Assessing uncertainty in the natural rate of interest: Info-gap as guide for monetary policy in the euro area

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Abstract
In this paper, we assume that the natural rate of interest is fundamentally uncertain. Based on a small-scale macroeconomic model, info-gap theory is used to rank different monetary policy strategies in terms of their robustness against this uncertainty. Applied to the euro area, we find that an inert or patient monetary strategy is more robust to natural rate uncertainty than a strategy that follows an estimated Taylor rule. An actively responsive monetary strategy is least robust. Our analysis presents a methodology that is applicable in a wide range of policy analyses under deep uncertainty.

KEYWORDS
Info-gaps, Knightian uncertainty, monetary policy, monetary strategy, satisficing

1 INTRODUCTION

The decline of interest rates to the lower bound has stirred the debate on the use of the natural rate ($r^*$) in monetary policy. In the literature, $r^*$ is the real interest rate that serves as a benchmark for the real policy rate in the steady state. According to Wicksell (1898), $r^*$ is the interest rate at which the (global) demand for and supply of capital are in balance. From this perspective, it can also be interpreted as the equilibrium interest rate that corresponds to the marginal product of capital. Using the natural rate of interest as the real short-term interest rate plays a prominent role in modern, New-Keynesian macroeconomic models. According to Woodford (2003), $r^*$ in these models is the rate at which the economy is in equilibrium and prices are fully flexible. The natural rate of interest is not necessarily constant in this equilibrium, but can fluctuate due to all kinds of shocks. If the economy is not in equilibrium and prices are not fully flexible, the market interest rate can deviate from $r^*$. In the spirit of Wicksell, this will lead to inflationary or deflationary pressure.

This concept of $r^*$ based on the real short-term interest rate allows it to be applied to monetary policy. By looking at the difference between the true real short-term market rate and the natural rate, or the “interest rate gap,” the central bank can make a judgement on its monetary stance, that is, the degree to which it eases or tightens monetary policy. That is why $r^*$ is part of the Taylor rule, which describes the standard reaction of the central bank to output and inflation. In equilibrium, the output gap is closed and there is price stability, implying that the policy rate equals the natural rate. The main complication in the use of $r^*$ for monetary policy is that it is an unobservable variable. It is a theoretical concept and there are several definitions in existence. For instance, $r^*$ can be defined as real long-term interest rate where there is equilibrium on the capital markets, or as the real short-term interest rate consistent with equilibrium in the economy.

Since $r^*$ cannot be directly observed, the empirical literature uses several methods to estimate proxies of $r^*$. For instance, $r^*$ is estimated by time series models: Semi-structural models and general equilibrium models (see Bonam, van Els, van den End, de Haan, & Hindrayanto, 2018, for an overview). A widely accepted model to estimate $r^*$ is the semi-structural model developed by Holston, Laubach, and Williams (2017). This
model relates the natural rate of interest to the potential economic output, while filtering $r^*$ out of the observable macroeconomic data series. In general, the outcomes for $r^*$ are sensitive to the model assumptions. Research shows that estimations of the natural rate can be highly inaccurate and can vary widely depending on the model specification applied (Beyer and Wieland (2017)).

The unobservable nature of $r^*$ and the large modelling and estimation uncertainties underscore that $r^*$ goes with fundamental uncertainty. According to Knight (1921), this type of uncertainty describes events with unknown or objectively un-measurable probabilities. Keynes (1921) also argued against probabilizing the unknown. Because an unknown uncertainty is indeterminate, he considered that it cannot be modelled with probabilistic random variables. This contrasts to measurable uncertainty or risk, which can be quantified based on known probability distributions of events. In case of Knightian uncertainty, however, the data distribution might be unknowable, either intrinsically or because of practical limitations. This goes with an unknown event space and indeterminate outcomes, because there are no laws that govern the process. This is a different type of uncertainty than those related to measurement problems (e.g., end-of-sample problems with regard to estimating potential output). Empirical research uses approximations of $r^*$, with each econometric method raising special problems of its own. The resulting variety and uncertainty of the value of $r^*$ pose significant challenges for practical monetary policy. Assuming fundamental uncertainty with regard to $r^*$, therefore, raises the question whether $r^*$ can be used by the central bank as benchmark for the policy rate in equilibrium.

There are several risk management strategies to deal with uncertainty in monetary policy. A well-known strategy to deal with measurable uncertainty (risk) is Brainard’s attenuation principle (1967). This assumes that, with uncertainty, the central bank should respond to shocks more cautiously and in smaller steps than in conditions without uncertainty. It would call for policy gradualism and less aggressive responses to economic shocks. Brainard referred to uncertainty about the monetary transmission mechanism, but also data uncertainty is associated with policy gradualism in the literature (e.g., by Aoki, 2003). There are also risk management strategies that call for a more aggressive response by the central bank, for instance, if there is uncertainty about the persistence in the rate of inflation (Tetlow, 2018). More aggressive policy measures could then be needed to prevent an adverse shock from destabilizing inflation expectations. When the policy rate is at the effective lower bound, several studies conclude that it is optimal for the central bank to raise inflation expectations by committing to keep interest rates low for a time in the future (Eggertsson & Woodford, 2003).

Two main strategies for managing un-measurable, or Knightian uncertainty, have emerged in the literature: Robust control and info-gap. Robust control insures against the maximally worst outcome (min-max) as defined by the policymaker (see Hansen & Sargent, 2008; Hansen, Sargent, Turmuhambetova, & Williams, 2006; Williams, 2008). As Hudgins and Crowley (2019, p.1511) explain, robust control “accounts for imprecise system models or error models,” and employs “a minimax approach that achieves a robust design by minimizing a performance index under the worst possible disturbances.” Olalla and Gómez (2011) applied the robust control tool to a Neo-Keynesian model to study the effect of model uncertainty in monetary policy. Typically, policies derived through min-max are more aggressive by comparison to those derived under no uncertainty. Intuitively, when mechanisms at work are poorly understood, aggressive policies allow decision makers to learn about them. The literature has raised two objections to this: (i) policymakers do not like experimenting for the purposes of learning; (ii) worst events are rare and hence poorly known. It is odd, therefore, to design policies for events about which one knows the least (Sims, 2001).

However, the most important drawback in our view is that robust control does not account for the fundamental choice between robustness against uncertainty, on the one hand, and aspiration for good performance outcomes, on the other. This is where the alternative approach, info-gap, makes an important contribution by mapping explicitly this trade-off (Ben-Haim, 2006, 2010). If the central bank adopts an ambitious inflation target, it needs to compromise on the degree of confidence in achieving that target (Ben-Haim & Demertzis, 2008, 2016). Conversely, if the central bank requires high confidence in achieving specified goals, it needs to moderate how ambitious these goals are. Info-gap theory quantifies this intuitive trade-off. Ben-Haim, Demertzis, and van den End (2018) applied info-gap to evaluate different monetary policy reaction functions. They show that a traditional Taylor rule, in which the inflation and output gap are targeted, is preferred over more complex reaction functions that include financial variables. Our paper builds on that approach, while it adds value by relating the uncertainty to the natural rate of interest (data uncertainty) and not to the parameters and shocks (model uncertainty). This allows us to evaluate different monetary strategies to deal with natural rate uncertainty, which contributes to the policy debate about the influence of $r^*$ on the effectiveness of monetary policy.
2 | CONTRIBUTION TO THE LITERATURE

