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2003

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### **citation for published version (APA)**

Koopman, S. J., & Lucas, A. (2003). *Business and Default Cycles for Credit Risk*. (Discussion Papers; No. 03-062/2). Tinbergen Instituut (TI). <http://www.tinbergen.nl/discussionpapers/03062.pdf>

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TI 2003-062/2

Tinbergen Institute Discussion Paper

# Business and Default Cycles for Credit Risk

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# Business and Default Cycles for Credit Risk\*

*First version: January 23, 2003*

*This version: July 29, 2003*

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## Abstract

Various economic theories are available to explain the existence of credit and default cycles. There remains empirical ambiguity, however, as to whether or these cycles coincide. Recent papers suggest by their empirical research set-up that they do, or at least that defaults and credit spreads tend to co-move with macro-economic variables. If true, this is important for credit risk management as well as for regulation and systemic risk management. In this paper, we use 1927–1997 U.S. data on real GDP, credit spreads, and business failure rates to shed new light on the empirical evidence. We use a multivariate unobserved components framework to disentangle credit from business cycles. It turns out that cyclical co-movements arise between default rates, but not real GDP. There is, however, a contemporaneous correlation between real GDP and default rates. Regarding the longer term evolution of the series, credit spreads influence default rates and real GDP, but not vice versa. This corroborates some of the empirical findings in the recent literature on the correlation between macro-variables and default rates. It also suggests the use of credit spreads besides or instead of economic growth rates to forecast the dynamics of future default rates.

**Key words:** credit cycles; business cycles; defaults; credit risk; procyclicality; multivariate unobserved component models.

**JEL Codes:** C19; G21.

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\*We thank Hashem Pesaran and Allan Timmerman for useful discussions. All errors are our own.

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# 1 Introduction

Credit risk research has considerably gained momentum over the last decade, see for example Caouette, Altman, and Narayanan (1998) and Allen and Saunders (2003) for an overview.<sup>1</sup> Spurred by regulatory developments, different classes of models have been put forward to measure, manage, and price credit risk. In this paper we study the dynamic behavior of two important determinants of credit risk, namely the default rate and the credit spread, in their relation to business cycle developments. We use a multivariate unobserved components approach to disentangle long-term patterns from shorter term cyclical patterns. We are particularly interested in testing whether cycles in credit risk factors coincide with business cycles. To answer this question, our model explicitly allows for direct estimation of lead and lag times between the different series under consideration.

Early credit risk models focus on the prediction of the likelihood to default (credit scoring) using, e.g., Altman's Z-score, logit and probit models, and neural networks, see Altman (1983) and Caouette et al. (1998). These models usually emphasize the cross-sectional rather than the time-series dimension of the sample to distinguish 'good' from 'bad' companies. The time-series or dynamic behavior of credit risk, however, has become increasingly important over the last few years both among academics, practitioners, and regulators. Three reasons for this appear important.

First, the market for credit risk has become much more liquid, see for example Patel (2003). Asset backed securities like Collateralized Bond and Loan Obligations (CBOs and CLOs), as well as credit derivatives, allow financial institutions to mitigate their credit risk exposure without breaking up client-relationships. Appropriate pricing and hedging of these new generation credit instruments, however, requires an adequate description of the dynamic behavior of interest rates, default and recovery rates, and credit spreads. Typical examples include Jarrow and Turnbull (1995), Jarrow, Lando, and Turnbull (1997), and Duffie and Singleton (1999), but see also the earlier work of Merton (1974). To identify the dynamic behavior of the relevant economic variables, one can either use directly observed historical data on the variables themselves, or use implied models based on prices of liquid credit sensitive instruments like credit default swaps, see for example Duffie and Singleton (1999). The increased flexibility in managing a portfolio of credits through derivatives or securitizations complements the well-known credit scoring methodology. Moreover, it entails a shift in attention from cross-sectional, point-in-time predictions of default to a dynamic credit man-

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<sup>1</sup>See also the collection of papers at <http://www.defaultrisk.com>.

agement perspective.

A second reason for the attention for credit risk dynamics lies in the adoption of a portfolio perspective to credit risk, see Gupton, Finger, and Bhatia (1997), Credit Suisse (1997), and Wilson (1997a,b). Whereas the models of, e.g., Jarrow and Turnbull (1995) and Duffie and Singleton (1999) can in principle be used both for single-name and multi-name credit risky instruments, there is a crucial difference as to the type of risk that is important. Making the standard distinction between idiosyncratic and systematic risk, it is the systematic risk that is most important at a portfolio level, see for example Jarrow, Lando, and Yu (2000), Frey and McNeil (2001), Lucas, Klaassen, Spreij, and Straetmans (2001), and Giesecke and Weber (2003). The idiosyncratic risk can be largely diversified. Portfolio models like *CreditMetrics* of Gupton, Finger, and Bhatia (1997) and *CreditRisk<sup>+</sup>* of Credit Suisse (1997) pay little attention to the dynamic behavior of the systematic risk factor, though extensions of these models are possible, see Finger (1999) and Li (1999). An exception is the *CreditPortfolioView* model, see Wilson (1997a,b). Systematic credit risk factors are usually thought to correlate with macro-economic conditions. This appears both from theoretical models on real business cycles, like for example Williamson (1987), Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), and Kwark (2002), and from empirical evidence, see for example Wilson (1997a,b), Nickell, Perraudin, and Varotto (2000), Bangia, Diebold, Kronimus, Schagen, and Schuermann (2002), and Kavvathas (2001). There is of course much experience in modeling the dynamic behavior of macro-economic variables. If, therefore, a link can be established between the macro-economic environment and systematic credit risk factors, knowledge on the state and direction of macro variables may help in assessing portfolio credit risk over time.<sup>2</sup>

