. $B \succ A$, $C \succ B$ and $D \succ C$). In short, we predicted that:

$$H_2: p(DA) > p(CA) = p(DB) > p(BA) = p(CB) = p(DC)$$

where p(DA) is the proportion of choices of D over A, and so forth.

Exhibit 5. Choice frequency of all choice patterns

Rank	Pattern	Frequency	Type
1	ABCD	16 (18%)	T*
2	BACD	8 (9%)	T*
	(ABCD)	8	I_4
	(DABC)	8	I_4
5	(BCDA)	7 (8%)	I_4
6	BCAD	5 (6%)	T*
7	(CADB)	4 (5%)	I_4
8	DCBA	3 (3%)	T*
	(ADBC)	3	I_4
10	(ABC)D	2 (2%)	I_3
	ABDC	2	T
	(BACD)	2	I_4
	CABD	2	T
	(BCAD)	2	I_4
	BDAC	2	T
16	ACBD	1 (1%)	T
	A(BCD)	1	I_3
	(ACBD)	1	I_4
	(ABD)C	1	I_3
	CBAD	1	T
	(DCB)A	1	I_3
	DACB	1	T
	(CBAD)	1	I_4
	C(DBA)	1	I_3
	CBDA	1	T*
	(DACB)	1	I_4
	(ADBC)	1	I_4
	(DCBA)	1	I_4
	(CDAB)	1	I_4

Methods

We tested 88 university students from the Free University, Amsterdam. Each subject made six binary choices, between all pairs of the four alternatives A(7/1), B(8/2), C(9/4) and D(10/7). The alternatives were written on separate pages and stapled into a booklet. Several versions of these booklets were prepared, with the choices shuffled within booklets. To minimize memory effects each of the six test choices were separated by two irrelevant filler alternatives.

The booklets were filled out in small groups of about 10. Each group was told that one of their number would be selected at the end of class. The winner would then have a single pair selected from their choices (including the filler items) and they would receive their preferred outcome from that pair at the appointed time. The amount would be placed in an envelope and sent to their address by first-class mail so that it would arrive in time.

Results

Exhibit 5 lists every observed choice pattern and its frequency. Our first question is whether our data can be explained by the non-stochastic core theories of exponential or hyperbolic discounting. Forty-eight percent (42) of the responses were transitive. Of these, the majority — 79% (33) — were the T* patterns specifically predicted by a core theory that assumes no indifference between

Exhibit 6. Frequency of each choice order for transitive and intransitive patterns

	Pattern type		
Choice	Both	Intransitive	Transitive
D (10/7) ≻ A (7/1)	43% (38)[a]	67% (31)	17% (7)
C (9/4) ≻ A (7/1)	38% (33)	44% (21)	29% (12)
D (10/7) ≻ B (8/2)	27% (24)	43% (20)	10% (4)
B (8/2) ≻ A (7/1)	31% (27)	15% (7)	48% (20)
C (9/4) ≻ B (8/2)	18% (16)	15% (7)	21% (9)
D (10/7) ≻ C (9/4)	17% (15)	15% (7)	19% (8)

[a]Number of choices of each type.

alternatives. A χ^2-test confirmed that, among the transitive subset of responses, T* patterns were over-represented relative to non-T* patterns: $\chi^2(1) = 48.2$, $p < 0.001$.

Intransitivity ratio

Our first hypothesis concerned *IR*, which cannot exceed 1.5 for any stochastic specification of an alternative-based choice model. The observed *IR* was 6.7 — 87% (40) of the 46 intransitive patterns were I_4 and 13% were I_3 — far from what would be expected on the basis of chance, $\chi^2 = 13.9$, $p < 0.001$. We conclude, therefore, that our results cannot be accounted for by any stochastic specification, and so we turn to an attribute-based explanation.

Lexicographic-semiorder rule

Exhibit 6 depicts the proportion of choices of each type. Recall that, if people choose according to the lexicographic-semiorder principle with amount as the dominant dimension, the proportion choosing each alternative would be ordered as follows:

$$p(DA) > p(CA) = p(DB) > p(BA) = p(CB) = p(DC)$$

The first column of figures in Exhibit 6 shows the aggregate results for the entire experiment. While these proportions approach that predicted, they are by no means clear-cut. If, however, the choices are divided according to whether they are transitive or intransitive, then the predicted pattern emerges very clearly for the intransitive responses, and disappears completely for the transitive ones. For intransitive choices, when the difference in amount is only Dfl 1 (AB, BC, CD), then people almost always choose the earliest payoff. As the difference increases, so does the likelihood of choosing the larger payoff. A series of χ^2-tests confirmed that all the predicted differences indicated by ' > ' are significant at $p < 0.001$, and none of the differences indicated by '=' even approach significance. An additional, more qualitative, analysis supports the same conclusion. Recall that in Tversky's first experiment, subjects chose between all pairs of adjacent alternatives, and made one choice between the most distant alternatives. If we restrict our consideration to four choice pairs only, the three adjacent pairs (AB, BC, CD) are the maximally distant pair (AD), then a lexicographic-semiorder pattern like Tversky's would be revealed by a large proportion of choices of the form:

$$A \succ B,\ B \succ C,\ C \succ D,\ \text{but}\ D \succ A$$

There are four choice patterns which have this form. These are (ABCD), (BCDA), (DABC) and (CDAB), all I_4 patterns. These four patterns alone make up over 50% of the observed intransitive patterns. Indeed, the first three patterns mentioned are the most frequently chosen intransitive patterns.

Among those who made transitive choices, the overwhelming preference was for the smaller-sooner payoff, and there was no particular disposition to choose D over A. Only one choice pair, A and B, stood out because the proportion choosing the smaller-sooner A and the larger-later B was identical. A series of χ^2 analyses confirmed that p(BA) was greater (at $p < 0.01$) than all the other choice probabilities, which did not differ among themselves. This supports the idea that transitive choices may often arise from the use of a consistent discount function. Of the seven T* patterns listed above, six of them involve a choice of B over A. As demonstrated in Appendix A, these six patterns are predicted by all values of r up to 0.143 (14% per *week*, or 910% per year) or any value of k up to 0.167. Of the 20 transitive choices that involved a choice of B over A, 85% (17) were T* patterns. Only the most popular T* alternative, ABCD, involved a choice of A over B.

We cannot necessarily conclude, however, that all transitive choices are based on the application of a consistent discount rate. While the common T* pattern ABCD can arise from a very high discount rate, it can also arise from consistent application of the lexicographic-semiorder rule. The ABCD pattern would be chosen by decision makers whose first-priority attribute was delay (i.e. they always took the least delayed alternative first), or whose first-priority attribute was amount, but who viewed a difference of Dfl 3 as insignificant. Likewise, another T* pattern, DCBA, would be chosen by decision makers whose first-priority attribute is money, and who view all the differences in amount on offer as significant (i.e. they always take the largest amount). This analysis places 19 of the 33 T* choices into a kind of interpretive limbo, since they are fully consistent with both alternative-based and attribute-based choice processes.

To summarize, there is strong evidence that the observed pattern of intransitive responses conform to those expected from the application of a lexicographic-semiorder rule with delay as the first-priority attribute. About half of the transitive choices are also consistent with this rule, although they can also be explained in other ways. Although subjects' responses are by no means random, they are quite heterogeneous, and we do not know whether there are two populations (discounters, and lexicographic-semiorderers), or one population using a lexicographic-semiorder rule with a substantial stochastic component. Clearly, this must be the basis for further research.[6]

We conclude by examining the implications that this study and others have for the kinds of theories and psychological processes we invoke to explain intertemporal choice.

DISCUSSION

There have been many recent articles attempting to specify the form of the individual discount function. The typical finding is that hyperbolic discounting fits observed behavior better than exponential discounting (e.g. Benzion, Rapaport and Yagil, 1986; Kirby, 1997; Overton and MacFadyen, 1998). There is considerable doubt, however, whether the psychological processes underlying this behavior actually draw on a personal discount function. This doubt arises because another line of research, concerned with factors that influence discount rates and with qualitative effects in intertemporal choices (reviewed in Loewenstein and Thaler, 1989; Loewenstein and Prelec, 1992; Roelofsma,

[6] There is a similarity between our results, however, and those reported by Tversky (1969). He either preselected subjects on the basis of their disposition to give intransitive responses to the specific experimental set-up that he used (Experiment 1) or he both preselected subjects and customized his materials to them (Experiment 2). In his Experiment 1, which is particularly pertinent to us, he found that for his pre-constructed set of materials, slightly less than half of his original pool of subjects gave the desired intransitive responses. In our study, slightly over half of our subjects gave intransitive responses.

