Unemployment, Growth and the Organisation of Work

Henri L.F. de Groot\textsuperscript{a}
Anton B.T.M. van Schaik\textsuperscript{b}

\textsuperscript{a} Department of Spatial Economics, Faculty of Economics and Business Administration, Vrije Universiteit Amsterdam, and Tinbergen Institute,
\textsuperscript{b} Department of Economics, Tilburg University, PO Box 90153, 5000 LE Tilburg.
The Tinbergen Institute is the institute for economic research of the Erasmus Universiteit Rotterdam, Universiteit van Amsterdam and Vrije Universiteit Amsterdam.

Tinbergen Institute Amsterdam
Keizersgracht 482
1017 EG Amsterdam
The Netherlands
Tel.: +31.(0)20.5513500
Fax: +31.(0)20.5513555

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31.(0)10.4088900
Fax: +31.(0)10.4089031

Most TI discussion papers can be downloaded at http://www.tinbergen.nl
Unemployment, Growth and the Organisation of Work

A Dual Labour Market Perspective

Henri L.F. de Groot and Anton B.T.M. van Schaik

a Department of Spatial Economics, Vrije Universiteit, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands

b Department of Economics, Tilburg University, PO Box 90153, 5000 LE Tilburg, The Netherlands

Abstract
We develop a two-sector endogenous growth model with a dual labour market resulting from the presence of an effort extraction function in one sector. Effort of workers can be influenced by pay and monitoring. This results in an endogenous non-competitive wage differential between sectors and a monitoring intensity that is a source of fixed costs for the firm. Growth is driven by investments in R&D performed in the high-wage sector. Unemployment is determined by the costs and benefits of waiting for a high-paid job. The wage structure, growth, and unemployment are shown to depend on the way effort is extracted.

JEL codes: E24, J21, J53, O41

Keywords: endogenous growth, unemployment, effort extraction, dual labour market

1. Introduction
Wages differ considerably across broad sectors of the economy, even after controlling for age, education, occupation, gender, and workplace characteristics (cf. OECD, 1994). There are certain common elements in the estimates of these differences for a number of countries, e.g., manufacturing pre-eminently being the large sector paying a relatively high non-competitive wage premium, whereas the agricultural sector pays the lowest wages. The apparent willingness of employers in imperfectly competitive product markets to share rents with their workers introduces friction in the market mechanism: the unemployed may prolong their job search in the hope of entering high-wage sectors, and workers displaced from these sectors may have very high

---

1 Corresponding author: Henri L.F. de Groot, Department of Spatial Economics, Vrije Universiteit, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands, Email: hgroot@feweb.vu.nl, tel. +31 20 444 6168, fax. +31 20 444 6004.
replacement rates and hence very high reservation wages when benefits are based on previous earnings (cf. Kletzer, 1992). In this view, unemployment is determined by outweighing the costs and benefits of waiting for a high-paid job.

Our starting point is an endogenous growth model with a traditional and a high-tech sector. The duality of the economy results in a segmented non-Walrasian labour market. Our model predicts that relative nominal wages are rigid. Labour is homogeneous, but employers in the high-tech sector are willing to pay efficiency wages for rent-sharing reasons. Thus, workers obtain a sector-specific wage rate. The existence of these rents in the imperfectly competitive high-tech sector of the economy is the benefit that gives people an incentive to wait for high-paid jobs. We generalise the well-known theoretical concept of an efficiency wage relation, in which only the wage rate features, by introducing the concept of an effort-extraction function (see also Bowles, 1985, and Mehta, 1998). The basic idea here is that employers have several means of 'extracting' effort from their employees. One is by monitoring and supervising the effort of employees, another is to pay relatively high wages. Introducing this basic idea in this paper allows us to study the effects of for example different organisations of work by firms on growth and unemployment in a consistent framework. Firms will optimally set the wage and monitoring intensity as to maximise their profits. This is shown to result in a trade-off between paying high wages and intensive monitoring. The monitoring intensity and wage level that result from this optimising behaviour are shown to be crucial for both the growth and unemployment performance of an economy.

Our model extends the available literature on growth and unemployment in several respects. First, our focus is mainly on distortions in the supply of labour causing equilibrium unemployment, whereas most of the available studies focus on distortions in demand. Second, we model unemployment as resulting from (extended) efficiency wage considerations playing a role in one sector only. Third, we address the problem in a general equilibrium model with a segmented labour market, characterised by endogenously determined non-competitive wage differentials. Finally, we explicitly model growth as requiring (research) labour, where the intensity with which R&D is performed is determined on the basis of optimising behaviour of firms.

We proceed as follows. Section 2 presents the model. It discusses household behaviour, firm behaviour and the labour market of the model. Section 3 presents the solution of the model. Section 4 looks in detail at the properties of the model. It discusses the consequences for growth and unemployment of institutionally determined differences in effort extraction functions, capturing different ways of organising work. We present our conclusions in section 5.
2. A model of R&D and unemployment in a dual economy

The economy comprises two sectors. There is perfect competition in the product market for traditional goods and monopolistic competition in the product market for high-tech goods. Each firm in the high-tech sector produces a unique brand of the high-tech good. There are \( N \) high-tech firms, indexed \( i = 1, \ldots, N \). In section 3, we elaborate on the determination of the number of firms. We assume that a high-tech firm only holds a negligibly small market share, so that competition is monopolistically à la Chamberlin. Growth stems from research done in the high-tech sector. Labour is homogeneous and can be employed in one of the two sectors or can be unemployed. Workers earn a sector-specific wage, while unemployed people get unemployment benefits. In this section, we will present the full model. Only the equations constituting the final model are numbered. Where there is no danger of confusion, time indices have been omitted.

2.1 Households

We assume identical infinite-lived households. Household behaviour is formulated as a three-stage budgeting problem. In the first stage, households maximise inter-temporal utility\(^2\)

\[
U_0 = \int_0^\infty \frac{C^{1-\rho}}{1-\rho} e^{-\theta t} \, dt \quad \text{s.t.} \quad \dot{A}_t = rA_t + I_{nt} - C_{P_t},
\]

where \( C \) is a composite good, \( 1/\rho \) is the inter-temporal elasticity of substitution, and \( \theta \) is the subjective discount rate. The dynamic budget constraint describes the development of financial assets (\( A \)) over time (\( \dot{A}_t = dA_t/dt \)). Households spend income on consumption (\( CP_c \)) and obtain income by working (\( I_w \)), and by receiving rental income (\( rA \)), over financial assets accumulated in the past.\(^3\) Households have Cobb-Douglas preferences over the two goods. In the second stage of the optimisation problem, they maximise

\[
C = X^{\sigma}Y^{1-\sigma} \quad \text{s.t.} \quad XP_X + YP_Y = CP_C \quad (0 < \sigma < 1),
\]

where \( Y \) is the traditional good, \( X \) is a bundle of varieties of the high-tech good, and \( P_Y \) and \( P_X \) are the corresponding prices. In addition, households have CES-preferences over the high-tech goods (cf. Dixit and Stiglitz, 1977), so in the final step they maximise

\(^2\) All the maximizations are stated on a macroeconomic level. We think of each household as being made up of a continuum of individuals. We will return to the exact determination of household income in a later stage of the paper. For the moment it is important that, irrespective of how household income is determined, we can derive the consumption-savings decision.