The third reason for the interest in the dynamics of credit risk lies in regulatory developments, see Basle Committee on Bank Supervision (2003). In the new proposals of the Basle Capital Accord, banks have to link their capital requirements directly to the creditworthiness of counterparties. The creditworthiness is assessed through default probabilities and collateral values. Default probabilities can be taken implicitly from ratings issued by the official rating agencies, or explicitly from banks' own internal rating models. A major concern with the new regulatory framework is that it may lead to pro-cyclical capital requirements, see Basel Committee on Banking Supervision (2002), and in this way to exacerbated business cycle fluctua-

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<sup>2</sup>The reverse may also be true: knowledge on the state of credit risk markets may help to predict macro-economic developments, see for example Kwark (2002) and Guha and Hiris (2002).

tions. The argument is that during an upswing of the economy, banks may lower their capital levels. Such a decrease in capital may be spurred by risk sensitive capital requirements based on recent estimates of default probabilities, see Altman and Saunders (2001) and Borio, Furfine, and Lowe (2001). As a result, capital levels may be too low at the peak of the cycle to cope with the subsequent downswing. The capital accumulation during the downswing may also be too slow. Moreover, the increases in capital may result in a credit crunch and thus worsen already adverse economic conditions, see Laeven and Majnoni (2002). The issue of pro-cyclicality highlights the need to assess whether ratings, default rates and spreads, and other credit risk drivers are pro-cyclical or not. The empirical evidence appears inconclusive. Whereas Altman and Saunders (2001) find ratings lagging the business cycle, D’Amato and Furfine (2003) claim that business cycle conditions influence new ratings much more than existing ratings. Moreover, using a theoretical model, Gorton and He (2003) show that credit cycles may have their own dynamics distinct from business cycles. See also Das, Freed, Geng, and Kapadia (2001) for empirical evidence.

Given the importance of dynamic credit risk modeling and the controversy on the exact relation of credit risk drivers with the state of the business cycle as mentioned above, we set out in this paper to build a multivariate time-series model for business failure rates, credit spreads, and real GDP growth. Empirical models that link default rates to macro variables can be found in Wilson (1997a,b), Nickell et al. (2000), Bangia et al. (2002), Kavvathas (2001), and Pesaran, Schuermann, Treutler, and M.Weiner (2003). The general conclusion of these models is that defaults probabilities tend to be higher in recession states, see also Allen and Saunders (2003). Empirical evidence linking credit spreads to the business cycle can be found in for example Fama and French (1989), Chen (1991), and Stock and Watson (1989). There, the general conclusion is that risk premia on bonds contain a countercyclical component and that credit spreads are good predictors for future business cycle conditions.

Our paper contributes to the literature in three ways. First, we build a trivariate model including both default rates and spreads in their relation to economic growth. Though bivariate analyses using either spreads or default rates in a combination with economic growth rates have been more prevalent in the literature, the empirical evidence mentioned earlier suggests that an analysis based on all three series simultaneously is more appropriate. In this way, we can investigate the claimed lead-relationship of credit spreads over growth, the (in)congruence between credit and business cycles, and the dynamics of default rates in one unified framework. The joint behavior of these series can moreover be used as an input to credit risk models in much

the same vein as in Pesaran et al. (2003). Our second contribution lies in the fact that we use an unobserved components model, see Harvey (1989) and Durbin and Koopman (2001). In this way, we are able to disentangle long-term (co)-movements from short-term cyclical movements in a clear and interpretable way. By focusing on the time-series dimension of our series, we also complement the existing literature by considering a long time span of data: 1927–1997. By contrast, papers like Nickell et al. (2000) and Bangia et al. (2002) focus much more on the cross-sectional dimension to estimate their models, typically using time series of 20 to 25 years for a large number of companies. Our longer time span allows for repeated observations on business cycles and therefore helps to test for the presence and co-variation of cyclical patterns in credit risk factors. The importance of the time-series dimension in credit risk analysis is also stressed in Gordy and Heitfield (2002). Finally, we use the recent approach by Rünstler (2002) to estimate lead and lag relationships between the cyclical movements in the three series directly from the data. A common approach to testing for lead/lag-relationships is by replacing explanatory variables in empirical models by their leads or lags. The difficulty with such an approach is that the models with different lags are difficult to compare. Moreover, statistical inference on the lead or lag-length is generally hard unless an explicitly Bayesian perspective is adopted. These problems are resolved in the parameterization suggested in Rünstler (2002). Here, we obtain a direct estimate of the lead/lag time with corresponding standard errors for inference purposes. This is particularly interesting in our present context, where claimed lead times of macro indicators are the main drivers for their inclusion in credit risk portfolio models.

Our empirical findings reveal a rich and diverse view on the dynamic relations between the three series considered. We distinguish between contemporaneous, cyclical, and long-term relations. The cyclical pattern in the series appears to be common to default rates and credit spreads, but not real GDP growth. Over the longer term, there is a feedback from past credit spreads to present and future real GDP growth, but not vice versa. There is, however, also a contemporaneous effect in the sense that the shock to the long-term component of real GDP is strongly negatively correlated with a shock to default rates. This may explain findings as in Nickell et al. (2000) and Bangia et al. (2002). Such correlations, however, should not be mistaken for evidence that default rates and real GDP are also co-cyclical in the sense that the business cycle coincides with a default cycle. This holds even after we allow for possible phase shifts in the cycle between the different series. To capture cyclical co-movements, credit spreads appear more promising as conditioning variables than economic growth variables. This is the more interesting given that credit spreads are more timely than GDP



growth and can be read directly from financial market information. The discrepancy between cyclical movements in default rates and credit spreads on the one hand, and GDP growth on the other, also has a possible impact on the pro-cyclicality issue mentioned earlier. Though a thorough investigation of this issue is beyond the scope of the current paper, the empirical patterns emerging from our analysis illustrate that more research is needed to uncover the intricate dynamic relations between credit and default cycles and their impact on the pro-cyclicality debate.

