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Initial conditions for dipole evolution beyond the McLerran-Venugopalan model

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Abstract

We derive the scattering amplitude $N(r)$ for a QCD dipole on a dense target in the semi-classical approximation. We include the first subleading correction in the target thickness arising from $\sim \rho^4$ operators in the effective action for the large-x valence charges. Our result for $N(r)$ can be matched to a phenomenological proton fit by Albacete *et al.* over a broad range of dipole sizes r and provides a definite prediction for the A-dependence for heavy-ion targets. We find a suppression of $N(r)$ for finite A for dipole sizes a few times smaller than the inverse saturation scale, corresponding to a suppression of the classical *bremsstrahlung* tail.

In this paper we derive the dipole scattering amplitude $N(r)$ [1] on a dense target in the semi-classical approximation. We restrict to a perturbative expansion of the Wilson lines valid at short distances r but include the first subleading (in density) correction arising from $\sim \rho^4$ operators in the effective action for the large-x valence charges.

Our result may prove useful for a better theoretical understanding of the A-dependence of the initial condition for high-energy evolution of $N(r)$, as discussed in more detail below.

Let us first recall the setup for the McLerran-Venugopalan (MV) model [2]. The "valence" charges with large light-cone momentum fraction x are described as recoilless sources on the light cone with $\rho^a(x_\perp, x^-)$ the classical color charge density per unit transverse area and longitudinal length x^-. Kinetic terms for ρ are neglected since transverse momenta are assumed to be small. In the limit of a very high density of charge ρ the fluctuations of the color charge, by the central limit theorem, are described by a Gaussian effective action

$$S_{MV}[\rho] = \int d^2\mathbf{x}_\perp \int_{-\infty}^{\infty} dx^- \; \frac{\rho^a(x^-, \mathbf{x}_\perp)\, \rho^a(x^-, \mathbf{x}_\perp)}{2\mu^2(x^-)} \; . \tag{1}$$

Here, $\mu^2(x^-)dx^-$ is the density of color sources per unit transverse area in the longitudinal slice between x^- and $x^- + dx^-$ and

$$\int_{-\infty}^{\infty} dx^- \mu^2(x^-) \sim g^2 A^{1/3} \tag{2}$$

is proportional to the thickness $\sim A^{1/3}$ of the target nucleus. This action can be used to evaluate the expectation value of the dipole operator

$$\begin{aligned} D(r) \;&\equiv\; \frac{1}{N_c}\langle \mathrm{tr}\; V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp)\rangle & (3)\\ &=\; \exp\left(-\frac{g^4 C_F}{8\pi} \int dx^- \mu^2(x^-)\; r^2 \log\frac{1}{r\Lambda}\right) & (4)\\ &=\; \exp\left(-\frac{Q_s^2\, r^2}{4} \log\frac{1}{r\Lambda}\right) \;, & (5) \end{aligned}$$

where $r \equiv |\mathbf{x}_\perp - \mathbf{y}_\perp|$ and Λ is an infrared cutoff on the order of the inverse nucleon radius; the explicit r-dependence has been obtained in the limit $\log 1/(r\Lambda) \gg 1$. The scale Q_s denotes the saturation momentum at the rapidity of the sources. For details of the calculation we refer to refs. [3–5] and to the appendix below.

To go beyond the limit of infinite valence charge density one considers a "random walk" in the space of SU(3) representations constructed from the direct product of a large number of fundamental charges [6]. The effective action describing color fluctuations then involves a sum of the quadratic, cubic, and quartic Casimirs which can be written in the form [7]

$$S[\rho] = \int d^2\mathbf{v}_\perp \int_{-\infty}^{\infty} dv_1^- \left\{ \frac{\rho^a(v_1^-,\mathbf{v}_\perp)\rho^a(v_1^-,\mathbf{v}_\perp)}{2\mu^2(v_1^-)} - \frac{d^{abc}\rho^a(v_1^-,\mathbf{v}_\perp)\rho^b(v_1^-,\mathbf{v}_\perp)\rho^c(v_1^-,\mathbf{v}_\perp)}{\kappa_3} \right.$$
$$\left. + \int_{-\infty}^{\infty} dv_2^- \, \frac{\rho^a(v_1^-,\mathbf{v}_\perp)\rho^a(v_1^-,\mathbf{v}_\perp)\rho^b(v_2^-,\mathbf{v}_\perp)\rho^b(v_2^-,\mathbf{v}_\perp)}{\kappa_4} \right\} \ . \quad (6)$$

The coefficients of the higher dimensional operators are

$$\kappa_3 \sim g^3 A^{2/3} \ , \quad (7)$$

$$\kappa_4 \sim g^4 A \ , \quad (8)$$

and so involve higher powers of $gA^{1/3}$. In what follows we restrict to leading order in $1/\kappa_4$ and drop the "odderon" operator $\sim \rho^3$ from (6) since it does not contribute to the expectation value of the dipole operator at leading order in $1/\kappa_3$. The details of the calculation are shown in the appendix, here we just quote the final result for the T-matrix

$$\begin{aligned} N(r) &\equiv 1 - D(r) \\ &= \frac{Q_s^2 r^2}{4} \log\frac{1}{r\Lambda} - \frac{C_F^2}{6\pi^3}\frac{g^8}{\kappa_4}\left[\int_{-\infty}^{\infty} dz^- \mu^4(z^-)\right]^2 r^2 \log^3\frac{1}{r\Lambda} \quad , \quad (r^2 Q_s^2 < 1) \ . \end{aligned} \quad (9)$$

This now involves a new moment of the valence color charge distribution, namely $\int dx^- \mu^4(x^-)$. We have calculated the $\mathcal{O}(1/\kappa_4)$ correction analytically only in the "short distance" regime up to order $\sim r^2$. Recall, however, that the effective theory (1) or (6) does not apply to the DGLAP regime at asymptotically short distances. Our result could in principle be extended into the saturation region $r \gtrsim 1/Q_s$ by generating the color charge configurations $\rho^a(x^-,\mathbf{x}_\perp)$ non-perturbatively, numerically [8].

The dipole scattering amplitude for a proton target has been fitted in ref. [9] to deep-inelastic scattering data. The Albacete-Armesto-Milhano-Quiroga-Salgado (AAMQS) model for the initial condition for small-x evolution is given by

$$N_{\mathrm{AAMQS}}(r, x_0 = 0.01) = 1 - \exp\left[-\frac{1}{4}\left(r^2 Q_s^2(x_0)\right)^\gamma \log\left(e + \frac{1}{r\Lambda}\right)\right] \ , \quad (10)$$

with $\gamma \simeq 1.119$ [1]. This model *simultaneously* provides a good description of charged hadron transverse momentum distributions in $p+p$ collisions at 7 TeV center of mass energy [10].

[1] There are actually several fits, we refer to the AAMQS paper [9] for a more detailed discussion.

This is a rather non-trivial cross-check: the MV model initial condition (5) overshoots the LHC data by roughly an order of magnitude at $p_\perp \gtrsim 6$ GeV [10].

However, since the model (10) was introduced essentially "by hand" it is an open question how it extends to nuclei. This is a crucial issue for predicting the nuclear modification factor R_{pA} for $p+Pb$ collisions at LHC, and for heavy-ion structure functions which could be measured at a future electron-ion eIC collider [11]. One possibility is that the AAMQS modification of the MV model dipole is due to some unknown A-independent non-perturbative effect. Here, we explore another option, namely that for protons the effects due to the $\sim \rho^4$ operators may not be negligible.

