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# Transverse $\Lambda$ polarization in semi-inclusive deep inelastic scattering

M. Anselmino,[1] D. Boer,[2] U. D'Alesio,[3] and F. Murgia[3]
[1]*Dipartimento di Fisica Teorica, Università di Torino and INFN, Sezione di Torino, Via P. Giuria 1, I-10125 Torino, Italy*
[2]*Department of Physics and Astronomy, Vrije Universiteit Amsterdam, De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands*
[3]*INFN, Sezione di Cagliari and Dipartimento di Fisica, Università di Cagliari, C.P. 170, I-09042 Monserrato (CA), Italy*
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Following a previous description of $\Lambda$ and $\bar{\Lambda}$ polarization in unpolarized *p*-*p* interactions, within a perturbative QCD factorization scheme with new polarizing fragmentation functions, here we investigate the transverse polarization of $\Lambda$'s and $\bar{\Lambda}$'s produced in semi-inclusive DIS. Analytical expressions for both neutral and charged current exchange are given. Since quantitative predictions cannot be given at this stage and the comparison with existing data is not yet significant, we present the general formalism and a qualitative analysis displaying generic features of the $\Lambda$ and $\bar{\Lambda}$ polarization for specific scenarios. Different kinematical situations are considered, corresponding to experiments currently able to study $\Lambda$ production in semi-inclusive DIS.

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## I. INTRODUCTION

Transverse hyperon polarization in high energy, unpolarized hadron-hadron collisions has formed a long-standing challenge for theoretical models of hadronic reactions [1,2]. The straightforward application of perturbative QCD and collinear factorization in the study of these observables is not successful, since the transverse polarization generated perturbatively in the partonic cross section would be proportional to $\alpha_s m_q/\sqrt{\hat{s}}$, which cannot be responsible for hyperon polarizations which may reach well over 20%.

Therefore, we have recently proposed a new approach [3] to this problem based on perturbative QCD (PQCD) and its factorization theorems, and which includes polarization and intrinsic transverse momentum, $\boldsymbol{k}_\perp$, effects. It requires the introduction of a new type of leading twist fragmentation function (FF), one which is polarization and $\boldsymbol{k}_\perp$ dependent, the so-called polarizing FF [3,4].

Ideally, our approach could be tested by first extracting these new functions by fitting some available experimental data, and then using the same functions to give consistent predictions for other processes. The problem with such a procedure is the actual availability of experimental data in kinematical regions appropriate to the application of our scheme, which requires a high center of mass energy and high transverse momentum $\boldsymbol{p}_T$ of the produced hyperon.

In Ref. [3] we used *p*-*p* and *p*-Be data in the range $p_T \simeq 1$–$3$ GeV/*c* (and $\sqrt{s}\simeq 25$–$60$ GeV) to gain knowledge of the polarizing FF; we assumed this to be data resulting from a current quark fragmentation, ignoring possible target fragmentation mechanisms. However, this turns out to be a questionable assumption: a preliminary study of the unpolarized cross section—at least in the kinematical region where both polarization and cross-section data are available (which is only a subset of the total region where data on $\Lambda$ polarization have been published and used)—shows that the formalism employed in our paper [3] results in at most a few percents of the experimental values. This casts doubt on the obtained polarizing FF, which, although they are reasonable in shape and magnitude and describe the $\Lambda$ and $\bar{\Lambda}$ polarization quite well, should not be used to make predictions. All this is currently under further investigation and a full assessment will be published elsewhere.

In this paper we study transverse $\Lambda$ polarization, $P_T^\Lambda$, in unpolarized semi-inclusive deep inelastic scattering (SIDIS), $lp \to l' \Lambda^\uparrow X$. At present there exists only one small set of SIDIS data (from the NOMAD experiment [5,6]) to compare with, but the kinematics and statistics are such that any comparison would be inconclusive. Other data will become available in the near future (HERMES, COMPASS). We develop a full consistent formalism to compute and evaluate $P_T^\Lambda$, but, according to the above considerations, instead of using the functions obtained from Ref. [3] to give predictions, we shall only use those results as a basis for selecting different models for the polarizing FF. This will be discussed extensively below.

A similar approach based on new polarization and $\boldsymbol{k}_\perp$-dependent functions has already been applied to the study of transverse single spin asymmetries in inclusive particle production at large $x_F$ and medium to large $\boldsymbol{p}_T$ [7]. In this case the new leading twist functions are called the Sivers distribution function [8] and the Collins fragmentation function [9]. Also here future data are awaited to further test the description of the asymmetries in terms of those functions. But also for pion or photon production, PQCD calculations for the unpolarized cross sections can be a factor 100 smaller than data [10–13], even in the central rapidity region and at large $\boldsymbol{p}_T$ values; in those cases the discrepancy can be explained by the introduction of $\boldsymbol{k}_\perp$ effects in the *distribution* functions: these give a large, spin-independent, enhancing factor, which brings the cross sections in agreement with data. Such factors would not alter the calculation of the $\Lambda$ polarization, as they cancel in the ratio of cross sections. However, it is too early to draw a definite conclusion, and a more detailed study is in progress.

It should also be mentioned that there is an alternative approach based on perturbative QCD and its factorization theorems, namely the inclusion of higher twist functions of the type investigated by Qiu and Sterman [14] (see also [15] for an earlier, related study and [16] for a comprehensive review paper). These so-called soft gluon pole functions were applied to single spin asymmetries in prompt photon production [14,17], in the Drell-Yan process [18–20], and in pion production [21–23]. This method has also been applied to transversely polarized $\Lambda$ production in electron-positron annihilation [24] and recently in hadron-hadron scattering [25]. The latter study closely resembles our study of transversely polarized $\Lambda$ production using the polarizing FF and it is not excluded that there might be a connection between the two approaches (for indications, see [19,26]).

The process we are looking at is $lp \to l' \Lambda X$, where the $\Lambda$ in general has a transverse momentum compared to the lepton scattering plane. This transverse momentum is assumed to be much smaller than the hard scale $Q$ and allows one to become sensitive to the transverse momentum dependence of the fragmentation function. Strictly speaking no factorization theorem has been proven for this particular situation. But as explained in Ref. [9] one expects a generalized factorization theorem to hold here, similarly to the case of $e^+e^- \to h_1 h_2 X$ (in a sense the crossed channel) where hadrons $h_1$ and $h_2$ are in opposite back-to-back jets. These hadrons themselves are nearly, but in general not exactly, back to back, giving rise to a dependence on transverse momentum in the fragmentation functions. For this process the factorization theorem has been proven rigorously [27].

The main difference between this generalized factorization theorem and the usual collinear factorization theorems is the appearance of so-called Sudakov factors. These factors appear naturally since fixed order perturbation theory is not expected to give a proper description of the cross section at small $p_T$. Resummation of large logarithms then gives rise to these Sudakov factors (which are spin independent). In practice, this resummation has the effect of broadening and lowering the transverse-momentum-distribution and, as a consequence, transverse momentum dependent asymmetries [28]. At the moderate $Q^2$ values we are considering, the effect of resumming such Sudakov logarithms is expected not to be important (the numerical study in Ref. [28] supports this expectation) and its effect on the average $k_\perp$ should be moderate as well. Moreover, the Gaussian distributions employed in the present paper can effectively describe the broadening of a resummed distribution (an issue also discussed in Ref. [29]); therefore, we will not include such a refinement here. However, should future data require a higher average $k_\perp$ than a fit to $p$-$p$ data suggests, then such a resummation is most likely an important contribution. In the $p$-$p$ case [3] no large logarithm resummation aspects were considered, since the explicit $k_\perp$ distribution is less important (due to the large $p_T$), hence the use of a distribution highly peaked around the average $k_\perp$ suffices.

The outline of this paper is as follows. In Sec. II we will discuss the details of the polarizing fragmentation functions and recall our approach for $pp \to \Lambda^\uparrow X$ in order to set the notation. In Sec. III we will elaborate on a Gaussian model for the $\boldsymbol{k}_\perp$ dependence of the fragmentation functions and in Sec. IV we present our study of various electroweak SIDIS processes, where analytical expressions for both neutral and charged current exchange are given (this is done more completely in the Appendix).