\(^3\) In equilibrium, aggregate income from financial assets (\( rA \)) equals aggregate dividends paid by the firm. We will further elaborate on this in footnote 13 where we describe the savings-investment equilibrium.
\[ X = \left[ \sum_{i=1}^{N} x_i^{(e-1)/\epsilon} \right]^{e/(e-1)} \text{s.t.} \sum_{i=1}^{N} p_{si} = XP_x \quad (\epsilon > 1), \]

where \( x_i \) represents the consumed quantity of the high-tech good of brand \( i \), \( \epsilon \) is the elasticity of substitution between any two high-tech goods, \( N \) is the number of available varieties of the high-tech good, and \( p_{si} \) is the price of a single brand of the high-tech good of variety \( i \).

The three-step maximisation procedure yields five equations. In the first step, households decide how to divide total income between savings and consumption expenditures. This yields the Ramsey rule

\[ \frac{\dot{C}}{C} = \frac{1}{\rho} \left[ r - \frac{\dot{P}_C}{P_C} - \theta \right]. \quad (1) \]

This equation relates the growth rate of consumption to the determinants of the consumption-savings decision. It shows that the rate of growth is high if the real return on savings \( (r - \dot{P}_C / P_C) \) is large, if households are patient \( (\theta \) is low), and if households are willing to substitute inter-temporally \( (1/\rho \) is high).

In the second step, households decide how to divide the income they want to spend on consumption expenditures between high-tech and traditional goods. Given the Cobb-Douglas specification chosen above, this results in

\[ \frac{YP_t}{XP_x} = \frac{1-\sigma}{\sigma}, \quad (2) \]

\[ P_C = \left( \frac{P_X}{\sigma} \right)^\sigma \left( \frac{P_Y}{1-\sigma} \right)^{1-\sigma}. \quad (3) \]

Equation (2) tells us that a fixed fraction \( 1-\sigma \) of aggregate consumption expenditure \( CP_C \) is spent on traditional goods and a fixed fraction \( \sigma \) is spent on high-tech goods. Equation (3) is the definition of the macroeconomic price index.

In the last step, households decide how to divide the income they want to spend on high-tech goods among the \( N \) varieties of this good that are available. This yields the demand for a single variety of the high-tech good

\[ x_i = X \left( \frac{p_{si}}{P_x} \right)^{-\epsilon}, \quad (4) \]
\[ P_X = \left[ \sum_{i=1}^{N} P_{i}^{-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \]  

(5)

The price-elasticity of demand for any variety of the high-tech good is thus equal to \( \varepsilon \). From now on we employ the assumption of symmetry across firms in the high-tech sector, so that we may drop the subscript \( i \). Hence, \( X = xN^{\varepsilon/(\varepsilon-1)} \) and \( N = XP_x/xp_x \). Notice that, after employing the symmetry assumption, the equation for the circular flow (2) can be written as \( YP_x/Nxp_x = (1-\sigma)/\sigma \).

2.2 Firms

The traditional sector exhibits unitary labour productivity

\[ Y = L_Y. \]  

(6)

\( L_Y \) stands for the number of workers employed in this sector and \( Y \) is the production of traditional goods. Under perfect competition, the price of a traditional good equals labour cost

\[ P_Y = w_Y, \]  

(7)

where \( w_Y \) denotes the wage rate in the traditional sector.

High-tech firms employ direct labour \((L_x)\) with labour productivity \( h \) and effort \( e \), to produce \( x \) units of output

\[ x = ehL_x. \]  

(8)

According to this equation, the overall productivity of direct labour \((x/L_x)\) is composed of two factors, each determined differently. With respect to the effort \((e)\), we assume the existence of a generalised version of the efficiency wage relation that we used in van Schaik and de Groot, 1998. We will further label this relation the effort-extraction function. The effort of a worker in the high-tech sector crucially depends on two factors. The first is the wage he earns \((w_T)\) relative to the wage a worker earns in the traditional sector \((w_Y)\). The second is the (effective) amount of labour employed for monitoring or supervision \((S \equiv eL_o)\)

\[ e = -a + \left[ \frac{w_T}{w_Y} \right]^{\gamma_1} \left( 0 \leq \gamma_2 < \gamma_1 / \varepsilon < 1 / \varepsilon \right), \]  

(9)

where \( \gamma_1 \) and \( \gamma_2 \) are the effort-wage and effort monitoring elasticity, respectively.\(^4\) We call this the ‘supply of effort’.\(^5\) Following Akerlof, 1982, the main reason in our model for high-tech firms to pay

\(^4\) We use this terminology for presentational convenience. The ‘true’ or ‘correct’ elasticities are endogenous due to the constant term \( a \) in the effort extraction function. They equal \( \gamma \Omega /(-a+\Omega) \), where \( \Omega \equiv c(w_T/w_Y)^{\gamma_2}S^{\gamma_2}. \)

\(^5\) In the special case where \( \gamma_2 = 0 \), firms will be shown to employ no monitoring labour so \( S=0 \). For reasons of continuity, we assume that \( S^{\gamma_2} \) is equal to one when \( \gamma_2 = 0 \) (\( x' \) approaches 1 if \( x \) approaches zero from above).
efficiency wages is based on sociological considerations. The idea is that each worker has a certain perception of the amount of effort that a ‘fair’ employer can ask from him. The employer can influence this fair amount of effort by changing the wage he pays. The more he pays, the higher the worker’s notion of the fair amount of effort to be supplied to the employer. By paying high wages, the firm is thus able to raise the norms of a fair working day and the fair amount of effort to be supplied in exchange for that wage.

The importance of this type of sociological consideration in explaining various phenomena in the labour market is increasingly acknowledged (see, e.g., Fehr et al., 1993, Kahneman et al., 1986, and Solow, 1980). The assumption that the efficiency wage considerations are only present in the high-tech sector is related to the prevailing imperfect competition in this sector. As profits are made in this sector only, workers may find it fair to share in these profits and hence ask for a higher wage. In that case, it may be in the interest of the profit-maximising firm to offer a higher wage. This matches with the empirical literature in which the relation between the operation of an efficiency wage relation and some characteristics of the sector like the size of the firm, capital intensity or kind of competition, has been investigated (e.g., Arai, 1994, Brown and Medoff, 1989, Dickens and Katz, 1987, Gera and Grenier, 1994, Krueger and Summers, 1988, and van Reenen, 1996). In these studies, evidence is found for a significant wage premium for those people working in large, innovating firms and in firms that operate in situations of imperfect competition. This research has also revealed that (i) there is an inter-industry wage structure that is significant and persistent over time and (ii) this wage structure cannot be explained solely on the basis of standard competitive factors as differences in skills, working conditions, etc. The second factor that positively influences the effort exerted by workers is the monitoring intensity (see also Bowles, 1985). We conceive the elasticities of the effort-extraction function as an important institutional characteristic of the economy. They are characteristic of the way work is being organised within firms. The importance of institutional and organisational factors on the effort of workers has been stressed in (historical) studies on the relation between economic institutions and economic performance. The following passage (Lazonick, 1991, p. 35) is instructive:

To overcome restrictions of output and encourage workers to apply their effort to further the goals of the enterprise, employers had to assure the workers that promises of higher wages, better work conditions, and employment stability would be kept. Most capable of keeping such promises were those corporations that had already attained competitive advantage in their product markets. It was these corporations that were already generating value gains that could be shared with workers to an extent that other, less advantaged corporations could not. The most effective way to implement these incentives was by promising hard-working, loyal workers long-term employment security and a rising standard of living both
on and off the job.

