Quantum creep and pinning properties of oxygen-deficient YBa$_2$Cu$_3$O$_{x-n}$ films

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A high sensitivity capacitance torquemeter has been used for a comprehensive investigation of the induced current densities and dynamic relaxation rates in a YBa$_2$Cu$_3$O$_{x-n}$ film with nominal oxygen content varying between $x_n = 6.55$ and $x_n = 7.0$. The dynamic relaxation rate $Q$ does not extrapolate to zero at $T = 0$ K, indicating the presence of quantum creep. By changing the oxygen content of the film it is possible to investigate the quantum creep rate $Q(0)$ and the normal-state resistivity $\rho_n(0)$ at low temperature. Although $Q(0)$ increases monotonically with $\rho_n$, it is found that $Q(0)$ is not proportional to $\rho_n(0)$, in contrast to the predictions of a theory based on dissipative tunneling of collectively pinned single vortices [Blatter et al., Rev. Mod. Phys. 66, 1125 (1994)]. The experimental results imply that in YBa$_2$Cu$_3$O$_7$ quantum creep takes place in a transition regime between Hall tunneling and dissipative tunneling. For lower oxygen contents the quantum creep regime moves towards the dissipative limit. For each oxygen content the characteristic pinning energy $U_c(0)$ at $T = 0$ is obtained by a linear extrapolation to $T = 0$ K of the $T/Q$ versus $T$ curves. The critical current density $j_c$ at $T = 0$ is determined independently by a linear extrapolation of the measured $\ln(j_c) \propto U_c(0)$ with $p \approx 0.5$ is found, indicating single vortex pinning at higher temperatures. This is confirmed by a detailed analysis of the measured current densities and relaxation rates by means of the generalized inversion scheme developed by Schnack et al. [Phys. Rev. B 48, 13 178 (1993)]. For $x_n \geq 6.6$ at $B_e = 0.6$ T and for $x_n \geq 6.7$ at $B_e = 2.0$ T the calculated temperature dependence of $j_c$ and $U_c$ agrees remarkably well with a model based on three-dimensional single vortex pinning caused by spatial fluctuations in the charge carrier mean free path. At lower oxygen contents and higher magnetic fields the agreement gradually breaks down due to the increasing importance of thermal fluctuations. [S0163-1829(96)04925-9]

I. INTRODUCTION

Shortly after the discovery of the high-$T_c$ superconductors it was observed$^1$ that the relaxation rate of the magnetic moment does not vanish as the temperature approaches $T = 0$ K. This contradicts the linear temperature dependence predicted by the Anderson-Kim model.$^2$ By using a nonlinear dependence of the activation energy $U(j)$ on the current density $j$, Griessen$^3$ showed that the relaxation rate approaches $T = 0$ K as $S \sim T^{1/n}$ with $n \approx 3/2$. However, this does not explain the observation that the relaxation rate becomes almost independent of temperature below $T \approx 1$ K as observed in many high-$T_c$ superconductors, such as YBa$_2$Cu$_3$O$_7$,$^{4-8}$ Bi$_2$Sr$_2$CaCu$_2$O$_8$ and Bi$_2$Sr$_2$Ca$_2$Cu$_3$O$_{10}$ (Refs. 9–11), and Tl$_2$Ba$_2$Ca$_1$Cu$_2$O$_{8+}$.$^{12,13}$

The low-temperature saturation of the relaxation rate is indicative of quantum tunneling of vortices. Quantum creep of vortices in high-$T_c$ superconductors has been studied theoretically by Blatter and co-workers.$^{14,15}$ They calculated the quantum creep rate in bulk superconductors by applying weak collective pinning theory. At low magnetic fields single vortices are individually pinned by the collective action of many weak pinning centers. Segments of a typical length $L_c$, which is called the correlation length, tunnel independently. At high magnetic fields the tunneling object is a collectively pinned vortex bundle.

On the basis of the prediction of Blatter et al.$^{15}$ for the creep rate in the single vortex limit, one expects that quantum creep will be favored in samples with a strong anisotropy, high normal-state resistivity, and a short coherence length. The purpose of this work is to make a quantitative test of the dissipative tunneling theory$^{14,15}$ by using samples with a relatively high normal-state resistivity at $T_c$, which can unambiguously be extrapolated to low temperatures. For an optimal comparison we chose a single film of YBa$_2$Cu$_3$O$_{x-n}$ with various oxygen content. The advantage of working with one film is that only the effect of oxygen doping can be investigated since the addition and removing of oxygen can be done fully reversibly leaving the microstructure of the sample unchanged. As shall be shown below, the experimental results cannot be understood within a theory of purely dissipative tunneling. They can, however, be well explained by a theory which incorporates both dissipative and Hall drag terms.

Another purpose of this work is to investigate the relation between critical current density $j_c$ and pinning in samples

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with a variable concentration of weak pinning centers (oxygen vacancies). The induced current densities \( j_s \) and relaxation rates \( Q \) measured at sufficiently high temperature where the effects of quantum creep are negligible, are analyzed by means of the generalized inversion scheme (GIS) developed by Schnack et al.\(^{16} \) in order to determine the true critical current density \( j_c \) and the characteristic pinning energy \( U_c \). This method is based on the general assumption that the current dependence and the temperature dependence of the activation energy can be separated. Furthermore, one exploits the fact that a power law relates \( U_c \) and \( j_c \). With this method it is possible to calculate the temperature dependence of \( j_c \), which is in general clearly different from the measured current density \( j_s \), and \( U_c \), once the exponent \( p \) in the power-law relation \( U_c \propto [j_c]^p \) is known. The GIS allows also to identify the pinning regime [single vortex (SV), small bundle pinning (SB), large bundle pinning (LB)] and the microscopic origin of the pinning (\( \delta T_c \) pinning due to spatial fluctuations in \( T_c \) or \( \delta \xi \) pinning due to spatial fluctuations in the mean free path of the charge carriers).

This paper is organized as follows. The theory of quantum creep of vortices, including dissipative tunneling and Hall tunneling, is briefly reviewed in Sec. II. The sample preparation and oxygen loading procedure is described together with the experimental technique used to measure the current density sample and its relaxation rate in Sec. III. An overview of the experimental results is given in Sec. IV. The relation between the low-temperature normal-state resistivity and the quantum creep rate is discussed in Sec. V A, whereas the magnetic-field dependence of the quantum creep rate is discussed in Sec. V B. The pinning properties of \( \text{YBa}_2\text{Cu}_3\text{O}_y \) are considered in Sec. VI. A power-law relation between the characteristic pinning energy and the critical current density is found experimentally in Sec. VI A. The generalized inversion scheme is introduced and applied to measured data in Sec. VI B. The microscopic origin of pinning is discussed in Sec. VI C. The conclusions are given in Sec. VII.

II. THEORY OF QUANTUM CREEP

A. Dissipative quantum tunneling

At low temperatures the flux dynamics in high-\( T_c \) superconductors is governed by quantum creep of vortices through the pinning barriers. In contrast to thermally activated flux motion, this process is not only governed by the height of the pinning barrier but also by the time spent by the flux segment under the barrier during the tunneling process.\(^{14} \) Due to the coupling between the vortex and its environment, quantum creep is a dissipative process. The influence of the dissipation on the tunneling probability of a particle through a barrier has been evaluated by Caldeira and Leggett.\(^{17} \) The dissipative quantum creep relaxation rate \( Q_D \) can then readily be evaluated\(^{18} \) to be

\[
Q_D = \frac{\hbar}{\eta_x \chi_{\text{hop}}} j_c ,
\]

where \( 2 \chi_{\text{hop}} \) is the distance separating the metastable positions of the vortex segment before and after tunneling. \( \eta \) is the viscous drag coefficient, \( A \) is a numerical parameter of order unity, \( j_c \) is the critical current for which the energy barrier between two vortex configurations vanishes, and \( j_s \) is the superconducting current induced in the sample by sweeping the external magnetic field at a certain sweep rate \( dB_z / dt \). For \( j_s = j_c \) and noting that \( \chi_{\text{hop}} \) is of the order of \( \xi \), Eq. (1) becomes

\[
Q_D = \frac{\hbar}{\eta_x \chi_{\text{hop}}} j_c. \tag{2}
\]