We can match our result (9) approximately to the AAMQS model by choosing

$$\beta \equiv \frac{C_F^2}{6\pi^3} \frac{g^8}{Q_s^2\,\kappa_4} \left[\int_{-\infty}^{\infty} dz^- \mu^4(z^-)\right]^2 \simeq \frac{1}{100} \quad , \quad (A=1). \tag{11}$$

For nuclei, $\beta_A \sim A^{-2/3}$ since each longitudinal integration over z^- is proportional to the thickness $\sim A^{1/3}$ of the nucleus while $\mu^2(z^-)$ and $\mu^4(z^-)$ are A-independent. The scattering amplitude for a dipole in the adjoint representation with this β is shown in fig. 1. [2] One

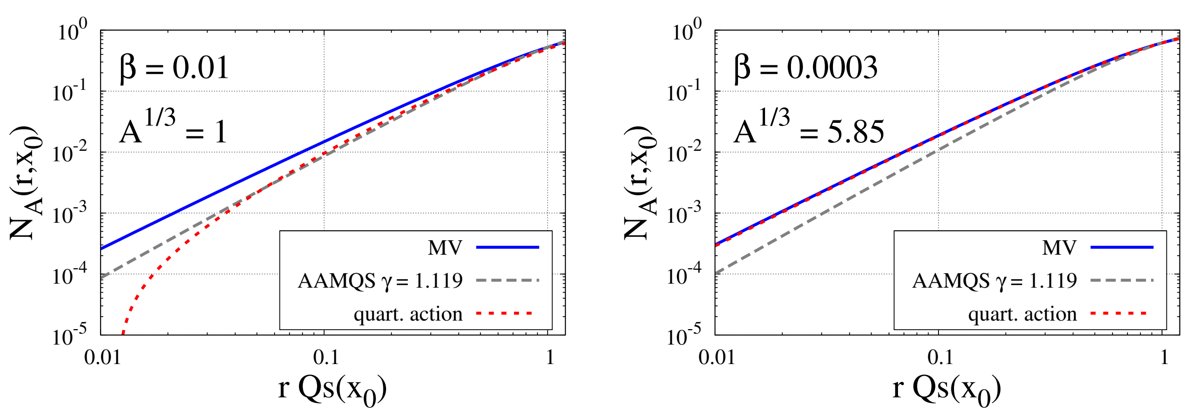

FIG. 1: Left: scattering amplitude for an adjoint dipole ($N_A = 2N - N^2$) on a proton, assuming $Q_s^2 = 0.168$ GeV^2 and $\Lambda^2 = 0.0576$ GeV^2. Right: same for a nucleus with $A = 200$ and $Q_s^2 \sim A^{1/3}$, $\beta_A \sim A^{-2/3}$.

observes that the dipole scattering amplitude derived from the quartic action is similar to the AAMQS model over a broad range, $rQ_S \gtrsim 0.04$. Discrepancies appear at very short

[2] To regularize the behavior at large r, for this figure we have replaced $\log \frac{1}{r\Lambda} \to \log(e + \frac{1}{r\Lambda})$ and assumed exponentiation of the $\mathcal{O}(r^2)$ expression. This does not affect the behavior at $rQ_s < 1$.

distances where none of the above can be trusted. A more careful and quantitative matching of β to the AAMQS fit beyond the leading $\log 1/r\Lambda \gg 1$ approximation should be performed in the future.

On the right, we plot the scattering amplitude for a nucleus with $A = 200$ nucleons, assuming that $Q_s^2 \sim A^{1/3}$ while $\beta_A \sim A^{-2/3}$. This illustrates how expectation values obtained with the quartic action converge to those from the MV model when the valence charge density is high (i.e., at large $A^{1/3}$). If our idea that the ρ^4 term in the action provides the explanation for the AAMQS model is indeed correct then their modification of the MV model should vanish like $\beta_A \sim A^{-2/3}$. This should be observable via R_{pA} at the LHC.

Acknowledgments

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I. APPENDIX

Expectation values of operators $O[\rho]$ are computed as

$$\langle O[\rho]\rangle \equiv \int \mathcal{D}\rho \; O[\rho] \; e^{-S[\rho]} \Big/ \int \mathcal{D}\rho \; e^{-S[\rho]} \; .$$

We work perturbatively in $1/\kappa_4$ and keep only the first term in the expansion. Then,

$$\begin{aligned}
\langle O[\rho]\rangle \;&\equiv\; \frac{\int \mathcal{D}\rho \; O[\rho] \; e^{-S_G[\rho]} \left[1 - \frac{1}{\kappa_4}\int d^2\mathbf{v}_\perp \int dv_1^- dv_2^- \rho^a_{v_1}\rho^a_{v_1}\rho^b_{v_2}\rho^b_{v_2}\right]}{\int \mathcal{D}\rho \; e^{-S_G[\rho]} \left[1 - \frac{1}{\kappa_4}\int d^2\mathbf{v}_\perp \int dv_1^- dv_2^- \rho^a_{v_1}\rho^a_{v_1}\rho^b_{v_2}\rho^b_{v_2}\right]} \\
&=\; \frac{\left\langle O[\rho]\left(1 - \frac{1}{\kappa_4}\int d^2\mathbf{v}_\perp \int dv_1^- dv_2^- \rho^a_{v_1}\rho^a_{v_1}\rho^b_{v_2}\rho^b_{v_2}\right)\right\rangle_G}{\langle 1 - \frac{1}{\kappa_4}\int d^2\mathbf{v}_\perp \int dv_1^- dv_2^- \rho^a_{v_1}\rho^a_{v_1}\rho^b_{v_2}\rho^b_{v_2}\rangle_G} \quad .
\end{aligned} \tag{12}$$

In lattice regularization the denominator evaluates to

$$\left\langle 1-\frac{1}{\kappa_4}\int d^2\mathbf{v}_\perp\int dv_1^- dv_2^-\,\rho^a_{v_1}\rho^a_{v_1}\rho^b_{v_2}\rho^b_{v_2}\right\rangle_G$$

$$=\;1-\frac{1}{\kappa_4}\frac{N_s}{\Delta\mathbf{v}_\perp}\left\{(N_c^2-1)^2\left[\int_{-\infty}^{\infty}dv^-\mu^2(v^-)\right]^2+2\,(N_c^2-1)\int_{-\infty}^{\infty}dv^-\mu^4(v^-)\right\}\quad,\tag{13}$$

where N_s denotes the number of lattice sites (the volume) and $\Delta x_\perp$ the transverse area of a single lattice site (square of lattice spacing). Also, we have used a local $\langle\rho\rho\rangle$ correlation function:

$$\langle\rho^a(x^-,\mathbf{x}_\perp)\,\rho^b(y^-,\mathbf{y}_\perp)\rangle=\delta^{ab}\mu^2(x^-)\delta(x^--y^-)\delta(\mathbf{x}_\perp-\mathbf{y}_\perp)\;.\tag{14}$$

A. Dipole Operator

We are interested in the expectation value of the dipole operator defined as

$$\hat{D}(\mathbf{x}_\perp,y_\perp)\equiv\frac{1}{N_c}\mathrm{tr}\;V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)\quad.\tag{15}$$

Here, V denotes a Wilson line

$$V(\mathbf{x}_\perp)=\mathcal{P}\exp\left\{-ig^2\int_{-\infty}^{\infty}dz^-\frac{1}{\nabla_\perp^2}\rho_a(z^-,\mathbf{x}_\perp)t^a\right\}\quad,\tag{16}$$

where

$$\frac{1}{\nabla_\perp^2}\rho_a(z^-,\mathbf{x}_\perp)=\int d^2\mathbf{z}_\perp G_0(\mathbf{x}_\perp-\mathbf{z}_\perp)\rho_a(z^-,\mathbf{z}_\perp)=-\frac{1}{g}A^+\quad,\tag{17}$$

is proportional to the gauge potential in covariant gauge. The matrices t^a are the generators of the fundamental representation of SU(3), normalized according to tr $t^at^b=\frac{1}{2}\delta^{ab}$.