In Sec. V we present some numerical results; since quantitative predictions cannot be made at this stage and comparison with existing data is not yet significant, we select simple scenarios for the polarizing FF and present a qualitative analysis displaying generic features of $\Lambda$ and $\overline{\Lambda}$ polarization. Different kinematical situations are considered, corresponding to experiments currently able to study $\Lambda$ production in SIDIS, i.e., NOMAD, HERMES, COMPASS, and E665.

## II. POLARIZING FRAGMENTATION FUNCTIONS

The main idea behind the polarizing FF is that a transversely polarized hyperon can result from the fragmentation of an *unpolarized* quark, as long as the hyperon has a nonzero transverse momentum compared to the quark (otherwise it would violate rotational invariance). This effect need not be suppressed by inverse powers of a large energy scale, such as the center of mass energy $\sqrt{s}$ in $p$-$p$ scattering or the momentum transfer $Q$ in SIDIS. Actually, in $p$-$p$ scattering it appears with an inverse power of the transverse momentum $p_T$ of the hyperon, which should be large enough for factorization to hold, but in practice is still much smaller than $\sqrt{s}$.

In order to be able to apply a factorization theorem to the process $pp \to \Lambda X$, one has to require that both $\sqrt{s}$ (the overall c.m. energy) and $p_T$ (the magnitude of the transverse momentum of the detected hadron in the $p$-$p$ c.m. frame) are large. Such a configuration originates from a large momentum transfer in the partonic interactions, so that perturbative calculations are justified. In semi-inclusive DIS processes, $lp \to l' \Lambda X$, one only needs to require the momentum transfer $Q$ to be large. Our results will be given in the virtual photon (or vector boson)-$p$ c.m. frame, and $\boldsymbol{p}_T$ is measured relatively to this direction; neglecting intrinsic motion in the incoming proton, in this frame $\boldsymbol{p}_T$ coincides with the hadron intrinsic $\boldsymbol{k}_\perp$, so that it can be small and one becomes sensitive to the precise $\boldsymbol{k}_\perp$ dependence of the polarizing FF. Therefore, in contrast to Ref. [3], where the $\boldsymbol{p}_T$ dependence of the observable was sufficiently described by evaluation of the polarizing FF at an *effective,* average $k_\perp^0(z)$, here we will assume an explicit Gaussian $k_\perp$ dependence of the polarizing FF (cf. the next section for details). This will allow us to investigate $lp \to l' \Lambda^\uparrow X$ for small (and in principle, after including hard perturbative corrections, also large) $p_T$. An important point is that one should restrict to the current fragmentation region, i.e., $x_F > 0$ (many models for the target fragmentation region exist, but we cannot compare our approach to those models, because they are simply not addressing the high energy QCD aspect of polarized $\Lambda$ production).

In order to set the notation we recall that in our approach for the $p\,p \to \Lambda^\uparrow X$ case, the transverse hyperon polarization in unpolarized hadronic reactions at large $p_T$ can be written as follows [3]:

$$P_T^\Lambda(x_F,p_T)=\frac{d\sigma^{pp\to\Lambda^\uparrow X}-d\sigma^{pp\to\Lambda^\downarrow X}}{d\sigma^{pp\to\Lambda^\uparrow X}+d\sigma^{pp\to\Lambda^\downarrow X}}$$
$$=\frac{\sum\int dx_a dx_b\int d^2\boldsymbol{k}_\perp f_{a/p}(x_a)f_{b/p}(x_b)d\hat\sigma(x_a,x_b;\boldsymbol{k}_\perp)\Delta^N D_{\Lambda^\uparrow/c}(z,\boldsymbol{k}_\perp)}{\sum\int dx_a dx_b\int d^2\boldsymbol{k}_\perp f_{a/p}(x_a)f_{b/p}(x_b)d\hat\sigma(x_a,x_b;\boldsymbol{k}_\perp)\hat D_{\Lambda/c}(z,\boldsymbol{k}_\perp)}, \tag{1}$$

where $f_{a/p}(x_a)$ and $f_{b/p}(x_b)$ are the usual unpolarized parton densities; $d\hat\sigma(x_a,x_b;\boldsymbol{k}_\perp)$ describes the lowest order partonic cross section with the inclusion of $\boldsymbol{k}_\perp$ effects; the $\Sigma$ takes into account all possible elementary interactions; $\hat D_{\Lambda/c}(z,\boldsymbol{k}_\perp)$ and $\Delta^N D_{\Lambda^\uparrow/c}(z,\boldsymbol{k}_\perp)$ are, respectively, the unpolarized and the polarizing FF [3,4] for the process $c\to\Lambda+X$. The polarizing FF is defined as

$$\Delta^N D_{h^\uparrow/a}(z,\boldsymbol{k}_\perp)\equiv\hat D_{h^\uparrow/a}(z,\boldsymbol{k}_\perp)-\hat D_{h^\downarrow/a}(z,\boldsymbol{k}_\perp)$$
$$=\hat D_{h^\uparrow/a}(z,\boldsymbol{k}_\perp)-\hat D_{h^\uparrow/a}(z,-\boldsymbol{k}_\perp), \tag{2}$$

and denotes the difference between the density numbers $\hat D_{h^\uparrow/a}(z,\boldsymbol{k}_\perp)$ and $\hat D_{h^\downarrow/a}(z,\ \boldsymbol{k}_\perp)$ of spin-1/2 hadrons $h$, with longitudinal momentum fraction $z$, transverse momentum $\boldsymbol{k}_\perp$ and transverse polarization $\uparrow$ or $\downarrow$, inside a jet originated from the fragmentation of an unpolarized parton $a$. From the above definition it is clear that the $\boldsymbol{k}_\perp$ integral of the function vanishes and that, due to parity invariance, the function itself vanishes in case the transverse momentum and transverse spin are parallel. Conversely, one can write

$$\hat D_{h^\uparrow/q}(z,\boldsymbol{k}_\perp)=\frac{1}{2}\hat D_{h/q}(z,k_\perp)$$
$$+\frac{1}{2}\Delta^N D_{h^\uparrow/q}(z,k_\perp)\frac{\hat{\boldsymbol{P}}_h\cdot(\boldsymbol{p}_q\times\boldsymbol{k}_\perp)}{|\boldsymbol{p}_q\times\boldsymbol{k}_\perp|} \tag{3}$$

for an unpolarized quark with momentum $\boldsymbol{p}_q$ which fragments into a spin-1/2 hadron $h$ with momentum $\boldsymbol{p}_h=z\boldsymbol{p}_q+\boldsymbol{k}_\perp$ ($\boldsymbol{p}_q\cdot\boldsymbol{k}_\perp=0$) and polarization vector along the $\uparrow=\hat{\boldsymbol{P}}_h$ direction; $\hat D_{h/q}(z,k_\perp)=\hat D_{h^\uparrow/q}(z,\boldsymbol{k}_\perp)+\hat D_{h^\downarrow/q}(z,\boldsymbol{k}_\perp)$ is the $k_\perp$-dependent unpolarized fragmentation function, where $k_\perp=|\boldsymbol{k}_\perp|$. The variable $z$, in the kinematical region considered in Ref. [3], is very close to the light-cone variable $\xi=p_h^+/p_q^+$.

Throughout the paper we will adopt also the following notation:

$$\Delta^N D_{h^\uparrow/q}(z,\boldsymbol{k}_\perp)\equiv\Delta^N D_{h^\uparrow/q}(z,k_\perp)\frac{\hat{\boldsymbol{P}}_h\cdot(\boldsymbol{p}_q\times\boldsymbol{k}_\perp)}{|\boldsymbol{p}_q\times\boldsymbol{k}_\perp|}$$
$$=\Delta^N D_{h^\uparrow/q}(z,k_\perp)\sin\phi, \tag{4}$$

where $\phi$ is the angle between $\boldsymbol{k}_\perp$ and $\boldsymbol{P}_h$, which, in our configuration (as explained later), is the difference between the azimuthal angles of $\boldsymbol{P}_h$ and $\boldsymbol{k}_\perp$, $\phi=\phi_{P_h}-\phi_{k_\perp}$.

Equation (1) is based on some simplifying assumptions (for a detailed discussion we refer to [3]): (1) The $\Lambda$ polarization is assumed to be generated in the fragmentation process; (2) The intrinsic $\boldsymbol{k}_\perp$ effects in the unpolarized initial nucleons are neglected; (3) The $\Lambda$ FF's also include $\Lambda$'s coming from decays of other hyperon resonances. We will make these same assumptions in the present SIDIS study.