The variable $h$ can be affected by the firm by doing R&D. Assuming that there is no uncertainty with respect to investment in knowledge, employing $L_r$ units of research labour yields an increase in technology equal to

$$
\dot{h} = \xi e L_r, 
$$

(10)

where $h$ stands for the stock of knowledge a firm possesses (and which has been built up in the past), and $\xi (> 0)$ is a productivity parameter. This specification of the knowledge base implies that knowledge is completely internal to the firm.\(^6\) Finally, firms have to employ a fixed amount of labour in efficiency units ($F$) before being able to produce. One may think here of a fixed amount of management required before production can be started. So we require $F \equiv e L_f$.

In maximising present discounted value, high-tech firms decide about labour input in the production department ($L_x$), labour input in the research department ($L_r$), the wage rate ($w_T$), and the monitoring intensity ($S$). This optimisation leaves us with five equations capturing the First Order Conditions of the firms' optimisation problem (see Appendix A for a derivation). In this approach, we determine the input of research labour on the basis of inter-temporally optimising behaviour of the firm. The first equation shows the wage-setting behaviour. Firms will pay higher wages as long as the increase in benefits related to the increase in efficiency more than offsets the increase in cost in the form of a higher wage bill. This comes down to the well-known Solow condition\(^7\)

$$
\frac{\partial e}{\partial w_T} \frac{w_T}{e} = 1. 
$$

(11)

For the monitoring intensity, we derive

---

\(^6\) Alternatively, we could assume that knowledge is only partly internal to the firm. As shown in van Schaik and de Groot, 1998, this does not affect the qualitative results. When knowledge is not completely internal to the firm, the incentive to engage in research is less, as the firm cannot fully appropriate the benefits that are generated through the research. This leads to a lower intensity of research (and therefore a lower growth rate) than when there are no knowledge spill-overs.

\(^7\) In van Schaik and de Groot, 1998, we assume that effort-extraction considerations only apply to production workers. Here, we assume that they apply to all high-tech workers. One can argue about the most appropriate assumption. In any case, only applying efficiency wage considerations to production workers yields a 'modified Solow condition'. According to this modified Solow condition, the endogenous effort-wage elasticity is larger than one in equilibrium. Increasing the wage by one percent should be accompanied with a more than one percent increase in effort, as the higher wage also has to be paid to research labourers and managers/fixed labour (of which the productivity is not affected by the wage setting behaviour).
\[
\frac{\partial e}{\partial S} \frac{S}{e} = \frac{L_a}{L_c + L_r + L_s + L_f} < 1.
\] (12)

Firms increase their monitoring intensity as long as the marginal revenue of doing so exceeds the marginal cost. This results in an equilibrium effort-monitoring elasticity that is smaller than one (see footnote 4). So a one percent increase in the monitoring intensity only needs to result in a less than one percent increase in effort since this higher effort not only applies to the monitoring labour itself but also to production workers, researchers and managers. Combining these two conditions and using the endogenous effort-wage and effort-monitoring elasticities (see footnote 4), we can derive

\[
e = \frac{a \gamma_2}{1 - \gamma_1}, \quad \omega = \frac{w_T}{w_Y} = \left[\frac{a}{\epsilon(1 - \gamma_1) S^\gamma_2}\right]^{\frac{1}{\gamma_2}} \text{ and } \frac{L_a}{L_c + L_r + L_s + L_f} = \frac{\gamma_2}{\gamma_1}.
\]

This result reveals that in maximising their profits, firms make a trade-off between eliciting effort via paying high wages (high \(\omega\)) and via intensive monitoring (high \(S\)). Depending on the relative effectiveness of the two available instruments, firms decide on how much to pay their workers and how much monitoring labour to employ. The amount of supervision labour as a fraction of the total labour force of a firm is equal to the ratio of the effort-monitoring and the effort-wage elasticity (\(\gamma_2/\gamma_1\)). An increase in firm size results in other words in an equi-proportionate increase in the amount of supervisors.

The third equation describes price-setting behaviour. Given the market power of high-tech firms, they will simply put a mark-up over their wage cost

\[
p_s = \frac{\epsilon}{\epsilon - 1} \frac{w_T}{e h}.
\] (13)

This relation shows that real wages in the high-tech sector \(w_T/p_s\) increase with labour productivity \(h\). Unit real labour costs \(w_T/e h p_s\) equal \((\epsilon - 1)/\epsilon\) and are therefore invariant with respect to labour productivity growth. The mark-up is inversely related to the elasticity of substitution between any two high-tech goods. The closer these goods form substitutes, i.e., the higher \(\epsilon\) is, the less market power firms have, and the lower the mark-up they can put on labour costs.

The fourth equation determines optimal research effort

\[
w_T = p_h \xi e h.
\] (14)

In this formula, \(p_h\) is the shadow price of the level of technology \(h\). It is a measure of the marginal value of an additional unit of \(h\) for the firm. According to this equation, a firm equalises the marginal revenue of doing research (consisting of an increase in the level of technology a firm can
use) with the marginal cost of R&D, i.e., the wage bill of a researcher. Combining equations (13) and (14) leads to \( p_h/p_x = (\varepsilon - 1)/\xi \). This relation shows that the price (of the input) of knowledge in terms of the price (of the output) of the final product will rise if it becomes relatively costly to generate new knowledge (\( \xi \) is low) and if high-tech goods form closer substitutes (higher \( \varepsilon \)).

Finally, we derive the dynamic equation

\[
r = \xi e L_x + e L_x \left( \frac{p_x}{p_h} \right) \frac{\varepsilon - 1}{\varepsilon} \left( \frac{\dot{p}_h}{p_h} \right).
\]

According to this equation, the marginal cost of an increase in \( h \) which consists of capital cost \( r \) should equal the marginal revenue of an increase in \( h \) which consists of an addition to the stock of knowledge, an increase in production, and a capital gains term, \( \dot{p}_h / p_h \).

### 2.3 Equilibrium unemployment in a segmented labour market

An essential characteristic of the model is its segmented labour market. The effort-extraction function operating in the high-tech sector leads to primary sector workers receiving a non-competitive rent (\( \omega > 1 \)). The existence of these rents is at the heart of the analysis to follow. Each individual within a household is striving for the highest possible pay-off (in terms of present discounted value). Hence, all individuals would like to be employed in the high-tech sector. The number of jobs in this sector is, however, limited since consumers want to spend their income on both high-tech and traditional goods (\( \sigma < 1 \)). We assume that at some exogenous rate \( \delta \), jobs in the high-tech sector become available. Upon being laid off, a worker faces two options. He can either decide to take a job in the traditional sector (these jobs are freely available), or he can join the pool of unemployed. In determining his optimal strategy, the worker has to take the following factors into consideration: (i) unemployment benefits are lower than the salaries in the traditional sector (\( b < w_Y \)), and (ii) the probability of being matched with a high-tech job when being in the traditional sector (\( \alpha q \)) is lower than when being unemployed (\( q \)). The process of weighing the two options that laid off high-tech workers face results in an endogenously determined probability (\( \eta \)) of

---

8 We restrict the parameters of the effort extraction function in such a way that a non-competitive wage differential results (i.e., \( a[l^\gamma(1-\gamma)]S_{l^\gamma} > 1 \)).