The paper is set-up as follows. In Section 2, we discuss the data and our modeling approach. The empirical results are contained in Section 3. Section 4 concludes.

## 2 Data and modeling approach

We use three data series in our analysis: real GDP growth, credit spreads, and business failure rates. The first series, real GDP, is taken from the data base of the Federal Reserve Bank of St.Louis (FRED). The series contains GDP in chained 1996 dollars for the period 1929–2002. From the same site, we also obtain Moody’s yields on Baa corporate bonds and the yield on government bonds with a maturity exceeding 10 years. These are used to construct annual credit spreads, defined as the difference between the two yields. We have credit spreads for the period 1925–1999. Our third series is from Dun and Bradstreet (1998) and contains U.S. business failure rates per 10,000 companies over the period 1927–1997. After 1997, the series was discontinued. Following the description of Dun and Bradstreet (1998), the numbers indicate businesses that ceased operations after assignment or bankruptcy; ceased operations with losses to creditors after such actions as foreclosure or attachment; voluntarily withdrew leaving unpaid debts; were involved in court actions such as receivership, reorganization or arrangement; or voluntarily compromised with creditors. As such, the business failure rate may be an underestimate of the default rate, because defaulting investment projects within a business may be compensated by well-performing projects within that same business, see also Kwark (2002). In this paper, however, we are not as much involved with the level of the default rate, but with its dynamic behavior over time and its co-variation with other variables included in the model. Given the difficulty in obtaining reliable default rate statistics from competing sources, we take the business failure rate as a proxy for describing default rate dynamics.<sup>3</sup> Combining all three series, our sample

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<sup>3</sup>One additional potential complication is the change in data collection by Dun & Bradstreet after 1984. This increased both the number of businesses and business failures. The

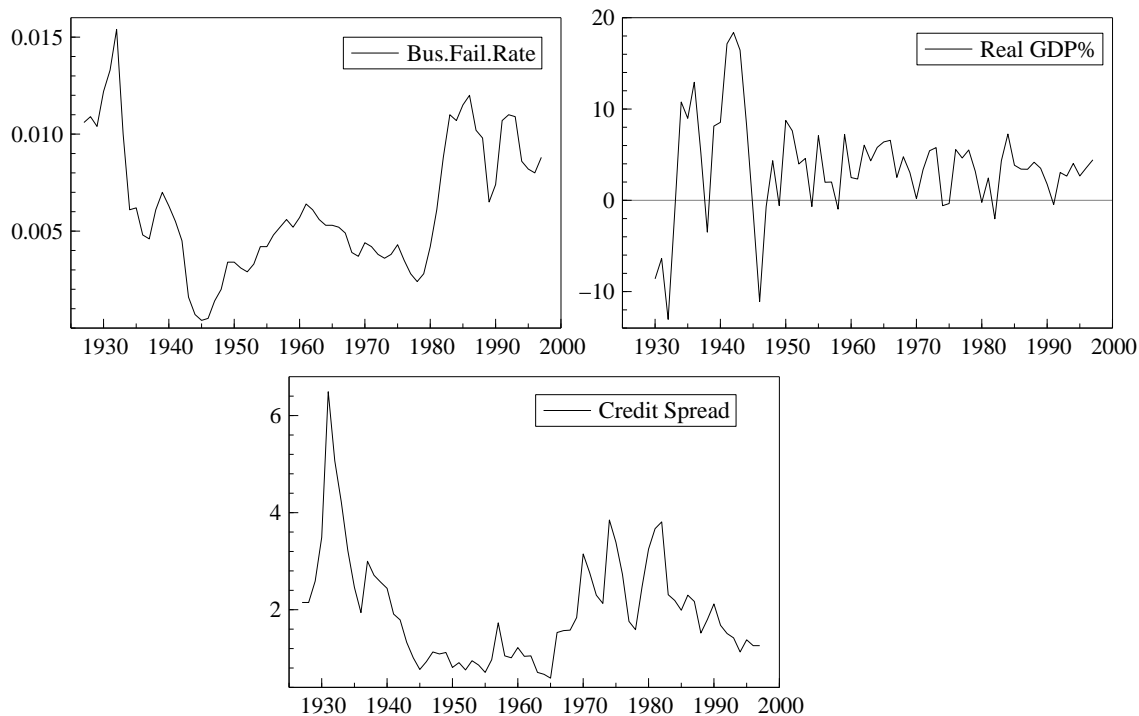


Figure 1: Business failure rate, Real GDP growth, and credit spread

runs from 1927 up to 1997, but with the real GDP growth observations from 1927 up to 1929 missing. The missing observations can be handled without difficulty in our estimation approach based on the Kalman Filter. The data is presented in Figure 1.

Though the series show some similarities, there are also marked differences. For example, all series show a much greater variability during the earlier years in the sample than in later years. Moreover, the long-term patterns are quite distinct between the various series. Whereas real GDP growth appears to fluctuate around a steady long-term mean, this is much less clear for the failure rate and the credit spread. Also, the credit spread and failure rate appear more congruent in the first half of the sample than the second half. This, however, is largely due to two outlying observations in 1931 and 1932 for the spread. In order not to corrupt the dynamic relations between the series by these two observations, we treat them as missing in our analysis.