Determining the overall sign of the $\Lambda$ polarization is important, hence we will specify the frame of choice whenever appropriate. In this respect, we would like to add this information to Eq. (4) of Ref. [3], which gives the relation between the function $D_{1T}^\perp$ defined in [4] and $\Delta^N D_{h^\uparrow/a}$ as will be used in the present paper; in terms of the angle $\phi$ defined in Eq. (4) the exact relation is

$$\Delta^N D_{h^\uparrow/a}(z,\boldsymbol{k}_\perp)=-2\frac{k_\perp}{zM_h}\sin\phi D_{1T}^\perp(z,k_\perp). \tag{5}$$

We would also like to note that since the polarizing FF are chiral-even functions, as opposed to the Collins FF for instance, the observable of interest here—transverse $\Lambda$ polarization in SIDIS—is independent of the lepton scattering plane. In other words, the $\Lambda$ polarization does not average to zero when one integrates over all directions of the backscattered lepton. This fact allows one to study the neutral current process $\nu p\to\nu\Lambda^\uparrow X$. Also, because we are dealing with chiral-even functions, charged current exchange processes—which select fixed helicities—may give access to them, in contrast to the case of chiral-odd functions.

## III. A GAUSSIAN MODEL FOR $k_\perp$-DEPENDENT FRAGMENTATION FUNCTIONS

The goal of this paper is to give a well defined formalism to compute the transverse $\Lambda$, $\bar\Lambda$ polarization in unpolarized semi-inclusive DIS and to present some qualitative and generic numerical results. As a first approach, we will take into account only leading twist (in the $1/Q$ power expansion) and leading order (in the coupling constant power expansion) contributions, looking at the process in the virtual boson-target nucleon c.m. reference frame (VN frame). Under these conditions the elementary virtual boson-quark scattering is collinear (and along the direction of motion of the virtual boson) and the transverse momentum of the final hadron with respect to the fragmenting quark, $\boldsymbol{k}_\perp$, coincides with the hadron transverse momentum, $\boldsymbol{p}_T$, as measured in the VN frame. Therefore, to study the $lp\to l'\Lambda^\uparrow X$ process and its dependence on the observed hadronic variables, a complete knowledge of the $\boldsymbol{k}_\perp$ dependence of the (unpolarized and polarizing) fragmentation functions is required. To this end we will consider a Gaussian model for the explicit $\boldsymbol{k}_\perp$ dependence of our FF. By imposing proper constraints on the polarizing fragmentation function $\Delta^N D_{\Lambda^\uparrow/q}(z,\boldsymbol{k}_\perp)$, we get an explicit Gaussian model for the FF, which enables us to investigate several interesting processes in the case of SIDIS. This procedure is described in detail in what follows.

For simplicity, we assume that the $\boldsymbol{k}_\perp$ dependence of the FF is the same for all flavors of the fragmenting quark, a choice which remains to be tested but that seems quite reasonable. We then write the unpolarized and polarizing FF for the $q \to \Lambda + X$ process in the following general form:

$$\hat{D}_{\Lambda/q}(z,\boldsymbol{k}_\perp) = \hat{D}_{\Lambda/q}(z,k_\perp) = \frac{d(z)}{M^2}\exp\left[-\frac{k_\perp^2}{M^2 f(z)}\right], \tag{6}$$

$$\Delta^N D_{\Lambda^\uparrow/q}(z,k_\perp) = \frac{\delta(z)}{M^2}\,\frac{k_\perp}{M}\exp\left[-\frac{k_\perp^2}{M^2\varphi(z)}\right], \tag{7}$$

where $M = 1\ \ \mathrm{GeV}/c$ is a typical hadronic momentum scale and $f(z)$, $\varphi(z)$ are generic functions of $z$, which we choose to be of the form $Nz^a(1-z)^b$. This simple behavior naturally allows us to take into account some general features of the $k_\perp$ dependence of the FF; the fast exponential decrease allows us to formally extend the integration region to the full range $0 < k_\perp < +\infty$ (rather than to a typical intrinsic $k_\perp$ range) making all analytical computations much easier, with negligible numerical differences.

Equations (6) and (7) must satisfy the positivity bound

$$\frac{|\Delta^N D_{\Lambda^\uparrow/q}(z,k_\perp)|}{\hat{D}_{\Lambda/q}(z,k_\perp)} = \frac{|\delta(z)|}{d(z)}\,\frac{k_\perp}{M}\exp\left[-\frac{k_\perp^2}{M^2}\left(\frac{1}{\varphi}-\frac{1}{f}\right)\right] \leqslant 1, \tag{8}$$

which, with $\varphi(z) = r f(z)$, implies $r < 1$ and

$$\frac{|\delta(z)|}{d(z)} \leqslant \left[\frac{2\,e}{f(z)}\,\frac{1-r}{r}\right]^{1/2}, \tag{9}$$

in order for Eq. (8) to hold true for each value of $z$ and $k_\perp$.

It is easily verified that the functions $d(z)$, $f(z)$ in Eq. (6) are simply related to the usual, unpolarized and $\boldsymbol{k}_\perp$-integrated FF and to the hadron mean squared transverse momentum inside the observed fragmentation jet, $\langle k_\perp^2(z)\rangle$:

$$D_{\Lambda/q}(z) = \int d^2\boldsymbol{k}_\perp \hat{D}_{\Lambda/q}(z,k_\perp) = \pi d(z) f(z), \tag{10}$$

$$\langle k_\perp^2(z)\rangle = \frac{\int d^2\boldsymbol{k}_\perp k_\perp^2 \hat{D}_{\Lambda/q}(z,k_\perp)}{\int d^2\boldsymbol{k}_\perp \hat{D}_{\Lambda/q}(z,k_\perp)} = M^2 f(z), \tag{11}$$

so that

$$d(z) = M^2\,\frac{D_{\Lambda/q}(z)}{\pi\langle k_\perp^2(z)\rangle}, \qquad f(z) = \frac{\langle k_\perp^2(z)\rangle}{M^2}\cdot \tag{12}$$

At present some experimental information on $\langle k_\perp^2(z)\rangle$ is available for pions but not yet for $\Lambda$ particles. However, several experiments, in different kinematical configurations, are studying or plan to study $\Lambda$ production in SIDIS and could give information on this observable.

To obey Eq. (9) in a most natural and simple way we can write

$$\frac{\delta(z)}{M^3} = \left[N_q\,\frac{z^\alpha(1-z)^\beta}{\alpha^\alpha\beta^\beta/(\alpha+\beta)^{(\alpha+\beta)}}\right]\frac{D_{\Lambda/q}(z)}{\pi[\langle k_\perp^2(z)\rangle]^{3/2}} \times[2e(1-r)/r]^{1/2} \tag{13}$$

with $\alpha,\beta > 0$, $|N_q| \leqslant 1$.

In this approach $\delta(z)$ is an unknown function depending on the parameters $\alpha$, $\beta$, $N_q$, and $r$, while $\varphi(z)$ is fixed by $r$, if we assume to know the functions $\langle k_\perp^2(z)\rangle$ and $D_{\Lambda/q}$.

If one were to demand consistency with the results of Ref. [3] $\delta(z)$ and $\varphi(z)$ could be fixed, allowing one to give predictions for $P_T^\Lambda$ in SIDIS. However, in the Introduction we have already expressed the problems of such an approach and we will not pursue this here.

Instead we will present the analytical formalism, which is meaningful and valid in the appropriate regions, and show some numerical results for different choices of $N_q,\alpha,\beta,r$ based on some of the qualitative features obtained from our earlier analysis of hadronic data and where we choose the expression of $\langle k_\perp^2(z)\rangle$ valid for pions at LEP energies.

Collecting all results, we can finally give explicit $z$ and $\boldsymbol{k}_\perp$-dependent expressions for the unpolarized and polarizing FF which we will use to investigate the consequences of the analytical expressions given in Sec. IV:

$$\hat{D}_{\Lambda/q}(z,k_\perp) = \frac{D_{\Lambda/q}(z)}{\pi\langle k_\perp^2(z)\rangle}\exp\left[-\frac{k_\perp^2}{\langle k_\perp^2(z)\rangle}\right], \tag{14}$$

$$\Delta^N D_{\Lambda^\uparrow/q}(z,k_\perp) = \frac{\delta(z)}{M^2}\,\frac{k_\perp}{M}\exp\left[-\frac{k_\perp^2}{r\langle k_\perp^2(z)\rangle}\right], \tag{15}$$

with $\delta(z)$ taken as in Eq. (13).

