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Residential Search and Mobility in a Housing Market Equilibrium Model

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Abstract

In this paper, we propose an equilibrium model for the housing market which provides an explanation for observed housing consumption of households over their lifetimes. The moving behavior of households is described as a stochastic dynamic process in which households' moving decisions depend on information which is obtained over time. Households move when the offer exceeds an endogenously determined threshold. On the basis of the households' moving behavior, the steady-state distribution of households over the housing stock is obtained. On the supply side of the market, landlords are looking for households to occupy their vacant dwellings. Their strategy is to set rents in a mixed strategy in order to profit from imperfect information. After formulating search behavior of households as well as the behavior of landlords, the market equilibrium is derived. We explore the sensitivity of the equilibrium to changes in the structural parameters.

Key Words: household behavior, residential mobility, housing markets

JEL Code: D1, R21

1. Introduction

The imperfect operation of the housing market has become a major issue in housing market analysis over the last decade. Indeed, it needs little explanation to understand that perfect housing markets are far removed from economic reality. The relatively high costs of housing construction, its durability, indivisibility, heterogeneity and locational fixity[1] all hamper the instantaneous adjustment of housing consumption to changes in preferences or socioeconomic circumstances. Indeed, the housing market can usually be characterized by:[2] (1) extensive search efforts by households in order to obtain a dwelling; (2) idiosyncratic preferences of households; (3) adjustment of housing consumption by moving; (4) frictions leading to vacancies;[3] (5) oligopolistic supply of dwellings, which gives suppliers market power in setting the price; (6) price dispersion; and, (7) relatively slow adjustment of supply in response to market changes.

The implications of this market structure are far-reaching, in that observed behavior does not only reveal households' preferences for housing, but also reveals housing market factors. The purpose of this paper is to show that search theory provides an explanation for

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observed housing consumption of households over their lifetimes. This paper formulates an equilibrium housing market model that ties these stylized facts[4] together, drawing on analogies between the housing market and (equilibrium models for) the labor market.

Numerous contributions to the housing market literature have used theories of search and matching in explaining vacancies, mobility or prices (see e.g., Hall, 1980; Arnott, 1989; Wheaton, 1990; Igarashi, 1991; Read, 1997; Anas, 1997; Igarashi and Arnott, 2000). This approach, denoted by Anas (1997) as the ''New Housing Economics'', typically treats some but not all aspects of the housing market. On the one hand, equilibrium models are developed with idiosyncratic differentiation in housing units and tastes, but ignoring residential mobility of existing market participants or assuming exogenous tenancy spells (cf. Arnott, 1989; Igarashi, 1991; Anas, 1997; Igarashi and Arnott, 2000). On the other hand, partial equilibrium models of the housing market are developed, which either formulate the households' search process, while assuming a certain price distribution (cf. Hall, 1980), or, alternatively, formulate suppliers' behavior with price dispersion, but with the behavior of households treated exogenously (cf. Read, 1997). A search model which explicitly takes account of residential mobility, based on behavior of both types of market participants, has to the best of our knowledge not yet been constructed for the housing market. Our aim is to formulate such a model.

Following recent contributions in search and posting games in equilibrium search models and matching models (cf. Burdett, 1990; Mortensen, 1990; Burdett and Mortensen, 1998; Mortensen, 1999), in this paper we formulate a housing market equilibrium model. The principal feature of this equilibrium search model is pure price dispersion in search equilibrium as a result of residential mobility. The Burdett–Mortensen equilibrium search model turns out to be a particularly appropriate framework for analyzing households' housing search and mobility process and its aggregate market outcome. In maximizing expected discounted utility over their lifetimes, households climb up the property ladder towards dwellings providing higher utility levels. Moreover, the model is particularly useful for analyzing different housing market states over the households' lifetimes. In the analysis, we distinguish between temporary accommodation and permanent accommodation, as (potential) households share a dwelling and do not instantaneously find a permanent dwelling. The residential spell in the dwelling is endogenously determined within the model. The model we now develop in this paper analyses, *inter alia*, market power, temporary and permanent accommodation, and residential mobility, but ignores idiosyncratic differentiation among housing units, tastes and search costs. One possible direction for future research would be to incorporate these idiosyncracies in the housing market equilibrium search model formulated in this paper. Another potential extension would be to let offer arrival rates depend on vacancies created, using a matching technology to determine the level of the offer arrival rates (cf. Mortensen and Pissarides, 1999).

The structure of the paper is as follows. Section 2 describes the basic equilibrium search housing market model with notation defined in Appendix A. After we have described the housing market, we present both the households' search and mobility behavior in the housing market, and the behavior of the landlords. Section 3 gives policy implications and comparative static results. Finally, Section 4 offers concluding remarks and a number of directions for future research.

2. Search and mobility in the housing market

2.1. Model description

We consider a housing market with households constantly entering and leaving the housing market. Each household arrives according to a Poisson process with constant rate θ, and depart with a Poisson process of the same rate,[5] so that the average population is constant over time. The measure of households is denoted by m, either temporarily m_T or permanently accommodated $(m - m_T)$. In this paper, temporary accommodation, refers to those households who share accommodation with another household, live in a slum, live in a motel or are otherwise temporarily accommodated, circumstances sometimes designated as homelessness.[6] Permanent accommodation refers to all other dwellings. These are units in which most households typically live, such as apartments, terraced, semi-detached or detached dwellings. Other interpretations are possible as well.[7]

Households do not initially have a permanent dwelling but have to search for it. The duration in temporary accommodation depends on the arrival rate of housing market opportunities and on the offered unit. We assume that landlords offer housing units with a given rent to searching households following a Poisson process at an exogenous, per household rate λ_T for households in temporary accommodation, and at an exogenous, per household rate λ_P for households in permanent accommodation. Upon receipt of an offer, the household has to decide whether or not to accept it, for which decision the household sets a reservation rule. We do not allow for bargaining. During the spell of residence we assume that the household experiences a constant level of utility. However, as households continuously search for better dwellings providing higher utility levels, they may subsequently move to other dwellings. Finally, due to urban renewal, each household might be forced to move to temporary accommodation, which happens following a Poisson process at a constant exogenous rate κ.

On the basis of these Poisson processes, the sojourn time in temporary and permanent accommodation can be obtained. For a household in temporary accommodation, new opportunities arrive following a Poisson process at a rate λ_T, so that the probability of an acceptable offer equals $\lambda_T\overline{F}(u^*)$, where $\overline{F} \equiv 1 - F(u)$, $F(u)$ is the associated utility distribution function, and u^* denotes the reservation utility value. As exits out of temporary accommodation also depend on the population death rate, the tenancy period in temporary accommodation is exponentially distributed with parameter $(\theta + \lambda_T\overline{F}(u^*))$. Likewise, for a household in permanent accommodation with associated utility level $u = u^1$, the tenancy period is exponential with parameter $\left(\kappa + \theta + \lambda_T\overline{F}(u^1)\right)$.