Together with the Bardeen-Stephen expression\(^{19} \)

\[
\eta = \Phi_0 B_c L_c / \rho_n (0)
\]

this leads to

\[
Q_D = \frac{\hbar^2}{2 \xi} j_c / j_0 . \tag{3}
\]

where \( \Phi_0 \) is the flux quantum, \( L_c \) is the length of the tunneling flux segment, and \( \rho_n (0) \) is the normal-state resistivity at \( T = 0 \text{ K} \). Note that this result is, except for a numerical factor close to unity, identical to the result obtained from a collective pinning theory of single vortices derived by Blatter and co-workers.\(^{14,15,20} \)

\[
Q_D = \frac{\hbar^2}{2 \xi} j_c / j_0 . \tag{4}
\]

The relaxation rate does not depend on the orientation of the magnetic field and is identical for in-plane and out-of-plane motion, although the viscous drag coefficients and the elasticities of the vortex are different for these two types of motion. The \( c \)-axis correlation length \( L_c = e \xi (j_0 / j_c)^{1/2} \) is independent of angle and type of motion. Combining this result with Eq. (4) reproduces Eq. (3), which demonstrates that this formula remains valid in the anisotropic case.

B. Hall tunneling

Recently, Feigel’man et al.\(^{21} \) proposed that high-\( T_c \) superconductors are not in the dissipative limit at low temperatures. Their analysis is, however, not based on a comparison of the kinetic and the dissipative terms in the Euclidean action describing the vortex motion. Instead of this, they estimate the electron mean free path \( l \) from a linear extrapolation of \( \rho_n (T \gg T_c) \) data which gives a low-temperature resistivity \( \rho_n (0) \approx 10 \mu \Omega \text{ cm} \) leading to \( l \approx 70 \text{ nm} \). This means that high-\( T_c \) superconductors might be in the superclean limit, satisfying the condition: \( l \gg \xi (\epsilon_F / \Delta) \) in which \( \epsilon_F \) is the Fermi energy and \( \Delta \) is the energy gap. In this case, the flux motion has to be described by the following equation of motion:\(^{22,23} \)

\[
\eta \ddot{\mathbf{v}}_v + \alpha \mathbf{v}_v \times \dot{\mathbf{z}} = \Phi_0 L_c \mathbf{j}_c \times \dot{\mathbf{z}} + \mathbf{F}_{\text{pin}}, \tag{5}
\]

where \( \eta \) and \( \alpha \) are the viscous and the Hall drag coefficients, \( \mathbf{v}_v \) is the vortex velocity, \( \mathbf{F}_{\text{pin}} \) is the pinning force, and \( \dot{\mathbf{z}} \) a unit vector parallel to the vortex. Feigel’man et al.\(^{21} \) use the analogy between vortex motion in a superconductor and non-dissipative flow in helium-II to derive the following expression for the Hall drag coefficient:

\[
\alpha = \pi \hbar n_s L_c , \tag{6}
\]
where \( n_s \) is the density of superconducting electrons. Finally, they calculate that the saddle-point solution of the Euclidean action \( S_E \) is given by

\[
\frac{S_{\text{eff}}}{\hbar} = \frac{\alpha}{\hbar} V_c, \tag{7}
\]

where \( V_c \) is the correlated volume of the vortex line segment or flux bundle. For a vortex line segment, this volume is approximated by \( V_c \approx \xi^2 L_c \). This leads finally to the following expression for the dynamic Hall relaxation rate:

\[
Q_H \approx \frac{\hbar}{\alpha \xi^2} = \frac{1}{\pi n_s \xi^2 L_c}. \tag{8}
\]

Note that, in contrast to dissipative quantum creep, the Hall creep rate does not depend on the low-temperature normal-state resistivity \( \rho_n(0) \). Measurements of the low-temperature relaxation rate \( Q(0) \) at various values of the oxygen concentration \( x_n \) can be used to determine the character of quantum creep, since \( \rho_n(0) \) increases markedly with decreasing \( x_n \).

C. Interpolation between dissipative and Hall tunneling

The approaches to the problem of quantum creep of vortices in high-\( T_c \) superconductors given in the previous section are limited to the dissipative limit and the superclean (Hall) limit. A more general approach has recently been given by Stephen \(^{24} \) for a parabolic potential of range \( x_c \) given by

\[
U(x) = \frac{1}{2} kL_c x^2, \quad |x| \leq x_c, \tag{9}
\]

\[
U(x) = -\infty, \quad |x| > x_c.
\]

The Euclidean action \( S_E \) of the tunneling process is approximately related to the mean-square quantum fluctuation \( \langle u_t^2 \rangle \) of the vortex position by \(^{25} \)

\[
S_E \approx \hbar \frac{x_c^2}{2\langle u_t^2 \rangle}. \tag{10}
\]

For this model the mean-square fluctuation \( \langle u_t^2 \rangle \) is given by \(^{24} \)

\[
\langle u_t^2 \rangle = \frac{\hbar \gamma}{2\pi kL_c} \int_0^{\omega_c} d\omega \coth(\beta\hbar\omega/2) \left( \frac{\omega - \omega_0}{\omega_0 - \omega} \right)^2 + \gamma^2, \tag{11}
\]

where \( \beta = 1/kT \) and \( \omega_c \) is a cutoff frequency probably related to the superconducting gap energy. The parameters \( \omega_0 \) and \( \gamma \) are given by

\[
\omega_0 = \frac{\alpha kL_c}{\alpha^2 + \eta^2} \tag{12}
\]

and

\[
\gamma = \frac{\eta kL_c}{\alpha^2 + \eta^2}, \tag{13}
\]

where \( \alpha \) and \( \eta \) are the Hall drag and the viscous drag coefficients, respectively. As discussed by Kopnin and Salomaa \(^{23} \) and Blatter et al. \(^{20} \) \( \alpha \) and \( \eta \) depend on \( \omega_B \tau \) as

\[
Q(0) \approx \frac{1}{\pi n_s \xi^2 L_c} \left[ \frac{1}{\omega_B \tau} + \frac{1}{2} + \frac{1}{\pi \arctan(\omega_B \tau)} \right]. \tag{17}
\]

The function \( f(\omega_B \tau) \) between square brackets in Eq. (17) is plotted in Figs. 1(a) and 1(b). From these figures it follows that \( f(\omega_B \tau) \) has the interesting property to be essentially

![Figure 1](https://example.com/figure1.png)
constant and equal to 1 for $\omega_B T > 1$. In the dissipative limit $\omega_B T \ll 1$ we recover Eq. (3), whereas in the pure Hall limit $\omega_B T \gg 1$, Eq. (17) reduces to Eq. (8).