This paper contributes to the literature by assessing various monetary policy strategies to cope with fundamental uncertainty with regard to \( r^* \). The assessment is based on info-gap theory. Several monetary strategies to deal with uncertainty about \( r^* \) can be distinguished. A monetary policy strategy is defined in this paper as a particular response of the central bank to deviations from the inflation and output objectives in a Taylor rule. Most approaches that call for expansionary monetary policy measures are based on the secular stagnation hypothesis. It assumes that \( r^* \) has fallen to negative levels due to deficient aggregate demand and a savings surplus, caused by structural developments like ageing and fallen labour productivity (see, for example, Summers, 2014). In that situation, the lower bound of interest rates impedes the ability of the central bank to stimulate the economy by lowering the policy rate (adjusted for inflation) to below \( r^* \). Alternatively, the central bank could raise the inflation target or pursue a price-level targeting strategy (Bernanke, 2017; Williams, 2017), thereby reducing the real interest rate. Inflation expectations could also be raised by keeping interest rates low for longer (Bernanke, Kiley, & Roberts, 2019; Yellen, 2018). In contrast, the financial cycle hypothesis argues that persistently low policy rates have contributed to a reduction in \( r^* \). This view assumes that accommodative monetary conditions encouraged a mis-allocation of production factors, and thus depressed production growth and thereby \( r^* \) (Borio, 2017). Following from this, a less expansionary monetary policy would be needed. Others argue that the fundamental uncertainty regarding the level of \( r^* \) constrains its practical usefulness as a benchmark for monetary policy (Bonam et al., 2018). Not only are model estimations of \( r^* \) beset with great uncertainty, there is no uniform definition of the natural rate of interest either.

This is in line with Tarullo (2017), who argues that policymakers ought to be cautious in basing their policy on non-observable variables. The various hypotheses illustrate that the view on \( r^* \) is a crucial—and highly uncertain—determinant of the preferred monetary policy strategy. Some studies take the uncertainty about \( r^* \) into account in monetary policy reaction functions. According to Williams and Ortandides (2007), taking into account natural rate uncertainty implies that in the optimal policy strategy the interest rate is more inert (higher degree of interest rate smoothing), reacts more aggressively to inflation and less strongly to the unemployment gap than would be optimal if agents had perfect knowledge of the economy. Their conclusions follow from Bayesian and min-max or robust control approaches. Kiley and Roberts (2017) propose a risk adjustment factor to \( r^* \) in the Taylor rule when shocks to the economy and a subsequent low level of \( r^* \) constrain the ability for monetary accommodation. The risk adjustment assumes that monetary policy is more accommodative, on average, than a Taylor rule would imply. Ferrero, Pietrunti, and Tiseno (2019) assume uncertainty about the value of the natural interest rate in a basic (two-equation) forward-looking new-Keynesian framework, in which the policy rate is set to maximize an objective function for the inflation and output gap. They conclude that optimal monetary policy should not react either more aggressively or more cautiously to exogenous shocks relative to the full information case. The rationale for this result is that the shocks affecting \( r^* \) do not interact with any endogenous variable in the model, so that the uncertainty can be integrated out without affecting the rest of the problem. Ferrero et al. (2019) seem to assume that their best estimate of the model is in fact the most robust to uncertainty. That is a basic and common confusion between a best putative model and a best response to uncertainty, and does not acknowledge deep uncertainty. We show that the central bank may err if it adopts a monetary policy strategy based on its best putative model, or best current understanding, and that a strategy that robustly satisfies critical requirements is preferable.

In this paper, we also recognize that the level and dynamics of variable \( r^* \) are fundamentally uncertain. Based on this, we evaluate various possible monetary strategies with the info-gap method for managing this uncertainty. This evaluates the trade-off of the potential loss for the central bank against the robustness of each strategy to fundamental uncertainty about \( r^* \). We use a small-scale macro model with a standard Taylor rule and a loss function that measures the deviation of the inflation and output gap from target (model 1). We also consider a variant model with time-varying parameters, to take into account structural shifts in \( r^* \) and the model relationships over time, particularly since the Great Financial Crisis (GFC) (model 2). Model 2 puts more weight on the more recent part of the sample, implying that a larger part of the monetary policy rate is explained by the inflation and output gap in the post-crisis period.

Based on both economic models, we use the info-gap method to assess four monetary policy strategies, testing their robustness against uncertainty in \( r^* \). The four strategies we evaluate are: (i) the historical response by the ECB, based on the estimated parameters of the Taylor rule; (ii) an inert or patient monetary strategy, in which the policy rate is kept constant at a low level; (iii) a strategy that is moderately responsive to the inflation and output gap; (iv) an actively responsive monetary policy.
strategy, which implies a stronger reaction to deviations from the policy targets than the historical response. In all strategies, except strategy 2, the central bank also responds to \( r^* \), which is the benchmark of the policy rate in the long run.

The outcomes show that, in both models (1 and 2), the inert or patient monetary policy strategy is most robust to fundamental uncertainty about \( r^* \), while achieving specified policy goals. This strategy performs best with regard to the trade-off between reaching the objectives (price stability and stable economic growth) and the robustness against uncertainty. The moderately responsive strategy ranks next and the response according to the estimated Taylor rule ranks third in the trade-off between performance (objectives) and robustness. An actively responsive strategy performs worst according to this trade-off criterion.\(^1\) The robust dominance of the inert strategy is most clear in the model with time-varying coefficients (model 2). This model is tailored to the post-crisis period, in which the inflation and output gap were negative most of the time. The info-gap approach, assuming uncertainty in \( r^* \), suggests that an inert or patient policy strategy suits this situation better than the other monetary strategies.

The rest of this paper is organized as follows. Section 3 presents the two economic models and Section 4 describes the policy strategies which the central bank can apply in the presence of fundamental uncertainty about \( r^* \). Section 5 describes the model outcomes in case of no uncertainty. Section 6 introduces the info-gap approach, which is used in Section 7 to evaluate the policy strategies. In Section 8, we assume uncertainty in two other model variables. Section 9 discusses the results in a wider policy context, and Section 10 concludes.

3 | MODEL

This section presents a small-scale macroeconomic model, in which \( r^* \) plays a role. The model is similar to the basic three-equation (forward looking) new-Keynesian model. The model has two variants, one with fixed parameters (model 1) and one with time-varying parameters (model 2). The model is used in the subsequent sections to simulate the effects of four different monetary policy strategies on key macroeconomic variables. The fixed parameter model (model 1) consists of a Phillips curve (Equation (1)), an aggregate demand curve (Equation (2)) and a traditional Taylor rule (Equation (3)):

\[
\tilde{x}_t = \alpha_1 + \beta_1 \tilde{x}_{t+1} + \beta_2 \hat{y}_t \\
\hat{y}_t = \alpha_2 + \beta_3 \hat{y}_{t-1} + \beta_4 \hat{r}_{t-2} \\
i_t = \alpha_3 r_t^* + \beta_5 \tilde{x}_{t+1} + \beta_6 \hat{y}_{t-1}
\]

Equation (1) is the forward-looking Phillips curve, with \( \tilde{x}_t \) as the inflation gap, \( \tilde{x}_{t+1} \) as the expected inflation gap and \( \hat{y}_t \) as the output gap. The inflation gaps are measured in terms of deviation from the target inflation rate, \( \pi \) (assumed to be 2% in the euro area). Expected inflation is defined as 1 year ahead expected inflation, based on the survey of Consensus Economics (see Table A1 in Appendix A for a detailed definition). The output gap is based on an economic approach that measures the gap relative to potential output based on an aggregate production function. Some other studies use a statistical approach to estimate the output gap (e.g., Brand & Mazelis, 2019), in which potential output and the output cycle are estimated by unobserved component models. In these approaches, the identification of trends and cycles is dependent on the model specification and priors used in the estimation process. In Section 8, we explicitly take into account the uncertainty in the output gap.