## IV. ANALYTICAL RESULTS

In this section we present the analytical expressions—in terms of the polarizing FF—for the transverse $\Lambda$, $\bar{\Lambda}$ polarization in semi-inclusive DIS, both for neutral and charged current interaction. Electroweak interference effects will be neglected, since they are hardly accessible by present or near-future experiments, at least in this context. We will apply the obtained expressions using different scenarios for the polarizing FF, but we expect them to be useful in the future, when either relevant SIDIS data are available or the polariz-

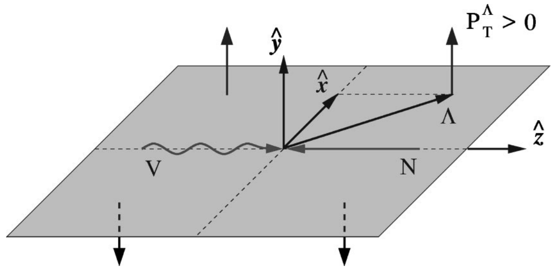


FIG. 1. Kinematical representation of the semi-inclusive hadron production process, as viewed in the virtual boson-target proton c.m. reference frame (VN frame). The $\hat{z}$ axis is taken along the virtual boson direction of motion; the $\hat{x}$ axis is along the direction of the hadron transverse momentum, $\boldsymbol{p}_T \equiv \boldsymbol{k}_\perp$ ; the $\hat{y}$ axis is chosen to form a right-handed frame. The direction of *positive* transverse (with respect to the production plane) polarization $\boldsymbol{P}_T$, in the four quadrants, is shown explicitly by the arrows.

ing FF have been extracted from other processes.

We give the expression of the polarization as a function of the usual DIS variables, $x = Q^2/2p\cdot q$, $y = p\cdot q/p\cdot l$, and $z_h = p\cdot p_h/p\cdot q$, where $l$, $p$, $q$, $p_h$ are the 4-momenta of the initial lepton, the proton, the virtual boson and the observed hadron, respectively. Notice that for SIDIS processes in the VN frame $z_h$ coincides with the light-cone variable $\xi = p_h^+/p_q^+$, where $p_q$ is the four-momentum of the fragmenting quark, $p_q = (W/2)(1,0,0,1)$ and $W = [(1-x)\,ys]^{1/2}$ is the total energy in the VN frame. When appropriate, the observables will be averaged over the kinematical range of the variables effectively covered by the particular experimental situation considered.

As already discussed, we will present our results in the virtual boson-proton c.m. reference frame (VN frame). More specifically, we take the $\hat{z}$ axis of our frame along the direction of motion of the exchanged virtual boson; the $\hat{x}$ axis is chosen along the transverse momentum $\boldsymbol{p}_T$ of the observed hadron with respect to the $\hat{z}$ axis. Since at leading twist and leading order the fragmenting quark moves collinearly to the virtual boson, $\boldsymbol{p}_T$ coincides with the hadron transverse momentum with respect to the fragmenting quark, $\boldsymbol{k}_\perp$ . In this configuration, the transverse (with respect to the hadron production plane) polarization is measured along the $+\hat{y}$ axis, and the angle between the hadron intrinsic transverse momentum and its polarization is fixed, $\phi = \pi/2$. For further clarity we show in Fig. 1 our kinematical configuration: the up and down arrows show explicitly the direction of *positive* polarization for Λ's produced in the four quadrants.

The transverse Λ polarization is given by

$$P_T^\Lambda(x,y,z_h,p_T) = \frac{d\sigma^{\Lambda^\uparrow} - d\sigma^{\Lambda^\downarrow}}{d\sigma^{\Lambda^\uparrow} + d\sigma^{\Lambda^\downarrow}}, \tag{16}$$

with

$$d\sigma^{\Lambda^{\uparrow(\downarrow)}} = \frac{d\sigma^{lp\to l'\Lambda^{\uparrow(\downarrow)}X}}{dx\,dy\,dz_h\,d^2\boldsymbol{p}_T} = \sum_{i,j} f_{q_i/p}(x)\frac{d\hat{\sigma}^{lq_i\to l'q_j}}{dy}\hat{D}_{\Lambda^{\uparrow(\downarrow)}/q_j}(z_h,p_T), \tag{17}$$

where $\hat{D}_{\Lambda^{\uparrow(\downarrow)}/q}$ is defined by Eqs. (3) and (4), with $\sin\phi = +(-)1$; $i$, $j$ indicate different quark flavors and the sum includes both quark and antiquark contributions; $(l,l')$ stands for $(l^\mp,l^\mp)$, $(\nu,\nu)$, $(\bar{\nu},\bar{\nu})$ (NC contributions) and $(\nu,l^-)$, $(\bar{\nu},l^+)$, $(l^-,\nu)$, $(l^+,\bar{\nu})$ (CC contributions); in the NC case the partonic cross sections $d\hat{\sigma}^{lq_i\to l'q_j}/dy$ include a $\delta_{ij}$ term.

Analytical expressions for the polarization can be derived from Eqs. (16) and (17) by inserting the explicit formulas of the elementary cross sections; the results are formally very similar to the ones presented in the case of longitudinal hadron polarization (with unpolarized initial target) in Ref. [30]. Clearly, some appropriate modifications have to be performed in order to adapt those results to the case of transverse polarization, with the inclusion of $\boldsymbol{k}_\perp$ effects. In particular, $\boldsymbol{k}_\perp$-integrated and longitudinally polarized FF, $\Delta D_{\Lambda/q}(z_h)$, must be substituted by the corresponding polarizing FF, $\Delta^N D_{\Lambda^\uparrow/q}(z_h,k_\perp)$. Furthermore, the sign of some of the terms in the numerator of the polarization changes from negative to positive, a fact which can be understood by a careful inspection of the formulas derived in Ref. [30]. The complete formulas can also be obtained from Ref. [31] by expressing the function $D_{1T}^\perp$ in terms of $\Delta^N D_{\Lambda^\uparrow/q}$ using Eq. (5).

We list the full results for the transverse Λ and $\bar{\Lambda}$ polarization, only neglecting electroweak interference effects, in the Appendix. In some cases the full expressions are quite involved and not too informative. In this section we present and discuss, for the processes of phenomenological interest, approximate expressions which are valid with high accuracy for Λ particles and are much simpler. These expressions are obtained by neglecting terms containing nonleading quark contributions both in the partonic distributions and fragmentation functions. Isospin symmetry is assumed to hold, that is we take $(\Delta^N)D_{\Lambda/d} = (\Delta^N)D_{\Lambda/u}$ .

A similar argument holds also for the $\bar{\Lambda}$ case, at least in the region $z_h > 0.2$, where nonleading (or sea) quark contributions to the FF are relatively small. However, since the $\bar{\Lambda}$ case always involves at the same time contributions of the type $q\,(\Delta^N)D_{\Lambda/\bar{q}}$ and $\bar{q}\,(\Delta^N)D_{\Lambda/q}$ , with combined sea and valence contributions, these approximations should be taken with more care, also depending on the different ranges of the SIDIS variables $x$, $y$, and $z_h$ .

Notice finally that, both in the Appendix and in the following simplified expressions, some factors resulting from the elementary partonic cross sections have canceled out in the numerator and the denominator of the polarization. When averaging the polarization over some kinematical range, those factors must, of course, be taken into account.

We consider separately different processes.

### A. $l^\mp p \to l^\mp \Lambda^\uparrow X$

This case is of interest for several experimental setups, e.g., HERMES, H1, and ZEUS at DESY, COMPASS at CERN, E665 at SLAC. One gets

$$P_T^\Lambda(x,y,z_h,p_T)=\frac{\sum_q e_q^2 f_{q/p}(x)[d\hat{\sigma}^{lq}/dy]\Delta^N D_{\Lambda^\uparrow/q}(z_h,p_T)}{\sum_q e_q^2 f_{q/p}(x)[d\hat{\sigma}^{lq}/dy]\hat{D}_{\Lambda/q}(z_h,p_T)}$$
$$\simeq\frac{(4u+d)\Delta^N D_{\Lambda^\uparrow/u}+s\Delta^N D_{\Lambda^\uparrow/s}}{(4u+d)\hat{D}_{\Lambda/u}+s\hat{D}_{\Lambda/s}}, \tag{18}$$

where in the second line, as discussed above, we have neglected nonleading, doubly suppressed, contributions, originating from sea quarks both in distribution and fragmentation functions; we have switched to the notation $f_{q/p}(x)\to q(x)$ and have assumed isospin symmetry to hold, that is $\hat{D}_{\Lambda/d}\equiv\hat{D}_{\Lambda/u}$, and similarly for the polarizing fragmentation functions.