### III. EXPERIMENTAL METHODS

#### A. Sample preparation

The YBa$_2$Cu$_3$O$_{x_n}$ (YBCO) film is prepared using in situ 90° off-axis magnetron dc sputtering from a single stoichiometric YBCO target. During sputtering the total chamber pressure is 400 mTorr, the gas is an 80% Ar/20% O$_2$ mixture and the MgO (100) substrate is at a temperature of 740 °C. X-ray diffraction as well as Rutherford backscattering spectroscopy channeling measurements show that the film is grown epitaxially on the MgO (100) substrate, with a channeling minimum yield smaller than 7%. The layers are purely c-axis oriented and have narrow rocking curves [full width at half height (005) $=0.3°$]. This in situ prepared film has a $T_c(R=0) = 89$ K and a transition width of 1 K. The critical current density $J_c$ at an applied field $B = 0.5$ T is $2 \times 10^{11}$ A/m$^2$ at 4.2 K and $4 \times 10^8$ A/m$^2$ at 77 K determined with torque magnetometry. The thickness of the film is 450 nm which is inferred from the sputtering time and the growth rate. More details about the preparation and characterization have been published elsewhere.$^{26}$

The oxygen content of the film is adjusted to the desired value by using the oxygen pressure-temperature ($P$-$T$) phase diagram.$^{27}$ In order to reduce the oxygen content in a controlled way, the sample is inside a stoichiometric YBCO ceramic box. Starting in an oxygen atmosphere of 10 Torr (1.33 kPa) at an annealing temperature $T_{an}(x_n)$, $[T_{an}(x_n)]$ varying between 435 °C for $x_n = 6.9$ and 545 °C for $x_n = 6.65$] the film is slowly cooled down (2–3 °C/min), while the O$_2$ pressure is decreased in order to follow the corresponding oxygen content line in the ($P$-$O_2$-$T$) phase diagram.$^{28}$ Finally, the film is quenched to room temperature in $P(O_2) = 75$ mTorr (7.5 Pa) and from a quench temperature $T_q(x_n)$ varying between $T_q = 360$ °C for $x_n = 6.9$ and $T_q = 444$ °C for $x_n = 6.65$. Before each oxygen content changing procedure, the film is fully oxidized by annealing it during 15–30 min at $T=500$ °C and $P(O_2) = 750$ Torr (100 kPa). After four oxygen treatments the $T_c$ of the fully oxidized film has decreased with approximately 1 K, which indicates that the oxygen depletion process is very reproducible. Furthermore, this small decrease of $T_c$ demonstrates that the quality of the sample is only weakly influenced by the oxygen treatments.

#### B. Oxygen content and the normal-state resistivity

Figure 2 shows the resistivity versus temperature for the YBa$_2$Cu$_3$O$_{x_n}$ film with different nominal oxygen contents $x_n$, as used in the torque measurements described below. The resistivity of the film is measured with a standard dc four-probe technique using wire bonding contacts. After each resistance measurement the contacts are removed from the sample. The film has a sharp transition when it is fully loaded with oxygen, whereas the transitions become broader as the oxygen content is lowered. The transition is unusually broad if the oxygen content is lowered to $x_n=6.55$, which indicates an inhomogeneous oxygen distribution in the film. This is probably due to the rather large thickness, 450 nm, of the film used in this work.

The critical temperature $T_c$ at 5% of the transition is shown as a function of the oxygen content in Fig. 3. For comparison the plot also contains data on two other YBa$_2$Cu$_3$O$_{x_n}$ films produced with the same method. These $T_c$ values were obtained with a superconducting quantum interference device (SQUID) magnetometer and an ac susceptometer.$^{29}$ The $T_c$ values obtained from the films are in good agreement with the data from bulk samples. In con-
The capacitance magnetometer used in this work is similar to that used in Ref. 32 and consists of two parallel capacitor plates, connected by phosphorus bronze springs. The base plate is attached firmly to a revolving sample holder, while the detection plate which carries the sample moves freely. A torque exerted on this detection plate rotates it over a small angle $\delta \theta$ towards a new equilibrium position. This angular displacement and the corresponding change in capacitance are proportional to the torque, because the springs obey Hooke’s law. The torque magnetometer is calibrated by means of a small coil through which a known current is passed.

For the conversion of magnetic moment into induced current density it is assumed that a homogeneous current distribution is set up in the sample. This seemingly crude assumption is justified by magneto-optical experiments of Koblischka et al.\textsuperscript{33} and Schuster et al.\textsuperscript{34} and more recently by Koblischka et al.\textsuperscript{35} Furthermore its validity has also been shown by numerical calculations of Schnack et al.\textsuperscript{36} and Brandt.\textsuperscript{37} These calculations show that the flux profile remains virtually linear during conventional relaxation.

A homogeneous current distribution in a rectangular sample leads to the following expression for the magnitude of the magnetic moment $M$:

$$M = \frac{1}{4} a^2 d (b - a/3) j_s, \quad b \gg a,$$

where $a$ and $b$ are the lengths of the sides of the sample, $d$ is the thickness of the sample along the $c$ axis, and $j_s$ is the current density flowing through the entire sample. In the derivation of this equation it has been assumed that the current flows in the $ab$ plane. This is justified by the geometry of the sample $d \ll a, b$ and also because $j_c \gg j_s$.

The experiments were performed in the bore of a 7 T Oxford Instruments superconducting NbTi magnet mounted in a low-loss helium cryostat. Temperature control and stabilization within 0.1 K is realized by simultaneously regulating the flow of liquid helium through the heat exchanger and the voltage supplied to a heater coil. In contrast to a conventional flow cryostat the liquid helium necessary for the temperature regulation is taken from an internal reservoir which is also used for cooling the magnet.

In this work the dynamic relaxation rate

$$Q = d \ln j_s / d \ln (dB_s / dt)$$

is determined from measurements of $j_s$ at various sweep rates $dB_s / dt$ of the magnetic field. As shown by Jirsa et al.\textsuperscript{39} and Püstü et al.\textsuperscript{40} the dynamic relaxation rate $Q$ is closely related to the conventional relaxation rate $R = -d \ln M / d \ln t$ where $M$ is the magnetic moment of the sample. This observation is supported by numerical calculations by Schnack et al.\textsuperscript{36}
Minor hysteresis loops (MHL) have been made around central field values $B_e = 0.6$ T, $B_e = 2.0$ T, and $B_e = 6.9$ T, respectively. The width of the loops has been chosen large enough to fully penetrate the sample to make sure that it is in the critical state at the field where the width of the hysteresis loop is measured. MHL’s have been recorded at sweep rates $dB_e/dt = 0.6, 1.25, 2.5, 5, 10, 20,$ and $40$ mT/s.

**IV. EXPERIMENTAL RESULTS**

An example of MHL’s is shown in Fig. 5. During these measurements the temperature was stabilized at 15 K. The oxygen content of the YBa$_2$Cu$_3$O$_{x_n}$ sample was 6.85 in this case. Apart from the steep slopes at the edges of the MHL’s, the torque and also the magnetic moment and the average induced current density are only weakly dependent on the field. This indicates that the sample has reached the critical state during the MHL’s. We checked that a large hysteresis loop from $B_e = 0$ T to $B_e = 7$ T and back gives the same value for the hysteresis at the same field and sweep rate as used in the MHL. The hysteresis loops tend to merge with a common curve directly after the reversal of the field sweep direction. The magnetic moment thus becomes independent on $dB_e/dt$, indicating that the sample is evolving from one critical state into the other one.

From Fig. 5 it is clear that the torque hysteresis and thus the average induced current density $j_s$ is reduced by the same amount whenever the sweep rate is lowered by a factor of 2. This shows that $j_s$ varies linearly with $\ln(dB_e/\text{dt})$ and that the relaxation rate $Q$ is only weakly dependent on the sweep rate. We have determined $Q$ as a function of temperature and external field for samples with nominal oxygen contents $x_n = 6.55$, $x_n = 6.6$, $x_n = 6.7$, $x_n = 6.85$, and $x_n = 7.0$, respectively.

The dynamic relaxation rate is displayed in Figs. 6(a), 6(b), and 6(c) for $B_e = 0.6$, 2, and 6.9 T, respectively, and all oxygen contents mentioned above. It is clear that the relaxation rate does not extrapolate to zero at $T=0$ K, but remains finite. The nonzero extrapolation $Q(0)$, which is attributed to quantum creep, increases with decreasing oxygen content. This is shown in more detail in Fig. 7 from which one also observes that the quantum creep rate increases with increasing field.

Furthermore, the width of the hysteresis loops is used to obtain $j_s$. Values of $j_s$ obtained at $B_e = 0.6$, 2, and 6.9 T, are shown in Figs. 8(a), 8(b), and 8(c), respectively. For all $x_n$...
the current density decreases approximately exponentially with temperature, except at low temperatures \( T < 10 \) K and at high temperatures close to the irreversibility line. The deviation at high temperatures is caused by thermal depinning whereas the saturation at low temperatures is due to quantum creep. Extrapolation of the linear part of the \( \ln j_s \) versus \( T \) curve leads to \( j_s(0) \) which is the current density that would be observed in the absence of quantum creep. There is no thermally activated flux motion at \( T = 0 \) K and therefore \( j_s(0) \) is equal to the critical current density \( j_c \) defined by the condition that the activation energy \( U = U(j_c, T, B_e) \) for thermal hopping of vortices vanishes for \( j_s = j_c \), i.e., \( U(j_c, T, B_e) = 0 \). The values for \( j_s(0) \) are shown in Fig. 9 as a function of the oxygen content for \( B_e = 0.6 \) and 2.0 T.