The housing units are offered by risk-neutral landlords maximizing expected discounted profits. Free entry is assumed as well, which drives the value of a vacant unit to zero. Landlords set a (time-invariant) rent in a non-cooperative equilibrium to gain from imperfect information. Equilibrium in the market is determined from the search and mobility behavior of the household and the behavior of the landlord, as well as steady-state housing market conditions.

In the remainder of this section, we fully describe the moving behavior of households, and characterize the steady-state housing market, the landlords' behavior and the resulting market equilibrium.

2.2. Residential search and mobility

The moving behavior of households can be described as a stochastic dynamic process in which the households' moving decisions depend on information which is obtained over time. The risk-neutral agents evaluate housing opportunities by means of a utility function u with the rent r as its argument:

$$u = u(r). \tag{1}$$

The utility function $u: R_+ \to R_+$ is continuously differentiable, decreasing in r. Under the given assumptions for the preferences u, it can be shown[8] that a functional equation exists for the sequence problem.

Expected lifetime wealth depends on the current accommodation, either temporary or permanent, and possible moves, either forced or not, to other dwellings in the future. Let $u^o = u(r^o)$ be the instantaneous utility in temporary accommodation. The residential choice and mobility decision process of households is best understood when formulating lifetime wealth in discrete time. This functional equation for a household in temporary accommodation, denoted by V_T, for a short period of time Δt, can be written as

$$\begin{aligned} V_T(r^o) = \frac{1-\theta\Delta t}{1+\rho\Delta t}\,&[u(r^o)\Delta t + \lambda_T \Delta t E_{r_x}\,(\max\{V_P(r_x), V_T(r^o)\}) \\ &+ (1-\lambda_T\Delta t)V_T(r^o) + o(\Delta t)], \end{aligned} \tag{2}$$

where E is the expectations operator with respect to the rent offer r_x; u^o the (exogenous) instantaneous utility in temporary accommodation; $1/(1+\rho)$ the (exogenous) discount rate; θ the (exogenous) death rate; and λ_T the (exogenous) offer arrival rate; V_P value in a permanent resident, and $o(\Delta t)$ the value in the event of more than one offer in period Δt. Thus, for the short period of time in which the household remains in the market, an event with probability $1-\theta\Delta t$, it receives a housing offer with probability $\lambda_T\Delta t$, and more than one offer with probability $o(\Delta t)$ (cf. Albrecht et al., 1991). For future reference, we rewrite the functional equation in continuous time by multiplying both sides by $(1+\rho\Delta t)$, dividing through Δt and letting $\Delta t \to 0$ to obtain

$$(\theta+\rho)V_T(r^o) = u(r^o) + \lambda_T[E_{r_x}(\max\{V_P(r_x), V_T(r^o)\}) - V_T(r^o)]. \tag{3}$$

Further interpretation of the wealth equation reveals that for a household in a temporary residence with an instantaneous utility level u^o, lifetime utility depends on the discounted level of instantaneous utility u^o and future gains in lifetime utility due to new residential offers. The second term in brackets indicates the expected utility gain. As the wealth equation shows, lifetime value depends on the offered dwelling as well, as households choose the maximum of the offer, given their expectations concerning the offer distribution and the current residence. Likewise, the value function for a household in a permanent residence, denoted by V_P, can be formulated:

$$(\theta+\rho)V_P(r) = u(r) + \lambda_P[E_{r_x}(\max\{V_P(r_x), V_P(r)\}) - V_P(r)] \\ + \kappa[V_T(r^o) - V_P(r)], \tag{4}$$

where λ_P is the associated (exogenous) offer arrival rate, and κ the (exogenous) rate of urban renewal. Note that, since a household in a permanent residence can be forced to move at rate κ into temporary accommodation due to urban renewal, lifetime utility is influenced by this event.

Comparison of these two value functions suggests that for a household in a permanent residence the optimal strategy is to accept any dwelling that is strictly better than the present one, whereas for a household temporarily accommodated, acceptance requires the value of a permanent dwelling to be higher than the value of the temporary accommodation, i.e.,

$$V_P(r^*) = V_T(r^o). \tag{5}$$

The following proposition gives the optimal strategy of a household searching for a residence.

Proposition 1: For a household residing in temporary accommodation, the optimal reservation rule is

$$u(r^*) = u(r^o) + (\lambda_T - \lambda_P)\int_{u(r^*)}^{\overline{u}} \frac{\overline{F}(u)}{\rho+\kappa+\theta+\lambda_P\overline{F}(u)}\mathrm{d}u, \tag{6}$$

while for a household in a permanent residence the optimal strategy is to accept every offer strictly better than the existing residence:

$$u^*(r) = u(r).$$

Proof (see also Mortensen and Neumann, 1988). To arrive at the optimal reservation rule for a household in temporary accommodation, let r^* be the reservation rent s.t. $V_P(r^*) = V_T(r^o)$. Then, from (3)–(5) the reservation rule equals

$$u(r^*) = u(r^o) + (\lambda_T - \lambda_P)[E_{r_x}(\max\{V_P(r_x), V_P(r)\}) - V_P(r)] \\ + \kappa[V_T(r^o) - V_P(r)], \tag{i}$$

which can be rewritten as

$$u^*(r) = u(r^o) + (\lambda_T - \lambda_P) \int_{u^*}^{\bar{u}} [V_P(r) - V_P(r^*)] \, \mathrm{d}F(u) \tag{ii}$$

$$= u(r^o) - (\lambda_T - \lambda_P) \int_{u^*}^{\bar{u}} [V_P(r) - V_P(r^*)] \, \mathrm{d}\overline{F}(u), \tag{iii}$$

where u and r are related by $u = u(r)$. Now, since the variable of integration is u and u is monotonically decreasing in r, we can write $r = u^{-1}(u)$. Then, integration by parts gives

$$u(r^*) = u(r^o) - (\lambda_T - \lambda_P) \left(\left[(V_P(r) - V_P(r^*)) \overline{F}(u) \right]_{u^*}^{\bar{u}} - \int_{u^*}^{\bar{u}} V_P{}'(r) \frac{\partial r}{\partial u} \overline{F}(u) \, \mathrm{d}u \right)$$

$$= u(r^o) + (\lambda_T - \lambda_P) \int_{u^*}^{\bar{u}} V_P{}'(r) \frac{1}{u'(r)} \overline{F}(u) \, \mathrm{d}u. \tag{iv}$$

From (4)

$$(\rho + \theta) V_P(r) = u(r) + \lambda_P \int_{u^*}^{\bar{u}} [V_P(r_x) - V_P(r^*)] \, \mathrm{d}F(u(r)) + \kappa [V_T(r^o) - V_P(r)]$$

$$V_P{}'(r) = \frac{u'(r)}{\rho + \theta + \kappa + \lambda_P \overline{F}(u)}. \tag{v}$$

Substituting $V_P{}'(r)$ into (iv) yields (6). □

Interpretation of the reservation utility expression for the temporarily accommodated households shows that when households decide whether or not to accept a dwelling, they take market opportunities into account. Interestingly, households in temporary accommodation do not necessarily accept the very first opportunity they receive, but rather accept the offer that increases lifetime utility. Thus, whether or not they postpone residential mobility depends on housing market factors.