The full expressions of the Appendix show that a similar simplified expression holds also for the $l^\mp p\to l^\mp\overline{\Lambda}^\uparrow X$ processes, with the exchange $q(x)\leftrightarrow\overline{q}(x)$ (which may imply different cancellation effects between the $u$, $d$, and $s$ polarizing FF in the numerator) and with some additional terms in the denominator coming from nonleading quark contributions to the unpolarized FF (which in turn may imply a stronger overall suppression factor as compared to the $\Lambda$ case).

### B. $\nu p\to\nu\Lambda^\uparrow X$

This process is of interest for the planned neutrino factories [32,33], and is currently under investigation by the NOMAD Collaboration at CERN. Since there is no measurement of the final lepton here, care must be taken in the definition of the appropriate kinematical range, which must exclude regions where our perturbative approach is unreliable (low $Q^2$ and $W^2$ regimes, etc.). The full expression of $P_T^\Lambda$ is cumbersome; we give here the much simpler expression found when nonleading quark contributions are neglected:

$$P_T^\Lambda(x,y,z_h,p_T)\simeq\frac{[u(1-8C)+d(1-4C)]\Delta^N D_{\Lambda^\uparrow/u}+s(1-4C)\Delta^N D_{\Lambda^\uparrow/s}}{[u(1-8C)+d(1-4C)]\hat{D}_{\Lambda/u}+s(1-4C)\hat{D}_{\Lambda/s}}$$
$$=\frac{(Ku+d)\Delta^N D_{\Lambda^\uparrow/u}+s\Delta^N D_{\Lambda^\uparrow/s}}{(Ku+d)\hat{D}_{\Lambda/u}+s\hat{D}_{\Lambda/s}}, \tag{19}$$

where $C=\sin^2\theta_W/3\simeq0.077$, $K=(1-8C)/(1-4C)\simeq0.55$ and terms quadratic in $C$, which only introduce a weak dependence on $y$ into $K$, have also been neglected.

An analogous expression holds for $\overline{\nu}p\to\overline{\nu}\Lambda^\uparrow X$ processes, with a factor $K$ ranging now between 0.78 and 4 for $y=0$ and $y=1$, respectively. Again, similar expressions hold also for $\nu p\to\nu\overline{\Lambda}^\uparrow X$ and $\overline{\nu}p\to\overline{\nu}\overline{\Lambda}^\uparrow X$ cases, with the exchange $q(x)\leftrightarrow\overline{q}(x)$ and with some additional terms in the denominator coming from nonleading quark contributions to the unpolarized FF, which must be taken into account.

### C. $\nu p\to l^-\Lambda^\uparrow X$

This case is again of interest for neutrino factories and for the NOMAD experiment, which quite recently has published results for $\Lambda$ and $\overline{\Lambda}$ polarization [5,6]. One finds

$$P_T^\Lambda(x,y,z_h,p_T)=\frac{(d+Rs)\Delta^N D_{\Lambda^\uparrow/u}+\overline{u}(\Delta^N D_{\Lambda^\uparrow/\overline{d}}+R\Delta^N D_{\Lambda^\uparrow/\overline{s}})(1-y)^2}{(d+Rs)\hat{D}_{\Lambda/u}+\overline{u}(\hat{D}_{\Lambda/\overline{d}}+R\hat{D}_{\Lambda/\overline{s}})(1-y)^2}, \tag{20}$$

where $R=\tan^2\theta_C\simeq0.056$. Notice that neglecting sea-quark contributions in the partonic distributions and in the FF leads to a remarkably simple expression, which gives direct access to the polarizing FF for $u$ quarks:

$$P_T^\Lambda(x,y,z_h,p_T)\simeq\frac{\Delta^N D_{\Lambda^\uparrow/u}}{\hat{D}_{\Lambda/u}}. \tag{21}$$

The same expression is true for the case $l^+p\to\overline{\nu}\Lambda^\uparrow X$, which may be of interest for the H1 and ZEUS experiments at HERA, for COMPASS at CERN, and E665 at SLAC.

Similar expressions also hold for the processes $\overline{\nu}p\to l^+\overline{\Lambda}^\uparrow X$ and $l^-p\to\nu\overline{\Lambda}^\uparrow X$ again with some additional terms (different in the two cases) in the denominators coming from the nonleading quark contributions to the unpolarized FF.

### D. $\bar{\nu} p \to l^+ \Lambda^\uparrow X$

This case is very close to the previous one, with obvious modifications:

$$P_T^\Lambda(x,y,z_h,p_T) = \frac{(1-y)^2 u(\Delta^N D_{\Lambda^\uparrow/d} + R\Delta^N D_{\Lambda^\uparrow/s}) + (\bar{d} + R\bar{s})\Delta^N D_{\Lambda^\uparrow/\bar{u}}}{(1-y)^2 u(\hat{D}_{\Lambda/d} + R\hat{D}_{\Lambda/s}) + (\bar{d} + R\bar{s})\hat{D}_{\Lambda/\bar{u}}}. \tag{22}$$

Again, when sea-quark contributions are neglected and isospin symmetry is invoked, we find the very simple expression

$$P_T^\Lambda(x,y,z_h,p_T) \simeq \frac{\Delta^N D_{\Lambda^\uparrow/u} + R\Delta^N D_{\Lambda^\uparrow/s}}{\hat{D}_{\Lambda/u} + R\hat{D}_{\Lambda/s}}. \tag{23}$$

The same expression holds for the $l^- p \to \nu \Lambda^\uparrow X$ case. Once more, similar expressions also hold for the processes $\nu p \to l^- \overline{\Lambda}^\uparrow X$ and $l^+ p \to \overline{\nu} \overline{\Lambda}^\uparrow X$ apart from some additional terms (again different in the two cases) in the denominators coming from the nonleading quark contributions to the unpolarized FF.

The above results, Eqs. (18)–(23), relate measurable polarizations to different combinations of (known) distribution functions, (less known) unpolarized fragmentation functions, and (unknown) polarizing fragmentation functions; the different terms have relative coefficients which depend on the dynamics of the elementary partonic process and/or on the relevance of *s* quark contributions in the partonic distribution functions (the latter can be modulated according to the specific *x* region explored).

This large variety of possibilities gives a good opportunity to investigate and test the relevant properties of the unpolarized and polarizing Λ FF, by measuring the hyperon transverse polarization. In some special cases, Eq. (21), experiment offers direct information on these new functions.

## V. NUMERICAL ESTIMATES

We derive some numerical results for $P_T^\Lambda$ and $P_T^{\bar{\Lambda}}$, according to a few plausible scenarios for the fragmentation functions given in Eqs. (14) and (15). In the numerical calculations we have utilized the full expressions of the polarization given in the Appendix, with the only simplification $\Delta^N D_{\Lambda^\uparrow/\bar{q}} = 0$: these full expressions differ from the approximated ones (18)–(23) by extra terms in the denominators which, as we have explicitly checked, give negligible contributions to the numerical results presented here.

Let us consider the parameters $\alpha, \beta, N_q$, and $r$ appearing in Eqs. (15) and (13), and their possible values. Looking only qualitatively at the data on transverse Λ polarization in hadronic reactions we expect the following general features: (1) Negative contributions from up and down quarks ($N_{u,d} < 0$) and positive from strange quarks ($N_s > 0$) in order to have $P^\Lambda < 0$ and $P^{\bar{\Lambda}} \simeq 0$;[1] based on this one expects the Λ polarization in semi-inclusive DIS, where the *u* quark contribution is enhanced by the charge, to be negative. (2) A polarizing FF peaked at large *z* to explain the increase in magnitude of the polarization with $x_F$ at fixed $p_T$ [thus implying large values of $\alpha$, while $\beta \simeq O(1)$]. (3) A Gaussian shape similar for unpolarized and polarizing FF to explain the large values of the polarization [which means $r \simeq O(1)$].