G_0 is the static propagator which inverts the 2-dimensional Laplacian:

$$\frac{\partial^2}{\partial\mathbf{z}_\perp^2}G_0(\mathbf{x}_\perp-\mathbf{z}_\perp)=\delta(\mathbf{x}_\perp-\mathbf{z}_\perp)\quad;\tag{18}$$

$$G_0(\mathbf{x}_\perp-\mathbf{z}_\perp)=-\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^2}\frac{e^{i\mathbf{k}_\perp\cdot(\mathbf{x}_\perp-\mathbf{z}_\perp)}}{\mathbf{k}_\perp^2}\quad.\tag{19}$$

With this propagator we can write the Wilson line as

$$V(\mathbf{x}_\perp)=\mathcal{P}\exp\left\{-ig^2\int_{-\infty}^{\infty}dz^-\int d^2\mathbf{z}_\perp\,G_0(\mathbf{x}_\perp-\mathbf{z}_\perp)\,\rho_a(z^-,\mathbf{z}_\perp)\,t^a\right\}\quad.\tag{20}$$

The correlator $\langle V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)\rangle$ for a Gaussian (MV) action has already been calculated before, see for example ref. [5]. The result is:

$$\langle V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)\rangle_G=\exp\left\{-\frac{g^4}{2}(t^at_a)\left[\int_{-\infty}^{\infty}dz^-\mu^2(z^-)\right]\int d^2\mathbf{z}_\perp\left[G_0(\mathbf{x}_\perp-\mathbf{z}_\perp)-G_0(\mathbf{y}_\perp-\mathbf{z}_\perp)\right]^2\right\}\;.\tag{21}$$

Note that this is diagonal in color (proportional to $\mathbb{1}_{3\times 3}$). To calculate the expectation value of the dipole operator with the new action, we first expand the Wilson lines order by order in the gauge coupling g^2,

$$\begin{aligned} V(\mathbf{x}_\perp) \;=\; & 1 - ig^2 \int d^2\mathbf{z}_{\perp 1} G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 1}) \int_{-\infty}^{\infty} dz_1^- \rho^a(z_1) t^a \\ & - g^4 \int d^2\mathbf{z}_{\perp 1} d^2\mathbf{z}_{\perp 2} G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 1}) G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 2}) \int_{-\infty}^{\infty} dz_1^- \int_{z_1^-}^{\infty} dz_2^- \rho^a(z_1)\rho^b(z_2) t^a t^b \\ & + \cdots \end{aligned} \tag{22}$$

$$\begin{aligned} V^\dagger(\mathbf{y}_\perp) \;=\; & 1 + ig^2 \int d^2\mathbf{u}_{\perp 1} G_0(\mathbf{y}_\perp - \mathbf{u}_{\perp 1}) \int_{-\infty}^{\infty} du_1^- \rho^a(u_1) t^a \\ & - g^4 \int d^2\mathbf{u}_{\perp 1} d^2\mathbf{u}_{\perp 2} G_0(\mathbf{y}_\perp - \mathbf{u}_{\perp 1}) G_0(\mathbf{y}_\perp - \mathbf{u}_{\perp 2}) \int_{-\infty}^{\infty} du_1^- \int_{-\infty}^{u_1^-} du_2^- \rho^a(u_1)\rho^b(u_2) t^a t^b \\ & + \cdots \end{aligned} \tag{23}$$

For brevity we only write the terms up to $\mathcal{O}(g^4)$ but below we shall actually require terms up to $\mathcal{O}(g^8)$.

To zeroth order in g, the expectation value of the correlator is just 1:

$$\mathcal{O}\left(g^0\right) = 1 \quad .$$

The order g^2 contribution is zero because $\langle \rho^a(z) \rangle = 0$:

$$\mathcal{O}\left(g^2\right) = 0 \quad .$$

B. Order g^4

The first non-trivial contribution arises at $\mathcal{O}(g^4)$ and is given by the sum of expectation values of three terms (two from each Wilson line and one mixed term):

$$- g^4 \int d^2\mathbf{z}_{\perp 1} d^2\mathbf{z}_{\perp 2} G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 1}) G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 2}) \int_{-\infty}^{\infty} dz_1^- \int_{z_1^-}^{\infty} dz_2^- \langle \rho^a(z_1)\rho^b(z_2) \rangle t^a t^b \quad , \tag{24}$$

$$- g^4 \int d^2\mathbf{u}_{\perp 1} d^2\mathbf{u}_{\perp 2} G_0(\mathbf{y}_\perp - \mathbf{u}_{\perp 1}) G_0(\mathbf{y}_\perp - \mathbf{u}_{\perp 2}) \int_{-\infty}^{\infty} du_1^- \int_{-\infty}^{u_1^-} du_2^- \langle \rho^a(u_1)\rho^b(u_2) \rangle t^a t^b \quad , \tag{25}$$

$$g^4 \int d^2\mathbf{z}_{\perp 1} \int d^2\mathbf{u}_{\perp 1} G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 1}) G_0(\mathbf{y}_\perp - \mathbf{u}_{\perp 1}) \int_{-\infty}^{\infty} dz_1^- \int_{-\infty}^{\infty} du_1^- \langle \rho^a(z_1)\rho^b(u_1) \rangle t^a t^b \quad . \tag{26}$$

Using (12) and (21) the first term becomes (we will divide by the normalization factor later):

$$
\begin{aligned}
&- \frac{g^4}{2} t^a t_a \int_{-\infty}^{\infty} dz^- \mu^2(z^-) \int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp - \mathbf{z}_\perp) \\
&+ \frac{g^4}{\kappa_4} \int d^2\mathbf{z}_{\perp 1} d^2\mathbf{z}_{\perp 2} d^2\mathbf{v}_\perp G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 1}) G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 2}) \\
&\quad \times \int_{-\infty}^{\infty} dz_1^- \int_{z_1^-}^{\infty} dz_2^- \int dv_1^- dv_2^- \langle \rho^a_{z_1} \rho^b_{z_2} \rho^c_{v_1} \rho^c_{v_1} \rho^d_{v_2} \rho^d_{v_2} \rangle t^a t^b \ .
\end{aligned} \tag{27}
$$

All possible contractions for the second term in the above expression are

$$
\begin{aligned}
\langle \rho^a_{z_1} \rho^b_{z_2} \rho^c_{v_1} \rho^c_{v_1} \rho^d_{v_2} \rho^d_{v_2} \rangle = {}& \langle \rho^a_{z_1} \rho^b_{z_2} \rangle \left[\langle \rho^c_{v_1} \rho^c_{v_1} \rangle \langle \rho^d_{v_2} \rho^d_{v_2} \rangle + 2 \ \langle \rho^c_{v_1} \rho^d_{v_2} \rangle \langle \rho^c_{v_1} \rho^d_{v_2} \rangle \right] \\
& + 4 \ \langle \rho^a_{z_1} \rho^c_{v_1} \rangle \left[\langle \rho^b_{z_2} \rho^c_{v_1} \rangle \langle \rho^d_{v_2} \rho^d_{v_2} \rangle + 2 \ \langle \rho^b_{z_2} \rho^d_{v_2} \rangle \langle \rho^c_{v_1} \rho^d_{v_2} \rangle \right] ,
\end{aligned} \tag{28}
$$

and are shown diagrammatically in fig. 2. Using (14) to perform the contractions and

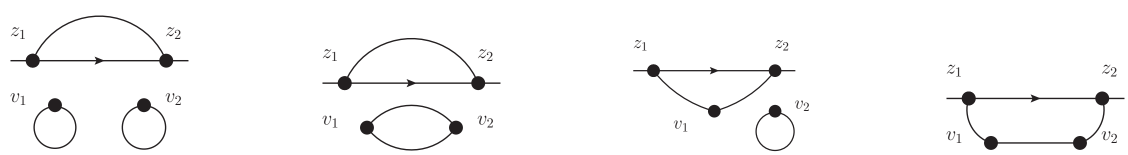


FIG. 2: $\langle V \rangle$ at order g^4/κ_4.