Equation (2) links aggregate demand to the real interest rate gap \( \tilde{r} \), which equals the nominal policy or shadow rate \( (i_t) \) minus the natural rate in nominal terms \( (r^*) \) and the inflation gap.\(^2\) In Equation (2), we take the 2-quarters lagged interest rate, to allow for lags in the monetary transmission mechanism (higher-order lags are used as instruments in the estimation of the model and so the information of higher-order lags is implicitly included, see Section 5; the coefficient of the real rate gap \( \beta_4 \) is not sensitive to changing the lag length to one instead of two quarters). A reduction of policy rate, \( i_t \), has an expansionary effect on output and vice versa. We assume that the policy rate \( i_t \) is not bound by zero; when it does drop below zero it reflects a shadow rate. A shadow rate is a model construct because it is a non-observable variable. Some shadow rates are proxies for unconventional monetary policy measures, or central bank balance sheet policy (e.g., Lombardi & Zhu, 2018; Wu & Xia, 2016). The shadow rate of Krippner (2013), which we use in our model, reflects both conventional and unconventional monetary policy measures. It measures the market expectations about the future policy rate (also influenced by forward guidance of the central bank) and the effect of quantitative easing on the term premium component of bond yields. This is measured by the expected lift-off of the policy rate from the zero lower bound, in terms of an option-value of holding cash. The option-value increases when the interest rate is closer to the zero lower bound (the strike price of the option) and when it is expected to remain close to zero for a longer period of time. In contrast, the option-value decreases when the expected date of policy rate lift-off is expected to come nearer. Since the
shadow rate is based on forward interest rates it introduces a forward-looking element in the model, reflecting financial market expectations. Being a model-construct, each version of the shadow rate is liable to model risk. Bauer and Rudebusch (2016) and others show that shadow-rate term structure models produce markedly different estimates of the shadow rate, depending on the details of their specification. This implies that the shadow rate can actually not be treated as data. To account for this, in Section 8, we assume that the shadow rate is fundamentally uncertain.

Equation (3) is a forward-looking standard Taylor rule, which assumes that the ECB reacts to the expected future inflation gap and the output gap, in addition to changes in $r$. The estimation of the rule implies that the estimated coefficients reflect the extent to which the ECB reacted to those variables in the sample period. Therefore, it is a description of the historical policy (which may guide future policy strategies) and not a policy rule that optimizes a loss function, such as the one presented below. The natural rate ($r^*$) features in Equation (3) and in Equation (2) through $\hat{r}$. In the model, $r$ is (implicitly) made up of the real interest rate plus inflation due in the steady state. We include a time-varying natural rate, proxied by the trend in the long-term euro average government bond yield (Figure 1). Approximating $r^*$ as the trend in the long-term interest rate is in line with other time-series approaches for $r^*$ (e.g., Del Negro, Giannone, Giannoni, & Tambalotti, 2017; Johannsen & Mertens, 2016). Including the long-term interest rate in the Taylor rule assumes that this rate would eventually stabilize the economy, and thus it focuses on long-run stabilization (see, for instance, Roberts, 2018). While the shadow rate and $r^*$ are different interest rate concepts, they both reflect the expected future policy rate (the shadow rate of Krippner is based on short- and long-term forward interest rates).

To account for time-variation in the parameters, model 2 is estimated with a dummy variable, distinguishing the period since the GFC from the pre-crisis period. This captures the structural shifts in $r^*$ and changes in other variables and relationships in the model. Since the GFC, several studies, have questioned the validity of the Phillips curve, due to the missing disinflation after the GFC and the missing inflation in more recent years, when inflation did not respond to improving economic conditions (e.g., in Bœeica & Sokol, 2019). This indicates changes in the Phillips curve slopes. To account for this, we include a crisis dummy in each equation of the model, plus the dummy in interaction with the variables in Equation (1a) and with $r^*$ in Equation (3a). The full model specification with the dummy variables is presented by Equations (1a), (2a) and (3a) below.

\[
\begin{align*}
\hat{x}_t &= \alpha_1 + \beta_1 \hat{x}_{t+1}^e + \beta_2 \hat{y}_t + \gamma_1 d_t + \gamma_2 d_t \hat{x}_{t+1}^e + \gamma_3 d_t \hat{y}_t \quad (1a) \\
\hat{y}_t &= \alpha_2 + \beta_3 \hat{y}_{t-1} + \beta_4 \hat{r}_{t-2} + \gamma_4 d_t \\
i_t &= \alpha_3 r^*_t + \beta_5 \hat{x}_{t+1}^e + \beta_6 \hat{y}_{t-1} + \gamma_5 d_t r^*_t \quad (3a)
\end{align*}
\]

The outcome of models 1 and 2 is evaluated by a loss function, which weights the objectives of monetary policy. We specify a standard quadratic loss function (Rotemberg & Woodford, 1997; Woodford, 2003),

\[L = \lambda_\pi \hat{x}_t^2 + \lambda_y \hat{y}_t^2 \quad (4)\]

It assumes that monetary policy attempts to manage losses that are caused by inflation, and output being away from their target (see Table A1 in Appendix A for a detailed definition). The central bank chooses an interest rate (and/or unconventional monetary policy measures as reflected in the shadow rate) to obtain adequately low values for the quadratic loss function. Coefficients, $\lambda$, are the relative weights with which the central bank pursues its objectives. Preference parameters in Equation (4) take values that proxy the ECB preferences (and the estimated coefficients of the Taylor rule, see Section 5), where $\lambda_\pi=1.5$ and $\lambda_y=0.5$. Since the Taylor rules in Equations (3) and (3a) include the estimated coefficients, they reflect the historical response of the central bank over the sample period. This is not necessarily the most optimal loss minimizing reaction function. In Section 4, we define some alternative response functions and test their performance in terms of loss as well as robustness against uncertainty in Sections 5–7.

![FIGURE 1](https://onlinelibrary.wiley.com/doi/10.1002/ijfe.2318) Interest rate variables (nominal values)
4 | MONETARY POLICY STRATEGIES

In this section, we distinguish four possible monetary policy strategies for the central bank to deal with fundamental uncertainty about the estimated natural rate (r∗), and later with uncertainty in the output gap (j) and the shadow rate (δ). Since r∗ captures all kinds of economic trends, such as in productivity growth and saving and investment patterns, the uncertainty about r∗ captures uncertainty about trends in the economy in general. The policy strategy is assumed to last for 2 years beyond 2018Q4. Based on that, the model generates out-of-sample predictions of the economic variables and the loss function for the period 2018Q4–2020Q3. Taking 2018Q3 as the starting point of the simulations implies that the policy response has to deal with economic conditions that do not necessarily reflect an equilibrium situation. Therefore, the analysis mimics a real-time policy decision, for which the central bank takes into account all available information and the conditions at the starting point. As a decision theory, info-gap can guide the central bank in choosing a robust policy strategy based on known information and on uncertain parameters such as r∗.