To describe the dynamic behavior of the three series as well as their in-  


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 failure rate, however appears relatively unaffected. Formal testing revealed no statistical evidence of a structural changes in the failure rate as of 1984.

terdependencies, we introduce an unobserved components model, see Harvey (1989) and Durbin and Koopman (2001). Our basic specification<sup>4</sup> is

$$y_t = c + \gamma_t + A\psi_t + \varepsilon_t, \quad \varepsilon_t \stackrel{\text{i.i.d.}}{\sim} N(0, \Sigma_\varepsilon) \quad t = 1, \dots, n, \quad (1)$$

where  $c$  is a vector of constants, and  $y_t$  represents the time series observation vector. The observation vector is given by

$$y_t = \begin{bmatrix} y_t^D \\ y_t^S \\ y_t^R \end{bmatrix} = \begin{bmatrix} \text{business failures (DEFAULT)} \\ \text{credit spreads (SPREAD)} \\ \text{real GDP growth \% (RGDP)} \end{bmatrix}, \quad t = 1, \dots, n, \quad (2)$$

where  $t = 1$  for 1927 and  $t = n = 71$  for 1997. The stationary component  $\gamma_t$  is a persistent long-term component, whereas component  $\psi_t$  is for cyclical medium term dynamics. The irregular component  $\varepsilon_t$  is included to allow for measurement noise in the observations. Components  $\gamma_t$  and  $\varepsilon_t$  are vectors while the cycle component  $\psi_t$  is a univariate unobserved variable common to all time series in  $y_t$ . The vector  $A$  contains unknown scaling constants and is sometimes referred to as a vector of factor loadings. The irregular component  $\varepsilon_t$  is normally distributed and its elements are mutually and serially uncorrelated.

The stationary vector  $\gamma_t$  is modelled as a vector autoregressive process of order two, in short VAR(2), given by

$$\gamma_t = \Gamma_1 \gamma_{t-1} + \Gamma_2 \gamma_{t-2} + \eta_t, \quad \eta_t \stackrel{\text{i.i.d.}}{\sim} N(0, \Sigma_\eta), \quad (3)$$

where coefficient matrices  $\Gamma_1$  and  $\Gamma_2$  and variance matrix  $\Sigma_\eta$  are fixed and unknown. The disturbance vector  $\eta_t$  is mutually uncorrelated with other disturbances in the model for all time periods. More general processes within this class of models can also be considered; see, for example, Lutkepohl (1991) for details on vector autoregressive moving average models. The coefficient matrices can be constrained to have no roots outside the unit circle so that component  $\gamma_t$  is forced to be stationary; see Ansley and Kohn (1986). However such reparameterizations were not necessary for the empirical analysis presented in the next section.

Various specifications for the stationary cycle component  $\psi_t$  can be considered. For example, the cycle can be modelled as an autoregressive process of order 2, in short AR(2), with the polynomial autoregressive coefficients

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<sup>4</sup>We have also experimented with competing model specifications and comment on our findings at the end of the next section when discussing the robustness of our empirical results.

selected in the complex range. To enforce this restriction we can represent the model as a trigonometric process, that is

$$\begin{pmatrix} \psi_t \\ \psi_t^* \end{pmatrix} = \phi \begin{bmatrix} \cos \lambda & \sin \lambda \\ -\sin \lambda & \cos \lambda \end{bmatrix} \begin{pmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{pmatrix} + \begin{pmatrix} \omega_t \\ \omega_t^* \end{pmatrix}, \quad \begin{array}{l} \omega_t \stackrel{\text{i.i.d.}}{\sim} \text{N}(0, \sigma_\omega^2), \\ \omega_t^* \stackrel{\text{i.i.d.}}{\sim} \text{N}(0, \sigma_\omega^2), \end{array} \quad (4)$$

with frequency  $\lambda$  and persistence parameter  $|\phi| < 1$ . The disturbances  $\omega_t$  and  $\omega_t^*$  are serially and mutually uncorrelated and normally distributed with common variance  $\sigma_\omega^2$ . This stochastic cycle specification generates a stationary cyclical process with a period of  $p = 2\pi/\lambda$ . The factor loading vector  $A$  scales the common cycle for the individual series. For the identification of  $\sigma_\omega^2$ , the first element of  $A$  is restricted to unity.

The cycle is restricted to be common to all series in  $y_t$ . However it is well known that lead and lag relationships between cyclical dynamics of macroeconomic time series may exist. This indicates that cycles are possibly more correlated when phase shifts have taken place. Such phase shifts for the cycle component can be incorporated within model (4) by following a suggestion of Rünstler (2002). It follows from the construction of the stochastic cycle process (4) and a standard trigonometric identity that the cycle process  $\psi_t$  is shifted  $\xi$  time periods to the right (relative to  $\psi_t$  itself and for  $\xi > 0$ ) by considering

$$\cos(\lambda\xi)\psi_t + \sin(\lambda\xi)\psi_t^*,$$

for  $t = 1, \dots, n$ . For obvious identification purposes we restrict the cycle shift to the range  $-\frac{1}{2}\pi \leq \lambda\xi \leq \frac{1}{2}\pi$ . The common cycle can therefore be shifted for any time series  $i$  by considering the model specification

$$y_t^i = c^i + \gamma_t^i + A^i \{ \cos(\lambda\xi^i)\psi_t + \sin(\lambda\xi^i)\psi_t^* \} + \varepsilon_t, \quad i = D, S, R,$$

where  $a^i$  denotes element  $i$  of any vector  $a$  as for  $y_t$  in (2). It is therefore assumed that  $\xi$  is a  $3 \times 1$  vector with the restriction that one element, say  $\xi^D$  for the default series, is restricted to zero. This particular series associates itself with the reference cycle. The sign of the coefficient in  $\xi$  determines whether the cycle is leading (positive) or lagging (negative). More statistical implications of the multiple phase shift cycle component are discussed by Rünstler (2002) and more general specifications are discussed by Koopman and Azevedo (2003) for the synchronization and convergence of multiple cycles.