dividing also by the normalization factor (13) leads us to[3]

$$
\begin{aligned}
&\frac{1}{1 - \frac{1}{\kappa_4} \frac{N_s}{\Delta \mathbf{v}_\perp} \left\{ (N_c^2-1)^2 \left[\int_{-\infty}^{\infty} dv^- \mu^2(v^-) \right]^2 + 2 \ (N_c^2-1) \int_{-\infty}^{\infty} dv^- \mu^4(v^-) \right\} } \times \\
&\left\{ - \frac{g^4}{2} t^a t_a \int_{-\infty}^{\infty} dz^- \mu^2(z^-) \int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp - \mathbf{z}_\perp) \times \right. \\
&\left\{ 1 - \frac{1}{\kappa_4} \frac{N_s}{\Delta \mathbf{v}_\perp} \left[(N_c^2-1)^2 \left[\int_{-\infty}^{\infty} dv^- \mu^2(v^-) \right]^2 + 2 \ (N_c^2-1) \int_{-\infty}^{\infty} dv^- \mu^4(v^-) \right] \right\} + \\
&\left. 2 \frac{g^4}{\kappa_4} \frac{t^a t_a}{\Delta \mathbf{v}_\perp} \int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp - \mathbf{z}_\perp) \left[(N_c^2-1) \int_{-\infty}^{\infty} dz^- \mu^2(z^-) \int_{-\infty}^{\infty} dv^- \mu^4(v^-) + 2 \int_{-\infty}^{\infty} dz^- \mu^6(z^-) \right] \right\}
\end{aligned} \tag{29}
$$

The third line in the above expression cancels[4] the normalization factor once the latter is

[3] One has to be careful with performing the integration over z_2^-: $\int_{z_1^-}^{\infty} dz_2^- \delta(z_1^- - z_2^-) = 1/2$. If x^- is discretized, z_2^- should be placed ahead of z_1^- by at least half a lattice spacing Δx^-. Similarly, when expanding a single Wilson lines to order g^6: $z_3^- \geq z_2^- + \Delta x^-/2 \geq z_1^- + \Delta x^-$.

[4] This is the standard cancellation of disconnected diagrams.

expanded to leading order in $1/\kappa_4$ so that the previous expression simplifies to

$$
\begin{aligned}
&- \frac{g^4}{2} t^a t_a \int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp - \mathbf{z}_\perp) \\
&\quad \times \left\{ \int_{-\infty}^{\infty} dz^- \mu^2(z^-) \right. \\
&\quad \left. - \frac{4}{\kappa_4 \Delta \mathbf{v}_\perp} \left[(N_c^2 - 1) \int_{-\infty}^{\infty} dz^- \mu^2(z^-) \int_{-\infty}^{\infty} dv^- \mu^4(v^-) + 2 \int_{-\infty}^{\infty} dz^- \mu^6(z^-) \right] \right\} \ . \qquad (30)
\end{aligned}
$$

The correction is absorbed into a renormalization of $\int_{-\infty}^{\infty} dz^- \mu^2(z^-)$ in order that the two-point function $\langle \rho\rho \rangle$ remains unaffected by the quartic term in the action:

$$
\begin{aligned}
&\int_{-\infty}^{\infty} dz^- \tilde{\mu}^2(z^-) = \\
&\int_{-\infty}^{\infty} dz^- \mu^2(z^-) - \frac{4}{\kappa_4 \Delta \mathbf{v}_\perp} \left[(N_c^2 - 1) \int_{-\infty}^{\infty} dz^- \mu^2(z^-) \int_{-\infty}^{\infty} dv^- \mu^4(v^-) + 2 \int_{-\infty}^{\infty} dz^- \mu^6(z^-) \right] \qquad (31)
\end{aligned}
$$

Finally, the expectation value of $V(x_\perp)$ to order g^4 can simply be written as

$$
- \frac{g^4}{2} t^a t_a \ \int_{-\infty}^{\infty} dz^- \tilde{\mu}^2(z^-) \int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp - \mathbf{z}_\perp) \ \ . \qquad (32)
$$

Similarly, $\langle V^\dagger(y_\perp) \rangle$:

$$
- \frac{g^4}{2} t^a t_a \ \int_{-\infty}^{\infty} dz^- \tilde{\mu}^2(z^-) \int d^2\mathbf{u}_\perp G_0^2(\mathbf{y}_\perp - \mathbf{u}_\perp) \ \ . \qquad (33)
$$

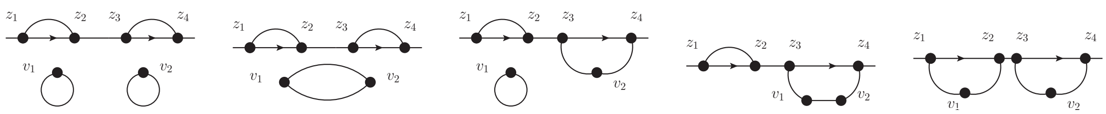
The mixed term will be the same as the previous terms except with a positive sign and without the factor of 1/2 which originated from the path ordering (z_1^- and u_1^- are not ordered relative to each other):

$$
g^4 t^a t_a \ \int_{-\infty}^{\infty} dz^- \tilde{\mu}^2(z^-) \int d^2\mathbf{z}_\perp G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) G_0(\mathbf{y}_\perp - \mathbf{z}_\perp) \ \ . \qquad (34)
$$

Summing (32), (33) and (34), we obtain the complete result at order g^4:

$$
\mathcal{O}(g^4) = - \frac{g^4}{2} t^a t_a \ \int_{-\infty}^{\infty} dz^- \tilde{\mu}^2(z^-) \int d^2\mathbf{z}_\perp \left[G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) - G_0(\mathbf{y}_\perp - \mathbf{z}_\perp) \right]^2 \ \ . \qquad (35)
$$

This is identical to the result obtained in the Gaussian theory once the two-point function $\langle \rho\rho \rangle \sim \mu^2$ has been matched. This was to be expected, of course, since only two-point functions of ρ arise at $\mathcal{O}(g^4)$. Note, also, that (35) vanishes as $\mathbf{y}_\perp \to \mathbf{x}_\perp$.



(a) (b) (c)

FIG. 3: Order g^8/κ_4 contribution to $\langle V \rangle$.

C. Order g^8

Next, we consider order g^8. There are all in all five terms: two of order g^8 from the expansion of a single Wilson line, two mixed terms (order g^2 from the first line and g^6 from the second line and vice versa), and one term from multiplying g^4 terms from both Wilson lines.

1. g^8 from $V(\mathbf{x}_\perp)$

First, we calculate $\langle V(\mathbf{x}_\perp) \rangle$ at order g^8. In the Gaussian theory

$$\frac{g^8}{8}(t^a t_a)^2 \left[\int_{-\infty}^{\infty} dz^- \mu^2(z^-)\right]^2 \left[\int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp - \mathbf{z}_\perp)\right]^2 \quad .$$

Again, using (12), the correction is

$$- \frac{g^8}{\kappa_4} \int d^2\mathbf{z}_{\perp 1} d^2\mathbf{z}_{\perp 2} d^2\mathbf{z}_{\perp 3} d^2\mathbf{z}_{\perp 4} \int d^2\mathbf{v}_\perp G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 1}) G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 2}) G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 3}) G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 4})$$
$$\times \int_{-\infty}^{\infty} dz_1^- \int_{z_1^-}^{\infty} dz_2^- \int_{z_2^-}^{\infty} dz_3^- \int_{z_3^-}^{\infty} dz_4^- \int_{-\infty}^{\infty} dv_1^- \int_{-\infty}^{\infty} dv_2^- \langle \rho^a_{z_1} \rho^b_{z_2} \rho^c_{z_3} \rho^d_{z_4} \rho^e_{v_1} \rho^e_{v_1} \rho^f_{v_2} \rho^f_{v_2} \rangle t^a t^b t^c t^d \quad (36)$$

All possible contractions for $\langle \rho^a_{z_1} \rho^b_{z_2} \rho^c_{z_3} \rho^d_{z_4} \rho^e_{v_1} \rho^e_{v_1} \rho^f_{v_2} \rho^f_{v_2} \rangle$ are shown diagrammatically in fig. 3. The disconnected diagrams 3a give the following contribution:

$$- \frac{g^8}{8}(t^a t_a)^2 \left[\int_{-\infty}^{\infty} dz^- \mu^2(z^-)\right]^2 \left[\int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp - \mathbf{z}_\perp)\right]^2$$
$$\times \frac{1}{\kappa_4} \frac{N_s}{\Delta \mathbf{v}_\perp} \left[(N_c^2 - 1)^2 \left[\int_{-\infty}^{\infty} dv^- \mu^2(v^-)\right]^2 + 2\,(N_c^2 - 1) \int_{-\infty}^{\infty} dv^- \mu^4(v^-)\right] , \quad (37)$$

which will cancel the normalization factor.