The reservation expression reveals that, if the offer arrival rate for households temporarily accommodated (λ_T) is smaller than that of households having a permanent dwelling (λ_P), then the reservation value will be lower than u^o. So, in a housing market where temporarily accommodated households have great difficulties in obtaining an offer, dwellings which yield lower utility than u^o will be accepted. The reason is that when $\lambda_T < \lambda_P$, it is more attractive for the household to first accept a dwelling, and subsequently move to one with a higher utility level, than to wait for another offer while being temporarily accommodated. On the other hand, if offer arrival rates for both states are the same, a household will set the reservation value equal to u^o. If the housing market is such that the arrival rate for households having a dwelling is smaller than that for households not having a dwelling of their own, then the reservation value will be higher than u^o.

For the households residing in the permanent housing stock, the optimal strategy is to accept all offers that are strictly better. Since we neglect search or moving costs for the moment and assume a continuous search process, it is clear that the household will accept every offer that exceeds the current utility level. Note, however, that households experiencing a higher level of utility in the current situation are less likely to move, since they are more fastidious about a dwelling. Mobility on the housing market would be even lower if search and moving costs are introduced. In that case, households would not necessarily move to a dwelling with a higher utility level, because the gain in utility has to exceed a certain positive threshold level.

Now that we have derived the optimal strategy of a household, we are able to analyze the behavior of households in temporary accommodation in more detail, by checking the signs of the first derivatives of the reservation value u^* of equation (6). For example, a rise in the number of opportunities or a rise in the search efficiency, represented by a shift in the number of offers λ_T, will make the household unambiguously more fastidious, with a larger reservation utility value u^* $(\partial u^*/\partial\lambda_T > 0)$ (see Appendix B). Clearly, increasing the market opportunities for a household, by giving it more choice, will make it more choosy. In addition, if the household's temporary accommodation is more convenient in that it yields a higher utility value, represented by a upward shift in the base line utility u^o, this will also result in a higher utility value $(\partial u^*/\partial u^o > 0)$. A rise in the discount rate ρ, reflecting the anxiousness of the household, in the housing market turnover rate θ, or in the rate of urban renewal κ, all affect the reservation utility in the same order of magnitude. Depending on housing market opportunities for households in both temporary accommodation and permanent accommodation, the reservation utility will either increase or decrease. If the number of market opportunities is higher for households in temporary accommodation, such that $\lambda_T > \lambda_P$, a household for which ρ increases becomes more anxious and will become less choosy $(\partial u^*/\partial\rho < 0)$, and likewise for an increase in the housing market turnover rate, and in the urban renewal rate $(\partial u^*/\partial\theta < 0, \partial u^*/\partial\kappa < 0)$. Increasing the search efficiency for households in permanent dwellings, represented by an upward shift of λ_P, will lead to either a higher or a lower reservation utility, depending on the magnitude of all parameters, and remains therefore ambiguous.

2.3. *Steady-state housing market dynamics*

Based on the acceptance behavior of households, in this section, we will specify housing market flows into and out of the different stocks. Our stock-flow representation of the housing market in Figure 1 shows that these flows depend on specific housing market conditions. In our stock-flow scheme, we show three stocks for the two states. We consider one stock for temporary dwellings, and two for permanent dwellings, in order to allow for housing careers within the state of permanent dwellings. Clearly, the number of moves could be extended indefinitely.

Figure 1 shows that θm households enter temporary accommodation upon entering the housing market. Over time, households in temporary accommodation may move to dwellings from the permanent housing stock. If dwellings with a utility level less than u

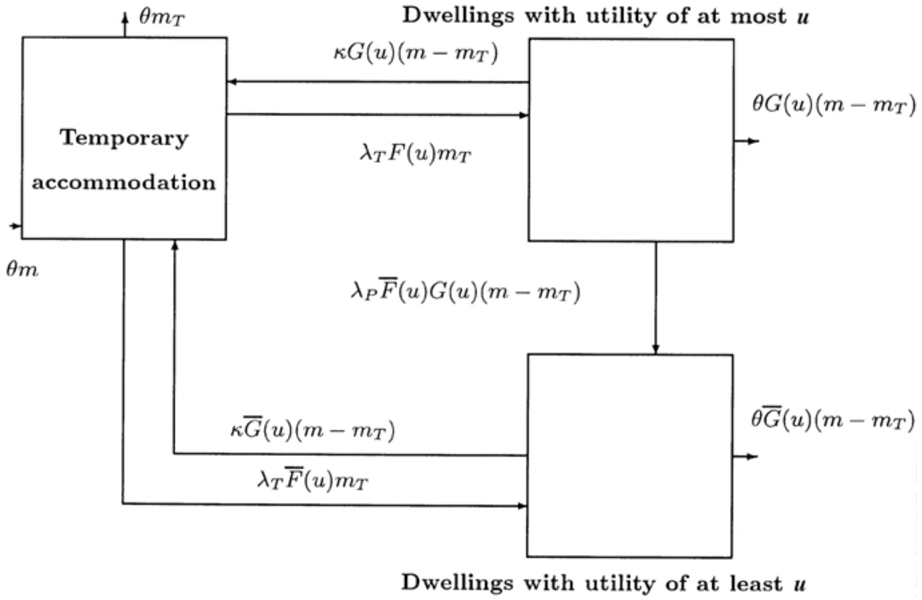

Figure 1. A stock-flow scheme of the housing market.

are offered more regularly, households may move to dwellings with lower utility more frequently. In addition, since households continuously search for better dwellings, households may subsequently move to dwellings with a utility level greater than u. Thus, the model is consistent with the notion of ‘‘housing career’’, where households start in a less preferred dwelling and subsequently move to a dwelling offering a higher utility level. As a result of the residential move, a dwelling with a lower utility level will then become vacant, which subsequently will be offered on the market, thus creating a vacancy chain.[9] Though we will not explore the issue of vacancy chains in the remainder of this paper, these can be taken into account quite easily. Note that the (stochastic) length of the vacancy chain, i.e., the number of moves after a vacancy occurs, depends on the utility level of the successive units which become vacant, as the chain stops if an agent in temporary accommodation accepts a unit. For a vacant unit with a given utility level u, households in both temporary accommodation and permanent accommodation with utility less than u compete for it. Thus, the probability of a vacancy chain of, for example length 2, equals the probability that a household in permanent accommodation with utility less than u accepts the vacant unit, times the probability that a household in temporary accommodation accepts the unit with utility less than u.

Moreover, as Figure 1 reveals, due to urban renewal, households might be forced to move. As households are assumed to maximize their lifetime wealth, households do not move to dwellings with a lower utility level.

Using these housing market dynamics, together with the assumption that the housing market will settle down at a steady-state, we are able to specify the distribution of households over the housing stock, as formulated in the next proposition.