9 We are confronted in this model with the problem of incorporating a non-Walrasian labour market structure in a dynamic general equilibrium model (see, e.g., Danthine and Donaldson, 1990, and Gali, 1995, for a discussion of these problems in the context of a real business cycle model). Though the construction that we use here of having a representative household (making the consumption-savings decision) being composed out of a continuum of individuals aimed at achieving the highest possible pay-off (in terms of present discounted value) is admittedly somewhat artificial, it allows us to embody the relation between unemployment and endogenous growth in a general equilibrium framework.
entering one of the two states (i.e., the state of unemployment or traditional sector employment). The outcome for this probability is such that ex-ante laid off workers (which are distributed randomly) are indifferent between the two options they face.

Figure 1 presents a stylised interpretation of the labour market flows. The assumption that the unemployed have a higher probability of being matched with a job in the high-tech sector than workers in the traditional sector ($\alpha < 1$) is important in our model and often used as a simple and useful working hypothesis in the literature on unemployment in dual labour markets (e.g., Bulow and Summers, 1986, Burda, 1988, Calvo, 1978, Harris and Todaro, 1970, McCormick, 1990, Taubman and Wachter, 1986).

To formalise the determination of the labour market equilibrium, we now introduce three value functions (Bellman equations; see, e.g., Pissarides, 1990). Let $V_Y$, $V_U$, and $V_T$ denote the present discounted value of expected income streams of a worker in the traditional sector, an unemployed person, and a worker in the high-tech sector, respectively. The worker in the traditional sector earns a wage rate of $w_Y$ and in unit time he expects to get a job in the high-tech sector with probability $\alpha q$, which gives him a surplus of $V_T - V_Y$ over his current position. $V_Y$ thus satisfies

$$rV_Y = w_Y + \alpha q(V_T - V_Y),$$

where $rV_Y$ is, in a perfect capital market, the valuation put on having a job in the traditional sector (this job may be seen as an asset). This valuation equals the return on the traditional sector job. Similarly, we derive
The workers discount their income at the nominal interest rate $r$ as they can freely save and borrow in the financial market at the nominal interest rate. In equilibrium, it is required that the value of a job in the traditional sector equals the value of being unemployed

$$V_T = V_U. \quad (19)$$

In addition, we impose two flow-equilibrium conditions, guaranteeing a constant allocation of labour over the three states

$$\delta \eta L_T = \alpha q L_T, \quad (20)$$

$$\delta (1-\eta) L_T = q U. \quad (21)$$

Note that we can neglect flows between traditional-sector employment and unemployment because, in equilibrium, there is no incentive to alternate between equilibrium strategies that have been chosen.\(^{10}\) Employment in the high-tech sector equals

$$L_T = N(L_\alpha + L_T + L_\omega + L_f). \quad (22)$$

Finally, we have to impose a stock-equilibrium condition

$$L = L_T + L_T + U, \quad (23)$$

so total labour supply $L$ is either employed in one of the two sectors or unemployed. This labour market block of the model yields a relationship between the unemployment rate and the number of high-tech workers as a function of the relative wage differential $\omega$, the unemployment benefit $b$, the acceptance rate of a worker from the traditional sector $\alpha$, and the interest rate $r$.

By combining the above relations (equations (16) – (19)), we can derive the matching probability of an unemployed person with a job in the high-tech sector as a function of the rate of interest\(^{11}\)

$$q = \frac{(1-b)(r+\delta)}{\omega (1-\alpha) - (1-\alpha b)}.$$  

---

\(^{10}\) Take, for example, a worker in the traditional sector. Working in that sector has some value for him, and this value consists of current and future earnings. In equilibrium, this value is the same as the value that unemployed workers derive from being unemployed. Now suppose that a traditional sector worker moves to the pool of unemployed. The effect of that move is that the value of being unemployed goes down as more unemployed people compete for the available high-tech jobs, reducing the inflow rate into the high-tech sector $q$. The strategy of moving from traditional-sector employment to the unemployment pool will therefore not be chosen in equilibrium (and vice versa).

\(^{11}\) An economically meaningful solution requires $0 < q < 1$ so $\omega > [(1-b)(r+\delta)+(1-\alpha b)]/(1-\alpha)$.
This reveals a positive relation between $q$ and $r$. We can also derive a relation between the number of unemployed and the number of high-tech workers. This relation follows from the stock and flow equilibria on the labour market (equations (20) – (23))\footnote{To avoid corner solutions (in which all labour would be employed and divided over the two sectors) we restrict parameters to cases in which $U > 0$.}

$$\frac{U}{L} = \frac{\delta + \alpha q}{q(1-\alpha)} \frac{L_e}{L} - \frac{\alpha}{1-\alpha}.$$

According to this equation, the unemployment rate $U/L$ is positively related to the number of high-tech workers and negatively related to the outflow rate out of unemployment (and thereby to the interest rate). This can be understood as follows. As more high-tech jobs become available (\textit{ceteris paribus}), the number of high-tech jobs opening up as a result of lay-offs increases. For a given matching probability of unemployed people, this increases the attractiveness of waiting for a high-tech job as an unemployed job seeker. The unemployment rate will rise accordingly. An increase in the interest rate decreases the unemployment rate since a higher interest rate increases the importance attached to current payments. As being unemployed yields a relatively low current pay-off as compared to a traditional sector job, being in the traditional sector becomes relatively more attractive, reducing the unemployment rate (\textit{ceteris paribus}). The model is thus characterised by a (partial) negative relation between growth (formally, the interest rate which, as we will see in the next section, positively depends on the growth rate) and unemployment.

The resulting unemployment in our model has to be thought of as wait unemployment. That is, part of the labour force is deliberately queuing up for the high-paid jobs. In the dual structure that we have in our model, it is impossible to call this type of unemployment either voluntary or involuntary. It is voluntary in the sense that the unemployed could, in principle, choose to be employed in the traditional sector. It is involuntary, however, as all the unemployed people are willing to accept a job in the high-tech sector, but are not offered such a job because of the rationing in that sector.

3. The steady state of the model

In this section, we will elaborate on the steady state equilibrium of the model. The system can be solved after defining a numéraire (alternatively, we could solve the model in relative prices), and after taking into account the definitions for the growth rates that link the levels of consumption, the price index of consumption, the level of technology and the shadow price of the level of technology with their respective growth rates. Furthermore, we need one more equation to determine the
number of firms. The number of firms follows from a zero-profit condition according to which firms enter or leave the market as long as excess profits are non-zero (the free entry regime). The system jumps to a steady-state growth equilibrium as there are no predetermined rigidities and as there are constant returns to scale with respect to knowledge.

The free-entry equilibrium is characterised by a zero-profit condition in the high-tech sector

$$\pi = xp_s - (L_x + L_r + L_s + L_f) w_T = 0.$$  

Using the price equation (13) and the production function (8), this condition can be written as

$$\frac{\varepsilon}{\varepsilon - 1} = \frac{L_x + L_r + L_s + L_f}{L_x} \equiv R.$$  

$R$ will further be denoted as the firm's 'fixed cost ratio' and equals the mark up. It measures total firm size ($L_x + L_r + L_s + L_f$) in relation to the size of the production department ($L_x$). The closer goods from different firms are substitutes, the lower the mark-up will be. A lower mark-up implies that the fixed costs that the firms can afford in relation to their output are lower.