The connected diagrams from fig. 3b again renormalize $\mu^2(z^-)$ to $\tilde{\mu}^2(z^-)$:

$$\left[\int_{-\infty}^{\infty} dz^- \tilde{\mu}^2(z^-)\right]^2 = \left[\int_{-\infty}^{\infty} dz^- \mu^2(z^-)\right]^2 - \frac{8}{\kappa_4 \Delta \mathbf{v}_\perp}\left[(N_c^2-1)\left[\int_{-\infty}^{\infty} dz^- \mu^2(z^-)\right]^2 \int_{-\infty}^{\infty} dv^- \mu^4(v^-) + 2\int_{-\infty}^{\infty} dz^- \mu^2(z^-)\int_{-\infty}^{\infty} dv^- \mu^6(v^-)\right] \tag{38}$$

Finally, the diagrams 3c give the correction

$$-\frac{g^8}{\kappa_4}(t^a t_a)^2 \left[\int_{-\infty}^{\infty} dz^- \mu^4(z^-)\right]^2 \int d^2\mathbf{z}_\perp G_0^4(\mathbf{x}_\perp - \mathbf{z}_\perp) \ . \tag{39}$$

That leads us to the complete $\mathcal{O}(g^8)$ contribution to $\langle V(x_\perp)\rangle$:

$$\frac{g^8}{8}(t^a t_a)^2 \left[\int_{-\infty}^{\infty} dz^- \tilde{\mu}^2(z^-)\right]^2 \left[\int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp - \mathbf{z}_\perp)\right]^2 - \frac{g^8}{\kappa_4}(t^a t_a)^2 \left[\int_{-\infty}^{\infty} dz^- \mu^4(z^-)\right]^2 \int d^2\mathbf{z}_\perp G_0^4(\mathbf{x}_\perp - \mathbf{z}_\perp) \ . \tag{40}$$

The expectation value of $V^\dagger(\mathbf{y}_\perp)$ is obtained from the above by substituting $\mathbf{x}_\perp \to \mathbf{y}_\perp$.

2. g^6 *from* $V(\mathbf{x}_\perp) \times$ g^2 *from* $V^\dagger(\mathbf{y}_\perp)$

Next is the mixed term obtained when multiplying the g^6 term from the x Wilson line with the g^2 term from the y Wilson line. In the Gaussian theory,

$$-\frac{g^8}{2}(t^a t_a)^2 \left[\int_{-\infty}^{\infty} dz^- \mu^2(z^-)\right]^2 \int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp - \mathbf{z}_\perp)\int d^2\mathbf{u}_\perp G_0(\mathbf{x}_\perp - \mathbf{u}_\perp) G_0(\mathbf{y}_\perp - \mathbf{u}_\perp) \ . \tag{41}$$

The correction is:

$$+\frac{g^8}{\kappa_4}\int d^2\mathbf{z}_{\perp 1} d^2\mathbf{z}_{\perp 2} d^2\mathbf{z}_{\perp 3} G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 1}) G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 2}) G_0(\mathbf{x}_\perp - \mathbf{z}_{\perp 3}) \int d^2\mathbf{u}_{\perp 1} G_0(\mathbf{y}_\perp - \mathbf{u}_{\perp 1}) \times \int d^2\mathbf{v}_\perp \int_{-\infty}^{\infty} dz_1^- \int_{z_1^-}^{\infty} dz_2^- \int_{z_2^-}^{\infty} dz_3^- \int_{-\infty}^{\infty} du_1^- \int_{-\infty}^{\infty} dv_1^- \int_{-\infty}^{\infty} dv_2^- \times \langle \rho^a_{z_1}\rho^b_{z_2}\rho^c_{z_3}\rho^d_{u_1}\rho^e_{v_1}\rho^e_{v_1}\rho^f_{v_2}\rho^f_{v_2}\rangle t^a t^b t^c t^d \ . \tag{42}$$

The possible contractions are given in fig. 4. As before, the disconnected diagrams 4a cancel against the normalization, while the diagrams in 4b renormalize μ^2 to $\tilde{\mu}^2$; finally the diagrams 4c will give the correction. Summing all these diagrams plus the Gaussian part, we get:

$$-\frac{g^8}{2}(t^a t_a)^2 \left[\int_{-\infty}^{\infty} dz^- \tilde{\mu}^2(z^-)\right]^2 \int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp - \mathbf{z}_\perp)\int d^2\mathbf{u}_\perp G_0(\mathbf{x}_\perp - \mathbf{u}_\perp) G_0(\mathbf{y}_\perp - \mathbf{u}_\perp) + 4\frac{g^8}{\kappa_4}(t^a t_a)^2 \left[\int_{-\infty}^{\infty} dz^- \mu^4(z^-)\right]^2 \int d^2\mathbf{z}_\perp G_0^3(\mathbf{x}_\perp - \mathbf{z}_\perp) G_0(\mathbf{y}_\perp - \mathbf{z}_\perp) \tag{43}$$

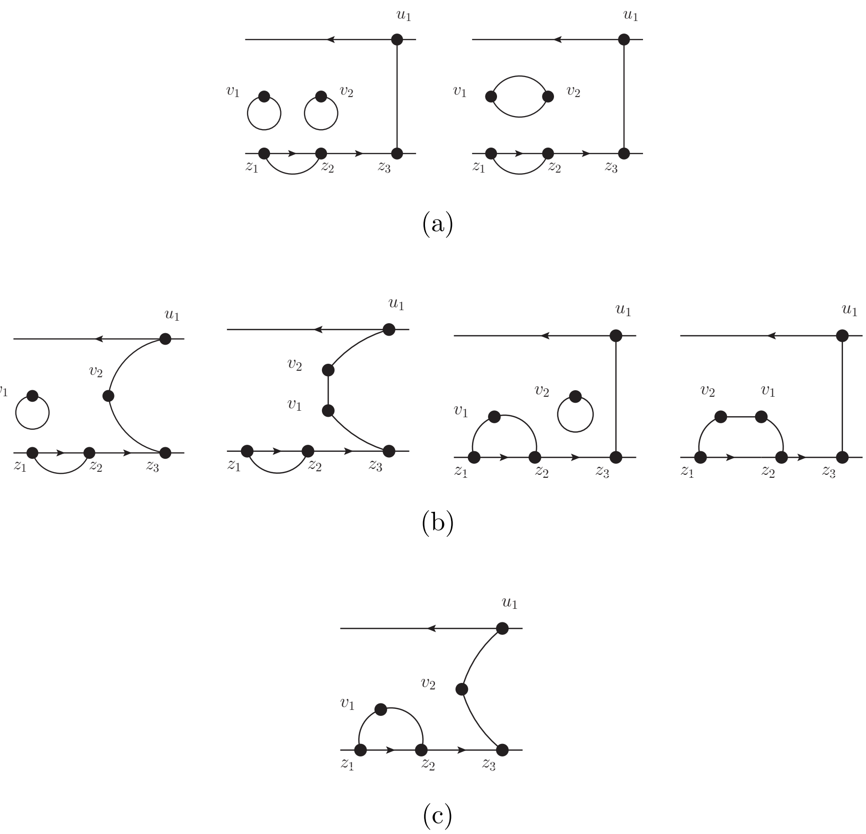


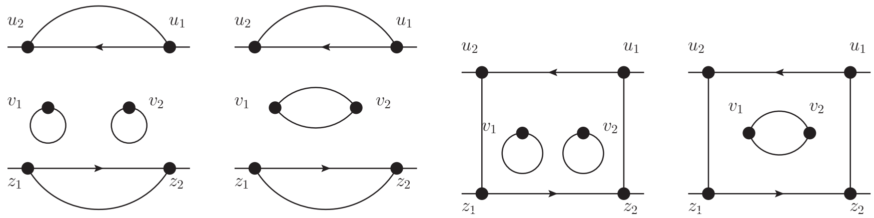
FIG. 4: $V(x_\perp)V^\dagger(y_\perp)$ at order g^8/κ_4 (g^6 from $V(\mathbf{x}_\perp) \times\ g^2$ from $V^\dagger(\mathbf{y}_\perp)$).