Proposition 2: In a steady-state, the number of households in temporary residence equals

$$m_T = \frac{(\kappa + \theta)m}{\kappa + \theta + \lambda_T}, \tag{7}$$

while the number of households in a permanent residence equals

$$m - m_T = \frac{\lambda_T m}{\kappa + \theta + \lambda_T}. \tag{8}$$

Only part of the total number of households in a permanent residence experience a utility of at most u, being equal to

$$G(u)(m - m_T) = \frac{(k + \theta)F(u)}{(\kappa + \theta + \lambda_P \overline{F}(u))} \frac{\lambda_T m}{(\kappa + \theta + \lambda_T)}. \tag{9}$$

Proof. To show this, note that since the housing market is in a steady-state, the time-rate change of m_T and $G(u)(m - m_T)$ is zero. Hence, for m_T we have

$$\begin{aligned} 0 = \frac{\mathrm{d}(m_T)}{\mathrm{d}t} &= \theta m + \kappa G(u)(m - m_T) + \kappa \overline{G}(u)(m - m_T) \\ &\quad - \theta m_T - \lambda_T F(u) m_T - \lambda_T \overline{F}(u) m_T \Leftrightarrow m_T = \frac{(\kappa + \theta)m}{\kappa + \theta + \lambda_T}. \end{aligned}$$

For

$$G(u)(m - m_T) : 0 = \frac{\mathrm{d}(G(u)(m - m_T))}{\mathrm{d}t} = \lambda_T F(u) m_T - (\kappa + \theta + \lambda_P \overline{F}(u))(m - m_T).$$

Then, substituting for m_T gives the expression for $G(u)(m - m_T)$. □

This proposition shows that the steady-state housing market does indeed depend on the housing market dynamics. As is easily seen, the number of households in temporary accommodation depends not only on the outflow, as represented by the offer arrival rate of offers and the death rate, but also on the inflow of households. Interestingly, in the steady-state there are still a number of households in temporary accommodation, as a result of both housing market dynamics and housing market imperfections. For the permanent dwellings, the number of households also depends on the value of the utility offer.

The analysis so far has a partial equilibrium character. The moving behavior of households together with the steady-state assumption has allowed us to compute the various stocks and flows in the market, conditional on a particular distribution of offers F. As the landlords' strategy is conditional on the households' behavior, the landlords' strategy should be made endogenous rather than taking it as exogenous. In the next section, we will endogenize the landlords' offer distribution.

2.4. *Supply of a dwelling*

On the supply side of the market, landlords are looking for households to occupy their vacant dwellings. The landlords' strategy is to set vacancies and the rent r of the unit in order to maximize the expected discounted profits. We assume that landlords know that households evaluate dwellings such that

$$u(r) = \alpha - \tau, \tag{10}$$

where α is a constant denoting the (fixed) housing services; and r denotes the rent. As a result, variation in utility is only the result of pure rent dispersion. The landlords set rents in a non-cooperative equilibrium to profit from imperfect information. We assume that the creation of vacancies does not affect the expected search time for a unit, so that λ_T and λ_P are independent of the number of vacancies, denoted by v. The creation of new vacancies, however, affects the rate at which new vacancies are filled.

The expected discounted profits for a landlord of a vacant unit are

$$\rho J_V(r) = \beta(r)(J_O(r) - J_V(r)) - c_V, \tag{11}$$

where $\beta(r)$ is the average rate at which vacancies are filled; $J_O(r) - J_V(r)$ the capital gain of having the vacancy filled; and $c_V > 0$ the per unit time cost of a vacant unit. The expected discounted profit of an occupied unit $J_O(r)$ which is a function of the profit per unit time and the period of occupation is

$$\rho J_O(r) = r + (\theta + \kappa + \lambda_P F_r(r))(J_V(r) - J_O(r)), \tag{12}$$

where r is the rent per unit time; θ the exogenous rate at which a household exits the market; κ the exogenous rate of urban renewal; and $\lambda_P F_r(r)$ the rate at which a household exits, because it finds a dwelling offering a higher utility level.

The average rate at which vacancies fill is endogenously determined by the demand side of the model. For a landlord offering, a unit at rent r, the filling rate is equal to the total number of households that will accept the unit over the number of vacancies, which is equal to the number of temporarily accommodated households plus the number of households with a utility of at most u:

$$\beta(r) = \frac{\lambda_T m_T + \lambda_P G_r(r)(m - m_T)}{v} \tag{13}$$

with v determined by the zero profit condition.

Proposition 3: Free entry of landlords drives the profit of a vacancy to zero, so that the equilibrium number of vacancies (for $\rho \to 0$[10]) is the solution to

$$c_V v = \max_u \left[\frac{\lambda_T m(\kappa+\theta)(\kappa+\theta+\lambda_P)}{(\kappa+\theta+\lambda_T)(\kappa+\theta+\lambda_P \overline{F}(u))} \frac{\alpha - u}{(\kappa+\theta+\lambda_P \overline{F}(u))} \right] \tag{14}$$

for every offer they make. □

Proof. Given that $J_V(r) = 0$, (12) gives

$$J_O(r) = \frac{r}{\rho+\theta+\kappa+\lambda_P F_r(r)},$$

so that (11) reads

$$c_V = \beta(r)\left(\frac{r}{\rho+\theta+\kappa+\lambda_P F_r(r)}\right).$$

In the derivation we use a result of the invariance principle of probability theory which states that if r is a continuous random variable with density function $f_r(r)$ and associated distribution function $F_r(r)$, then $u = u(r)$ is also a random variable with associated density $f(u) = f_r(\alpha - u)$ and distribution function $F(u) = \overline{F}_r(\alpha - u)$.

Using this result, and substituting for m_T and $G(u)$ in (13), thereby letting $\rho \to 0$, gives the resulting expression. □

Free entry thus drives the expected discounted profits, hereafter denoted by π, to the cost on the left-hand side of equation (14). In setting the price, a landlord makes a trade-off between tenancy period and revenues during occupation. Clearly, a higher rent will increase revenues during the occupation, but it will shorten the occupation period as the household will want to move to another cheaper unit. As a result, a landlord who sets higher rents will have the same expected discounted profits as one who sets lower rents. Thus, competition between landlords for a household to occupy a vacant dwelling, together with the assumption that households continuously search for a better dwelling, eliminates discontinuities in the offer distribution. In addition, landlords know the households' reservation level. Hence, landlords would never post rents such that utility is less than the households' reservation value, so that $u = u^*$ and $F(u^*) = 0$. As households are not instantaneously informed, landlords set rents in a mixed strategy to profit from imperfect information. The supplier's optimal strategy is formulated in the following definition of an equilibrium in mixed strategies.

Definition 4: The supplier's equilibrium in mixed strategies is characterized by $(F(u), \pi)$ on the support $[u^*, \bar{u}]$, such that profits π equal

$$\pi = \pi(u), \quad \forall\ u \in [u^*,\ \bar{u}] \text{ of } F(u),$$
$$\pi \leq \pi(u), \quad \forall\ u \notin [u^*,\ \bar{u}] \text{ of } F(u),$$

conditional on the reservation utility of households u^* and offer distribution $F(u)$.