We then fix $\beta = 1$, $\alpha = 6$, $r = 0.7$ and follow Ref. [7] using

$$\sqrt{\langle k_\perp^2(z) \rangle} = 0.61 z^{0.27} (1-z)^{0.2} \ \ \mathrm{GeV}/c, \tag{24}$$

as given by fitting the pion $k_\perp$ distribution inside jets at CERN $e^+e^-$ collider LEP energies.

We adopt for the unpolarized, $\boldsymbol{k}_\perp$-integrated, Λ FF, the $SU(3)$ symmetric parametrizations of Ref. [35] [Boros, Londergan, and Thomas (BLT)]. Other interesting sets of $SU(3)$ symmetric fragmentation functions were obtained in Ref. [36]. However, these sets do not separate between Λ and $\bar{\Lambda}$ and we cannot use them here, where we compute separately $P_T^\Lambda$ and $P_T^{\bar{\Lambda}}$; this was not a problem in Ref. [3] as, in the kinematical regions considered there for $pp \to \Lambda^\uparrow X$, it turned out that $P_T^\Lambda \simeq P_T^{\Lambda+\bar{\Lambda}}$. We have checked that, at least for HERMES and NOMAD kinematics (for which $d\sigma^\Lambda \gg d\sigma^{\bar{\Lambda}}$), the $\Lambda + \bar{\Lambda}$ fragmentation functions of Ref. [36] give here results close to those obtained with the symmetric BLT set.

$SU(3)$ breaking unpolarized FF are also available in the literature [37]; however, we do not aim here at discriminating among different FF, and we do not consider all possible cases. Such an analysis can only be attempted when much more experimental information will be available; at that stage the full analytical formalism discussed here can be used to learn about quark fragmentation processes into Λ's.

We look at qualitative differences of the results by considering for the polarizing FF two different models (keeping the BLT set of unpolarized FF): (1) a scenario with almost the same weight for up and strange quarks, with $N_u = N_d = -0.8$ and $N_s = 1$; (2) a scenario similar to the model of Burkardt and Jaffe [34] for the longitudinally polarized FF $\Delta D_{\Lambda/q}$, with $N_u = N_d = -0.3$ and $N_s = 1$.

We have adopted the Martin-Roberts-Stirling-Thorne 1999 (MRST99) [38] parametrization for the unpolarized proton distribution functions. We consider kinematical con-

[1] This feature is similar to what is expected for the longitudinally polarized FF, $\Delta D_{\Lambda/q}(z)$, in the well-known Burkardt-Jaffe model [34].

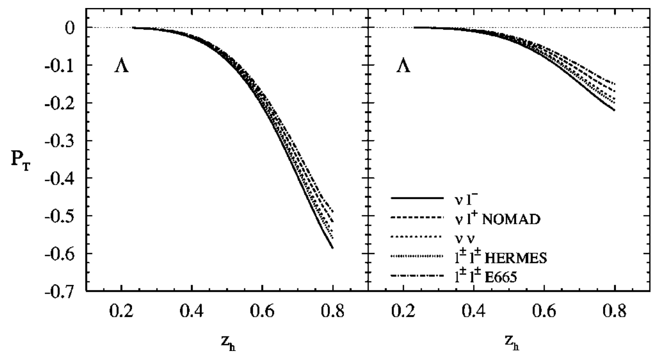


FIG. 2. Transverse $\Lambda$ polarization, $P_T^\Lambda$, vs $z_h$ and averaged over $p_T$, for several SIDIS production processes, with scenario 1 (on the left) and scenario 2 (on the right) for the polarizing FF (see the text for more details).

figurations typical of running experiments (HERMES at DESY, NOMAD at CERN, E655 at SLAC) that are presently measuring transverse $\Lambda$ polarization or plan to do it in the near future. The main kinematical cuts considered are (1) HERMES: $0.023 < x < 0.4$, $y < 0.85$, $1 < Q^2 < 10$ $(\mathrm{GeV}/c)^2$, $E_\Lambda > 4.5$ GeV, (2) NOMAD: $\langle x\rangle = 0.22$, $\langle y\rangle = 0.48$, $\langle Q^2\rangle = 9$ $(\mathrm{GeV}/c)^2$, and (3) E665: $10^{-3} < x < 10^{-1}$, $0.1 < y < 0.8$, $1.0 < Q^2 < 2.5$ $(\mathrm{GeV}/c)^2$, $E_\Lambda > 4$ GeV.

Since the $Q^2$ evolution of the polarizing FF is not under control at present, and the HERMES and NOMAD experiments involve a relatively limited range of $Q^2$ values, in our numerical calculations we have adopted a fixed scale, $Q^2 = 2$ $(\mathrm{GeV}/c)^2$. The unpolarized FF of Ref. [35] are given at a very low $Q^2$ value, and no evolution program is available at present from the authors; we have then performed the proper evolution to the adopted scale by using the evolution codes of Miyama and Kumano [39].

Our results are shown in Figs. 2 and 3. Figure 2 shows $P_T^\Lambda$ as a function of $z_h$, after $p_T$ average, for all processes considered in Secs. IV A–IV D and for kinematical configurations typical of the corresponding relevant experiments (as indicated in the legends); the two plots (left and right) correspond, respectively, to scenarios 1 and 2 of the polarizing FF.

The polarization is in general large in magnitude and negative: contributions from $\Delta^N D_{\Lambda^\uparrow/s}$ are always suppressed, either by the $s$ quark distribution [via factors like $s/(Ku+d)$] or by the standard model factor $R$, see Eqs. (18)–(23). Thus, the strange quark contribution is suppressed, unless one uses a $SU(3)$ asymmetric FF set for which $|\Delta^N D_{\Lambda^\uparrow/s}| \gg |\Delta^N D_{\Lambda^\uparrow/u,d}|$. This is reflected by the fact that in Fig. 2 all processes have similar polarizations, approximately given by the $p_T$-averaged ratio $\Delta^N D_{\Lambda^\uparrow/u}/\hat{D}_{\Lambda/u}$. The polarization is smaller in the right plot (scenario 2) simply because $|\Delta^N D_{\Lambda^\uparrow/u}|$ is smaller.

Figure 3 shows the corresponding results for the case of $\bar{\Lambda}$ SIDIS production; here the effect of cancellations between up (down) and strange contributions is more significant: notice, for instance, how $P_T(\nu p \to \nu \bar{\Lambda}^\uparrow X)$ is suppressed compared to the analogous process for $\Lambda$.

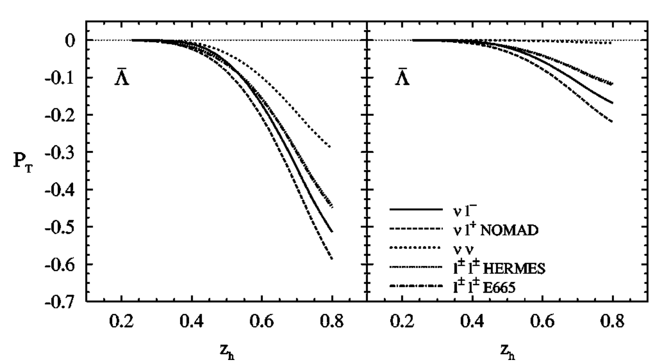


FIG. 3. Transverse $\bar{\Lambda}$ polarization, $P_T^{\bar{\Lambda}}$, vs $z_h$ and averaged over $p_T$, for several SIDIS production processes, with scenario 1 (on the left) and scenario 2 (on the right) for the polarizing FF (see the text for more details).

## VI. CONCLUSIONS

Single spin effects, which appear to be suppressed in leading twist collinear applications of PQCD factorization theorems, may instead reveal new interesting aspects of nonperturbative QCD. In Ref. [3] and in the present paper we have considered the long-standing problem of the transverse polarization of hyperons produced from *unpolarized* initial nucleons. Our approach is based on the use of a QCD factorization scheme, generalized to include intrinsic $\boldsymbol{k}_\perp$ in the fragmentation process: this allows us to introduce new spin dependences in the fragmentation functions of unpolarized quarks, which may cause the observed polarization of the produced hyperons.