Once again, a similar contribution (with $\mathbf{x}_\perp \leftrightarrow \mathbf{y}_\perp$) arises from $V(\mathbf{x}_\perp)$ at $\mathcal{O}(g^2)$ times $V^\dagger(\mathbf{y}_\perp)$ at order g^6.

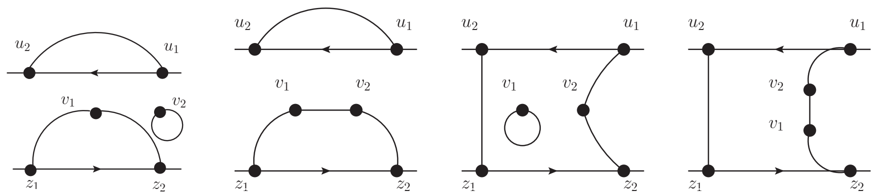
3. g^4 from $V(\mathbf{x}_\perp) \times\ g^4$ from $V^\dagger(\mathbf{y}_\perp)$

The last term to consider is the one obtained when multiplying $\mathcal{O}(g^4)$ from the x Wilson line with $\mathcal{O}(g^4)$ from the y Wilson line. The Gaussian contribution is:

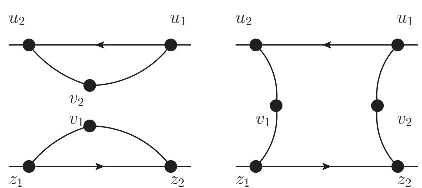
$$\frac{g^8}{4}\,(t^a t_a)^2 \left[\int_{-\infty}^{\infty} dz^- \mu^2(z^-)\right]^2 \int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp - \mathbf{z}_\perp) \int d^2\mathbf{u}_\perp G_0^2(\mathbf{y}_\perp - \mathbf{u}_\perp)$$
$$+\frac{g^8}{2}\,(t^a t_a)^2 \left[\int_{-\infty}^{\infty} dz^- \mu^2(z^-)\right]^2 \left[\int d^2\mathbf{z}_\perp G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) G_0(\mathbf{y}_\perp - \mathbf{u}_\perp)\right]^2 . \tag{44}$$



(a)



(b)



(c)

FIG. 5: Expectation value of $V(\mathbf{x}_\perp)$ at order g^4 times $V(\mathbf{y}_\perp)$ at order g^4.

For the quartic action we have to add

$$
\begin{aligned}
&-\frac{g^8}{\kappa_4}\int d^2\mathbf{z}_{\perp 1}d^2\mathbf{z}_{\perp 2}G_0(\mathbf{x}_\perp-\mathbf{z}_{\perp 1})G_0(\mathbf{x}_\perp-\mathbf{z}_{\perp 2})\int d^2\mathbf{u}_{\perp 1}d^2\mathbf{u}_{\perp 2}G_0(\mathbf{y}_\perp-\mathbf{u}_{\perp 1})G_0(\mathbf{y}_\perp-\mathbf{u}_{\perp 2})\\
&\quad\times\int d^2\mathbf{v}_\perp\int_{-\infty}^{\infty}dz_1^-\int_{z_1^-}^{\infty}dz_2^-\int_{-\infty}^{\infty}du_1^-\int_{-\infty}^{u_1^-}du_2^-\int_{-\infty}^{\infty}dv_1^-\int_{-\infty}^{\infty}dv_2^-\\
&\quad\times\langle\rho^a_{z_1}\rho^b_{z_2}\rho^c_{u_1}\rho^d_{u_2}\rho^e_{v_1}\rho^e_{v_1}\rho^f_{v_2}\rho^f_{v_2}\rangle t^at^bt^ct^d\ .
\end{aligned}
\tag{45}
$$

Following the same procedure as before, the diagrams from fig. 5 give:

$$
\begin{aligned}
&\frac{g^8}{4}(t^at_a)^2\left[\int_{-\infty}^{\infty}dz^-\tilde{\mu}^2(z^-)\right]^2\int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp-\mathbf{z}_\perp)\int d^2\mathbf{u}_\perp G_0^2(\mathbf{y}_\perp-\mathbf{u}_\perp)\\
&\quad+\frac{g^8}{2}(t^at_a)^2\left[\int_{-\infty}^{\infty}dz^-\tilde{\mu}^2(z^-)\right]^2\left[\int d^2\mathbf{z}_\perp G_0(\mathbf{x}_\perp-\mathbf{z}_\perp)G_0(\mathbf{y}_\perp-\mathbf{u}_\perp)\right]^2\\
&\qquad-6\,\frac{g^8}{\kappa_4}(t^at_a)^2\left[\int_{-\infty}^{\infty}dz^-\mu^4(z^-)\right]^2\int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp-\mathbf{z}_\perp)G_0^2(\mathbf{y}_\perp-\mathbf{z}_\perp)\ .
\end{aligned}
\tag{46}
$$

4. *Complete order g^8*

Combining eqs. (40+$x_\perp \leftrightarrow y_\perp$), (43+$x_\perp \leftrightarrow y_\perp$), and (46) we get the complete contribution to the expectation value of the dipole at order g^8:

$$\frac{1}{2}\left(\frac{g^4\,(t^a t_a)}{2}\right)^2 \left[\int_{-\infty}^{\infty} dz^- \tilde{\mu}^2(z^-)\right]^2 \left[\int d^2\mathbf{z}_\perp \left[G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) - G_0(\mathbf{y}_\perp - \mathbf{z}_\perp)\right]^2\right]^2$$
$$-\frac{g^8}{\kappa_4}\,(t^a t_a)^2 \left[\int_{-\infty}^{\infty} dz^- \mu^4(z^-)\right]^2 \int d^2\mathbf{z}_\perp \left[G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) - G_0(\mathbf{y}_\perp - \mathbf{z}_\perp)\right]^4 \ .$$

Note that this again vanishes as $\mathbf{y}_\perp \to \mathbf{x}_\perp$.

D. Complete expectation value of the dipole operator

Adding together the terms of order 1, g^4 and g^8, the expectation value of the dipole operator becomes

$$\begin{aligned}
D(x_\perp - y_\perp) \;&\equiv\; \langle \frac{1}{N_c}\mathrm{tr}\; V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)\rangle = \\
1 \;&-\; \frac{g^4}{2}C_F \int_{-\infty}^{\infty} dz^- \tilde{\mu}^2(z^-) \int d^2\mathbf{z}_\perp \left[G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) - G_0(\mathbf{y}_\perp - \mathbf{z}_\perp)\right]^2 \\
&+\; \frac{1}{2}\left(\frac{g^4 C_F}{2}\right)^2 \left[\int_{-\infty}^{\infty} dz^- \tilde{\mu}^2(z^-)\right]^2 \left[\int d^2\mathbf{z}_\perp \left[G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) - G_0(\mathbf{y}_\perp - \mathbf{z}_\perp)\right]^2\right]^2 \\
&-\; \frac{g^8}{\kappa_4}C_F^2 \left[\int_{-\infty}^{\infty} dz^- \mu^4(z^-)\right]^2 \int d^2\mathbf{z}_\perp \left[G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) - G_0(\mathbf{y}_\perp - \mathbf{z}_\perp)\right]^4 + \cdots \qquad (47)
\end{aligned}$$

where

$$C_F = \frac{N_c^2 - 1}{2N_c} \ .$$

To write this in a more compact form we introduce the saturation scale

$$Q_s^2 \equiv \frac{g^4}{2\pi}C_F \int_{-\infty}^{\infty} dz^- \tilde{\mu}^2(z^-) \ , \qquad (48)$$

so that

$$\begin{aligned}
D(r) \;&=\; 1 - \pi Q_s^2 \int d^2\mathbf{z}_\perp \left[G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) - G_0(\mathbf{y}_\perp - \mathbf{z}_\perp)\right]^2 \\
&+\; \frac{\pi^2}{2}Q_s^4 \left[\int d^2\mathbf{z}_\perp \left[G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) - G_0(\mathbf{y}_\perp - \mathbf{z}_\perp)\right]^2\right]^2 \\
&-\; \frac{g^8}{\kappa_4}C_F^2 \left[\int_{-\infty}^{\infty} dz^- \mu^4(z^-)\right]^2 \int d^2\mathbf{z}_\perp \left[G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) - G_0(\mathbf{y}_\perp - \mathbf{z}_\perp)\right]^4 \ . \qquad (49)
\end{aligned}$$

Here, $r = |\mathbf{x}_\perp - \mathbf{y}_\perp|$.