The model we will consider in the empirical study of the next section is given by

$$y_t = c + \gamma_t + A \odot \{ \cos(\lambda\xi)\psi_t + \sin(\lambda\xi)\psi_t^* \} + \varepsilon_t, \quad (5)$$

where  $y_t$  is as in (2),  $c$  is a constant vector,  $\gamma_t$  is a stochastic vector following the VAR(2) process (3),  $A$  is a vector of coefficients (factor loadings) with its first element equal to one,  $\lambda$  is the common cycle's frequency,  $\xi$  is a vector containing the shifts in time units with its first element equal to zero,  $\psi_t$  is a stochastic cycle and common to all elements in  $y_t$  and its associated stochastic variable  $\psi_t^*$  appears by construction for a shifted cycle, and  $\varepsilon_t$  is a vector representing the irregular component and accounting for possible noise in the measurement of  $y_t$ . All vectors are of dimension  $3 \times 1$  and the elements are associated with default, spread and GDP growth (in this order). Note that the notation of  $\odot$  in (5) is for element by element multiplication and further note that  $\cos(\lambda\xi)$  and  $\sin(\lambda\xi)$  are  $3 \times 1$  vectors.

The multivariate unobserved components model (5) can be put into the state space form

$$y_t = Z\alpha_t + \varepsilon_t, \quad \alpha_{t-1} = T\alpha_t + \nu_t, \quad (6)$$

where the state vector contains the mean parameters including the unobservables, that is  $\alpha_t = (c', \gamma_t', \psi_t, \psi_t^*)'$ . The system matrices  $Z$  and  $T$  are constructed according to the specifications implied by model (5). The state disturbance vector  $\nu_t$  contains the disturbances  $\eta_t$ ,  $\omega_t$  and  $\omega_t^*$ . The unknown coefficients of the model ( $\Gamma_1$ ,  $\Gamma_2$ ,  $\Sigma_\eta$ ,  $\phi$ ,  $\lambda$ ,  $\sigma_\omega^2$ ,  $A$ ,  $\xi$ , and  $\Sigma_\varepsilon$ ) can be estimated by numerically maximising the log-likelihood function of the model for a given set of observations  $y_1, \dots, y_n$ . The log-likelihood function can be computed via the Kalman filter; see, for example, Durbin and Koopman (2001) for details of the Kalman filter and associated methods and techniques. Once the parameters are estimated, the unobserved components  $\gamma_t$  and  $\psi_t$  can be extracted from the observations using the Kalman filter and the associated smoother. These estimates, together with confidence intervals, can be graphically presented. Diagnostic statistics and graphs can be obtained as by-products of the Kalman filter and can be used to test the underlying assumptions of the model such as normality and independence of the disturbances. Finally, standard goodness-of-fit statistics can be computed for each equation of the multivariate model.

### 3 Empirical results

We start our empirical modeling exercise with a simple VAR analysis. After some experimentation, it turned out that the dynamics in the data are captured adequately and parsimoniously given the limited number of observations by a VAR model of order 2. The estimation results are presented in Table 1.

Table 1: Parameter Estimates

The table contains parameter estimates for a VAR(2) model

$$y_t = c + \Gamma_1 y_{t-1} + \Gamma_2 y_{t-2} + \eta_t, \quad \eta_t \stackrel{\text{i.i.d.}}{\sim} N(0, \Sigma_\eta),$$

with  $y_t$  containing Dun and Bradstreet (1998) business failures (DEFLT), credit spreads (SPRD), and real GDP growth (RGDP), respectively. The failure rates are transformed using a probit transformation. The  $Q(k)$  statistics have a  $\chi^2$  distribution with  $k$  degrees of freedom, while the normality test has a  $\chi^2(2)$  distribution. Parameter significance is denoted by <sup>a</sup> (20%), <sup>b</sup> (10%), or <sup>c</sup> (5%).

	DEFLT	SPRD	RGDP%
$c$	-2.515	1.847	3.104
DEFLT(-1)	1.227 <sup>c</sup>	-0.229	21.917 <sup>c</sup>
SPRD(-1)	0.027	1.059 <sup>c</sup>	-3.794 <sup>c</sup>
RGDP(-1)	-0.003	-0.002	0.543 <sup>c</sup>
DEFLT(-2)	-0.329 <sup>b</sup>	0.251	-20.869 <sup>b</sup>
SPRD(-2)	-0.044 <sup>a</sup>	-0.244	4.635 <sup>c</sup>
RGDP(-2)	-0.003	-0.005	0.148
VAR	0.005	0.01	-0.098
variance	0.01	0.271	-0.801
matrix ( $\Sigma_\eta$ )	-0.098	-0.801	13.668
$R^2$	0.91	0.67	0.42
Normality	9.51	38.37	7.55
1 <sup>st</sup> order autocorr.	0.112	-0.087	0.145
Q(10)	6.07	10.06	16.16
Q(15)	11.67	11.80	18.07
Log-lik		-46.14	
#par		18	

Looking at the model's diagnostics, there is ample evidence of non-normality, especially in spreads. The Box-Pierce  $Q$  statistics, however, are all insignificant. The VAR(2) model shows some indication of cyclical movements, as 4 of the 6 characteristic roots are complex. The VAR specification is, however, less suited to concentrate on cycles of specific frequencies (e.g., business cycle frequencies). Moreover, it is harder in a simple VAR framework to test whether cyclicality is a common feature across different series. We therefore regard the VAR modeling stage only as a preliminary analysis. The VAR model does show, however, that the dynamic features of the data can be modeled adequately with only a limited number of lags.