We will now derive the full solution of the model. To start with, notice that in the steady state it holds by definition that

$$\frac{\dot{h}}{h} = \frac{\dot{x}}{x} = \frac{\dot{X}}{X} \text{ and } 0 = \frac{\dot{Y}}{Y}.$$  

Labour productivity in the high-tech sector grows at a constant rate, denoted by $g$. Output of high-tech goods also grows at rate $g$, while output of traditional goods is constant. In addition, from equations (2) and (3) it can be derived that the steady state circular flow equilibrium is characterised by

$$g = \frac{\dot{P}_Y}{P_Y} - \frac{\dot{P}_s}{P_s} = \frac{1}{1 - \sigma} \left( \frac{\dot{P}_C}{P_C} - \frac{\dot{P}_s}{P_s} \right) = \frac{1}{\sigma} \frac{\dot{C}}{C}.$$  

Since households spend a constant fraction $\sigma$ on high-tech goods, the macroeconomic rate of growth is $\sigma g$, whereas the relative price $P_d/P_s$ increases at the rate $g$. Taking the price of the traditional good as numéraire ($P_T = 1$), this implies that the price of a high-tech good decreases at the rate $g$.

$$g = \frac{r - \theta}{\sigma (\rho - 1)}.$$  

The equilibrium growth- and interest rate can be found by confronting investment behaviour from
the firms with savings behaviour from households.\textsuperscript{13} Savings behaviour satisfies the Ramsey rule. This will be called the warranted or required rate of growth. A second relation between the rate of growth and the interest rate follows from producer behaviour\textsuperscript{14}

\[
g = r \left( R - 1 - \frac{\gamma_2 R}{\gamma_1} \right) - \xi F.
\]

This will be called the planned rate of growth. The solution to the model is depicted in Figure 2. In this figure, the line $WW$ represents the warranted rate of growth, while the line $PP$ represents the planned rate of growth. The slope of these curves are $1/[\sigma(\rho-1)]$ and $R-1-\gamma R/\gamma$, respectively.

![Figure 2. Equilibrium growth and interest rate](image)

Stability of the model is guaranteed if the warranted rate of growth intersects the planned rate of growth.

\textsuperscript{13} In this economy, aggregate income equals wage income $I_w (= w_T L_T + w_T L_f)$ plus total dividends ($ND$). Dividends equal high-tech output ($Nxp$) minus production costs ($Nw_T (L_x + L_e + L_f)$) and are paid by high-tech firms. They equal income from financial assets $rA$. Investments by high-tech firms equal $Nw_T L_e$. Savings amount to aggregate income minus consumption expenditure ($I_w + ND - YP - Nxp$). Using the definition for dividends, savings thus amount to $Nw_T L_e$. So, in equilibrium, aggregate investments equal aggregate savings.

\textsuperscript{14} The dynamic equation governing producer behaviour (equation 15) can be written as $L_\delta = r/\xi e$ (using the steady state definition, the definition of the growth rate (equation 1), and equations (13) and (14) from which we derive that $p_e / p_\delta = -g$). The effort wage elasticity is $\gamma \Omega(-a+\Omega)$ (see footnote 4) and is equal to one (see equation 1) from which we can solve for $\Omega$. Substituting this solution for $\Omega$ in equation (12), according to which $\gamma \Omega(-a+\Omega) = L_x / RL_\delta$ (see also footnote 4), we derive that $\gamma / \gamma = L_x / (RL_\delta)$. It therefore holds that the 'fixed cost ratio' $R$ equals $1+Rg/r + R\gamma/\gamma + \xi F/\xi e$. Rewriting this expression yields $g = r(R-1-\gamma R/\gamma) - \xi F$. 

14
growth from above, which holds if \((R-1-\gamma R/\gamma) > 1/(\sigma(\rho-1))\). An economically meaningful steady state equilibrium is characterised by positive growth and interest rates. We can formulate this requirement as \(\xi F > (R-1-\gamma R/\gamma)\theta\). So for the growth and interest rates to be positive, the traditional fixed costs need to be large enough. Using that \(R = \epsilon/(\epsilon-1)\), these conditions can be written as \(\xi F/\theta > [\gamma-\epsilon \gamma]/[\gamma/(\epsilon-1)] > 1/[(\sigma(\rho-1)] > 0\). The details of the solution can be found in Appendix B. In the next section, we will discuss the properties of the model in more detail.

4. The properties of the model

In this section, we will focus attention on the comparative static results that are obtained by changing the fixed costs \((F)\), and the effort-monitoring elasticity \((\gamma_2)\). The equilibrium interest and growth rate follow from confronting the planned and warranted rate of growth as derived in section 3, and using \(R = \epsilon/(\epsilon-1)\). This yields

\[
r = \frac{\xi F \sigma (\rho - 1) - \theta}{\sigma (\rho - 1) Q - 1}
\]

and

\[
g = \frac{\xi F - \theta Q}{\sigma (\rho - 1) Q - 1}
\]

where \(Q = \frac{\gamma_1 - \epsilon \gamma_2}{\gamma_1 (\epsilon - 1) > 0}\).

The equilibrium monitoring intensity and relative wage are then derived as

\[
S = \frac{\epsilon \gamma_2}{\epsilon - 1} \frac{\sigma (\rho - 1) F - \theta \xi}{\sigma (\rho - 1) Q - 1}
\]

and

\[
\omega = \left[ \frac{a}{(1 - \gamma_1) c S^{\gamma_1}} \right]^{\frac{1}{\gamma_1}},
\]

and the equilibrium number of firms as

\[
N = \frac{\epsilon L}{1-a} \left[ \frac{\frac{1 - \sigma}{\gamma_1 \sigma} \frac{1 - \epsilon}{\epsilon - 1} \frac{\sigma (\rho - 1) F - \theta \xi}{\sigma (\rho - 1) Q - 1} - \frac{\delta [\alpha(1-\alpha) - (1-\alpha)],]}{[\gamma_1 + \gamma_2] \sigma (\rho - 1) Q - 1} + \frac{\delta [\alpha(1-\alpha) - (1-\alpha)]}{(1-\alpha)(1-b) \sigma (\rho - 1) Q - 1} \right].
\]

An important remark with respect to the solution for the growth rate is that under free entry the equilibrium rate of growth does not depend on the size of the labour force \(L\). This result is important in the light of the ongoing debate on the importance of scale effects in models of endogenous growth (e.g., Jones, 1995 and Young, 1998).

We now turn to the comparative static characteristics of the model. They are presented in Table 1. An increase in the fixed cost requirement \((F)\) unambiguously increases the growth and the interest rate. This is explained since large fixed costs will leave limited room for firms with non-negative profits. As a consequence, (remaining) high-tech firms will be larger and will have larger market shares. This increases their potential to spread the (quasi) fixed costs of R&D over a large output and thus increases their incentive to engage in R&D. This will result in large growth rates,
and relatively large firms that employ more labour in all activities they perform (i.e., production, research, monitoring, and managing). This result reveals the Schumpeterian character of our model. The increased monitoring intensity ($S$) will be accompanied by a reduction in the relative wage ($\omega$). We can unambiguously derive that employment in the high-tech sector increases (see Appendix B). In other words, the increase in firm size will always outweigh the reduction in number of firms. Due to the terms of trade effect that is associated with the decline in the relative wage, the ratio of traditional sector employment to high-tech employment will fall (so the economy becomes more high-tech in both absolute and relative terms). Also, wait-unemployment as a fraction of high-tech employment ($U/L_T$) declines due to the fact that (i) the relative wage rate declines which implies that the return to waiting is smaller and (ii) the interest rate is larger which increases the importance attached to current payments and thus makes waiting for a future high-paid job less attractive. Still, the effect on the level of traditional employment and unemployment cannot unambiguously be derived. Making the assumption that terms of trade effects do not dominate, traditional sector employment increases along with high-tech employment and unemployment declines.