V. QUANTUM CREEP IN YBa\(_2\)Cu\(_3\)O\(_x\)_n

A. Quantum creep and the role of the normal-state resistivity

Since Eqs. (3) and (4) predict that \( Q(0) \) is proportional to the normal-state resistivity \( \rho_n(0) \) at low temperatures, we have plotted in Fig. 10 \( Q(0) \) versus the normal-state resistivity values shown in Fig. 4(b) with \( \rho_n(0) = 41 \mu\Omega \) cm for \( x_n = 7.0 \) as discussed above. We observe that \( Q(0) \) increases approximately linearly with \( \rho_n(0) \), but with a finite intercept at low resistivities. At first sight one could try to explain this by claiming that the \( c \)-axis correlation length \( L_c \) depends on

![FIG. 8. Temperature dependence of the induced current density \( j_s \), measured at a sweep rate \( dB_e/dt = 40 \) mT/s. The applied magnetic-field is \( B_e = 0.6 \) T (a), 2.0 T (b), and 6.9 T (c). Each graph contains the data for the various oxygen contents \( x_n \) used in this work.](image1)

![FIG. 9. Oxygen content dependence of the critical current density \( j_s(0) \) at \( T = 0 \) K and \( B_e = 0.6 \) T and 2.0 T. The \( j_s(0) \) values are determined by a linear extrapolation to \( T = 0 \) K of the observed linear temperature dependence of \( \ln j_s(T) \).](image2)

![FIG. 10. Quantum creep rate \( Q(0) \) of a 450 nm YBa\(_2\)Cu\(_3\)O\(_x\)_n film displayed as a function of the low-temperature normal-state resistivity \( \rho_n(0) \) at \( B_e = 0.6 \) T, 2.0 T, and 6.9 T. The corresponding values of the nominal oxygen content are indicated in the graph. The solid lines are linear fits to the data. Note that these lines do not extrapolate to \( Q(0) = 0 \) at \( \rho_n = 0 \) \( \mu\Omega \) cm.](image3)
the current density $j_c(0) = j_c$. Furthermore, the variation of $\sqrt{\ln \kappa}$ is approximately 5% and thus not of major importance. For the evaluation of $L_c$, we use that $L_c$ is given by

$$L_c = \left( \frac{\Phi_0}{4 \pi \xi^2} \frac{e^2 \kappa}{\mu_0 j_c} \right)^{1/2} \approx e \xi (j_0 / j_c)^{1/2},$$

(20)

where $j_0 = \Phi_0 / 3 \sqrt{3} \pi \mu_0 \xi$ is the depairing current density. However, this equation is only valid in the single vortex regime. An extensive proof that $\text{YBa}_2\text{Cu}_3\text{O}_6$ is in the single vortex regime, for all oxygen contents used, even at $B = 7$ T is given below. Because the critical current density is at most weakly field dependent in the single vortex regime, we have used the current density $j_c(0)$ obtained at $B_c = 0.6$ T, which is the lowest field value used in the experiment, to evaluate $L_c$. Furthermore, the use of field-dependent values of $j_c(0)$ for the evaluation of $L_c$ would lead to an increase of $L_c$ with increasing field. This is in contradiction with the measured data displayed in Fig. 10. This leads finally to a correlation length which is only weakly dependent on the oxygen content. The values for $L_c$ obtained from Eq. (20) and the experimental data described above are given in Table I.

From Eqs. (3) and (4) and the weak variation of $L_c$ with oxygen content follows that the quantum creep rate should be proportional to the normal-state resistivity in the purely dissipative limit. From Fig. 10 it can be seen that the quantum creep rate increases approximately linearly with the normal resistivity. However, the quantum creep rate is not proportional to the normal-state resistivity $\rho_n(0)$, since there is a finite intercept at low resistivities. This leads to the important conclusion that the vortex system in oxygen-deficient $\text{YBa}_2\text{Cu}_3\text{O}_6$ cannot be in a purely dissipative limit since a finite intercept is in contradiction with Eq. (3).

An estimate of $\omega_B \tau$ and thus a determination of the nature of the vortex motion can be obtained from a comparison of the measured quantum creep rates $Q(0)$ with the Stephen interpolation formula in Eq. (17). From $n, \approx \lambda^{-2}$ follows that the prefactor on the right-hand side of Eq. (17) varies as $\kappa^2 = \lambda^2 / \xi^2$. Together with the measured quantum creep rates $Q(0) = 0.02$ at $x_n = 7.0$ and $Q(0) = 0.063$ at $x_n = 6.55$ and $B_c = 2$ T it follows that the function $f$ between the square brackets in Eq. (17) increases by a factor of 6 upon oxygen removal. The change of $\omega_B \tau$ upon oxygen removal can be evaluated from the variation of $\rho_n(0)$ and $\kappa$ with oxygen content. This leads to the observation that $\omega_B \tau$ increases by a factor of 19 when the oxygen content decreases from $x_n = 7.0$ to $x_n = 6.55$. The values of $\omega_B \tau(x_n = 6.55)$ and $\omega_B \tau(x_n = 7.0)$ can be found by simultaneously solving

$$f(\omega_B \tau(x_n = 6.55)) / f(\omega_B \tau(x_n = 7.0)) = 6,$$

$$\omega_B \tau(x_n = 7.0) = 19 \times \omega_B \tau(x_n = 6.55).$$

(21)

This leads to $\omega_B \tau(x_n = 6.55) = 0.14 \pm 0.01$ and $\omega_B \tau(x_n = 7.0) = 2.6 \pm 0.2$. Repeating this procedure at $B_c = 0.6$ T and $B_c = 6.9$ T leads within experimental error to the same estimates for $\omega_B \tau(x_n = 6.55)$ and $\omega_B \tau(x_n = 7.0)$.

An upper estimate for $\omega_B \tau(x_n = 6.55)$ follows from the observation that the function $f$ between square brackets in Eq. (17) increases with a factor of 6 upon oxygen removal. Using the fact that $f(\omega_B \tau) > 1$ for all values of $\omega_B \tau$, one sees from Fig. 1 that in the extreme case where $\omega_B \tau(x_n = 7.0)$ would be infinite, then $\omega_B \tau(x_n = 6.55)$ would need to be 0.18 in order to have a factor of 6 between the values of the function $f$. This leads to the condition $\omega_B \tau(x_n = 6.55) \leq 0.18$.

This analysis can be performed in a more accurate way by fitting the measured quantum creep rates to the interpolation

<table>
<thead>
<tr>
<th>$x_n$</th>
<th>$\xi$ (nm)</th>
<th>$\lambda$ (nm)</th>
<th>$\gamma$</th>
<th>$j_c$ (A/m²)</th>
<th>$L_c$ (nm)</th>
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<tr>
<td>7.0</td>
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</tr>
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<td>2.3</td>
</tr>
<tr>
<td>6.7</td>
<td>2.43</td>
<td>180</td>
<td>15.6</td>
<td>$4.46 \times 10^{10}$</td>
<td>2.0</td>
</tr>
<tr>
<td>6.6</td>
<td>2.80</td>
<td>193</td>
<td>22.1</td>
<td>$3.83 \times 10^{10}$</td>
<td>1.5</td>
</tr>
<tr>
<td>6.55</td>
<td>3.06</td>
<td>202</td>
<td>26.6</td>
<td>$4.47 \times 10^{9}$</td>
<td>3.6</td>
</tr>
</tbody>
</table>
down to $x_n = 6.55$. In the single vortex limit, the quantum creep rate as well as the critical current density should not depend on the average distance between the vortices, since in this case they move independently. However, it is still possible that the correlation length of the tunneling vortex segment is field dependent due to a field-dependent anisotropy.