These new functions, the polarizing FF, are supposed to describe universal features of the hadronization process, which is factorized in a similar way as the usual $\boldsymbol{k}_\perp$-integrated fragmentation functions. If correct, this idea should allow for a consistent phenomenological description of hyperon polarization in different processes: once the polarizing FF are extracted from some sets of data, their use in other processes should yield genuine predictions. With present experimental knowledge this does not appear to be feasible, unfortunately.

In Ref. [3] we have parametrized in a simple way the new functions and, by fitting all data on $pp \to \Lambda^\uparrow X$ and $pBe \to \Lambda^\uparrow X$ with $p_T > 1$ GeV/$c$, we have obtained explicit expressions for the polarizing FF; however, as the study of the unpolarized cross section reveals, these data seem not to be in the appropriate kinematical region yet, so that the extracted polarizing FF cannot be safely used to give true predictions. Data which unambiguously originate from kinematical regions where PQCD and factorization theorems can be applied, with a clear signature of current quark fragmentation, are still awaited. Therefore, the previously obtained expressions have been utilized here only in order to construct different scenarios for the polarizing FF and, hence, for the transverse $\Lambda$ polarization in SIDIS processes, $lp \to l' \Lambda^\uparrow X$. In this respect our present results should not be interpreted as predictions. But our results do display some interesting qualitative features that seem to be generic and are worth testing, for instance the $\bar{\Lambda}$ polarization is in most cases comparable in size to the $\Lambda$ polarization.

We would like to stress once more that our approach is describing a different kinematical region compared to the models based on target fragmentation, which aim at explaining $\Lambda$ polarization produced in low $p_T$ data in $p$-$p$ collisions and at $x_F<0$ data in semi-inclusive DIS. In contrast, our present investigation applies to the production of $\Lambda$'s in semi-inclusive DIS for $x_F>0$. Relevant data are expected to be available soon and a comparison with our analytical results will then be very useful in determining the polarizing FF.

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## APPENDIX

We present here the full analytical expressions for the transverse polarization, $P_T$, of $\Lambda$ and $\bar{\Lambda}$ hyperons produced in semi-inclusive DIS, at leading order in the coupling constants and in a $1/Q$ expansion; we neglect, for $(l^\pm,l^\pm)$ processes, electroweak interference effects. Our kinematical configuration is defined in the virtual boson-target proton c.m. reference frame, explained in detail in the text (see Fig. 1). The polarization is a function of the usual DIS variables $x=Q^2/2p\cdot q$, $y=p\cdot q/p\cdot l$, and $z_h=p\cdot p_h/p\cdot q$, where $l$, $p$, $q$, and $p_h$ are the 4-momenta of the initial lepton, the proton, the virtual boson, and the observed hadron, respectively. It depends also on $p_T$ ($\equiv k_\perp$ in our configuration), through the FF. For simplicity, all these dependences are not indicated in the following equations. Similarly, the $Q^2$ evolution is also implicit. We use the notation $q=u,d,s$ for the partonic distributions and in all cases we refer to $\Lambda$ fragmentation functions, since $(\Delta^N)D_{\bar{\Lambda}/q,\bar{q}}\equiv(\Delta^N)D_{\Lambda/\bar{q},q}$. The factors $(1-y)^2$ result from the elementary partonic cross sections involving leptons and quarks (or antiquarks) with opposite helicities. In all expressions we always put first terms which contain the dominant fragmentation functions. $R$ and $C$ are typical standard model factors, $R=\tan^2\theta_C\simeq0.056$, and $C=\sin^2\theta_W/3\simeq0.077$.

An inspection of the full set of equations shows that there are simple ways of relating the various processes: the polarizations for $\Lambda$ and $\bar{\Lambda}$ involving the same leptons are simply connected by exchanging $(\Delta^N)D_{\Lambda/q}$ with $(\Delta^N)D_{\Lambda/\bar{q}}$; both for $\Lambda$ and $\bar{\Lambda}$, the polarizations involving, respectively, $(\nu,l^-)$, $(\nu,\nu)$, $(l^-,\nu)$, and $(\bar{\nu},l^+)$, $(\bar{\nu},\bar{\nu})$, $(l^+,\bar{\nu})$ processes are related by exchanging $q$ with $\bar{q}$ and $(\Delta^N)D_{\Lambda/q}$ with $(\Delta^N)D_{\Lambda/\bar{q}}$; both for $\Lambda$ and $\bar{\Lambda}$, the polarizations involving "crossed" processes like $(\nu,l^-)$ and $(l^+,\bar{\nu})$ are connected by exchanging the position of the $(1-y)^2$ factors in the terms involving quark and antiquark distributions. This can be easily understood since one process and its "crossed" one both correspond to the exchange of the same $W$ boson, but with a negative and positive helicity lepton, respectively.

### 1. Λ polarization

(a) $\nu p\to l^-\Lambda^\uparrow X$

$$P_T=\frac{(d+Rs)\Delta^ND_{\Lambda^\uparrow/u}+(1-y)^2\bar{u}(\Delta^ND_{\Lambda^\uparrow/\bar{d}}+R\Delta^ND_{\Lambda^\uparrow/\bar{s}})}{(d+Rs)\hat{D}_{\Lambda/u}+(1-y)^2\bar{u}(\hat{D}_{\Lambda/\bar{d}}+R\hat{D}_{\Lambda/\bar{s}})}. \tag{A1}$$

(b) $\bar{\nu}p\to l^+\Lambda^\uparrow X$

$$P_T=\frac{(1-y)^2u(\Delta^ND_{\Lambda^\uparrow/d}+R\Delta^ND_{\Lambda^\uparrow/s})+(\bar{d}+R\bar{s})\Delta^ND_{\Lambda^\uparrow/\bar{u}}}{(1-y)^2u(\hat{D}_{\Lambda/d}+R\hat{D}_{\Lambda/s})+(\bar{d}+R\bar{s})\hat{D}_{\Lambda/\bar{u}}}. \tag{A2}$$

(c) $l^-p\to\nu\Lambda^\uparrow X$

$$P_T=\frac{u(\Delta^ND_{\Lambda^\uparrow/d}+R\Delta^ND_{\Lambda^\uparrow/s})+(1-y)^2(\bar{d}+R\bar{s})\Delta^ND_{\Lambda^\uparrow/\bar{u}}}{u(\hat{D}_{\Lambda/d}+R\hat{D}_{\Lambda/s})+(1-y)^2(\bar{d}+R\bar{s})\hat{D}_{\Lambda/\bar{u}}}. \tag{A3}$$

(d) $l^+p\to\bar{\nu}\Lambda^\uparrow X$

$$P_T=\frac{(1-y)^2(d+Rs)\Delta^ND_{\Lambda^\uparrow/u}+\bar{u}(\Delta^ND_{\Lambda^\uparrow/\bar{d}}+R\Delta^ND_{\Lambda^\uparrow/\bar{s}})}{(1-y)^2(d+Rs)\hat{D}_{\Lambda/u}+\bar{u}(\hat{D}_{\Lambda/\bar{d}}+R\hat{D}_{\Lambda/\bar{s}})}. \tag{A4}$$

(e) $l^{\mp}p \to l^{\mp}\Lambda^{\uparrow}X$

$$P_T = \frac{4u\Delta^N D_{\Lambda^\uparrow/u} + d\Delta^N D_{\Lambda^\uparrow/d} + s\Delta^N D_{\Lambda^\uparrow/s} + 4\bar{u}\Delta^N D_{\Lambda^\uparrow/\bar{u}} + \bar{d}\Delta^N D_{\Lambda^\uparrow/\bar{d}} + \bar{s}\Delta^N D_{\Lambda^\uparrow/\bar{s}}}{4u\hat{D}_{\Lambda/u} + d\hat{D}_{\Lambda/d} + s\hat{D}_{\Lambda/s} + 4\bar{u}\hat{D}_{\Lambda/\bar{u}} + \bar{d}\hat{D}_{\Lambda/\bar{d}} + \bar{s}\hat{D}_{\Lambda/\bar{s}}}. \tag{A5}$$