In an attempt to provide clear evidence on the existence and commonality

of credit cycles and business cycles in our data, we consider the decomposition model explained in the previous section. Our model disentangles the series into a cyclical movement common to all three series, and a VAR(2) model used to capture any remaining dynamic patterns in the data. Upon estimation, the parameter estimates revealed that the VAR(2) component in the decomposition model had a (near) singular covariance matrix. This can be explained by looking at the plain vanilla VAR(2) model from Table 1. In that model, there are three shocks, i.e., the three elements of  $\eta_t$ . In our decomposition model, there is an additional independent set of shocks for the cyclical component. These shocks take over the role of one of the elements of  $\eta_t$  in the standard VAR(2) model of Table 1. To reduce instability in the estimation process, and to retain comparability in the number of shocks between the standard VAR model and our decomposition model, we therefore impose a reduced rank condition on the variance matrix  $\Sigma_\eta$  of the VAR component. As mentioned, this restriction is congruent with the empirical data.

As explained in the previous subsection, we estimate two different types of models. In the first model specification, the cyclical movement is restricted to be synchronous across all three series. In our second specification, we use Rünstler (2002) to allow for asynchronicity, meaning that the cycle in one series may lead or lag the cycle in the other series. We estimate the lead/lag time directly from the data. Moreover, we estimate both types of models with and without restrictions to eliminate any potential redundant variables. In this way we try to come up with a more parsimonious model representation given the limited number of observations. The results are presented in Table 2.

We start our discussion with the unrestricted model without phase shifts. The likelihood value is higher than for the VAR(2) model from Table 1: it increases by 3 upon adding 5 additional parameters. This implies that the unrestricted model without shifts is not parsimonious, and we will try to further reduce the model later on. Looking at the fit of the different equations, we see that the fits of the business failure and spread series increase most. The fit of the real GDP equation, however, deteriorates. The diagnostics of this decomposition model are somewhat better than in Table 1. Especially the non-normality in the business failure equation has reduced significantly.

The component model as estimated in Table 2 produces a cycle with a period of about 10 years. The cycle is persistent in the sense that its dampening factor  $\phi$  is about 0.89 and consistent across the model specifications considered. The period of 10 years matches the period of 11.7 years that follows from the plain vanilla VAR model in Table 1, but the latter has a lower cyclical persistence in the sense of a corresponding characteristic root

Table 2: Parameter Estimates

The table contains parameter estimates for the model

$$\begin{aligned}
 y_t &= c + \gamma_t + A \odot (\cos(\lambda\xi)\psi_t + \sin(\lambda\xi)\psi_t^*) + e_t, & e_t &\stackrel{\text{i.i.d.}}{\sim} N(0, \Sigma_e), \\
 \gamma_t &= \Gamma_1\gamma_{t-1} + \Gamma_2\gamma_{t-2} + \eta_t, & \gamma_t &= (\gamma_t^D, \gamma_t^S, \gamma_t^R)', & \eta_t &\stackrel{\text{i.i.d.}}{\sim} N(0, \Sigma_\eta), \\
 \begin{pmatrix} \psi_t \\ \psi_t^* \end{pmatrix} &= \phi \begin{pmatrix} \cos(\lambda) & \sin(\lambda) \\ -\sin(\lambda) & \cos(\lambda) \end{pmatrix} \begin{pmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{pmatrix} + \begin{pmatrix} \omega_t \\ \omega_t^* \end{pmatrix}, & (\omega_t, \omega_t^*)' &\stackrel{\text{i.i.d.}}{\sim} N(0, \sigma_\omega^2 I_2),
 \end{aligned}$$

with  $y_t$  containing Dun and Bradstreet (1998) business failures (DEFLT), credit spreads (SPRD), and real GDP growth (RGDP), respectively. The failure rates are transformed using a probit transformation. The variance matrix  $\Sigma_e$  of the irregular component is consistently estimated equal to zero. It is therefore omitted from the table. The  $Q(k)$  statistics have a  $\chi^2$  distribution with  $k$  degrees of freedom, while the normality test has a  $\chi^2(2)$  distribution. Parameter significance is denoted by <sup>a</sup> (20%), <sup>b</sup> (10%), or <sup>c</sup> (5%).