Table 1. Comparative static results

<table>
<thead>
<tr>
<th>$g$</th>
<th>$r$</th>
<th>$S$</th>
<th>$\omega$</th>
<th>$L_\alpha$</th>
<th>$L_r$</th>
<th>$L_T$</th>
<th>$L_T$</th>
<th>$U$</th>
<th>$N$</th>
<th>$BB$ $^{15}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>n</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>v</td>
<td>v</td>
<td>n</td>
</tr>
</tbody>
</table>

Note: The signs in the cells indicate the signs of the derivatives of the respective variables with respect to the parameters under consideration. A ‘v’ indicates that the variable follows a U-shaped pattern, while a ‘n’ indicates that it follows a hump-shaped pattern. Details on the comparative statics w.r.t. $\gamma_2$ can be found in Appendix B.

Finally, we consider the effects of differences in the effort-monitoring elasticity. We consider differences in this elasticity as representative of differences in the way work is organised. In our view, these differences provide a potential explanation for observed differences in non-competitive wage differentials and the bureaucratic burden, but also for differences in the growth- and unemployment performance of an economy. In the remainder of this section we will analyse how growth, relative wages, unemployment and the sectoral allocation of labour develop as the effort-monitoring elasticity increases. Since we proceed with a focus on differences in the effort-supervi-

$^{15}$ $BB$ represents the bureaucratic burden and is defined as $BB \equiv (L_\alpha + L_T)/(L_\alpha + L_T + L_r + L_T) = (L_\alpha + L_T)/RL_r$. 

16
sion elasticity and assume $\gamma_1$ to be constant, we can consider the combinations of the relative wage and monitoring intensity that result from optimising behaviour as combinations that are required to extract a certain (constant) level of effort from workers (note that the effort level equals $\alpha \gamma_1/[1 - \gamma_1]$ and is thus independent of $\gamma_2$). In our model, monitoring labour is an additional source of (quasi) fixed costs for firms. This implies that the more attractive it becomes for firms to use monitoring labour (i.e., the larger $\gamma_2$) as a means of eliciting effort from workers, the larger the fixed costs will be and, analogous to the logic with respect to increases in $F$, the larger the growth and interest rate will be. Along with this increase, the production- and research departments will become larger in size. The effects on the relative wage and the allocation of labour over the three states on the labour market are non-monotonic. As the effort-monitoring elasticity increases, firms will initially not only increase the amount of monitoring labour they employ, but also the (relative) wage they are willing to pay. In other words, the process of effort extraction initially becomes less effective as $\gamma_2$ increases in that both more monitoring labour and higher wages are required to extract a certain amount of effort. Only when the effort-monitoring elasticity surpasses some critical level, relative wages start to decline (see Appendix B; of course, for given elasticities, the result that high wages are traded off against high monitoring intensities stands upright).

The increase in the relative wage will initially make unemployment such an attractive option that unemployment will increase (even though the increased interest rate makes waiting relatively costly). As the growth rate increases along with $\gamma_2$, waiting will ultimately become so expensive that unemployment will decline. This is reinforced once the relative wage starts to decline. The development of the size of both the high-tech and the traditional sector follows a U-shaped pattern (the mirror-image of unemployment which follows a hump-shaped pattern; see Appendix B for details on the (relative) development of the allocation of labour). Ultimately, we are left with a picture in which countries with a low effort-monitoring elasticity are characterised by low growth, low unemployment, a low non-competitive wage differential, and a high bureaucratic burden. At the other end of the spectrum are countries with high growth rates, low non-competitive wage differentials, low unemployment rates, and a low bureaucratic burden. In intermediate cases, we have countries with high relative wages, high unemployment rates and intermediate rates of growth. The bottom-line of this exercise is that once we start to study empirically the relation between growth and unemployment in a cross-section of countries, one should not be too surprised to find a partial correlation between growth and unemployment that is neither clearly positive nor negative. Differences in institutions like the organisation of work need to be controlled for in a proper and complete way in empirical studies.
A final general remark with which we conclude this section is that in all the comparative static exercises that we discussed, unemployment and high-tech employment move in opposite directions. This is important in the light of an often heard critique on the standard Harris-Todaro type of dual labour market models. Lindbeck and Snower, 1991, criticize the Harris-Todaro types of models for this feature as it is inconsistent with empirical evidence. Our general equilibrium framework turns out to overcome this unattractive feature. This result shows the importance of a sound general equilibrium framework in which also demand and supply considerations are taken into account when analysing the effects of, for example, policy changes (Lindbeck and Snower, 1991, point at the importance of these general equilibrium effects but do not model them explicitly).

5. Conclusion
In this paper, we studied the implications of different ways of organising work for growth and unemployment. The model can best be characterised as a dynamic general equilibrium model with a non-Walrasian labour market structure. Investment in R&D is a major source of fixed cost and therefore of excess profits in imperfectly competitive product markets. The innovative aspect of the paper is that incumbent firms are assumed to be willing to share excess profits with their workers due to the presence of an effort-extraction function. Firms trade off high wages against intensive monitoring. This results in a dual economy with high-paying jobs in the growth-generating high-tech sector and low-paying jobs in the traditional sector.

Changes in the way work is organised within firms turned out to affect growth and unemployment via various channels. The extent to which firms rely on paying high wages relative to intensive monitoring was shown to be an important determinant for both growth and unemployment. The more firms rely on paying high wages, the larger the non-competitive rents will be that workers are searching for, and hence the larger equilibrium unemployment will be. Intensive monitoring is a source of fixed costs for firms. Due to the Schumpeterian character of the model in which large market shares have a positive influence on the incentives of firms to engage in R&D, the monitoring intensity is thus an important determinant of the rate of growth. We finally concluded that countries relying heavily on monitoring can thereby afford the payment of low relative wages in the process of effort extraction and are characterised by high growth, low unemployment, and a low bureaucratic burden.

This paper shows that controlling for labour market institutions in a broad sense, including factors related to, for example, the organisation of work, is of crucial importance when empirically studying the relation between growth and unemployment. The negative relation between growth
and unemployment that we found in our theoretical model may remain unnoticed in empirical research due to cross-country differences that have not been taken into account. One should therefore not be too surprised that the partial relation between growth and unemployment is neither clearly positive nor negative (see also Bean and Pissarides, 1993, Nickell and Layard, 1997, for an overview of theoretical and empirical studies on growth and unemployment). Although an empirical investigation on the relation between growth and unemployment is beyond the scope of this paper, we think that this is an interesting way to go and may yield new insights.