E. Explicit evaluation of $D(r)$ to leading $\log 1/r$ accuracy

It is useful to obtain an explicit expression for $D(r)$ in the limit $\log 1/r\Lambda \gg 1$, where Λ is an infrared cutoff on the order of the inverse nucleon radius.

The first non-trivial term in eq. (49) gives

$$\pi Q_s^2 \int d^2\mathbf{z}_\perp \left[G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) - G_0(\mathbf{y}_\perp - \mathbf{z}_\perp)\right]^2$$
$$= Q_s^2 \int_0^\infty \frac{dk}{k^3}\left[1 - J_0(kr)\right] \simeq \frac{1}{4} r^2 Q_s^2 \log\frac{1}{r\Lambda}\ , \tag{50}$$

in the leading log $1/r\Lambda \gg 1$ approximation.

Next, we need to compute the integral

$$\int d^2\mathbf{z}_\perp \left[G_0(\mathbf{x}_\perp - \mathbf{z}_\perp) - G_0(\mathbf{y}_\perp - \mathbf{z}_\perp)\right]^4\ . \tag{51}$$

From eq. (19) for the propagator,

$$\int d^2\mathbf{z}_\perp G_0^4(\mathbf{x}_\perp - \mathbf{z}_\perp) = \frac{1}{(2\pi)^8}\int d^2\mathbf{z}_\perp \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 d^2\mathbf{k}_3 d^2\mathbf{k}_4 \frac{1}{\mathbf{k}_1^2\mathbf{k}_2^2\mathbf{k}_3^2\mathbf{k}_4^2} e^{i(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)\cdot(\mathbf{x}_\perp - \mathbf{z}_\perp)} \tag{52}$$
$$= \frac{1}{(2\pi)^6}\int d^2\mathbf{K} d^2\mathbf{k} d^2\mathbf{Q} d^2\mathbf{q} \frac{\delta(\mathbf{K}+\mathbf{Q})}{\left(\frac{\mathbf{K}}{2}+\mathbf{k}\right)^2\left(\frac{\mathbf{K}}{2}-\mathbf{k}\right)^2\left(\frac{\mathbf{Q}}{2}+\mathbf{q}\right)^2\left(\frac{\mathbf{Q}}{2}-\mathbf{q}\right)^2} \tag{53}$$
$$= \frac{1}{(2\pi)^6}\int d^2\mathbf{K}\left[\int d^2\mathbf{k} \frac{1}{\left(\frac{\mathbf{K}}{2}+\mathbf{k}\right)^2\left(\frac{\mathbf{K}}{2}-\mathbf{k}\right)^2}\right]^2 . \tag{54}$$

We regularize the integral in the square brackets by introducing a cutoff Λ:

$$\int d^2\mathbf{k} \frac{1}{\left(\left(\frac{\mathbf{K}}{2}+\mathbf{k}\right)^2 + \Lambda^2\right)\left(\left(\frac{\mathbf{K}}{2}-\mathbf{k}\right)^2 + \Lambda^2\right)} = \frac{2\pi}{\mathbf{K}^2}\log\frac{\mathbf{K}^2}{\Lambda^2}\ . \tag{55}$$

Then,

$$\int d^2\mathbf{z}_\perp G_0^4(\mathbf{x}_\perp - \mathbf{z}_\perp) = \frac{1}{(2\pi)^4}\int \frac{d^2\mathbf{K}}{\mathbf{K}^4}\log^2\frac{\mathbf{K}^2}{\Lambda^2}$$
$$= \frac{1}{(2\pi)^3}\int_\Lambda^\infty \frac{d\mathbf{K}}{K^3}\log^2\frac{\mathbf{K}^2}{\Lambda^2} = \frac{1}{(2\pi)^3}\frac{1}{\Lambda^2}\ . \tag{56}$$

Following the same procedure for $\int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp - \mathbf{z}_\perp) G_0^2(\mathbf{y}_\perp - \mathbf{z}_\perp)$ we arrive at:

$$\int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp - \mathbf{z}_\perp) G_0^2(\mathbf{y}_\perp - \mathbf{z}_\perp) = \frac{1}{(2\pi)^4}\int \frac{d^2\mathbf{K}}{\mathbf{K}^4} e^{i\mathbf{K}\cdot(\mathbf{x}_\perp - \mathbf{y}_\perp)}\log^2\frac{\mathbf{K}^2}{\Lambda^2}$$

$$= \frac{1}{(2\pi)^3}\int_\Lambda^\infty \frac{d^2\mathbf{K}}{K^3} J_0(\mathbf{K}r)\log^2\frac{\mathbf{K}^2}{\Lambda^2} = \frac{1}{(2\pi)^3}\left(\frac{1}{\Lambda^2} + \frac{1}{3}r^2\log^3(r\Lambda)\right) + \mathcal{O}\left[r^2\log^2(r\Lambda)\right]\ .$$

Similarly,

$$\int d^2\mathbf{z}_\perp G_0^3(\mathbf{x}_\perp - \mathbf{z}_\perp) G_0(\mathbf{y}_\perp - \mathbf{z}_\perp) =$$

$$\frac{1}{(2\pi)^5} \int \frac{d^2\mathbf{K}}{\mathbf{K}^2} e^{i\frac{\mathbf{K}}{2}\cdot(\mathbf{x}_\perp - \mathbf{y}_\perp)} \log\frac{\mathbf{K}^2}{\Lambda^2} \int d^2\mathbf{q} \frac{e^{i\mathbf{q}\cdot(\mathbf{x}_\perp - \mathbf{y}_\perp)}}{\left(\frac{\mathbf{K}}{2}+\mathbf{q}\right)^2 \left(\frac{\mathbf{K}}{2}-\mathbf{q}\right)^2} \ . \tag{57}$$

Using:

$$\begin{aligned} &\int d^2\mathbf{q} \frac{e^{i\mathbf{q}\cdot(\mathbf{x}_\perp - \mathbf{y}_\perp)}}{\left(\frac{\mathbf{K}}{2}+\mathbf{q}\right)^2 \left(\frac{\mathbf{K}}{2}-\mathbf{q}\right)^2} \\ \approx &\int d^2\mathbf{q} \frac{1 + i\mathbf{q}\cdot\mathbf{r}}{\left(\frac{\mathbf{K}}{2}+\mathbf{q}\right)^2 \left(\frac{\mathbf{K}}{2}-\mathbf{q}\right)^2} = \frac{2\pi}{\mathbf{K}^2} \log\frac{\mathbf{K}^2}{\Lambda^2} + \mathcal{O}(Kr) \ , \end{aligned} \tag{58}$$

we get

$$\begin{aligned} &\int d^2\mathbf{z}_\perp G_0^3(\mathbf{x}_\perp - \mathbf{z}_\perp) G_0(\mathbf{y}_\perp - \mathbf{z}_\perp) \\ = \frac{1}{(2\pi)^3} \int_\Lambda^\infty \frac{d^2\mathbf{K}}{K^3} J_0\left(\frac{1}{2}Kr\right) \log^2\frac{\mathbf{K}^2}{\Lambda^2} &= \frac{1}{4}\frac{1}{(2\pi)^3} \left(\frac{4}{\Lambda^2} + \frac{1}{3} r^2 \log^3(r\Lambda)\right) \ , \end{aligned} \tag{59}$$

so that, finally,

$$\begin{aligned} D(r) &= 1 - \frac{r^2 Q_s^2}{4} \log\frac{1}{r\Lambda} + \frac{1}{6\pi^3}\frac{g^8}{\kappa_4} C_F^2 \left[\int_{-\infty}^{\infty} dz^- \mu^4(z^-)\right]^2 r^2 \log^3\frac{1}{r\Lambda} && (60) \\ &= 1 - \frac{r^2 Q_s^2}{4} \log\frac{1}{r\Lambda} + \beta\, r^2 Q_s^2 \log^3\frac{1}{r\Lambda} \ . && (61) \end{aligned}$$

β has been defined in eq. (11).