In the Ginzburg-Landau theory the anisotropy is independent of field and temperature. This is in contrast with the behavior of a stack of Josephson coupled superconducting layers for which Daemen et al. have demonstrated that the anisotropy increases as the magnetic field increases due to the field dependence of the $c$-axis penetration depth $\lambda_c$. An increase of the anisotropy leads to a shorter correlation length and consequently to a larger quantum creep rate, which has been observed on bulk and powdered Bi$_2$Sr$_2$CaCu$_2$O$_{10}$ compound samples by Moehlecke and Kopelevich. Therefore, the observed increase of the quantum creep rate with increasing field does not necessarily contradict single vortex pinning in three dimensions.

In the case of tunneling of two-dimensional (2D) vortex pancakes the correlation length is equal to the distance between the CuO planes which is, of course, inherently field independent. This leads to a field-independent quantum creep rate which has been observed in the highly anisotropic materials Bi$_2$Sr$_2$CaCu$_2$O$_{8+}\delta$ (Refs. 10 and 11) and Tl$_2$Ba$_2$CaCu$_2$O$_{8+}\delta$.

At high magnetic fields where the vortices form (super) bundles the quantum creep rate decreases with increasing field. This is caused by a growth of the flux bundle and therefore an increased activation energy and Euclidean action as the field increases. For small bundles the quantum creep rate depends on the magnetic field as $Q(0) \propto B^{-1}\exp(-B_{e<0}^{1/2})$ whereas for large bundles $Q(0) \propto B^{-13/2}$. This rather strong decrease of $Q(0)$ with increasing field is in contrast with the results shown in Fig. 10. This means that the increase of the quantum creep rate cannot be explained by pinning of flux (super)bundles, neither in the local limit (small bundles), nor in the nonlocal limit (large bundles). This indicates that in the field regime investigated in this work quantum creep occurs in the single vortex limit. Furthermore, the crossover field $B_{c<0}$ separating single vortex pinning and small bundle pinning is determined by the condition $L_c = \gamma^{-1}a_0$ where $a_0 = (\Phi_0/B_{e<0})^{1/2}$ is the average distance between the vortices. Together with $L_c \approx 2.5$ nm and $\gamma = 5.6$ for $x_n = 7.0$ this gives $B_{c<0} \approx 11$ T, confirming that for YBa$_2$Cu$_3$O$_7$ quantum creep takes place in the single vortex limit. However, due to the increase of the anisotropy with decreasing oxygen content, $B_{c<0}$ shifts to lower values and for $x_n=6.55$, $\gamma = 26$ leads to $B_{c<0} \approx 0.5$ T. This would imply that for YBa$_2$Cu$_3$O$_{x_n}$ with reduced oxygen content, the tunneling object is not a single vortex segment but a vortex bundle. This is not consistent with the observed weak increase of $Q(0)$ with increasing field.

VI. PINNING PROPERTIES OF YBa$_2$Cu$_3$O$_{x_n}$

In this section we discuss the relaxation data by means of two different approaches. First, in Sec. VI A we use a collective pinning theory which assumes a particular current dependence of the activation energy. This approach, which is certainly not adequate for the entire temperature range, can
be used to determine values of the characteristic activation energy \( U_c \) at low temperature. In a second approach we use the generalized inversion scheme of Schnack (GIS) et al.\(^{16} \) to determine the temperature dependence of \( U_c \) and the true critical current density \( j_c \).

### A. Current-dependent activation energy

For intermediate temperatures the flux motion in the \( \text{YBa}_2\text{Cu}_3\text{O}_x \) film is well described in terms of thermally activated vortex hopping. The central parameter in this theory is the activation energy \( U(j,T,B_e) \) the flux line or flux bundle has to overcome in order to move from a given configuration to an adjacent metastable pinning configuration. The influence of the current density \( j \) on the activation energy has been investigated theoretically by Feigel’man et al.\(^{47} \) for the case of vortices (or vortex bundles) pinned by the collective action of randomly distributed weak pinning centers. They followed a scaling approach to obtain the size of the tunneling object. Together with the free-energy functional, this leads finally to a power-law relation

\[
U(j) \approx j^{-\mu}.
\]

Since \( U(j) \) should vanish for \( j = j_c \), it is now general practice to use the following interpolation relation:

\[
U(j) = \frac{U_c}{\mu} \left[ \left( \frac{j}{j_c} \right)^{\mu} - 1 \right].
\]

This equation contains the linear \((\mu = -1)\) and the logarithmic current dependence \((\mu = 0)\) as special cases. The value of the exponent \( \mu \) is determined by the pinning regime (single vortex, small bundles, or large bundles) and the dimensionality of the system. Theoretical values of \( \mu \) are given in the left-hand part of Table II.

Using the fact that during a sweep of the applied magnetic field the activation energy\(^{36} \) is given by \( U(j_c,T,B_e) = CkT \), where for a disk-shaped sample

\[
C = \ln(2v_0B_e/R(dB_e/dt))
\]

in which \( v_0 \) is an attempt velocity and \( R \) is the radius of the sample, one can evaluate the ratio \( j_s/j_c \) using Eq. (23) leading to

\[
j_s = j_c \left[ 1 + \frac{\mu kT}{U_c} \ln \left( \frac{2v_0B_e}{RdB_e/dt} \right)^{-1/\mu} \right].
\]

From this equation the dynamic relaxation rate

\[
Q = d\ln j_s/d\ln(dB_e/dt)
\]

can be calculated by taking the normalized derivative of \( j_s \) with respect to \( \ln(dB_e/dt) \). Finally, by using the expression for \( C \) given above, one finds\(^{39} \)

\[
\frac{T}{Q} = \frac{U_c}{k} + \mu C T,
\]

by using the expression for \( C \) given above, one finds\(^{39} \)

\[
FIG. 13. Temperature dependence of \( T/Q \) in an oxygen-deficient \( \text{YBa}_2\text{Cu}_3\text{O}_x \) film measured at \( B_e = 0.6 \) T (a), \( 2.0 \) T (b), and \( 6.9 \) T (c) at various values of the nominal oxygen content \( x_n \).

where \( k \) is Boltzmann’s constant. In Fig. 13 \( T/Q \) is displayed as a function of temperature for all oxygen contents considered in this work at \( B_e = 0.6, 2.0, \) and \( 6.9 \) T. The deviations at low and high temperatures are due to quantum creep and thermal depinning, respectively. The crossover from quantum creep to thermally activated creep takes place at \( T \approx 6 \) K, which is in agreement with values reported in the literature.\(^{4,6,12} \)

It is, in principle, possible to use the slope \( \mu C \) in the \( T/Q(T) \) graphs to identify the pinning regime. For this a value for \( C \) is required to evaluate \( \mu \). As shown below \( C \) can be determined by means of the GIS. The values of \( \mu \) determined from the slopes \( \mu C \) of the \( T/Q(T) \) graphs and the values of \( C \) found by means of the GIS (see Sec. VI B) are listed in Table III. Note that the values found for \( x_n = 6.85 \) are in good agreement with \( \mu = 0.95 \) at \( B_e = 1 \) T, reported by Ossandon et al.\(^{50} \)

<table>
<thead>
<tr>
<th>( x_n )</th>
<th>( B_e ) (T)</th>
<th>( C )</th>
<th>( \mu ) from ( C )</th>
<th>( \mu ) from ( U(j) )</th>
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<tbody>
<tr>
<td>7.0</td>
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</tr>
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<td>0.6</td>
<td>18</td>
<td>0.34</td>
<td>0.32</td>
</tr>
</tbody>
</table>