	Unrestr.; No shifts			Restr.; No shifts			Unrestr.; Shifts			Restr.; Shifts		
Period ( $2\pi/\lambda$ )	9.880 <sup>c</sup>			9.769 <sup>c</sup>			9.947 <sup>c</sup>			9.839 <sup>c</sup>		
Damp.factor ( $\phi$ )	0.897 <sup>c</sup>			0.898 <sup>c</sup>			0.898 <sup>c</sup>			0.894 <sup>c</sup>		
Variance ( $\sigma_\omega^2$ )	0.008 <sup>c</sup>			0.009 <sup>c</sup>			0.009 <sup>c</sup>			0.009 <sup>c</sup>		
	DEFLT	SPRD	RGDP%	DEFLT	SPRD	RGDP%	DEFLT	SPRD	RGDP%	DEFLT	SPRD	RGDP%
Cycle load ( $A$ )	1	3.685 <sup>b</sup>	-2.575	1	2.010 <sup>a</sup>	—	1	3.854 <sup>c</sup>	-1.49	1	2.575 <sup>a</sup>	-0.671
Cycle shift ( $\xi$ )	—	—	—	—	—	—	-0.497	—	—	-0.134	—	—
$c$	-2.510	1.924	2.741	-2.509	1.867	2.930	-2.526	1.921	2.849	-2.500	1.809	2.620
$\gamma_{t-1}^D$	1.813 <sup>c</sup>	-2.532	19.673	1.873 <sup>c</sup>	—	—	1.862 <sup>c</sup>	-2.565	15.209	1.886 <sup>c</sup>	—	—
$\gamma_{t-1}^S$	0.060	0.918 <sup>c</sup>	-3.859 <sup>a</sup>	—	0.867 <sup>c</sup>	-1.943 <sup>c</sup>	0.065	0.935 <sup>c</sup>	-4.383 <sup>a</sup>	0.015	0.990 <sup>c</sup>	-3.912 <sup>b</sup>
$\gamma_{t-1}^R$	0.002	-0.021	0.562 <sup>a</sup>	—	—	0.554 <sup>c</sup>	0.003	-0.021	0.487	—	—	0.481 <sup>c</sup>
$\gamma_{t-2}^D$	-0.841 <sup>c</sup>	2.622	-20.145	-0.906 <sup>c</sup>	—	—	-0.887 <sup>c</sup>	2.677	-16.006	-0.918 <sup>c</sup>	—	—
$\gamma_{t-2}^S$	-0.058 <sup>a</sup>	-0.085	4.549 <sup>b</sup>	—	—	2.378 <sup>c</sup>	-0.063 <sup>a</sup>	-0.109	5.074 <sup>c</sup>	-0.018 <sup>a</sup>	-0.137	4.776 <sup>c</sup>
$\gamma_{t-2}^R$	-0.001	0.003	0.104	—	—	—	-0.001	0.004	0.125	—	—	0.128
VAR	0.001	0.000	-0.104	0.001	—	-0.115	0.001	0.000	-0.095	0.001	—	-0.099
Variances	0.000	0.221	-0.828	—	0.252	—	0.000	0.228	-0.868	—	0.261	—
matrix ( $\Sigma_\eta$ )	-0.104	-0.828	15.091	-0.115	—	17.838	-0.095	-0.868	14.202	-0.099	—	15.725
$R^2$	0.93	0.69	0.36	0.94	0.66	0.33	0.93	0.68	0.4	0.94	0.66	0.34
Normality:	2.208	38.053	6.738	0.813	37.376	8.413	1.834	33.882	5.033	0.7	42.133	5.02
1 <sup>st</sup> order autocorr.	0.006	-0.051	0.158	-0.098	0.070	0.027	0.008	-0.042	0.143	-0.082	-0.035	0.062
Q(10):	9.191	9.924	14.777	10.523	12.205	12.875	9.042	9.244	13.865	10.308	10.522	14.896
Q(15):	12.411	11.284	19.082	14.588	13.462	17.030	12.316	11.081	18.017	14.829	12.084	16.584
Log-lik	-43.12			-47.75			-42.83			-46.79		
#par	23			10			24			16		



of about 0.69.

In our decomposition model we can directly assess the loading ( $A$ ) of the cyclical factor in each of the three series under consideration. We normalize the loading for the business failure equation to unity. The spread equation has a positive loading for the cyclical component. If the failure cycle tends upwards and failure rates increase, the cyclical component in credit spreads also increases. This agrees with intuition and also follows from theoretical models like that of Kwark (2002). The cycle has a negative loading in the real gdp growth equation. Again, this agrees with intuition: high growth regimes coincide with low failure rates, see also Nickell et al. (2000) and Bangia et al. (2002). The loading is, however, insignificant with a  $t$ -value far below 0.5.

The remaining dynamic patterns as captured by the VAR(2) component  $\gamma_t$  also show some interesting features. This is seen most clearly if we first eliminating a number of insignificant variables from the VAR component  $\gamma_t$  in our model. The estimation results are presented under the heading (restr.; no shifts) in Table 2. All parameters remain stable compared to their unrestricted estimates. There appear to be both dynamic and contemporaneous relations between the different variables in the system. We first focus on the dynamic ones. The spread component only depends on its own lag. Lags of the other two series turn out to be insignificant. By contrast, real GDP is also explained by lagged spreads. This corroborates earlier empirical findings regarding the explanatory power of credit spreads for business cycles, see the references mentioned earlier. Business failures are explained by their own past.

It is also interesting to note that there appears to be no direct (significant) link from past real GDP growth to failure rates or vice versa. This may be somewhat puzzling given the empirical evidence in for example Nickell et al. (2000) and Bangia et al. (2002). The puzzle is largely resolved, however, if we consider the variance matrix  $\Sigma_\eta$  of the VAR component  $\gamma_t$ . As mentioned earlier,  $\gamma_t$  is driven by a bivariate shock process. A third and independent shock enters the system through the cyclical component  $\psi_t$ . The bivariate nature of the VAR innovation comes out in the reduced rank of  $\Sigma_\eta$  as shown in Table 2. The structure of  $\Sigma_\eta$ , however, is remarkable. Out of the two shocks, one enters the spread equation, while the other is shared by the default and the real GDP equation. This again underlines the special role of credit spreads for the dynamic relation between credit risk factors and the real economy. Furthermore, the fact that the second shock is shared by the default and real GDP series may explain the correlations found in Nickell et al. (2000) and Bangia et al. (2002). The covariance between the default and real GDP innovation is negative, implying that a positive shock to real

GDP is matched by a decrease in default rates. This is in line with the evidence in papers mentioned. The main implication of this finding is that though there may be a contemporaneous correlation between defaults and real GDP, the dynamic (cyclical) pattern in defaults is picked up much more by credit spreads. Consequently, credit spreads as conditioning variables to predict future default rates are at least as valuable as more commonly used variables such as economic growth rates, compare Nickell et al. (2000) and Kavvathas (2001). This is the more relevant given the timeliness of credit spread information vis-à-vis information on real GDP.