Using this definition of the suppliers' equilibrium, together with the equal profit condition, we arrive at the following non-cooperative equilibrium in mixed strategies.

Proposition 5: The supplier's optimal mixed strategy is represented by an equilibrium utility distribution

$$F(u) = \frac{\kappa + \theta + \lambda_P}{\lambda_P}\left(1 - \sqrt{\frac{\alpha - u}{\alpha - u^*}}\right), \tag{15}$$

which is continuous for $u[u^*, \bar{u}]$.

Proof. We give an exposition of how to arrive at the firm's equilibrium, of its existence, and of the uniqueness of the equilibrium distribution, and the continuity on its support. The argument to prove that F is continuous on $[u^*, \bar{u}]$ is by contradiction. If $F(\cdot)$ were such that it had a discontinuity, then it would be profitable for a supplier to lower the rent r (with $r = \alpha - u$) by an infinitesimally small amount and increase profits. However, $F(\cdot)$ then would never be the supplier's optimal reply, given the offers of other suppliers. The derivation of a candidate for $F(\cdot)$ runs as follows. As in equilibrium every offer gives the same steady-state profit on its support by the equal profit condition, together with the boundary condition $F(u^*) = 0$, we are able to solve for the distribution $F(\cdot)$:

$$\begin{aligned}\pi(u^*) = \pi(u) \Leftrightarrow (\alpha - u^*)\ m\lambda_T(\kappa + \theta)/(\kappa + \theta + \lambda_T)(\kappa + \theta + \lambda_P)\\ = (\alpha - u)\ m\lambda_T(\kappa + \theta)(\kappa + \theta + \lambda_P)/(\kappa + \theta + \lambda_T)\big(\kappa + \theta + \lambda_P\overline{F}(u)\big)^2,\end{aligned}$$

which we can solve for $F(\cdot)$ To complete the proof of uniqueness it needs to be checked that profits are not larger anywhere outside the support $[u^*, \bar{u}]$. An offer $u > \bar{u}$ would not increase profits, as it decreases revenues but does not attract additional households to compensate the loss in rent per unit time, since $F(\bar{u}) = 1$. An offer $u < \bar{u}$ would attract no tenant at all, and hence it too would not increase profits. □

A further interpretation of the equilibrium offer distribution shows that the supplier's offer of at most u depends—besides on the offer and the reservation rule of the household—on the dynamics of the housing market as represented by the rate of households entering and leaving the housing market (θ), the urban renewal rate (κ), or the offer arrival rates of dwellings with a utility level of at least u (λ_P). A rise in the urban renewal rate, or a rise in the rate at which households enter and quit the market, leads for both cases (for a given u^*) to more searchers in temporary accommodation, resulting in more monopoly power of the supplier, which unambiguously leads to a shift in $F(\cdot)$ to $F'(\cdot)$, such that $F(\cdot)$ stochastically dominates $F'(\cdot)$ ($F(u)/\kappa > 0$ and $F(u)/\theta > 0$) (see Appendix C). As $F(u) = Pr(u_x < u)$ increases, this implies that higher offers are less likely, so that suppliers lower their offer (by increasing the rent) as a result of more temporarily accommodated households. An increase in search efficiency for households in a permanent dwelling (λ_P) leads to less power for the supplier and subsequently to offers with higher utility values, making the offer distribution less concentrated ($\partial F(u)/\partial\lambda_P < 0$). Note, however, that the offer distribution does not depend directly on λ_T. But for the market equilibrium as characterized in the next section, we will see that the offer distribution will depend on λ_T. The interaction from the reservation utility value u^* to $F(\cdot)$ makes $F(\cdot)$ depend on λ_T.

2.5. *Dispersed market equilibrium*

Now that we have formulated the search behavior of households and the price setting behavior on the supply side, we can derive the market search equilibrium, which we define as follows:

Definition 6: A housing market search equilibrium is characterized by (u^*, $F(\cdot)$, π), such that (a) u^* is the reservation utility level of a household, and (b) ($F(\cdot)$, π) is the supplier's equilibrium.

Using this definition, we can now derive the market search equilibrium as a function of the structural parameters, as formulated in the next proposition.

Proposition 7: The equilibrium offer distribution of utility for $u[u^*, \bar{u}]$ is

$$F(u) = \frac{\kappa + \theta + \lambda_P}{\lambda_P}\left(1 - \sqrt{\frac{\alpha - u}{\alpha - u^*}}\right),$$

and the equilibrium reservation utility is

$$u^* = \frac{(\kappa + \theta + \lambda_P)^2 u^o + (\lambda_T - \lambda_P)\lambda_P\alpha}{(\kappa + \theta + \lambda_P)^2 + (\lambda_T - \lambda_P)\lambda_P} \tag{16}$$

Proof. To arrive at the reservation utility level, we substitute (15) into equation (6), and integrate $F(\cdot)$ over the support $[u^*, \bar{u}]$. Using the fact that the highest utility offered $(\bar{u})$ satisfies $F(\bar{u}) = 1$, we are able to derive the supremum of the support:

$$1 = \frac{\kappa + \theta + \lambda_P}{\lambda_P}\left(1 - \sqrt{\frac{\alpha - \bar{u}}{\alpha - u^*}}\right) \Leftrightarrow \bar{u} = \left(1 - \left(\frac{\kappa + \theta}{\kappa + \theta + \lambda_P}\right)^2\right) \alpha + \left(\frac{\kappa + \theta}{\kappa + \theta + \lambda_P}\right)^2 u^*.$$

Then, substituting for the support, one obtains the equilibrium reservation utility level. □

After the derivation of the market equilibrium, we are now able to further characterize the market equilibrium, as given in the next proposition.

Proposition 8: The steady-state distribution of utility equals

$$G(u) = \frac{(\kappa + \theta)}{\lambda_P}\left(\frac{1}{\sqrt{\frac{\alpha - u}{\alpha - u^*}}} - 1\right). \tag{17}$$

Proof. To obtain the equilibrium distribution of utility experienced, we substitute the equilibrium offer distribution into the steady-state distribution (see Proposition 7): $G(u) = (\kappa + \theta)F(u)/(\kappa + \theta + \lambda_P\overline{F}(u))$, which gives, after rewriting, $G(u)$. □

The equilibrium distribution of utility experienced reveals that newly arrived offers drawn from $F(\cdot)$ are less likely to be higher than x compared with the steady-state distribution of utility $G(\cdot)$, as $G(x)$ stochastically dominates $F(x)$ (i.e., if the integral of $G(x)$ over the range $[0, x]$ is less than or equal to that of $F(x)$ for any x).