\( \mu \) is required to evaluate \( \mu \). As shown below \( C \) can be determined by means of the GIS. The values of \( \mu \) determined from the slopes \( \mu C \) of the \( T/Q(T) \) graphs and the values of \( C \) found by means of the GIS (see Sec. VI B) are listed in Table III. Note that the values found for \( x_n = 6.85 \) are in good agreement with \( \mu = 0.95 \) at \( B_e = 1 \) T, reported by Ossandon et al.\(^{50} \)
According to these values one would conclude that for $x_\text{r}=6.85$ and $x_\text{r}=7.0$ our sample is in a regime intermediate between single vortex ($\mu=1/7$) and small bundles ($\mu=3/2$). The following experimental results are, however, in favor of the single vortex regime. To discuss this point we note that a rather general relation between $U_c$ and $j_c$ has been derived by Schnack et al.\textsuperscript{47} and Wen et al.\textsuperscript{51} For example, for a collectively pinned single vortex $U_c\approx j_c \Phi_0 L_c \xi$ where the correlation length $L_c \approx j_c^{1/2}$ in the 3D regime. This means that $U_c \approx j_c^{p \mu}$. In the 2D regime $L_c$ is given by the thickness of the layers and consequently $U_c \approx j_c^{p \mu}$. The pinning barrier for a flux bundle is in the collective pinning theory given by $U_c\approx j \Phi_0 V_p r_p$ where $V_p$ is the volume of the flux bundle and $r_p$ is the range of the potential. A scaling approach has been used to obtain the volume of the flux bundle.\textsuperscript{47} This leads finally to a power-law relation between $U_c$ and $j_c$ in the regime of flux bundles.\textsuperscript{16} One can generally write

$$U_c(T) \approx [j_c(T)]^p.$$ \hspace{1cm} (26)

The values for the exponent $p$ are displayed in the right-hand side of Table II. Although both exponents $\mu$ and $p$ are used to relate pinning energies to current densities, their physical origin is very different. The exponent $\mu$ describes the relation between the activation energy and the current density when other parameters such as temperature, external field, and oxygen concentration are kept constant. This $U(j)$ relation is partially scanned during a relaxation experiment. The exponent $p$ states that $U_c$ and $j_c$ do not change independently if the external parameters $T$ and $B_c$ or the disorder in the sample are varied.

Extrapolation of the linear part of the $T/Q$ curves to $T=0$ K gives the values of the characteristic pinning energy $U_c$, which are shown as a function of oxygen content and magnetic field in Fig. 14. Because $U_c$ and $j_c$ are determined independently it is possible to map $U_c$ as a function of $j_c$ by varying the oxygen concentration and the oxygen ordering of the sample. Data obtained at the same magnetic field can be used for checking Eq. (26). This is displayed in Fig. 15 from which one obtains $p=0.51$ at $B=0.6$ T, $p=0.48$ at $B=2.0$ T. At $B=6.9$ T one finds the value $p=0.29$. These values imply that below $B=2$ T the system is certainly in the single vortex regime where each vortex is pinned by the collective action of random weak pinning centers since in this case theory predicts $p=1/2$ in contrast to $p=-3/2$ and $p=-1/2$ for small and large (super) bundles, respectively. At high fields the system is gradually switching from the single vortex regime to the bundle regime and $p$ decreases ($p=0.29$ at $B_c=6.9$ T).

From the data displayed in Fig. 15 one can obtain for the prefactor $C$ in the power-law relation $U_c=C_j^{p/2}$ the value $C=(5.5 \pm 0.8) \times 10^{-27}$ J m/A$^{1/2}$. Combination of the relation $U_c\approx j \Phi_0 L_c \xi$ with the estimate $L_c \approx \xi^{3/2} j_c^{1/2}$ derived previously leads directly to $C=(0.25 \Phi_0^{3/2} \xi^{3/2} \gamma \mu_0)^{1/2}$. However, before applying this relation to experimental results it is important to realize that it is not an exact relation as it is derived from a dimensional estimate. This point is further illustrated by calculating the prefactor $\tilde{C}$ in a different way. This is done in the next section where $U_c$ and $j_c$ are expressed in terms of the pinning parameter $\delta$. Elimination of this parameter gives finally $\tilde{C}=(0.65 \Phi_0^{3/2} \xi^{3/2} \gamma \mu_0)^{1/2}$. This result differs from the previously obtained expression for $C$ by a factor of 2.6.

Using the values $\lambda = 146$ nm, $\xi = 1.6$ nm, and $\gamma = 5.6$ for fully oxidized YBa$_2$Cu$_3$O$_{6+\delta}$ gives $\tilde{C}=1.6 \times 10^{-27}$ J m/A$^{1/2}$. Application of the second equation for $\tilde{C}$ gives $C=4.3 \times 10^{-27}$ J m/A$^{1/2}$, which is in good agreement with the experimentally observed value. However, the ratio $\xi^{3/2} \gamma \mu_0$ is not independent of the oxygen content but decreases with a factor of 2.5 when the oxygen content is decreased from 7.0 to 6.55. A decreasing $\tilde{C}$ would lead to a downward curvature in the $U_c(j_c)$ diagram, which is not observed here. Considering the uncertainty in the theoretically obtained $\tilde{C}$ and the experimental values of $\xi$ and $\lambda$, we conclude that the agreement with the experimental value is satisfactory.
B. The generalized inversion scheme

A more accurate way to determine the pinning regime and the dimensionality of the system is by using the generalized inversion scheme (GIS) developed by Schnack et al.\textsuperscript{16} This is a method to calculate the true critical current density and the activation energy from a set of induced current densities ($j_s$) and dynamic relaxation rates ($Q$) measured as a function of temperature at a fixed external field. It also gives information about the exponent $p$, leading to a determination of the pinning regime and dimensionality.

In contrast to the well-known inversion method of Maley et al.\textsuperscript{52} the GIS takes the temperature dependence of $U_c$ and $j_c$ into account, without assuming an explicit analytic expression describing the temperature dependence of $U_c$ and $j_c$. The starting point of the GIS is the assumption that the temperature and current dependence of the activation energy can be separated in the form

$$U(j_c, T; B_x) = g(T; B_x) \times f(J),$$  \hspace{1cm} (27)

where $J = j_c / j_s(T)$ in which $j_c$ is the true critical current density for which the activation energy vanishes. During a sweep of the applied magnetic field the activation energy is given by\textsuperscript{16}

$$U(j, T; B_x) = CkT.$$  \hspace{1cm} (28)

The parameter $C$ can be estimated from the measured data, since at low temperatures $g$ and $j_c$ are nearly temperature independent, leading to

$$C = \lim_{T \to 0} \frac{1}{Q} \frac{d\ln j_c}{d\ln T}.$$  \hspace{1cm} (29)

It is important to realize that this relation is only valid in the regime of thermally activated flux motion. Direct application of this relation to measured data would lead to an underestimate of $C$, because $d\ln j_c / d\ln T = (Tj_c d(j_c / dT)$ becomes zero in the low-temperature limit, while $Q$ remains finite due to quantum creep. As discussed in the previous section, for the extrapolation of $Q$ down to low temperatures, we use the linear temperature dependence of $T/Q$ which extrapolates to a finite value at $T = 0$ K. The extrapolation of $j_c$ is more difficult. It might be tempting to use a linear extrapolation of $\ln j_c$, since the data displayed in Fig. 8 suggest that $\ln j_c$ decreases linearly with temperature. However, as discussed recently by Wen et al.\textsuperscript{51} it follows from collective pinning theory that $\ln j_c$ does not vary linearly with temperature. The slope $d\ln j_c / dT$ is nearly constant at intermediate temperatures, but increases sharply below $\approx 10$ K. Unfortunately, this behavior is fully masked by quantum creep. Therefore it follows that extrapolation of the linear temperature dependence of $\ln j_c$ leads to an underestimation of both $j_c$ and $d\ln j_c / dT$ at $T = 0$ K and consequently by application of Eq. (29) on experimental data as outlined above, to an underestimation of $C$. In this work $C$ has been determined by using the fact that at low temperatures $j_c(T)$ and $g(T)$ are to first order independent of the temperature.