(f) $\nu p \to \nu\Lambda^{\uparrow}X$

$$\mathrm{Num}(P_T) = [(1-4C)^2 + (1-y)^2 16C^2]u\Delta^N D_{\Lambda^\uparrow/u} + [(1-2C)^2 + (1-y)^2 4C^2](d\Delta^N D_{\Lambda^\uparrow/d} + s\Delta^N D_{\Lambda^\uparrow/s})$$
$$+ [(1-y)^2(1-4C)^2 + 16C^2]\bar{u}\Delta^N D_{\Lambda^\uparrow/\bar{u}} + [(1-y)^2(1-2C)^2 + 4C^2](\bar{d}\Delta^N D_{\Lambda^\uparrow/\bar{d}} + \bar{s}\Delta^N D_{\Lambda^\uparrow/\bar{s}}),$$

$$\mathrm{Den}(P_T): \Delta^N D_{\Lambda^\uparrow/q} \to \hat{D}_{\Lambda/q}. \tag{A6}$$

(g) $\bar{\nu}p \to \bar{\nu}\Lambda^{\uparrow}X$

$$\mathrm{Num}(P_T) = [(1-y)^2(1-4C)^2 + 16C^2]u\Delta^N D_{\Lambda^\uparrow/u} + [(1-y)^2(1-2C)^2 + 4C^2](d\Delta^N D_{\Lambda^\uparrow/d} + s\Delta^N D_{\Lambda^\uparrow/s}) + [(1-4C)^2$$
$$+ (1-y)^2 16C^2]\bar{u}\Delta^N D_{\Lambda^\uparrow/\bar{u}} + [(1-2C)^2 + (1-y)^2 4C^2](\bar{d}\Delta^N D_{\Lambda^\uparrow/\bar{d}} + \bar{s}\Delta^N D_{\Lambda^\uparrow/\bar{s}}),$$

$$\mathrm{Den}(P_T): \Delta^N D_{\Lambda^\uparrow/q} \to \hat{D}_{\Lambda/q}. \tag{A7}$$

Notice that often one can safely neglect terms involving sea (or nonleading) quark contributions both in the partonic distributions and fragmentation functions, which in some cases leads to very simple expressions for the polarization. Some examples are presented and discussed in Sec. IV. However, in the cases involving $\bar{\nu}$'s, because of the terms $(1-y)^2$, these approximate expressions should be taken with some care in the region $y \simeq 1$. Further simplifications result when assuming isospin invariance, $(\Delta^N)D_{\Lambda/u} \equiv (\Delta^N)D_{\Lambda/d}$.

### 2. $\bar{\Lambda}$ polarization

(a) $\nu p \to l^{-}\bar{\Lambda}^{\uparrow}X$

$$P_T = \frac{(1-y)^2\bar{u}(\Delta^N D_{\Lambda^\uparrow/d} + R\Delta^N D_{\Lambda^\uparrow/s}) + (d+Rs)\Delta^N D_{\Lambda^\uparrow/\bar{u}}}{(1-y)^2\bar{u}(\hat{D}_{\Lambda/d} + \hat{D}_{\Lambda/s}) + (d+Rs)\hat{D}_{\Lambda/\bar{u}}}. \tag{A8}$$

(b) $\bar{\nu}p \to l^{+}\bar{\Lambda}^{\uparrow}X$

$$P_T = \frac{(\bar{d}+R\bar{s})\Delta^N D_{\Lambda^\uparrow/u} + (1-y)^2 u(\Delta^N D_{\Lambda^\uparrow/\bar{d}} + R\Delta^N D_{\Lambda^\uparrow/\bar{s}})}{(\bar{d}+R\bar{s})\hat{D}_{\Lambda/u} + (1-y)^2 u(\hat{D}_{\Lambda/\bar{d}} + R\hat{D}_{\Lambda/\bar{s}})}. \tag{A9}$$

(c) $l^{-}p \to \nu\bar{\Lambda}^{\uparrow}X$

$$P_T = \frac{(1-y)^2(\bar{d}+R\bar{s})\Delta^N D_{\Lambda^\uparrow/u} + u(\Delta^N D_{\Lambda^\uparrow/\bar{d}} + R\Delta^N D_{\Lambda^\uparrow/\bar{s}})}{(1-y)^2(\bar{d}+R\bar{s})\hat{D}_{\Lambda/u} + u(\hat{D}_{\Lambda/\bar{d}} + R\hat{D}_{\Lambda/\bar{s}})}. \tag{A10}$$

(d) $l^{+}p \to \overline{\nu\Lambda}^{\uparrow}X$

$$P_T = \frac{\bar{u}(\Delta^N D_{\Lambda^\uparrow/d} + R\Delta^N D_{\Lambda^\uparrow/s}) + (1-y)^2(d+Rs)\Delta^N D_{\Lambda^\uparrow/\bar{u}}}{\bar{u}(\hat{D}_{\Lambda/d} + R\hat{D}_{\Lambda/s}) + (1-y)^2(d+Rs)\hat{D}_{\Lambda/\bar{u}}}. \tag{A11}$$

(e) $l^{\mp}p \to l^{\mp}\bar{\Lambda}^{\uparrow}X$

$$P_T = \frac{4\bar{u}\Delta^N D_{\Lambda^\uparrow/u} + \bar{d}\Delta^N D_{\Lambda^\uparrow/d} + \bar{s}\Delta^N D_{\Lambda^\uparrow/s} + 4u\Delta^N D_{\Lambda^\uparrow/\bar{u}} + d\Delta^N D_{\Lambda^\uparrow/\bar{d}} + s\Delta^N D_{\Lambda^\uparrow/\bar{s}}}{4\bar{u}\hat{D}_{\Lambda/u} + \bar{d}\hat{D}_{\Lambda/d} + \bar{s}\hat{D}_{\Lambda/s} + 4u\hat{D}_{\Lambda/\bar{u}} + d\hat{D}_{\Lambda/\bar{d}} + s\hat{D}_{\Lambda/\bar{s}}}. \tag{A12}$$

(f) $\nu p \to \nu \overline{\Lambda}^{\uparrow} X$

$$\mathrm{Num}(P_T) = [(1-y)^2(1-4C)^2 + 16C^2]\bar{u}\Delta^N D_{\Lambda^\uparrow/u} + [(1-y)^2(1-2C)^2 + 4C^2](\bar{d}\Delta^N D_{\Lambda^\uparrow/d} + \bar{s}\Delta^N D_{\Lambda^\uparrow/s})$$
$$+ [(1-4C)^2 + (1-y)^2 16C^2]u\Delta^N D_{\Lambda^\uparrow/\bar{u}} + [(1-2C)^2 + (1-y)^2 4C^2](d\Delta^N D_{\Lambda^\uparrow/\bar{d}} + s\Delta^N D_{\Lambda^\uparrow/\bar{s}}),$$

$$\mathrm{Den}(P_T): \Delta^N D_{\Lambda^\uparrow/q} \to \hat{D}_{\Lambda/q}. \tag{A13}$$

(g) $\bar{\nu} p \to \overline{\nu \Lambda}^{\uparrow} X$

$$\mathrm{Num}(P_T) = [(1-4C)^2 + (1-y)^2 16C^2]\bar{u}\Delta^N D_{\Lambda^\uparrow/u} + [(1-2C)^2 + (1-y)^2 4C^2](\bar{d}\Delta^N D_{\Lambda^\uparrow/d} + \bar{s}\Delta^N D_{\Lambda^\uparrow/s})$$
$$+ [(1-y)^2(1-4C)^2 + 16C^2]u\Delta^N D_{\Lambda^\uparrow/\bar{u}} + [(1-y)^2(1-2C)^2 + 4C^2](d\Delta^N D_{\Lambda^\uparrow/\bar{d}} + s\Delta^N D_{\Lambda^\uparrow/\bar{s}}),$$

$$\mathrm{Den}(P_T): \Delta^N D_{\Lambda^\uparrow/q} \to \hat{D}_{\Lambda/q}. \tag{A14}$$

Notice that contrary to the case of Λ particle production, here the leading and nonleading quark contributions are mixed between partonic distributions and fragmentation functions, with terms of the type $q\,(\Delta^N)D_{\Lambda/\bar{q}}$ and $\bar{q}\,(\Delta^N)D_{\Lambda/q}$. Which terms are dominating depends on the kinematic range considered ($x$ and $z_h$ values). Moreover, the $(1-y)^2$ factors can also be relevant, for large $y$ values. Therefore, it is not easy to find approximate expressions for $P_T$; in any case, their range of validity is limited to particular kinematical configurations and has to be considered with care.


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