1996), has revealed a number of anomalous effects which appear inconsistent with discount rate theories, including:

(1) The *magnitude effect*: larger amounts are discounted at a lower rate than smaller amounts (Thaler, 1981; Green, Myerson and McFadden, 1997).
(2) The *gain–loss asymmetry*: gains are discounted at a higher rate than losses (Thaler, 1981).
(3) *Good-specific effects*: the observed discount rate is highly dependent on what is being discounted. These effects include
   - The *virtue–vice* effect: the inferred discount rate is higher for 'vices', which give immediate benefits in exchange for delayed costs, than for 'virtues' (Read, Loewenstein and Kalyanaraman, 1999; Read and Van Leeuwen, 1998).
   - *Visceral effects*: goods which evoke appetitive or 'visceral' responses (e.g. food, sex) yield much greater discount rates than other goods (Loewenstein, 1996).
(4) *Sequence effects*: the inferred discount rate depends on whether goods are evaluated in isolation, or as part of a sequence of events (Ariely and Zauberman, 2000; Loewenstein and Prelec, 1993; Chapman, 1996).

Decision makers appear to have as many discount rates as choice situations into which they can be placed. Moreover, different measures of discount rates are either uncorrelated, or are correlated weakly or idiosyncratically (e.g. Chapman and Coups, 1998; Chesson and Viscusi, 2000; Vuchinich and Simpson, 1998).

This point is amplified by trying to explain the results of our experiment with a discount-rate theory. Any single choice could be based on one of a range of compatible discount rates. A rate of 14.3% per week or more, for instance, could explain a choice of A over B (see Exhibit B1).[7] Rarely, however, were the six very similar choices made by a single decision maker in the space of two or three minutes consistent with a single discount rate. Moreover, even the majority of those T* choices consistent with a single discount rate were plausibly explained by a non-discount-rate based choice procedure, such as an attribute-based heuristic of 'always taking the early money unless the delayed money is much more'.

The study of intertemporal choice is currently undergoing a change in emphasis, as has already occurred in the study of decision making under risk and certainty. Rather than searching for the holy grail of a single utility function, researchers now take the more pragmatic view that preferences are constructed based on the circumstances of their expression (e.g. Camerer, 1989). The resultant choice models may not be as elegant as a single mathematical function, but they account for more behavior. Likewise, the study of intertemporal choice is moving, as it needs to move, from a focus on finding the 'correct' discounting function to an emphasis on the process of intertemporal choice. This study, which shows how the application of a heuristic decision rule gives a better account of observed choice than does discounted utility, is one contribution to this enterprise.

APPENDIX A: DERIVING A CHOICE OF PATTERN FROM A SET OF BINARY CHOICES

Label each alternative *a*, *b*, *c* and *d*. The analysis is not dependent on which alternative is given which label. After the six choices are made, indicate for each option the number of times that it is chosen over its alternative. For example, given the following pattern:

[*ab*, *ac*, *da*, *bc*, *db*, *cd*]

[7] One of the prima facie arguments against the discount rate account of intertemporal choice is the truly massive discount rates that have to be invoked to explain laboratory-based choices.

a and *d* are both preferred twice, *b* and *c* are both preferred once. We refer to this as a 2,2,1,1 pattern, because two alternatives are preferred twice, and two are preferred once. There are four possible choice patterns for four alternatives, and each pattern corresponds to a level of intransitivity:

3,2,1,0 — a T pattern
2,2,2,0 — an I_3 pattern, in which one alternative is dispreferred to all others
3,1,1,1 — an I_3 pattern, in which one alternative is preferred to all others
2,2,1,1 — an I_4 pattern

We will refer to options according to the number of times they are preferred. Thus, in the example, *a* and *d* are both 2-preferred, and *b* and *c* are 1-preferred. We will also refer to *intersection* choices, or choices between a pair of options that are at the same level of preference. Thus, in the example, *da* is an intersection choice between two 2-preferred options, and *bc* is an intersection choice between two 1-preferred options.

Depending on which choice pattern is obtained, the following procedures are used to determine the ordering of the alternatives:

(1) Transitive 3,2,1,0 patterns are a non-cyclical ordering with the 3-preferred item first, the 2-preferred item second, the 1-preferred item third, and the 0-preferred item fourth. For instance, pattern 33: [*ba*, *cb*, *dc*, *ca*, *db*, *da*] is the pattern *dcba*.

(2) Three-step intransitive 3,1,1,1 patterns are a cyclical ordering in which the 3-preferred item is chosen over all others. There are three steps to determine the pattern. First, write down the 3-preferred option and to the right of it put parentheses with space for the remaining three options. Second, in two adjacent spaces in the parentheses write down two of the remaining three options in the order of their preference. Third, put the remaining option in the remaining space. For instance, take pattern 13: [*ab*, *cb*, *dc*, *ac*, *bd*, *ad*].
Step 1: *a* (. . .)
Step 2: *a* (*cb* .)
Step 3: *a* (*cbd*)
Notice that this choice set is consistent with three orders: *a*(*cbd*), *a*(*bdc*), *a*(*dcb*). The latter ordering is given in Exhibit 1.