1. Historical reaction function. This assumes that the central bank follows a similar strategy in the out-of-sample period as the average strategy in the in-sample period. Hence the estimated Taylor rules (Equations (3) and (3a)) also determine the future policy strategy. This reflects a central bank that opts for a steady hand in times of great uncertainty. One reason for this strategy can be to bolster the credibility of the central bank. Market participants know the historical reaction function, and by following this function, the central bank may confirm market expectations, enhancing its credibility.

2. Inert or patient strategy. This strategy assumes that the policy rate, i, is kept constant at the level of 2018Q3 for two more years. The reason for such a strategy could be that the policymaker thinks r∗ is probably at a lower level than estimated and uncertain anyway. The strategy can be viewed as a version of the precautionary principle or an extreme variant of the attenuation principle, which leads to an unchanged policy until the policymaker knows the change is beneficial. Keeping i constant would seem to ignore insight and understanding that suggest the need for change. However, in times of great uncertainty, this approach may be justified, assuming that model relations (and the underlying insight and understanding) based on the past do not hold for the future.

A similar argument was made by Coeuré (2018) to justify the ECB’s forward guidance to keep the key ECB interest rates at their present low levels. He motivated this “risk management” approach by saying that parameter and model uncertainties imply that the costs of committing a type II error – of failing to anticipate a much slower than usual return of inflation to pre-crisis levels – may be uncomfortably high. Moreover, Coeuré also stated that unwinding policy accommodation in a multidimensional policy space is terra incognita for both financial markets and policymakers. De Groot and Haas (2018) show that an increase in policy inertia strengthens the signalling channel of negative interest rates and so boosts economic activity and inflation.

This policy strategy differs from the other strategies as it implies that the uncertainty about r∗ only enters via r in the demand curve, Equations (2) and (2a). In the other policy strategies, the uncertainty about r∗ enters also via the Taylor rule (through i).

3. Modestly responsive. This strategy follows Brainard (1967), meaning that the policy reaction is less vigorous relative to the optimal policy where r∗ is known. It is based on the assumption that, when facing uncertainty, the central bank should respond to shocks more cautiously, and in smaller steps, than in conditions without uncertainty. It would call for policy gradualism and less aggressive responses to economic shocks (see Section 1). In the model, the modest response strategy is applied by reducing the estimated parameters β and in the Taylor rule by a multiplicative factor of 0.5. In our model simulations, the more cautious response is applied symmetrically over the business cycle, meaning that policy rates will be lowered less vigorously in a business cycle downturn (when output and inflation gaps are negative) and raised less aggressively in an upturn (when output and inflation gaps are positive).

4. Actively responsive. This strategy prescribes a reaction function that is relatively sensitive to the output gap and the inflation gap. In our model simulations, such a response is applied symmetrically over the business cycle, meaning that policy rates are lowered in a business cycle downturn and raised in an upturn. Therefore, an actively responsive strategy can be viewed as optimal to prevent a worse outcome in terms of the robust control concept. The actively responsive reaction is applied in the model by raising the estimated parameters, β and β, in the Taylor rule by a multiplicative factor of 2.
5 | Putative Outcomes

We estimate the models assuming no fundamental parameter uncertainty (“putative model,” which is an expression used to describe the generally assumed model). Both models are estimated as a system of equations by generalized method of moments (GMM, see Hamilton, 1994). The system estimator uses more information than a single equation estimator (i.e., the contemporaneous correlation among the error terms across equations) and, therefore, produces more precise estimates. We do not impose cross-equation restrictions. GMM takes into account the interdependencies among the equations in the model, while controlling for the endogeneity of regressors and for the correlation between the lagged dependent variables and the error terms. The model is estimated with quarterly data for the euro area over the period 1995–2018Q3 (see Table A1 in Appendix A for a detailed description of the data). We assume that the path of \( r^* \) beyond the sample period is fundamentally uncertain, which makes 2018Q3 a logical starting point for analysing possible monetary policy strategies.

To account for time-variation in the parameters, model 2 is estimated with a dummy variable \( (d_t) \), distinguishing the crisis period from the pre-crisis period. Dummy \( d_t = 1 \) from 2008q1 onward and \( d_t = 0 \) otherwise. The dummy is included in each equation of the model. The dummy also interacts with the variables in Equation (1a) and with \( r^* \) in Equation (3a).

5.1 | Fixed parameters: Model 1

The estimation outcomes of model 1 (fixed-parameter model) in column 1 of Table B1 in Appendix B show that most coefficients are significant and have the expected sign. The low J-statistic indicates that the model is well specified. The inflation gap has a significant relationship with the output gap in the estimated Phillips curve (Equation (1)). The coefficient of the real interest rate gap in the demand curve of Equation (2) (the interest rate channel of monetary policy) is negative, and significant, although it has a low value, which implies that monetary policy has a limited effect on the output gap and thereby on inflation. The Taylor rule estimates of Equation (3) show that monetary policy reacts more strongly to the (expected) inflation gap than to the output gap (given larger and significant coefficients) and in line with the ordering of objectives in the ECB. The estimated parameters for the inflation gap and the output gap are somewhat higher compared to what generally is assumed in the literature (1.5 and 0.5, respectively), these values we also include in the loss function as \( \lambda_x \) and \( \lambda_y \), respectively). The significant positive value of \( \alpha_3 \) implies that the policy rate has followed our proxy of the natural rate over the sample period, although only partially \((\alpha_3 \approx 0.6)\), implying that monetary policy has been accommodative, on average, in the sample period.

Figure 2 shows the dynamic outcomes of the output and inflation gaps, \( \dot{y}_t \) and \( \dot{\pi}_t \), the nominal shadow rate \( (i_t) \) and the real interest rate gap \( (\hat{r}) \), for model 1, strategy 1 (historical response). These are the conditional forecasts for two quarters in-sample \((t = 1 \text{ and } t = 2)\) and eight quarters out-of-sample \((t = 3 \text{ to } t = 10)\). In all out-of-sample simulations, the value of \( \hat{r} \) is kept constant at its 2018Q3 level, based on the assumption that the latest estimate of \( r^* \) is the best proxy for future observations, given the fundamental uncertainty about the level and dynamics of \( r^* \). If the central bank would follow the historical reaction function (strategy 1), the shadow rate \( i_t \) jumps from around \(-3\%\) in \( t = 1 \) and \( t = 2 \) (the actual in-sample shadow rate) to positive numbers in the out-of-sample period. This illustrates that the actual shadow rate in 2018Q3 – the last in-sample quarter – deviates substantially from what the (estimated) Taylor rule would suggest to the shadow rate. Thus, at the end of the in-sample period, monetary policy was much looser than it ought to be if the historical reaction rule had been followed. According to that rule, the shadow rate responds to the inflation gap (which is slightly negative at the start of the simulation period) and the output gap (which is slightly positive). The positive value of \( i_t \) from \( t = 3 \) onward also relates to the positive value of \( r^* \), which is partly tracked by the shadow rate in the Taylor rule. As a result of that jump, the real rate gap, \( \hat{r} \), quickly closes and stabilizes slightly below 0% in the out-of-sample period. The negative interest rates in \( t = 1 \) and \( t = 2 \) support output so that the output gap becomes increasingly positive in the first instance, after which it converges to around slightly below 0%. As a result, the inflation gap is closed in period 2 and remains positive until \( t = 6 \).