Appendix A. Derivation of equations (4) – (15)

On the producer side of the model we assume that high-tech firms compete monopolistically. Each firm, producing a unique brand of the high-tech good, is assumed to maximise its present discounted value:

\[
\max_{L_{st}, L_{st}, w_{st}, x_{st}} \int_0^{\infty} \left[ x_{st} p_{st} - (L_{st} + L_{st} + L_{st} + L_{st}) w_{st} \right] e^{-rt} dt,
\]

subject to (time indices have been omitted for reasons of clarity)

\[
x_{i} = h_{i} e_{i} L_{st}, \quad (A.2)
\]

\[
e_{i} = -a + c \left[ \frac{w_{ti}}{w_{t}} \right] S_{i}^{\gamma}, \quad (A.3)
\]

\[
\dot{h}_{i} = \xi_{i} e_{i} h_{i} L_{st}, \quad (A.4)
\]

\[
x_{i} = X \left( \frac{p_{st}}{P_{X}} \right)^{e}, \quad (A.5)
\]

\[
F_{i} = L_{fi} e_{i}, \quad (A.6)
\]

\[
S_{i} = L_{si} e_{i}. \quad (A.7)
\]

The 'current value' Hamiltonian corresponding to this optimisation problem is

\[
H_{i} = x_{st} p_{st} - \left( L_{st} + L_{st} + \frac{S_{i} + F_{i}}{e_{i}} \right) w_{st} + p_{hi} \xi_{i} e_{i} h_{i} L_{st}, \quad (A.8)
\]

where \( p_{hi} \) is the shadow price of the level of technology \( h_{i} \). This shadow price is a measure of the marginal value of an additional unit of \( h \) for the firm.

The first order conditions of this maximisation problem are

\[
\frac{\partial H}{\partial w_{ti}} = \frac{\partial x_{i}}{\partial e_{i}} \frac{\partial e_{i}}{\partial w_{ti}} p_{st} \left( \frac{e-1}{e} \right) - \left( L_{st} + L_{st} + \frac{S_{i} + F_{i}}{e_{i}} \right) w_{ti} + \frac{\partial e_{i}}{\partial w_{ti}} \frac{w_{ti} \left( S_{i} + F_{i} \right)}{e_{i}^{2}} + \frac{\partial e_{i}}{\partial w_{ti}} p_{hi} \xi_{i} e_{i} h_{i} L_{st} = 0
\]

\[
= h_{i} L_{st} \frac{\partial e_{i}}{\partial w_{ti}} p_{st} \left( \frac{e-1}{e} \right) - \left( L_{st} + L_{st} + \frac{S_{i} + F_{i}}{e_{i}} \right) w_{ti} + \frac{\partial e_{i}}{\partial w_{ti}} \frac{w_{ti} \left( S_{i} + F_{i} \right)}{e_{i}^{2}} + \frac{\partial e_{i}}{\partial w_{ti}} p_{hi} \xi_{i} e_{i} h_{i} L_{st} = 0
\]

\[
= h_{i} L_{st} \frac{\partial e_{i}}{\partial w_{ti}} p_{st} \left( \frac{e-1}{e} \right) - \left( L_{st} + L_{st} + \frac{S_{i} + F_{i}}{e_{i}} \right) w_{ti} + \frac{\partial e_{i}}{\partial w_{ti}} \frac{w_{ti} \left( S_{i} + F_{i} \right)}{e_{i}^{2}} + \frac{\partial e_{i}}{\partial w_{ti}} p_{hi} \xi_{i} e_{i} h_{i} L_{st} = 0
\]
We now invoke the symmetry assumption. From equation (A.11) it directly follows that firms engage in mark-up pricing (equation (13) in the text). Equation (A.12) yields the optimal R&D input (equation (14) in the text). Equation (A.13) is the dynamic equation governing the allocation of high-tech labour over time. Using equations (A.11) and (A.12) and rewriting yields equation (15) in the text. Finally, substituting equations (A.11) and (A.12) into equations (A.9) and (A.10) we get the set of ‘Solow-conditions’ (equations (11) and (12) in the text).

Appendix B. Solution of the complete model

The reduced system of equations from which we can solve the complete model consists of the equations\textsuperscript{16}:

\begin{align}
g &= r \left( R - 1 - \frac{\gamma_i R}{\gamma_1} \right) - \xi F, \quad (B.1) \\
g &= \frac{r - \theta}{\sigma(r - 1)}, \quad (B.2) \\
L_T &= NRL_i, \quad (B.3) \\
L_x &= \frac{r}{\xi e}, \quad (B.4) \\
L_y &= \frac{1 - \sigma}{\sigma} NL_i - \frac{e}{\varepsilon - 1} \omega, \quad (B.5) \\
U &= \frac{\delta + \alpha q}{q(1 - \alpha)} L_T - \frac{\alpha}{1 - \alpha} L, \quad (B.6) \\
q &= \frac{(1 - b)(r + \delta)}{\omega(1 - \alpha) - (1 - \alpha b)}, \quad (B.7) \\
L &= L_T + L_T + U, \quad (B.8) \\
e &= \frac{\alpha \gamma_i}{1 - \gamma_1}, \quad (B.9)
\end{align}

\textsuperscript{16} Equation (B.5) is derived using goods-market equilibrium according to which spending on the available goods is divided according to \((1 - \sigma)/\sigma = YP_T/Nx = L_T\omega/(\varepsilon - 1)/(NL_\omega e \varepsilon)\).
\[ \omega = \left[ \frac{a}{(1 - \gamma_1) e S \gamma_2} \right]^{\frac{1}{\gamma_2}} = \left[ \frac{a}{(1 - \gamma_1) e \left( \frac{\gamma_2 R}{\xi \gamma_1} \right)^{\gamma_2}} \right]^{\frac{1}{\gamma_2}}. \]  
(B.10)

\[ R = \frac{\varepsilon}{\varepsilon - 1}. \]  
(B.11)

Combining the planned and warranted rate of growth (equations (B.1) and (B.2)) we can derive the equilibrium rate of growth and the interest rate as

\[ g = \frac{\xi F - \theta \left( R - 1 - \frac{\gamma_2 R}{\gamma_1} \right)}{R - 1 - \frac{\gamma_2 R}{\gamma_1}} \sigma (\rho - 1) - 1 \]  
and

\[ r = \frac{\xi F \sigma (\rho - 1) - \theta}{R - 1 - \frac{\gamma_2 R}{\gamma_1}} \sigma (\rho - 1) - 1. \]  
(B.12)

The number of production workers now follows from equation (B.4)

\[ L_x = \frac{F \sigma (\rho - 1) - \theta}{e - \xi e} \frac{\xi}{R - 1 - \frac{\gamma_2 R}{\gamma_1}} \sigma (\rho - 1) - 1. \]  
(B.13)

Using equations (B.3), (B.5), (B.6), and (B.7), high-tech employment, traditional employment and unemployment can now be written as a function of the parameters of the model, the number of firms, \( N \), the relative wage, \( \omega \), and \( R \). Substituting the expressions for \( L_T, L_S, \) and \( U \) into equation (B.8), we can solve for the equilibrium number of firms as a function of \( R \) and the relative wage (which can also be written as a function of \( R \))

\[ N = \frac{e^{L_x/1-\alpha}}{F \sigma (\rho - 1) - \theta \left( R - 1 - \frac{\gamma_2 R}{\gamma_1} \right) \sigma (\rho - 1) - 1}. \]  
(B.14)