3. Comparative statistics and a numerical example

Now we have established the market search equilibrium, we are able to investigate the effects of housing market changes and policy changes on housing market characteristics. In this section, we will discuss the implications of these effects in turn. As we will show, a number of comparative statics results are obtained quite easily, as long as the reservation utility does not change in response to changes in the housing market (proofs are given in Appendix D). However, since suppliers set rents depending on the reservation utility level of households, comparative statics results are ambiguous as soon as the reservation utility value responds to changes in the housing market (i.e., it interacts with the offer distribution F). In order to evaluate changes in the steady-state search equilibrium of the housing market, we consider a numerical example.

3.1. Housing market information desks

We consider the effect of introducing institutions that aid housing market search, either publicly or privately owned. One could think of listing services by real estate agents on the Internet, local government information desks, or central matching agencies, which provide information on vacancies, market opportunities, average duration in temporary accommodation, housing units, and new construction plans.

We consider the situation in which dwellings are renovated once in 30 years, where the expected lifetime of a household is 50 years, and the initial duration of residence in both temporary and permanent accommodation is 4 years. Moreover, we assume that u^o the baseline utility of temporary accommodation equals 1,400, and the fixed rent at which permanent accommodation is offered is 800. Finally, let the number of households be 100,000. Hence, the base parameters are $(\theta, \kappa, \alpha, u^o, m, u) = (1/50, 1/30, 3,000, 1,400, 100,000, 2,200)$. Initially $\lambda_T = \lambda_P = (1/4)$. In the analysis, we assume that introducing housing market information desks is associated with changes in λ_T and λ_P. In the remainder of this section, we consider the effect of a change in λ_T or λ_P on $m_T, u, F(\cdot)$ and $G(\cdot)$.

An improvement in the search process leads to an upward shift in the offer arrival rate of dwellings for households in temporary accommodation (λ_T), and the offer arrival rate for households in a permanent dwelling (λ_P). Consider the first case, where households in temporary accommodation are offered more information by the introduction of an institution that aids search. As a result, households in temporary accommodation will find a dwelling more easily, so that the exit rate out of temporary accommodation will rise, and will lead to a decrease in the number of households in temporary accommodation $(\partial m_T / \partial \lambda_T < 0)$. As households are better informed, they become more fastidious and the reservation utility level will rise $(\partial u^* / \partial \lambda_T > 0)$. In the meantime, suppliers react to the increased reservation utility value by offering dwellings with a higher utility level. As a result, lower values of utility u become less likely $(\partial F / \partial \lambda_T < 0)$. With respect to the effect on the distribution of utility, partial comparative statics show quite different results (for a partial approach, see Read, 1997). If we consider, e.g., partial effects of λ_T on F and G (conditional on a given reservation value), comparative statics indicate no effect, whereas equilibrium effects show a negative effect. Table 1 gives our comparative statics results.[11]

If, on the other hand, households in permanent dwellings receive more housing market opportunities due to the introduction of a housing market institution, then λ_P will rise. An increase in λ_P will not necessarily lead to the same effects as for λ_T, as households both in a temporary and in a permanent dwelling react to an increase in λ_P. As households in temporary accommodation compare the offer arrival rate with the offer arrival rate when

Table 1. Equilibrium effects of institutions that aid housing search.

	m_T	$u*$	$F(u)$	$G(u)$
λ_T	$-$	$+$	$-$	$-$
λ_P	0	?	?	?

Table 2. Numerical results of increasing λ_P on equilibrium values.

For $\lambda_P > \lambda_T$	m_T	u^*	$F(u)$	$G(u)$
$\lambda_P = 1/36$	17,582	1,035	0.420	0.091
$\lambda_P = 1/24$	17,582	296	0.505	0.089
$\lambda_P = 1/60$	17,582	1,616	0.304	0.084
$\lambda_P = 1/54$	17,582	1,520	0.328	0.086

being in a permanent residence, comparative statics effects typically depend on the magnitude of the structural parameters κ, θ, λ_T, λ_P. The comparative statics result of λ_P on search equilibrium in Table 1 clearly shows the ambiguity of the effects. The effects on the offer distribution $F(\cdot)$ depends on the magnitude of the comparative statics result of the reservation utility, the sign of which depends on all the housing market dynamics. For $\lambda_P < (\kappa + \theta)\lambda_T/(2\kappa + 2\theta + \lambda_T)$, an upward shift in λ_P makes households more fastidious, and increases the reservation utility level. Table 2 gives the results of a numerical example which show the ambiguity of the effects on the housing market equilibrium.

As the comparative statics results have already indicated, the steady-state number of households in temporary dwellings is not affected by a change in λ_P. In addition, for a household currently in temporary accommodation, the reservation utility level might be lower than the current baseline utility level u^o. Interpretation of the table reveals that landlords do indeed react to the households' reservation value. If the households' reservation value decreases, suppliers on average offer dwellings with a lower utility value, as indicated by an increase in $F(\cdot)$. Using these results, we can investigate the consequences for the aggregate distribution of households over both temporary (m_T) and permanent accommodations, both with a utility of at most u: $(G(u)(m - m_T))$ and with a utility level of at least u: $\left(\overline{G}(u)(m - m_T)\right)$. These results are shown in Table 3.

As Table 3 shows, for $\lambda_P > \lambda_T$, increasing the search efficiency for households in a permanent dwelling (λ_P) will decrease the number of households in a dwelling with a utility level of at most u, but will increase the number of households in a dwelling with a utility greater than u. As a result, introducing housing market institutions affects the mobility rate either directly, or indirectly via the vacancy chain mechanism.

Now that we have examined the housing market search equilibrium in more detail, the question naturally arises: What have we learned about the housing market? We have shown that the housing market search equilibrium, characterized by the reservation utility and the equilibrium offer distribution, depends on the dynamics of the housing market, as

Table 3. Effects of housing market institutions on the aggregate distribution.

For $\lambda_P > \lambda_T$	$G(u)(m - m_T)$	$\overline{G}(u)(m - m_T)$
$\lambda_P = 1/36$	7,481	74,937
$\lambda_P = 1/24$	7,371	75,047
$\lambda_P = 1/60$	6,933	75,485
$\lambda_P = 1/54$	7,121	75,297

represented by κ, θ, λ_T, λ_P. Hence, residential search and mobility, and rent dispersion in the housing market all depend on these parameters. Moreover, the numerical example has shown the ambiguity of changes in the housing market search equilibrium as a result of complicated interactions via the reservation utility value. Our search equilibrium analysis, however, reveals that partial equilibrium models may easily yield misleading results as effects via the reservation utility are not incorporated. By using comparative statics together with numerical examples, we are, in principle, able to assess changes in the housing market.

4. Conclusion

This paper has formulated a basic housing market equilibrium search model with residential mobility, and captures most elements of the ''New Housing Economics'' in an integrated theoretical framework. Based on a housing market stock-flow model, we have derived the number of households in temporary accommodation and permanent residences in a steady-state housing market. In addition, we have formulated optimal acceptance behavior of the household, depending on housing market conditions. This reservation utility level is sensitive to changes in the housing market, such as the offer arrival rate of dwellings. Landlords set rents in a mixed equilibrium in order to profit from the imperfect operation of the housing market, thereby taking account of the households' reservation utility level. Once demand and supply had been formulated, a search market equilibrium was characterized, and then examined by means of comparative statics and some numerical examples. As compared with the partial equilibrium model of search, the equilibrium housing market model formulated here clearly shows that partial equilibrium analysis may yield misleading results, as an important feedback mechanism through the reservation utility level of households is not incorporated. For example, introducing housing market information desks in a partial equilibrium analysis would unambiguously make offers with low levels of utility less likely, as the offer distribution becomes less concentrated, whereas in a general equilibrium analysis the comparative statics effect would remain ambiguous.