A second relation between $U_c$ and $j_c$ is provided by the power-law relation discussed in the previous section. The function $g(T)$ can be associated with $U_c$ leading to

$$g(T) = U_c(T) \times [j_c(T)]^p \times G(T),$$  \hspace{1cm} (30)

where the explicit temperature dependence of the activation energy is given by the thermal function $G(T)$ and $B_x$ has been dropped in order to simplify the notation since we consider only data at a given fixed field $B_x$. This function can be derived from the temperature dependence of the prefactor in the power-law relation between $U_c$ and $j_c$. Therefore, $G(T)$ depends on the pinning regime (SV, SB, LB) and the dimensionality of the vortices in the sample (see Ref. 51). Following the method outlined by Schnack et al.\textsuperscript{16} this leads to

$$\frac{j_c(T)}{j_c(0)} = \exp \left[ \int_0^T \frac{CQ(T)(1 - d\ln G / d\ln T) + (d\ln j_c / d\ln T)}{1 + pCQ(T)} \frac{dT}{T} \right]$$  \hspace{1cm} (31)

and

$$g(T) = \exp \left[ \int_0^T \frac{CQ(T) + (d\ln j_c / d\ln T) + (1/p)(d\ln G / d\ln T)}{1 + pCQ(T)} \frac{dT}{T} \right].$$  \hspace{1cm} (32)

Until now, $p$ has been treated as a free parameter. However, from the discussion in the previous section, leading to the power-law relation which defines $p$, it follows that only the values $p = -3/2, -1/2, 0, 1/2,$ and $1$ are physically meaningful. In the analysis of the experimental data we have only used these values of $p$. The results of the GIS put more restrictions on $p$. This is obvious from an inspection of Fig. 16 where the calculated values for $j_c$ are shown together with the measured $j_c$ values. The $j_c$ values are calculated for $p = -3/2, -1/2, 0, 1/2,$ and $1$, respectively. In this example $B_x = 0.6$ T and $x_n = 7.0$. A choice of $p=0$ leads to $j_c > j_c$.
above a certain temperature. This would lead to the conclusion that the sample is in the flux-flow regime. This is in contradiction with the nearly linear dependence of \( j_v \) on \( \ln (dB_c/dt) \) observed at all temperatures \( T < T_{IP} \). The corresponding \( g(T) \) values are shown in Fig. 17. Choosing \( p \leq 0 \) leads to a rapid divergence of the function \( g(T) \). This is physically not acceptable since at \( T = T_c \) one expects \( U(T_c) = 0 \) and thus \( g(T_c) = 0 \).

Our choice of \( p \) is thus limited to \( p = 1/2 \), corresponding to 3D single vortex pinning and \( p = 1 \), corresponding to 2D single vortex pinning. From Figs. 16 and 17 it is clear that in both cases the temperature dependence of \( j_v(T) \) and \( U_c(T) \) is in good agreement with the theoretical predictions given by the (3D) and (2D) lines. Therefore, in this case it is not possible to determine the dimensionality of the vortex system by means of the GIS only (although the slightly better agreement for \( p = 1/2 \) suggests that the 3D single vortex regime is more appropriate). However, in the previous section it has been shown that \( U_c \) varies as \( j_v^p \) where \( p = 0.51 \) at \( B_c = 0.6 \) T and \( p = 0.48 \) at \( B_c = 2 \) T. Therefore, we conclude that the system is in the single vortex regime and that vortices behave as 3D objects, despite their short \( c \)-axis correlation length.

C. Microscopic origin of pinning

In the framework of collective pinning theory, the pinning potential is due to weak short-range disorder. In type-II superconductors, this disorder arises from spatial fluctuations of the critical temperature and/or the mean free path. These two types of pinning are called \( \delta T_c \) pinning and \( \delta l \) pinning, respectively. Blatter et al.\cite{20} use the expansion of the Gor'kov equations near \( T_c \) in the clean limit to relate the Ginzburg-Landau parameters \( \alpha \) and \( m \) to microscopic quantities. These are used to determine the dimensionless pinning parameters \( \delta_a \) and \( \delta_m \) for \( \delta T_c \) and \( \delta l \) pinning, respectively. This leads to a temperature dependence of the pinning parameter given by \( \delta_a \propto (1 - t)^{-1/2} \) and \( \delta_m \propto (1 - t)^{3/2} \) with \( t = T/T_c \). From the fact that the correlation length varies as \( (1 - t)^{-1/2} \) for temperatures close to \( T_c \) we infer \( \delta_a \propto \xi \) and \( \delta_m \propto \xi^3 \).

In the case of 3D single vortex pinning, Blatter et al.\cite{20} showed that all quantities characterizing the collectively pinned vortex line can be expressed in terms of the disorder parameter \( \delta \) as

\[
L_c = \xi \delta^{-1/3} \epsilon^{4/3},
\]

\[
U_c = \mu_0^{-1} B_c^2 \xi \delta^{1/3} \epsilon^{5/3},
\]

\[
j_c = j_0 \delta^{2/3} \epsilon^{-2/3},
\]

where \( \epsilon \) is the mass anisotropy ratio, \( B_c = \Phi_0/2 \sqrt{2} \pi \lambda \xi \) and \( j_0 = 4 B_c \sqrt{3} \sqrt{\mu_0 \lambda} \) is the depairing current density. Note that the definition of the disorder parameter used here is different from the more widespread disorder parameter \( \gamma \) defined by \( \gamma = f_{PB} n_i \xi^2 \) where \( f_{PB} \) is the force exerted on a single flux line by a weak pinning center and \( n_i \) is the density of these pinning centers. The last two expressions of Eq. (33) can be used to derive a \( \delta \)-independent power-law relation between \( U_c \) and \( j_c \):

\[
U_c = \left[ \frac{B_c^2 \xi \epsilon}{\mu_0 j_0} \right]^{1/2} (T, B_c) = G(T) j_c^{1/2}.
\]

The temperature dependence of \( j_c \) and \( U_c \) is found as follows. As a first step, in the case of 3D single vortex pinning, the thermal function \( G(T) \) defined by Eq. (30) can be derived from the temperature dependence of the prefactor in the power-law relation between \( U_c \) and \( j_c \). In Eq. (34). Comparison of Eqs. (30) and (34) leads straightforwardly to \( G(T) \propto B_c \xi \delta \propto (1 - t^2)^{1/2} \). With \( B_c \propto (1 - t^2) \), \( \xi \propto (1 + t^2)(1 - t^2)^{1/2} \), and \( \lambda \propto (1 - t^4)^{-1/2} \) and using \( G(0) = 1 \) one finds

\[
G_{SV}(t) = (1 - t^2)^{-1/4} (1 + t^2)^{3/4}.
\]

This relation does not depend on the microscopic origin of pinning. The derivation of \( G(t) \) proceeds in a similar way in the case of pinning of small and large superbundles (see Ref. 51).

With Eq. (33) it is possible to calculate \( j_c(t)/j_c(0) \) once a theoretical model for \( \delta \) has been chosen. In the case of \( \delta T_c \) pinning one finds

\[
\frac{j_c(t)}{j_c(0)} = (1 - t^2)^{7/6} (1 + t^2)^{5/6}
\]

and

\[
g(t) = (1 - t^2)^{1/3} (1 + t^2)^{5/3}.
\]

In the case of \( \delta l \) pinning one finds similarly

\[
\frac{j_c(t)}{j_c(0)} = (1 - t^2)^{5/2} (1 + t^2)^{-1/2}
\]

and

\[
g(t) = (1 - t^4).
\]

In other cases, the evaluation of the temperature dependence of \( j_c \) and \( U_c \) proceeds in a similar way.\cite{51} Until recently it
FIG. 18. Critical current density $j_c$ calculated with the GIS, using $p = 1/2$, from data obtained at the YBa$_2$Cu$_3$O$_{x_n}$ film at $B_c = 0.6$ T and $x_n > 6.6$ (a) and $B_c = 2.0$ T and $x_n > 6.7$ (b). The value $p = 0.5$ corresponds to 3D single vortex pinning. The full line corresponds to pinning due to spatial fluctuations in the charge carrier mean free path ($\delta l$ pinning) and the dashed line to pinning due to spatial variations in the critical temperature ($\delta T_c$ pinning).