Figure 3 shows the conditional forecasts for model 1, strategy 2. This inert or patient strategy implies that \( i_t \) remains constant at \(-3.18\%\) and that \( \hat{r} \) becomes even more negative during the out-of-sample period. As a result, \( \dot{y}_t \) becomes increasingly positive over the out-of-sample period, showing that this policy strategy does not stabilize the economy. The inert policy strategy successfully closes the inflation gap \( \hat{\pi}_t \) at the end of the simulation horizon. Figures 4 and 5 show the conditional forecasts for model 1, strategies 3 and 4. They look similar to the outcomes of strategy 1 (historical response). If the central bank would follow a modest response strategy, the output and inflation gaps both converge...
somewhat closer to zero than in the historical strategy and the responsive strategy, although both gaps do not close completely during the sample period, even though \( r \) becomes negative at the end of the simulation horizon. This is due to the low value of the interest rate gap coefficient in the demand equation, which limits the effect of monetary policy. This is specific to our small-scale economic model, which does not include the full set of monetary transmission channels through which the interest rate can influence aggregate demand and inflation.

Figure 6 compares the loss functions of the four policy strategies based on model 1; these are the outcomes of Equation (4). Not surprising, the loss of strategy 2 is the largest, since following an inert strategy implies that the output gap tends to deviate most from the policy target over the simulation horizon. While the other three strategies all result in a zero loss at \( t = 6 \) (when the output and inflation gaps are closed), strategy 3 (modestly responsive) results in the lowest loss at the end of the simulation period \((t = 10)\). The outcome of the historical reaction function (strategy 1) is close to the best performing monetary policy strategy.

5.2 | Time-varying parameters: Model 2

Model 2 allows for time-varying parameters, to take into account structural shifts in \( r^* \) and changes in other variables and relationships in the model. The inclusion of time-variability allows for higher coefficients in the post-crisis period, in which the inflation and output gap were negative most of the time. The estimated dummy coefficients of the Phillips curve indicate that the inflation gap was more persistent in the post-crisis period \((\gamma_1\) being significantly positive), while the inflation gap was less
sensitive to the output gap ($\gamma_3$ being significantly negative), see column 2 of Table B1 in Appendix B. The coefficient of the real interest rate gap in the demand curve of Equation (2a) has a larger negative value than in the fixed parameter model, which implies that monetary policy had somewhat larger effects on the output gap in the post-crisis period. Similar to model 1, the Taylor rule estimates of Equation (3a) in model 2 show that monetary policy reacts more strongly to the (expected) inflation gap than to the output gap. The estimated parameter for the inflation gap is somewhat higher, and for the output gap, somewhat lower compared to model 1. The significant negative value of $\gamma_5$ suggests that monetary policy has been more accommodative, on average, in the post-crisis period than in the pre-crisis period, as the coefficient value implies that the policy rate followed $r^*$ just to a limited extent. As in model 1, in the out-of-sample simulations, the value of $r^*$ is kept constant at its 2018Q3 level.

If the central bank would follow the historical reaction function (strategy 1) in model 2, the shadow rate, $i_t$, jumps from around $-3\%$ in $t = 1$ and $t = 2$ to nearly 2\% at $t = 3$, after which it gradually declines to negative values at the end of the simulation horizon (Figure 7). The real rate gap $\hat{r}$ follows a similar pattern. The inflation gap turns positive in the first out-of-sample quarter ($t = 3$) as a result of the positive output gap in the subsequent periods. The inflation gap and output gap converge to zero over the simulation horizon only in the inert strategy 2, in which $i_t$ remains low (Figure 8). In the other three strategies, the output gap, $\hat{y}_t$, becomes increasingly negative due to the lagged response to the jump in the real rate gap. The outcomes of $\hat{y}_t$ and $\hat{\pi}_t$ in strategy 3 (modestly responsive, Figure 9) are similar to those in strategy 1 (Figure 7). If the central bank would apply a responsive strategy (strategy 4), the shadow rate and real rate gap jump to higher levels at the first out-of-
6 | INFO-GAP APPROACH

6.1 | Background

This section introduces info-gap as a risk management strategy to deal with fundamental uncertainty about the natural rate of interest in monetary policy. As a starting point, we assume that for the central bank $r_t^*$ is an unobservable variable, for which it has a model-based series of values. Given a sequence of $r_t^*$ values together with other historical data, dynamic equations, (1), (2), (3) or (1a), (2a) and (3a), are used to predict the future inflation gap and output gap. These predictions underlie the loss function in Equation (4). The monetary policymaker wants the value of the predicted loss (expressed in terms of deviations from target) to be small, and, in any case, less than a critical value: A largest acceptable loss. Similarly, the central bank would like to know what values of critical loss are realistic and how confident it can be that these values will not be exceeded. Based on that it would like to explore the policy implications of these issues.

The challenge, however, is that $r_t^*$ is highly uncertain and can deviate either above or below the model-based values by an unknown amount. By nature, the deviation of $r_t^*$ is persistent, or slowly changing, and the rate of change is unknown. That is, its actual value over a time interval of $t$ quarters in the future may differ, perhaps greatly, from the anticipated values for that interval. $r_t^*$ can either increase or decrease and can even be negative.

The relevant question for the central bank is which monetary policy strategy (historical reaction, inert, modestly responsive, or responsive) is most robust to uncertainty in the magnitude and temporal behaviour of the deviation of $r_t^*$ from its modelled value, while assuring that the loss is acceptable.

6.2 | Info-gap approach to uncertainty

To address this question with an info-gap approach, we suppose that $r_t^*$ varies over a future interval of $T$ quarters according to the following relation, which is a generator model of future values of $r^*$ with uncertain parameters:

$$r_{1+j}^* = (1 + \epsilon_j)r_1^*, \quad j = 0\ldots T-1$$

Thus, $r_1^*$ is the first future value of $r^*$, $r_{1+j}^*$ is changes by a fraction of $\epsilon$ of $r_1^*$ for each quarter, and the sign of $r_{1+j}^*$ can change at most once during the future interval. $r_{1+j}^*$ increases over time if and only if $r_1^*$ and $\epsilon$ are both positive, or both negative.
model $m$ at time $t$, with uncertain deviation of the future $\hat{r}$, is defined as follows:

$$h(k, m, t, L_c) = \max \left\{ h : \left( \max_{(r', \varepsilon) \in U(h)} L(k, m, t, r', \varepsilon) \right) \leq L_c \right\} \quad (8)$$

Note that the robustness does not depend on either $r_1'$ or $\varepsilon$, both of which are uncertain. The robustness does depend on the estimated values of the variables, $\hat{r}_1'$ and $\hat{\varepsilon}$. The value of the robustness is the greatest fractional error in these estimates up to which the loss is acceptable. Thus, if the robustness is large then the dependence on the estimated values is small, and the loss is acceptable even if these estimates err greatly.