(3) Three-step intransitive 2,2,2,0 patterns are cyclical orderings in which the 0-preferred item is dispreferred to all others. The procedure is almost identical to that for 3,1,1,1 patterns except that this time the 0-preferred item goes to the right of the parentheses. For instance, take pattern 44: [*ab*, *cb*, *dc*, *ca*, *db*, *ad*].
Step 1: (. . .) *b*
Step 2: (*ca* .) *b*
Step 3: (*cad*) *b*
Again, this choice set is consistent with three orders: *cadb*, (*adc*)*b*, and (*dca*)*b*. The latter is given in Exhibit 1.

(4) Four-step intransitive 2,2,1,1 patterns. There are two intersection items in these sets, one 2-preferred intersection set, and one 1-preferred intersection set. The first pair of options in the ordering are the 2-preferred set, and the second pair are the 1-preferred set. Put parentheses around the items. For instance, take pattern 11: [*ba*, *bc*, *cd*, *ac*, *db*, *ad*]. The two intersection pairs are *ba* and *cd*.
Step 1: (*ba* . .)
Step 2: (*bacd*)

APPENDIX B: DERIVING THE T* PATTERNS

In this appendix we demonstrate that, for choice alternatives that do not all have the same present value for a single discount rate (a minor restriction on the alternatives), and if we assume that the decision maker is not indifferent between any of the alternatives, there will be

$$\binom{N}{2} + 1$$

T* patterns, where N is the number of alternatives being evaluated.

For each pair of alternatives, there will be a unique discount parameter, r or k, which will result in indifference between that pair. We can call these discount parameters 'crossover points' because they are the only points at which the patterns change. For two alternatives V_a with delay a and V_b with delay b, the exponential discounting crossover point is given by:

$$r = \left(\frac{V_a}{V_b}\right)^{1/a-b} - 1$$

This is obtained by setting

$$V_a\left(\frac{1}{1+r}\right)^a = V_b\left(\frac{1}{1-r}\right)^b$$

from equation (1) and then solving for r. Using an analogous procedure for equation (2), we obtain the hyperbolic discounting crossover point:

$$k = \frac{V_a - V_b}{aV_b - bV_a}$$

Because there is one crossover point for each pair of alternatives, and because a set of N alternatives contains

$$\binom{N}{2} = \frac{N!}{2(N-2)!}$$

pairs, there will be

$$\binom{N}{2}$$

crossover points. These crossover points divide up the range of possible discount parameters ($-\infty$ to ∞) into

$$\binom{N}{2} + 1$$

regions, with each region corresponding to a unique ordering of alternatives. Exhibit B1 shows the regions and the corresponding T* for the four alternatives in our study. Any discount parameter chosen from within one of these regions will yield the same choice order.

We made the assumption that people are not indifferent between any pair of alternatives. This assumption is almost certain to be valid if we treat individuals as having a single discount

Exhibit B1. The range of exponential (r) and hyperbolic (k) discounting parameters leading to each T* pattern

Parameter range				
r		k		
Lower	Upper	Lower	Upper	Pattern
$-\infty$	0.03577	$-\infty$	0.04347	DCBA
0.03577	0.04564	0.04347	0.05556	CDBA
0.04564	0.06066	0.05556	0.07143	CBDA
0.06066	0.06125	0.07143	0.07692	BCDA
0.06125	0.08738	0.07692	0.10526	BCAD
0.08738	0.14286	0.10526	0.16667	BACD
0.14286	∞	0.16667	∞	ABCD

parameter — whether they discount hyperbolically or exponentially — and are able to discriminate perfectly between alternatives on the basis of that discount parameter. Since indifference will only arise if the decision maker is operating based on one of seven exact points in the range $(-\infty, \infty)$, such indifference is extraordinarily unlikely. For instance, in order for a hyperbolic discounter to be indifferent between alternative C and D, their discount parameter would have to be *exactly* 0.04347 — below that value they would prefer D, above it they would prefer C.

If we relax the assumption that people will be indifferent between alternatives only if their discount rate is such a precise value — that is, if we allow them to be indifferent between alternatives that fall within the same range of values for a specified discount parameter, or between alternatives that have the same value for one discount parameter within a range of possible parameters — then we are introducing a *stochastic component* into choice. This paper is a test of whether this is how people make decisions.

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