The housing market model presented here has a stylized character, but turns out to be a promising framework for analyzing housing market issues. Further research could make it more realistic. For example, households' offer arrival rates could be endogenized as they may well depend on vacancies created. Furthermore, we have not made a clear distinction between the rented and owner-occupied sector of the housing market, ignoring the importance of credit-constraints and downpayments in the tenure choice of households when moving. Moreover, the operation of the vacancy chain mechanism could be more fully formalized in the model. Like welfare implications, these are left for future research. Another direction for future research would be to formulate and estimate an empirical housing market model. In such a model, one could impose steady-state conditions on the housing market model and use a continuous-time Markov Chain as the statistical model. Using data on individuals' housing market histories, which contain information on the duration of the different spells of residence in the dwellings, and the destinations of the moves, one would be able to estimate the parameters of the model.

Alternatively, one might broaden the perspective by formulating an integrated framework which analyzes residential search in relation to commuting and/or job mobility. In addition, one might analyze residential search from a spatial perspective along the lines of Read (1991). Space enters the decision problem with respect to both the neighborhood in which residences are examined and the geographic proximity of residential locations to workplace locations. As such, households would trade off the residential site and the journey to work.

Appendix A. Notational glossary

θ	housing market turnover rate
λ_T	offer rate of dwellings to a temporarily accommodated household
λ_P	offer rate of dwellings to a permanently accommodated household
u	utility
$f(u)$	density function of offers with utility u
$F(u)$	offer distribution function of u
$G(u)$	steady-state distribution of experienced utility u
α	housing services
r	rent set by the landlord
$f_r(\alpha - u)$	rent offer density function
$F_r(\alpha - u)$	rent offer distribution function
κ	rate of forced moves
$\overline{F}(u)$	1-$F(u)$
$\overline{G}(u)$	1-$G(u)$
$\underline{G}(u)(m - m_T)$	number of households in a dwelling with at most u
$\overline{G}(u)(m - m_T)$	number of households in a dwelling with at least u
m_T	number of households in temporary accommodation
m	number of households
u^*	reservation utility
$u(r)$	instanteneous utility for households in permanent accommodation
$V_T(\cdot)$	wealth equation for temporary accommodation
u^o	baseline utility level for households in temporary accommodation
r^o	baseline rent
ρ	discount rate
E_{r_x}	expectation operator w.r.t. r_x
$V_P(\cdot)$	wealth equation for permanent accommodation
π	profits for a supplier
$[u, \bar{u}]$	support of $f(\cdot)$, $F(\cdot)$, $\overline{F}(\cdot)$, $g(\cdot)$, $G(\cdot)$, $\overline{G}(\cdot)$
v	number of vacancies
J_V	asset equation of a vacant unit
J_O	asset equation of an occupied unit
β	rate at which vacancy is filled
c_V	cost of vacant unit

Appendix B. First derivatives of household's optimal strategy

The effects of a change in the parameters of the household's optimal strategy (for given $F(u)$) are derived from equation (6):

$$\frac{\partial u^*}{\partial \lambda_T} = \int_{u^*}^{\overline{u}} \overline{F}(u) Z(u)\, \mathrm{d}u > 0 \tag{B1}$$

with $Z(u) \equiv \frac{1}{\rho+\kappa+\theta+\lambda_P \overline{F}(u)}$

$$\frac{\partial u^*}{\partial \lambda_P} = -(\lambda_T - \lambda_P) \int_{u^*}^{\overline{u}} \overline{F}(u)^2 Z(u)^2\, \mathrm{d}u - \int_{u^*}^{\overline{u}} \overline{F}(u) Z(u)\, \mathrm{d}u, \tag{B2}$$

$$\frac{\partial u^*}{\partial \kappa} = \frac{\partial u^*}{\partial \theta} = \frac{\partial u^*}{\partial \rho} = -(\lambda_T - \lambda_P) \int_{u^*}^{\overline{u}} \overline{F}(u) Z(u)^2\, \mathrm{d}u. \tag{B3}$$

Appendix C. First derivatives of supplier's optimal strategy

The effects of a change in the parameters of the landlord's optimal strategy (for given u^*) are derived from equation (15):

$$\frac{\partial F(u)}{\partial \lambda_P} = -\frac{(\kappa+\theta)}{\lambda_P^2}\left(1 - \Omega(u)^{1/2}\right) < 0, \tag{C1}$$

with $\Omega(u) \equiv \left(\frac{(\alpha-u)}{(\alpha-u^*)}\right)$

$$\frac{\partial F(u)}{\partial \kappa} = \frac{\partial F(u)}{\partial \theta} = \frac{1}{\lambda_P}\left(1 - \Omega(u)^{1/2}\right) > 0. \tag{C2}$$

Appendix D. Comparative statics

The effects on the housing market search equilibrium of institutions that aid the search process for temporarily accommodated households can be obtained using equations (7), (15)–(17), respectively:

$$\frac{\partial m_T}{\partial \lambda_T} = -\frac{(\kappa+\theta)m}{(\kappa+\theta+\lambda_T)^2} < 0, \tag{D1}$$

$$\frac{\partial u^*}{\partial \lambda_T} = \frac{\lambda_P(\kappa+\theta+\lambda_P)^2(\alpha-u^o)}{[(\kappa+\theta+\lambda_P)^2+(\lambda_T-\lambda_P)\lambda_P]^2} > 0, \tag{D2}$$

$$\frac{\partial F(u)}{\partial \lambda_T} = -\frac{(\kappa+\theta+\lambda_P)}{2\lambda_P}\left(\frac{(\alpha-u)}{(\alpha-u^*)^2}(\Omega(u))^{-1/2}\left(\frac{\partial u^*}{\partial \lambda_T}\right)\right), \tag{D3}$$

$$\frac{\partial G(u)}{\partial \lambda_T} = -\frac{(\kappa+\theta)}{2\lambda_P}\left(\frac{(\alpha-u)}{(\alpha-u^*)}(\Omega(u))^{-3/2}\left(\frac{\partial u^*}{\partial \lambda_T}\right)\right) \tag{D4}$$

with $\Omega(u) \equiv ((\alpha-u)/(\alpha-u^*))$.