The central bank prefers policy strategy $k$ with model $m$, over policy strategy $j$ with model $n$ if and only if the former is more robust:

$$(k, m) > (j, n) \quad \text{if and only if} \quad \hat{h}(k, m, t, L_c) > \hat{h}(j, n, t, L_c) \quad (9)$$

Note that the preference may depend on the critical value of the loss, $L_c$. That is, there may be a preference reversal: The preference may change as the policymaker alters the critical value of the loss. This is expressed by crossing of the corresponding robustness functions when $\hat{h}(k, m, t, L_c)$ is plotted vs. $L_c$, as we will show subsequently.

### 7 | ROBUST POLICY STRATEGIES

#### 7.1 | Three properties of the robustness function

Figure 12 shows the robustness function, $\hat{h}(k, m, t, L_c)$, vs. the critical loss, $L_c$, based on model 1 for each of the four monetary policy strategies. The last observed quarter is 2018Q3, and the loss is evaluated at the end of 2020Q3, eight quarters in the future. As priors for the info-gap analysis, the central bank assumes that the estimated first future value of the natural rate $\hat{r}_1'$ equals the last known historical observation (0.71, at 2018Q3) and that its rate of change, $\hat{\varepsilon}$, equals 0.08, which is equal to the average (absolute) quarterly change of our proxy of $\hat{r}$. For illustration, this choice implies that $-0.08 \leq \varepsilon \leq 0.08$ in Equation (6) when $h = 1$. This section explores the robustness of these highly uncertain estimates.

Note that each robustness curve in Figure 12 starts off from the horizontal axis at a specific value of critical loss at which the robustness against uncertainty equals zero.
For instance, the curve for strategy 4 (responsive strategy) intersects the $L_c$ axis at 0.50. This means that robustness is zero for strategy 4 with this critical value of loss. A basic theorem of info-gap theory asserts, in the present context, that the value at which the robustness curve reaches the horizontal axis is precisely the predicted putative value of the loss. That is, if one adopts $L(k,m,8,r_1^*,\varepsilon^*)$, the putative best estimate of the loss at the eighth future quarter, as the critical value $L_c$, then the robustness to uncertainty in $r_1^*$ and $\varepsilon$ is precisely zero. This is the zeroing property: A best estimate has no robustness to uncertainty in the data and models upon which that estimate is based. This property means that this value of the loss (based on best estimates of the model) is not informative for the performance of the policy strategy if the central bank takes the uncertainty about the natural rate into account.

The second thing to note in the robustness curves of Figure 12 is that they are monotonically increasing functions, where the curve of strategy 2 is very nearly vertical. The positive slope of the curves expresses a trade-off: Greater robustness (a favourable property) is obtained only by accepting greater critical loss (which is not favoured). For instance, in Figure 12, we see that the robustness of strategy 4 (responsive) is nil at a critical loss of 0.50, while the robustness ($\tilde{h}$) equals 3.0 when the critical loss is 1.68. A robustness of 3.0 means that $r_1^*$ can deviate fractionally from its estimated value, $\tilde{r}_1$, by a factor of 3.0, and $\varepsilon$ can take any value in the interval $[-3.0\varepsilon, +3.0\varepsilon]$, without the loss exceeding the value 1.68. The zeroing property asserts that strategy 4 (with model 1) has no robustness for its putative loss of 0.50, while the trade-off property shows that the robustness ($\tilde{h}$) equals 3.0 only for the larger loss of 1.68.

The third property to observe in Figure 12 is that, as a result of the trade-off of robustness vs. loss, one can think of the slope of the robustness curve as a cost of robustness. High slope entails low cost of robustness. Figure 12 shows that the robustness curve for strategy 2 is very steep, compared with the other curves. This means that enhancing the robustness of the three other strategies requires greater increase of the critical loss than for strategy 2. For instance, we mentioned before that increasing the robustness of strategy 4 (with model 1), from 0 to 3.0, requires an increase of 1.18 in the critical loss (from 0.50 to 1.68). For strategy 2, the same increase in robustness requires a negligible increase of critical loss.

So a preliminary conclusion is that the putative loss of the inert, patient strategy 2 is greater than for the other strategies, but the cost of robustness is much smaller for this strategy. This suggests that an inert or wait-and-see strategy is superior, in terms of robustness, to the other monetary policy strategies if one can tolerate loss no less than the value at which strategy 2 intercepts the horizontal axis (namely, loss of 0.84). Keep in mind that the cost of robustness is a different property from the putative loss, precisely because trade-off and zeroing are distinct properties of the robustness function. A policy strategy can have greater putative loss, but lower cost of robustness than other strategies (e.g., strategy 2). This will result in intersection of the robustness curves, with perhaps startling reversal of preference between the policies. As we see in Figure 12, strategy 2 is putatively worse than all of the other strategies. However, because the robustness of putative outcomes is zero, and because strategy 2 has the lowest cost of robustness, strategy 2 is robust-preferred for all values of loss exceeding 0.84, which is only slightly greater than all of the other putative losses.

### 7.2 Robust dominance

Figure 13 shows the robustness curves for all four strategies in model 2. They indicate that strategy 2 (inert strategy) is more robust than all other strategies, for all values of critical loss shown, with model 2. In the sense of the preference relation of Equation (9), strategy 2 is robust dominant: preferred over all other strategies in model 2 (as indicated by the relative position of the robustness curve of strategy 2, which has putative loss of about 0.0035 and is to the left of the other strategies). So based on the robustness curves, we conclude that an inert monetary policy strategy is superior to the other monetary policy strategies. This implies that, given the economic state on which the model estimations are based, a wait-and-see attitude to deviations from the inflation and output objectives is preferred. This is a variant of Brainard’s attenuation principle, which assumes that, with uncertainty, the central bank should respond to shocks more
cautiously than in conditions without uncertainty. Our result also confirms the finding of Williams and Orphanides (2007), that, with natural rate uncertainty, the preferred policy strategy is a more inert interest rate policy (higher degree of interest rate smoothing). While their conclusions follow from Bayesian and min-max or robust control approaches, our conclusion is based on the info-gap approach, which does not require either probabilistic information or specification of a worse-case.

In Figure 13, the robust prioritization of the strategies is the same as the putative prioritization (indicated by the intersections of the robustness curves with the horizontal axis). This means that a putative outcome-optimizing central bank would have the same prioritization of the strategies as a robust-satisficing central bank. However, the prioritization is based on a different reasoning, which may lead to different anticipations about the future. The putative outcome-optimizer prefers strategy 2 because the models predict that it has the best outcome (lowest predicted loss). The robust-satisficer prefers strategy 2 because it is most robust to error in the models. Likewise, the putative optimizer prefers strategy 3 over strategy 1 because the predicted loss for strategy 3 is less than for strategy 1. The robust-satisficer agrees with this preference because strategy 3 robustly dominates strategy 1. Their prioritizations of the strategies are the same, but their anticipations of future outcomes are different. The outcome-optimizer assumes it is a reasonable anticipation that the loss of strategy 3 is about 1.95 (as seen by the horizontal intercept of strategy 3 in Figure 13). The robust-satisficer, however, takes into account that the robustness is zero for future loss being 1.95, so there is little confidence in achieving it; only greater loss can be robustly anticipated.