Performing a Fourier transform we obtain the transverse momentum dependence of the dipole for $k_\perp \gg Q_s \gg \Lambda$:

$$\begin{aligned} D(k_\perp) &\approx 2\pi\frac{Q_s^2}{k_\perp^4} - \frac{g^8}{\pi^2\kappa_4} C_F^2 \left[\int_{-\infty}^{\infty} dz^- \mu^4(z^-)\right]^2 \frac{1}{k_\perp^4} \log^2\frac{k_\perp^2}{\Lambda^2} && (62) \\ &= 2\pi\frac{Q_s^2}{k_\perp^4} - 6\pi\beta\frac{Q_s^2}{k_\perp^4} \log^2\frac{k_\perp^2}{\Lambda^2} \ . && (63) \end{aligned}$$

The first term was taken from appendix B of ref. [5]. The second term provides a correction to the classical *bremsstrahlung* tail for finite valence parton density.

F. Gluon density

One can define [4] a Weizsäcker-Williams like gluon density of a nucleus, in the limit of small $\mathbf{x}_\perp^2$, as

$$xG_A(x, \mathbf{x}_\perp^2) = \frac{(N_c^2 - 1)\,\pi R^2}{\pi^2 \alpha_s N_c} \frac{1}{\mathbf{x}_\perp^2} N(\mathbf{x}_\perp^2) \ . \tag{64}$$

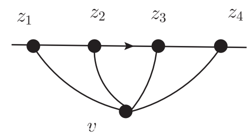


FIG. 6: $\langle V\rangle$ at order g^8/κ_4 for the action (67).

For the quartic action

$$N(x_\perp^2) = \frac{Q_s^2 x_\perp^2}{4} \log\frac{1}{x_\perp\Lambda} - \beta x_\perp^2 Q_s^2 \log^3\frac{1}{x_\perp\Lambda}\ . \tag{65}$$

The first term, via eq. (64), gives $xG_A \sim A$ [4]. The second term gives a correction

$$-\frac{(N_c^2-1)\,\pi R^2}{\pi^2\alpha_s N_c}\beta\, Q_s^2 \log^3\frac{1}{x_\perp\Lambda} \ \sim\ -A^{\frac{1}{3}}\ . \tag{66}$$

G. Form of the quartic term in the action

In this section we explain why the ρ's in the quartic term of the action should sit at two different points in the longitudinal direction in order that $N(r)$ vanishes as $\sim r^2$ as required by color transparency.

First, let us note that the correction to the dipole scattering amplitude due to the quartic term in the action results from the diagrams 3c, 4c and 5c. Now, let us consider the action:

$$\begin{aligned} S[\rho] \ =\ & \int d^2\mathbf{v}_\perp \int_{-\infty}^{\infty} dv^- \left\{ \frac{\rho^a(v^-,\mathbf{v}_\perp)\rho^a(v^-,\mathbf{v}_\perp)}{2\mu^2(v^-)} \right. \\ & \left. +\frac{\rho^a(v^-,\mathbf{v}_\perp)\rho^a(v^-,\mathbf{v}_\perp)\rho^b(v^-,\mathbf{v}_\perp)\rho^b(v^-,\mathbf{v}_\perp)}{\kappa_4} \right\}\ . \end{aligned} \tag{67}$$

The analogue of diagram 3c for $\langle V\rangle$ at order g^8 for this action is shown in fig. 6. However, this diagram vanishes due to the longitudinal path ordering in the Wilson line.

The same reasoning applies to the analogue of 4c shown in fig. 7. In this case the three points z_1^-, z_2^- and z_3^-, can not be connected simultaniously to one v^- point. The delta functions coming from this kind of contractions, $\delta(z_1^- - v^-)\delta(z_2^- - v^-)\delta(z_3^- - v^-)$, imply overlap of z_1^- and z_3^- which is not there in a path ordered integral.

That leaves us with only one type of diagram proportional to $1/\kappa_4$, shown in fig. 8. This diagram is not zero since there is no relative ordering between the points z^- and u^- on the two lines. This diagram has a constant r-independent contribution which does not cancel

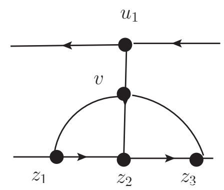


FIG. 7: Order g^6 from $V(\mathbf{x}_\perp)$ times order g^2 from $V^\dagger(\mathbf{y}_\perp)$ at order $1/\kappa_4$.

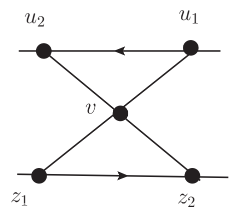


FIG. 8: Order g^4 from $V(\mathbf{x}_\perp)$ times order g^4 from $V^\dagger(\mathbf{y}_\perp)$ at order $1/\kappa_4$.

because of the missing diagrams 6 and 7. This is (only terms at order $1/\kappa_4$ are given)

$$\begin{aligned} D(r) \;&\sim\; 1 - \frac{3g^8}{\kappa_4}(C_F^2 - \frac{2}{N_c}) \int_{-\infty}^{\infty} dz^- \mu^8(z^-) \int d^2\mathbf{z}_\perp G_0^2(\mathbf{x}_\perp - \mathbf{z}_\perp) G_0^2(\mathbf{y}_\perp - \mathbf{z}_\perp) \\ &= 1 - \frac{3g^8}{(2\pi)^3\kappa_4}(C_F^2 - \frac{2}{N_c}) \int_{-\infty}^{\infty} dz^- \mu^8(z^-) \left(\frac{1}{\Lambda^2} - \frac{1}{3} r^2 \log^3\left(\frac{1}{r\Lambda}\right)\right) \; . \end{aligned} \tag{68}$$

Hence we see that for the action (67) that $N(r) = 1 - D(r)$ approaches a constant as $r \to 0$, in violation of color transparency.

For numerical (lattice gauge) computations of expectation values (12) it may be easier to integrate $\rho(x^-)$ and to drop the longitudinal path ordering,

$$\tilde\rho(\mathbf{x}_\perp) \;\equiv\; \int\limits_{-\infty}^{\infty} dx^- \rho(x^-, \mathbf{x}_\perp) \tag{69}$$

$$V(\mathbf{x}_\perp) \;=\; e^{ig^2 \frac{1}{\nabla_\perp^2} \tilde\rho(\mathbf{x}_\perp)} \; . \tag{70}$$

For a detailed discussion, we refer to ref. [8]. One would then consider the two-dimensional action

$$S[\tilde\rho] = \int d^2\mathbf{x}_\perp \left\{ \frac{\tilde\rho^a(\mathbf{x}_\perp)\tilde\rho^a(\mathbf{x}_\perp)}{2\mu^2} + \frac{\tilde\rho^a(\mathbf{x}_\perp)\tilde\rho^a(\mathbf{x}_\perp)\tilde\rho^b(\mathbf{x}_\perp)\tilde\rho^b(\mathbf{x}_\perp)}{\kappa_4} \right\} \; . \tag{71}$$

The diagrams from figs. 6 and 7 then do exist and cancel the r-independent contribution from fig. 8 so that again $N(r) \sim r^2$ at $r \to 0$.

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