FIG. 19. Temperature dependence of the normalized activation energy, calculated with the GIS, using $p = 1/2$, from data obtained for the YBa$_2$Cu$_3$O$_{x_n}$ film at $B_c = 0.6$ T and $x_n > 6.6$ (a) and $B_c = 2.0$ T and $x_n > 6.7$ (b). The full line corresponds to $\delta l$ pinning and the dashed line to $\delta T_c$ pinning. Note that the theoretical functions given by the solid and the dashed line are fully determined by the value of $T_c$ and do not contain fit parameters.

The current density $j_s(T)$ measured at a temperature $T$ corresponds to a current density $j_s = j_s(0) \times [j_s(T)/j_s(T_c)]$ at $T = 0$ K. According to Eq. (28) the activation energy $U[j_s(T), T]$ at a temperature $T$ is given by $U(j) = CkT$. The corresponding activation energy $U(j, 0)$ is then evaluated from $U[j_s(T), T] = U(j) \times X[j_s(T), 0]$. Measurements of $j_s$ and $U(j, T)$ at various temperatures are thus mapped onto a single curve, where the points at low current density are deduced from measurements at high temperature and vice versa. The $U(j, T = 0)$ curves for $x_n = 6.6, 6.7, 6.85$, and $7.0$ at $B_c = 0.6$ T and $x_n = 6.7, 6.85$, and $7.0$ at $B_c = 2.0$ T are shown in Figs. 20(a) and 20(b), respectively. The presentation in double logarithmic coordinates clearly shows that $U(j)$ cannot be written as $U \propto j^{-\mu}$ with a single valued, i.e., current independent exponent $\mu$ which is different from the exponent $\mu$ defined by Eq. (23). The solid lines are fits to Eq. (23) which takes the condition $U(j_s) = 0$ into account. For $x_n = 6.85$ and $7.0$ the $U(j)$ values obtained at high temperatures where $T/Q$ decreases with increasing temperature (see Fig. 13) have been excluded from the fitting procedure. In contrast to van der Beek et al. $^{52}$ who uses $U \propto j^{-\tilde{\mu}}$ instead of Eq. (23) we find that $U(j)$ can be satisfactorily described by a single, current independent value of $\mu$. The values of $\mu$, was not clear which pinning mechanism ($\delta T_c$ pinning or $\delta l$ pinning) was the dominant one. The GIS is a very powerful tool to obtain $j_s(t)/j_s(0)$ and $g(t)$, because it does not make any assumption about the type of pinning. The only parameter which has to be specified is the exponent $p$. From the discussion above it is clear that YBa$_2$Cu$_3$O$_{x_n}$ is in the 3D single vortex state, corresponding to $p = 1/2$, leaving no other possibilities.

The GIS has been applied to the values of $j_s(T)$ and $Q(T)$ measured at various nominal oxygen concentrations. The parameter $C$ in Eq. (28) has been obtained from the requirement that the temperature dependence of $j_s$ and $U_c$ is weak at low temperatures. The values of $C$ found in this way, which are displayed in Table III, are systematically slightly higher than the values of $C$ obtained by application of Eq. (29). The temperature dependence of $j_c$ at $B_c = 0.6$ and $2.0$ T is shown in Figs. 18(a) and 18(b), respectively. The corresponding temperature dependence of $U_c$ is shown in Fig. 19(a) ($B_c = 0.6$ T) and Fig. 19(b) ($B_c = 2.0$ T). Only the results for $x_n > 6.6$ at $B_c = 0.6$ T and $x_n > 6.7$ at $B_c = 2.0$ T are shown.

The current dependence of the activation energy at $T = 0$ can be calculated from the temperature dependence of $C_kT$ and $g$, shown in Figs. 18 and 19 in the following way.
fluctuations when \( T \) is maintained on YBa\(_2\)Cu\(_3\)O\(_7\) and YBa\(_2\)Cu\(_4\)O\(_8\) films.\(^5\) Recently on a twin-free DyBa\(_2\)Cu\(_3\)O\(_7\) single crystal,\(^5\) this result has also been obtained. The predictions for \( j_c(T) \) are in remarkably good agreement with the theoretical predictions Eqs.\(^38\) and\(^39\) and the calculated values of \( j_c(T)/j_c(0) \) and \( g(T) \) gradually breaks down at lower oxygen contents and higher magnetic fields. In these cases \( U_c(0) \) is lower than \( \approx 60 \) K and the effect of thermal fluctuations when \( T \) approaches \( T_{\text{irr}} \) is not negligible any more. The predictions for \( \delta T_r \) are in total disagreement with the observations. This result has also been obtained on YBa\(_2\)Cu\(_3\)O\(_7\) and YBa\(_2\)Cu\(_4\)O\(_8\) films\(^5\) and recently on a twin-free DyBa\(_2\)Cu\(_3\)O\(_7\) single crystal.\(^5\)

The disorder parameter \( \delta \) can be evaluated from Eq.\(^{33}\) leading to \( \delta = \gamma^{-1}(j_c/j_0)^3/2 \) where \( j_0 = \Phi_0/(3\sqrt{2}\pi\mu_0\lambda^2\xi) \). The variation of \( \delta \) with oxygen content is calculated from the values of \( \lambda, \xi, \gamma, \) and \( j_c \) given in Table I. This leads to \( \delta = 3.5 \times 10^{-3} \) for \( x_n = 7.0 \) and to \( \delta = 1.6 \times 10^{-5} \) for \( x_n = 6.55 \). Furthermore it is found that \( \delta \) decreases monotonically with decreasing oxygen content. This is somewhat surprising since disorder is expected to be proportional to the number of defects.

In this work the induced current density \( j_c \) and dynamic relaxation rate \( Q \) of an YBa\(_2\)Cu\(_3\)O\(_{x_n}\) film with adjustable oxygen content have been measured as a function of temperature and magnetic field. The relaxation rate does not extrapolate to zero at \( T = 0 \) K which is caused by quantum creep of the vortices through the energy barriers separating metastable vortex configurations. We observe that the quantum creep rate and the normal-state resistivity increase as the oxygen content is decreased. However, the quantum creep rate is not proportional to the normal-state resistivity which is predicted by collective pinning theory. This behavior cannot be explained by taking the influence of the correlation length \( L_c \) into account, since this is only weakly dependent on the oxygen content. The weak variation of \( Q(0) \) with \( x_n \) indicates that quantum creep occurs in a transition regime between dissipative tunneling and Hall tunneling. This implies that the average direction of vortex motion is neither parallel nor perpendicular to the current. The process of quantum creep shifts towards the dissipative limit for decreasing oxygen content. The weak increase of \( Q(0) \) with magnetic field indicates that quantum creep is certainly not involving vortex bundles.

An independent determination of \( j_c \) and the characteristic pinning energy \( U_c \) shows that the system remains in the single vortex limit at higher temperatures, where the vortices move due to thermal activation over the energy barriers. This is shown in more detail by calculating the true critical current density and the thermal activation energy by means of the generalized inversion scheme. This procedure diverges if one assumes pinning of vortex (super)bundles. For \( x_n \geq 6.6 \) at \( B_c = 0.6 \) T and for \( x_n \geq 6.7 \) at \( B_c = 2.0 \) T the calculated temperature dependences of the true critical curve density and the activation energy are in good agreement with theoretical predictions, showing that in these cases the pinning is due to fluctuations in the charge carrier mean free path. For lower oxygen contents and higher magnetic fields the agreement between the results of the inversion and the theoretical predictions becomes less good. This is possibly explained by the presence of thermal fluctuations, since in these cases the activation energy is strongly reduced. We have observed that the activation energy and the critical current density systematically decrease with decreasing oxygen content. This leads to the conclusion that oxygen vacancies are not the main source of pinning in YBa\(_2\)Cu\(_3\)O\(_{x_n}\) films.

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