7.3 How much robustness is enough?

Equation (9) establishes a criterion for preference between policy options based on the robustness to uncertainty. For example, in Figure 12, we see that, with model 1, strategy 2 (inert) is preferred over strategy 3 (modestly responsive), which is preferred over strategy 1 (historical reaction), for critical loss exceeding 0.84. Thus, strategy 2 would be robust-preferred over the other three strategies if loss exceeding 0.84 is acceptable. On the other hand, strategy 2 has zero robustness for loss less than 0.84 and the other strategies are, therefore, robust-preferred over strategy 2 for lower loss. Less loss is clearly better than greater loss, but how much robustness is enough? For instance, strategy 3 has robustness of 2.2 at a loss of 0.7. Is this enough robustness, and should strategy 3 be preferred over strategy 2, which has zero robustness at that level of loss?

This is a judgement for which various conceptual tools are available, such as analogical reasoning or consequence severity (Ben-Haim, 2006). Without going into much detail, the basic intuition derives from the info-gap model of uncertainty, Equation (6). With regard to $r^*$, robustness of 2.2 means that the fractional error of the estimated value can be as large as 2.2 without violating the corresponding performance requirement on the loss. More colloquially, this is a 220% robustness with respect to $r^*$, and similarly for robustness with respect to $\iota$. Contextual understanding and subjective judgement is needed to decide if this is adequate.

Suppose, hypothetically, that the central bank’s assessment is that the natural rate, $\hat{r}$, may deviate fractionally from its observed historical values by 50%, or 100%, or possibly even more in extreme situations. The central bank cannot know how much the future rate will deviate from its historical values. Nonetheless, it would be plausible to conclude that robustness of 220% is quite substantial. From Figure 12, we see that this implies that strategy 3 can be confidently relied upon to keep the loss below 0.7, and that only truly extreme deviations would violate this.

Alternatively, suppose that the central bank’s understanding is that the natural rate, $\hat{r}$, could deviate fractionally from its historical values by 100%, or 200%, or perhaps even more. Without knowing, of course, the degree of deviation, the central bank would likely conclude that robustness of 220% is at best moderate, but not large. In other words, the conclusion from Figure 12 would be that none of these policy alternatives can be confidently relied upon to keep the loss below the value of 0.7 (the critical loss with strategy 3 if $\hat{h} = 2.2$). One response would be to seek new policy alternatives. Alternatively, the central bank could consider whether a
greater critical loss is acceptable. For instance, if loss as large as 0.9 is acceptable, then Figure 12 shows that strategy 2 is strongly robust dominant over all of the other strategies.

The judgement of how much robustness against natural rate uncertainty is enough for the central bank, like most judgements of safety and reliability, is difficult and subjective. It may, for instance, depend on the inflation process and developments in the economy. If these are driven by shocks beyond the control of the central bank (e.g., global or demographic shocks that drive the natural rate), it cannot perfectly control inflation, and especially headline inflation. A greater tolerance of deviations of inflation from the objective or a longer horizon over which it is expected to be achieved could make communication easier about what monetary policy can deliver and so contribute to the central bank’s credibility. In such circumstances, the central bank might accept a higher critical loss (in terms of deviations from the policy objectives) to improve the robustness of its policy strategy. The robustness curves provide quantitative support for such deliberations. Nonetheless, the robust prioritization in Equation (9) is unambiguous, and if a decision must be made from a specified set of options, that decision is clear. The policymaker should always keep in mind, however, that the greatest available robustness may not be a solid guarantee of success. Shocks that go beyond the robustness can still happen, but within this horizon the maximum possible loss associated with a certain policy strategy is acceptable.

8 | UNCERTAINTY IN OTHER VARIABLES

In this section, we assume fundamental uncertainty in the output gap, \( \tilde{y}_t \), or the shadow rate, \( i_t \), in a similar way as we did for \( r^* \). To illustrate the impact of uncertainty in \( \tilde{y}_t \) or \( i_t \), the loss in model 2 (time-varying parameters) is calculated by treating all other variables as observed data. This method singles out the impact of output gap or shadow rate uncertainty on the model outcomes. We acknowledge that assuming uncertainty \( \tilde{y}_t \) or \( i_t \) in isolation underestimates the challenges posed by uncertainty about the model variables to the policymaker, but this approach allows us to show the impact of uncertainty, in particular, variables on the model outcomes.

8.1 | Output gap uncertainty

Uncertainty in the output gap, \( \tilde{y}_t \), is taken into account by adding the uncertain term, \( z_t \), to the aggregate demand Equation (2a), see Table B1 in Appendix B. \( z_t \) is determined as follows.

\[
z_{1+j} = (1 + qj)z_1, \quad j = 1, 2, \ldots
\]

\( q \) and \( z_1 \) are constant but uncertain. We represent info-gap uncertainty in \( q \) and \( z_1 \) with this info-gap model:

\[
U(h) = \left\{ z_1, q : \frac{z_1}{|z_1|} \leq h, \frac{|q|}{q} \leq h \right\}, \quad h \geq 0
\]

The putative values are \( \tilde{z}_1 = 0.37 \) and \( \tilde{q} = 0.3 \).

Figure 14 shows the robustness curves for all four strategies under output gap uncertainty. They indicate that strategy 2 (inert strategy) is clearly more robust than all other strategies, for all values of critical loss shown. In the sense of the preference relation of Equation (9), strategy 2 is **robust dominant**: preferred over all other strategies for any level of critical loss. This is similar to the outcomes under natural rate uncertainty in Section 7 and in line with findings in other literature (e.g., Williams & Orphanides, 2007). The robustness curves for strategies 1, 3, and 4 are all essentially the same for the range of critical loss shown.

8.2 | Shadow rate uncertainty

Uncertainty in the shadow rate, \( i_t \), is taken into account by adding the uncertain term, \( \eta_t \), to the Taylor rule Equation (3a), see Table B1 in Appendix B. \( \eta_t \) is determined as follows:

\[
\eta_{1+j} = (1 + \xi j)\eta_1, \quad j = 1, 2, \ldots
\]

\( \xi \) and \( \eta_1 \) are constant but uncertain. We represent info-gap uncertainty in \( \xi \) and \( \eta_1 \) with this info-gap model:

\[
U(h) = \left\{ \eta_1, \xi : \frac{\eta_1}{|\eta_1|} \leq h, \frac{|\xi|}{\xi} \leq h \right\}, \quad h \geq 0
\]

The putative values are \( \tilde{\eta}_1 = -3.18 \) and \( \tilde{\xi} = 0.4 \).

Figure 15 shows the robustness curves for all four strategies under shadow rate uncertainty. They also indicate that strategy 2 (inert strategy) is clearly more robust than all other strategies, for all values of critical loss shown. In the sense of the preference relation of Equation (9), strategy 2 is robust dominant, as before under natural rate uncertainty in Section 7 and under output gap uncertainty (Section 8.1). Likewise, the robustness curves of strategies 1, 3, and 4 are quite similar.
DISCUSSION

Info-gap theory ranks different monetary policy strategies in terms of their robustness against natural rate uncertainty, or uncertainty in the output gap or the shadow rate, while also achieving specified policy goals. This approach demonstrates the zeroing property of the robustness function, stating that putative predictions have no robustness and thus are a poor basis for policy selection. “Outcome optimization” is commonly implemented by using the best available knowledge to predict outcomes of policy alternatives and to select the policy whose predicted outcome is best, also in macro models that include the (inherently uncertain) natural rate of interest. However, monetary policy strategies based on such models often fail in the presence of Knightian uncertainty. The zeroing property of the robustness function demonstrates that the putative performance of a policy strategy – measured by a loss function and best estimates of an economic model – is not reliably informative if the central bank takes the uncertainty about the natural rate or other variables into account. As stated by the trade-off property of the robustness function, only worse-than-predicted (putative) outcomes have robustness to uncertainty. This suggests that the central bank must make judgments about the acceptability of this trade-off for each policy strategy.