The effects on the housing market search equilibrium of institutions that aid search for permanently accommodated households can be obtained using equations (15)–(17):

$$\frac{\partial u^*}{\partial \lambda_P} = \frac{(\kappa+\theta+\lambda_P)[(\kappa+\theta)(\lambda_T-2\lambda_P)-\lambda_T\lambda_P](\alpha-u^o)}{((\kappa+\theta+\lambda_T)^2+(\lambda_T-\lambda_P)\lambda_P)^2}, \tag{D6}$$

$$\begin{aligned}\frac{\partial F(u)}{\partial \lambda_P} = {} & \frac{1}{\lambda_P}\left(1-(\Omega(u))^{1/2}\right) - \frac{(\kappa+\theta+\lambda_P)}{\lambda_P^2}\left(1-(\Omega(u))^{1/2}\right) \\ & - \frac{(\kappa+\theta+\lambda_P)}{\lambda_P}\left(\frac{1}{2}\frac{(\alpha-u)}{(\alpha-u^*)^2}(\Omega(u))^{-1/2}\left(\frac{\partial u^*}{\partial \lambda_P}\right)\right),\end{aligned} \tag{D7}$$

$$\begin{aligned}\frac{\partial G(u)}{\partial \lambda_P} = {} & -\frac{(\kappa+\theta)}{\lambda_P^2}\left(\left(\frac{\alpha-u}{\alpha-u^*}\right)^{-1/2} - 1\right) \\ & - \frac{(\kappa+\theta)}{2\lambda_P}\left(\frac{(\alpha-u)}{(\alpha-u^*)^2}(\Omega(u))^{-3/2}\left(\frac{\partial u^*}{\partial \lambda_P}\right)\right).\end{aligned} \tag{D8}$$

The effects on the housing market search equilibrium of urban renewal programs can be obtained using the above-mentioned equations:

$$\frac{\partial m_T}{\partial \kappa} = \frac{\lambda_T m}{(\kappa+\theta+\lambda_T)^2} > 0, \tag{D10}$$

$$\frac{\partial u^*}{\partial \kappa} = -\frac{2(\kappa+\theta+\lambda_P)(\lambda_T-\lambda_P)\lambda_P(\alpha-u^o)}{((\kappa+\theta+\lambda_P)^2+(\lambda_T-\lambda_P)\lambda_P)^2}, \tag{D11}$$

$$\frac{\partial F(u)}{\partial \kappa} = \frac{1}{\lambda_P}\left(\left(1-(\Omega(u))^{1/2}\right). - \frac{(\kappa+\theta+\lambda_P)}{\lambda_P}\left(\frac{1}{2}\frac{(\alpha-u)}{(\alpha-u^*)^2}(\Omega(u))^{-1/2}\left(\frac{\partial u^*}{\partial \kappa}\right)\right)\right), \tag{D12}$$

$$\frac{\partial G(u)}{\partial \kappa} = \frac{1}{\lambda_P}\left(\left((\Omega(u))^{1/2}-1\right). - \frac{(\kappa+\theta)}{\lambda_P}\left(\frac{1}{2}\frac{(\alpha-u)}{(\alpha-u^*)^2}(\Omega(u))^{-3/2}\left(\frac{\partial u^*}{\partial \kappa}\right)\right)\right). \tag{D13}$$

The effects on the housing market search equilibrium of increased housing market turnover of households can be obtained using the above-mentioned equations:

$$\frac{\partial m_T}{\partial \theta} = \frac{\lambda_T m}{(\kappa+\theta+\lambda_T)^2} > 0, \tag{D15}$$

$$\frac{\partial u^*}{\partial \theta} = -\frac{2(\kappa+\theta+\lambda_P)(\lambda_T-\lambda_P)\lambda_P(\alpha-u^o)}{((\kappa+\theta+\lambda_P)^2+(\lambda_T-\lambda_P)\lambda_P)^2}, \tag{D16}$$

$$\frac{\partial F(u)}{\partial \theta} = \frac{1}{\lambda_P}\left(\left(1-(\Omega(u))^{1/2}\right). - \frac{(\kappa+\theta+\lambda_P)}{\lambda_P}\left(\frac{1}{2}\frac{(\alpha-u)}{(\alpha-u^*)^2}(\Omega(u))^{-1/2}\left(\frac{\partial u^*}{\partial \theta}\right)\right)\right), \tag{D17}$$

$$\frac{\partial G(u)}{\partial \theta} = \frac{1}{\lambda_P}\left(\left((\Omega(u))^{-1/2}-1\right) - \frac{(\kappa+\theta)}{\lambda_P}\left(\frac{1}{2}\frac{(\alpha-u)}{(\alpha-u^*)^2}(\Omega(u))^{-3/2}\left(\frac{\partial u^*}{\partial \theta}\right)\right)\right). \tag{D18}$$

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Notes

1. For interesting reviews see Quigley (1979), Arnott (1987) or Smith et al. (1988).
2. For a discussion of the search process in the housing market, see Clark (1982); for the role of idiosyncratic preferences in housing, Arnott (1989); for mobility and housing consumption adjustments, Hanushek and Quigley (1979); for the role of vacancies, Rietveld (1984); and, for supply considerations, Blank and Winnick (1953).
3. Vacancies may arise either as a result of intrinsic characteristics of housing (Arnott, 1989), or as a result of turnover and residential mobility (Rietveld, 1984; Wheaton, 1990).
4. Empirical evidence on housing mobility, vacancies and rent setting behavior, on which these stylized facts rest, can be found, in among others, Weinberg (1979), Eubank and Sirmans (1983), Hanushek and Quigley (1979), Weinberg et al. (1981), Rosen and Smith (1983) and Guasch and Marshall (1985).
5. Admittedly, this is a very simple representation of finite lifetimes of households. However, since the aggregate distribution of households is generally stable over time, and since there is no other satisfactory treatment of household formation and dissolution for the ease of exposition, we use Poisson processes implying exponential lifetimes. In addition, the assumption that the departure rate equals the households' dissolution rate is far from realistic, except for the case of single-person households.
6. In relation to this, Snickars (1978) argues that, in modeling homelessness, it should not be seen as a queue, but may be interpreted as households not having a dwelling of their own and, therefore, postponing household formation. As such, homelessness is equivalent to households being in temporary accommodation, whereby both interpretations assume some least preferred type of accommodation (see also Arnott, 1989).
7. For example, Van der Vlist (2000), who examines new households' residential careers, distinguishes between agents sharing and those not sharing a dwelling with a parent.
8. As Stokey and Lucas (1989) show, given the assumptions for the preferences, theorems 9.2 and 9.4 hold (cf. Stokey and Lucas, 1989: Chapter 9). Theorem 9.2 then provides the sufficient conditions for a solution to the functional equation to be the supremum function in that it equals the optimal solution of the sequence problem, whereas theorem 9.4 ensures that the optimal policy generated from the policy correspondence attains the supremum.
9. Welfare implications from a household's move and the resulting vacancy chain remain to be analyzed.
10. Since we cannot derive a closed-form solution if ρ is not negligible, we make the simplifying assumption that ρ is negligibly small.
11. For $\lambda_P < (\kappa + \theta)\lambda_T/(2\kappa + 2\theta + \lambda_T) \Rightarrow \partial u^*/\partial \lambda_P > 0$.

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