**Comparative Statics**

The comparative static characteristics as described in the text and in Table 1 with respect to \( r, g, S, L_x(= r/\xi e), L_T(= g/\xi e) \) and \( L_s = S/e \) are straightforwardly derived by taking first order derivatives. The comparative static results with respect to the bureaucratic burden can be derived by solving for the bureaucratic burden as

\[ BB = \frac{L_x + L_T}{R L_x} = \frac{\gamma_2}{\gamma_1} + \frac{\left[ R \left( \frac{\gamma_1 - \gamma_2}{\gamma_1} \right) \sigma (\rho - 1) - 1 \right]}{R \sigma (\rho - 1) - \frac{\theta}{\xi F}}. \]  
(B.15)

and taking derivatives with respect to the parameters under consideration. To consider the effects of a change in \( \gamma_2 \) on the relative wage as discussed in section 4, we derive from (B.10) that
Using equation (B.17), we know that at any point onwards, we cannot unambiguously conclude that $N$ is decreasing in $\gamma$. The second order derivative is negative so eventually $\omega$ becomes a declining function of $\gamma$. The comparative static characteristics of $\omega$ are as follows. Since the interest rate increases and the relative wage declines, we know from equation (B.17) that $NL_t$ increases so high-tech employment increases. The effects on unemployment and traditional employment are ambiguous. In the economically most reasonable case where inter-sectoral terms of trade effects do not dominate, $L_T$ increases and unemployment declines. We can, however, not preclude a priori that traditional sector employment declines and unemployment increases. The effects of $b$ do not depend on the sign of $\gamma$, since changes in $b$ leave the relative wage rate unaffected.

The effects of changes in $\gamma$ on the allocation of labour are non-monotonic. We know that $r$ is increasing in $\gamma$. We have also seen that $\omega$ reaches a maximum value at some $\gamma^*$. We define this value as $\omega^{\gamma^*}$. Starting from this point, we will now derive the relative position of the peaks and/or troughs of the sectoral labour shares in several steps. The derivatives of variables of interest w.r.t. $\gamma$ at different values of $\gamma$ are summarized in Table B.1 that is constructed on the basis of the following reasoning:

(i) Using equation (B.17), we know that at $\gamma^{\omega}$, $L_T (= RNL_o)$ is increasing since $\omega$ is constant and $r$ is increasing. So by using goods-market equilibrium (equation 1), we can conclude that also $L_T$ is increasing in $\gamma$ at $\gamma^{\omega}$. Unemployment is thus decreasing in $\gamma$ at $\gamma^{\omega}$.

(ii) When $\gamma < \gamma^{\omega}$, both $\omega$ and $r$ are increasing in $\gamma$. At low values of $\gamma$, the increase in the relative wage rate is strong relative to the increase in $r$, so $NL_t$ is decreasing. At some value for $\gamma$ which we define as $\gamma^{LT}$, $L_T (= NRL_o)$ reaches a minimum. At this point, $L_T$ is increasing in $\gamma$ since $\omega$ is increasing. Unemployment is thus decreasing in $\gamma$ at $\gamma^{LT}$.

(iii) The strong increase in $\omega$ at low levels of $\gamma$ exerts an upward pressure on unemployment, where unemployment reaches a maximum at a point we define as $\gamma^{LT}$. At this value, traditional sector employment is increasing in $\gamma$ since high-tech employment is decreasing.

(iv) $L_T$ reaches a minimum at $\gamma^{LT}$ which must be to the left of $\gamma^{LT}$.

The effects on $N$ follow by using that $N = L_T/RNL_o = L_T/\varepsilon Rr$. At low values of $\gamma$, $L_T$ is decreasing in $\gamma$, while $r$ is increasing in $\gamma$, so $N$ is unambiguously decreasing, until $L_T$ starts to increase. From this point onwards, we cannot unambiguously conclude that $N$ is decreasing in $\gamma$. 

\[ \frac{d\omega}{d\gamma} = -\frac{\omega}{\Gamma} \left[ \ln S + \frac{\sigma(\rho - 1) - (\varepsilon - 1)}{\sigma(\rho - 1)} \right]. \]  (B.16)

At low levels of $\gamma$, this derivative is positive ($\ln(S)$ tends to $-\infty$ as $\gamma$ approaches zero from above). So at small values of $\gamma$, $\omega$ is increasing in $\gamma$. The second order derivative is negative so eventually $\omega$ becomes a declining function of $\gamma$. The comparative static characteristics of $\omega$ are then easily derived as reported in Table 1.
Table B.1. Derivative of variables of interest w.r.t. γ₂: at different values of γ₂

<table>
<thead>
<tr>
<th></th>
<th>γ₂^{LY}</th>
<th>&lt;</th>
<th>γ₂^{U}</th>
<th>&lt;</th>
<th>γ₂^{LT}</th>
<th>&lt;</th>
<th>γ₂^{o}</th>
</tr>
</thead>
<tbody>
<tr>
<td>ω</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>r</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>LT</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>U</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>LY</td>
<td>−</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>N</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>?</td>
</tr>
</tbody>
</table>

Appendix C. Some numerical simulations

In this appendix, we perform some numerical experiments to get a feeling for the comparative static characteristics of the model and the sensitivity of the model with respect to parameter changes. We start from a set of base-line parameters that is given in Table C.1. These parameters result in \( g=3.176\%, \ \omega=1.08, \ U=10.99, \ L_r=51.69, \ L_Y=37.31, \ L_c=2.67, \ L_s=0.73, \ L_o=0.22, \ N=12.90 \) and \( q=0.127 \). Based on the constraints that we imposed in the main text (0<q<1, U>0, g>0, and stability of the model), we derived extreme bounds of the parameter values. These are given in Table C.1.

Table C.1. Base values parameters and extreme bounds

<table>
<thead>
<tr>
<th></th>
<th>Base</th>
<th>min</th>
<th>max</th>
<th></th>
<th>Base</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>ξ</td>
<td>0.02</td>
<td>0.0104</td>
<td>0.0323</td>
<td>γ₁</td>
<td>0.7</td>
<td>0.6925</td>
<td>0.9999</td>
</tr>
<tr>
<td>F</td>
<td>0.8</td>
<td>0.4143</td>
<td>1.0536</td>
<td>γ₂</td>
<td>0.04</td>
<td>0</td>
<td>0.0501</td>
</tr>
<tr>
<td>θ</td>
<td>0.02</td>
<td>0.0048</td>
<td>0.0386</td>
<td>a</td>
<td>0.925</td>
<td>0.9018</td>
<td>∞</td>
</tr>
<tr>
<td>σ</td>
<td>0.6</td>
<td>0.5553</td>
<td>1</td>
<td>c</td>
<td>3</td>
<td>0.0001</td>
<td>3.0773</td>
</tr>
<tr>
<td>ρ</td>
<td>6</td>
<td>5.5606</td>
<td>∞</td>
<td>α</td>
<td>0.25</td>
<td>0.0001</td>
<td>0.3790</td>
</tr>
<tr>
<td>ε</td>
<td>3</td>
<td>2.1001</td>
<td>3.1320</td>
<td>δ</td>
<td>0.05</td>
<td>0.0186</td>
<td>1</td>
</tr>
<tr>
<td>L</td>
<td>100</td>
<td>0.0001</td>
<td>∞</td>
<td>b</td>
<td>0.96</td>
<td>0.9268</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

Comparative static characteristics are presented by graphical means in Figures C.1 – C.3. These pictures show the impact of the respective parameters on the endogenous variables under consideration. Starting from the base-line, the figure reveals what values the endogenous variables take when one parameter of interest deviates from its base-line value. In the figures we put the value of the endogenous variable under consideration on the vertical axis. On the horizontal axis we depict the value of the parameter under consideration as a proportion of its base-line value (assuming all other variables remain unchanged).
References