As one of the obvious objections to our empirical findings concerns the synchronicity of the business cycle and credit cycle imposed in the first two models, we now turn to a model specification that allows for shifts in the cycle's phase across different equations. The results for the unrestricted and restricted model are in the right-hand half of Table 2. The phase shift of the cycle in the real GDP equation turned out to be insignificant and moreover, resulted in instabilities in the estimation process. This can be understood from the scant evidence on direct cyclicity in real GDP in our sample, see also Figure 2. The cycle in GDP mainly enters indirectly through its dependence on lagged credit spreads. We therefore restrict the phase shift in real GDP to zero in the remaining computations. The time shift in the spread cycle is negative. Its size of -0.49 or -0.13 implies a lag time of half a year or 1 to 2 months, but the parameter is not significant. We therefore conclude that there is no significant evidence of asynchronicity in the cyclical movements in our data set.

Figure 2 contains a graphical presentation of the model's fit (unrestricted; no shifts). Graphs for all 4 models considered look very similar. The VAR(2) component  $\gamma_t$  does well in picking up most of the long-term variation in the series, especially in real GDP growth. For credit spreads and failure rates, the cycle is needed to adequately model the remaining dynamics.

## 4 Conclusions

In this paper we used a multivariate unobserved components approach to describe the dynamic behavior of credit risk factors in their relation to the real economy. We depart from other approaches in credit risk modeling in that we focus on the time-series behavior rather than the cross section dimension of default related data. Moreover, we model credit spreads and business failure rates jointly with macro-economic developments. By adopting the unobserved components approach, we were able to disentangle medium term cyclical movements from longer term developments in credit risk factors. In

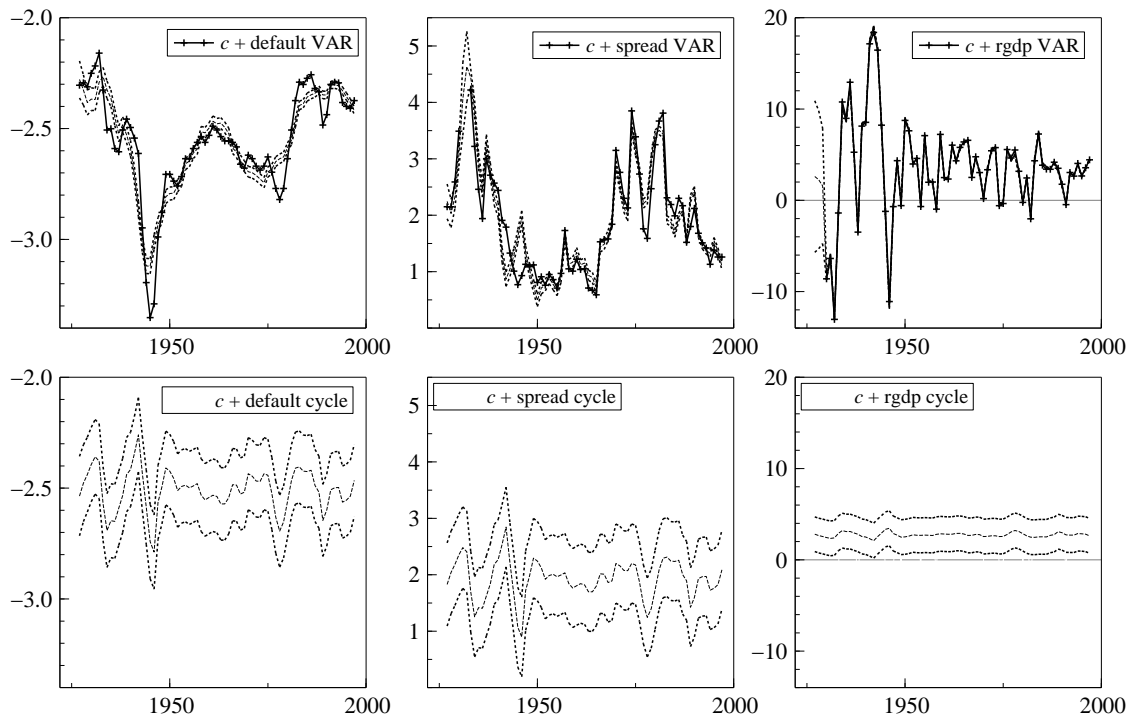


Figure 2: VAR(2) ( $c + \gamma_t$ ) and Cyclical components ( $c + \psi_t$ ) in Business failure rates, Real GDP growth, and the credit spread

This figure presents the fit (smoothed estimates) of the model estimated in Table 2. The top row of graphs indicates the Business failure series (top-left) indicated by the line with '+', and the VAR(2) component (solid line,  $c + \gamma_t$ ) with its 95% confidence band (dashed). The graphs for credit spreads and real GDP growth are in the middle and right-hand graphs, respectively. The bottom row of graphs contains the cyclical component ( $c + \psi_t$ ) of the three series considered, along with its 95% confidence band. Note that the top and bottom graphs have the same vertical scaling for each series.

this way, we could retrace some of the earlier empirical evidence on the relation between credit risk and the macro-economy. Though shocks to default rates were highly correlated with shocks to real GDP, the dynamics of default rates were much better captured by the credit spread. This held for both longer term patterns as well as for short term cyclical movements. There appeared no firm evidence of co-cyclicity in real GDP and business failures. Taken together, this casts doubt on the currently proposed use of economic growth rates in models predicting default rates. Given the analysis presented in this paper, perhaps a more promising route to predict default rate dynamics would be to condition on recent credit spread information rather than growth rates only.

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