An important caveat is that the outcomes are dependent on the model set-up (including the defined policy rule) and on the economy and sample period on which the estimations are based. This implies that the policy strategies will not necessarily be ranked similarly in all situations in which fundamental uncertainty about $r^*$ or other variables plays a role. For instance, the very low putative loss of strategy 2 in Figures 13 and 14 is as much a property of the specific economic conditions as of the strategy itself. Studies show that the effectiveness of monetary policy is state-dependent, being a function of uncertainty in markets and the economy. Conventional monetary policy is found to be less powerful in recessions (Tenreyro & Thwaites, 2016). In contrast, unconventional monetary policy measures, like Quantitative Easing, are found to be more effective in stressed market conditions compared to normal conditions. The impact is larger in times of stress because central bank measures can then alleviate binding frictions and higher risk premia (CGFS, 2019). These insights also apply to our model, since the shadow rate captures both conventional and unconventional monetary policy measures.

CONCLUSIONS

We use info-gap theory to evaluate different monetary policy strategies based on the trade-off between reaching the objectives (closing the inflation and output gaps) and robustness against uncertainty in the natural rate or other variables. The preferred policy is the strategy that maximizes the robustness and satisfices the policy outcome. This distinguishes our approach from robust control, which aims at minimizing the loss of the worst outcome, but does not account for the trade-off between robustness against uncertainty and performance of a policy strategy.

The application to the euro area, over a forecast horizon of 2 years, starting in 2018Q4, shows that an inert monetary policy strategy, which keeps the policy rate constant over the forecast horizon, is more robust to natural rate uncertainty than the historically followed reaction function and a modestly responsive policy strategy. An actively responsive strategy might be optimal to prevent a worse outcome (such as a recession or deflation) in terms of the robust control concept, but it turns out to be least robust in the info-gap approach when applied to...
the economic situation in the euro area. There are possible constellations, in which an inert policy strategy can have greater (worse) putative loss, but lower (better) cost of robustness than other strategies. This results in crossing of the robustness curves, implying that an inert policy ultimately turns out to be the preferred strategy. The preference ranking of the monetary policy strategies holds both in a fixed and in a time-varying parameter model.

An important caveat is that the outcomes are dependent on the model set-up (including the defined policy rule) and on the economy and sample period on which the estimations are based. The ranking could also be different if the policymaker considers other policy rules, or assumes that another model would better describe the economy, for instance, a more complex new-Keynesian model instead of the small-scale model that we use. Simulations based on another policy rule and economic model will generate different dynamics, but info-gap is also applicable to rank policy strategies based on more complex models. Moreover, the consequences of fundamental uncertainty in other variables or model equations, such as the slope of the Phillips curve, could be studied. Such applications of info-gap are left for further research.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request. Data citation [Model_variables] Jan Willem van den End, Yakov Ben-Haim; 2019; Model_variables (available from the corresponding author upon reasonable request).

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ENDNOTES

1 This ranking of policy strategies is ordinal.

2 The correction for the inflation gap makes the difference between the nominal policy rate and the nominal natural rate a real rate gap, which is not influenced by differences in (implied) expected inflation rates included in the policy rate and natural rate.

3 By taking the nominal natural rate, we abstain from assumptions about the inflation rate that can be used to calculate the real natural rate and so implicitly take the uncertainty about this deflator into account in the info-gap analysis.

4 Taking the euro average long-term bond yield as proxy for \( r^* \) assumes country-specific natural rates in the euro area, reflecting, for instance, country-specific differences in potential growth among other factors.

5 The model is estimated by heteroscedasticity and autocorrelation consistent GMM (HAC), applying pre-whitening to soak up the correlation in the moment conditions. Lags of the variables (nearest lags) are used as instruments.

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APPENDIX A.

**TABLE A1** Definition of model variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_t )</td>
<td>inflation gap (realized inflation rate - Inflation objective of 2%)</td>
</tr>
<tr>
<td>( \pi_{e,t+1} )</td>
<td>expected inflation gap (1 year ahead expected inflation based on the survey of Consensus Economics - Inflation objective of 2%)</td>
</tr>
<tr>
<td>( y_t )</td>
<td>output gap (real GDP – potential GDP) / potential GDP; source: ECB</td>
</tr>
<tr>
<td>( l_t )</td>
<td>relevant policy rate (Eonia rate until 2008, shadow rate from 2008 onward, source: Krippner)</td>
</tr>
<tr>
<td>( r_t )</td>
<td>real interest rate gap (( l_t - r^{\text{C3}}<em>t ) – ( \pi</em>{e,t+1} )); see the explanation below.</td>
</tr>
<tr>
<td>( r^{\text{C3}}_t )</td>
<td>HP filtered 10 years euro average nominal government bond yield (lambda 1,600)</td>
</tr>
<tr>
<td>( d_t )</td>
<td>crisis dummy (( d_t = 1 ) in 2008q1–2018q3; ( d_t = 0 ) in 1995q1–2007q4)</td>
</tr>
<tr>
<td>( z_{t,\gamma} )</td>
<td>uncertain driving term in Equation (2a) to account for output gap uncertainty in Section 8.</td>
</tr>
<tr>
<td>( \eta_{t,\gamma} )</td>
<td>uncertain driving term in Equation (3a) to account for shadow rate uncertainty in Section 8.</td>
</tr>
</tbody>
</table>

The real rate gap is defined as the gap between the (shadow) policy rate and the natural rate in real terms.

\[
\tilde{r}_t = l_t - \tilde{r}^*_t = \tilde{r}^{\text{C3}}_t + \pi_{e,t+1} - \tilde{r}^*_t,
\]

\[
\tilde{r}^*_t = \text{HP filtered 10 years euro average nominal government bond yield (lambda 1,600)};
\]

\[
\pi_{e,t+1} = \pi_{e,t+1} - \pi^{\text{C3}}_t \quad \Rightarrow \quad \tilde{r}_t = l_t - \tilde{r}^*_t,
\]

which defines the real rate gap purely in real terms.

---

**APPENDIX B.**

**TABLE B1** Estimation outcomes

<table>
<thead>
<tr>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed coefficients</td>
<td>Time-varying coefficients</td>
</tr>
<tr>
<td><strong>Phillips curve</strong></td>
<td></td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>( -0.11 )</td>
</tr>
<tr>
<td>( (0.12) )</td>
<td>( (0.14) )</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>( 0.92*** )</td>
</tr>
<tr>
<td>( (0.12) )</td>
<td>( (0.74) )</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>( 0.21*** )</td>
</tr>
<tr>
<td>( (0.04) )</td>
<td>( (0.09) )</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>( 0.39** )</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>( 1.24 )</td>
</tr>
<tr>
<td>( \gamma_3 )</td>
<td>( -0.39*** )</td>
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<tr>
<td><strong>IS curve</strong></td>
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<tr>
<td>( \alpha_2 )</td>
<td>( -0.15 )</td>
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<tr>
<td>( (0.11) )</td>
<td>( (0.05) )</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>( 0.98*** )</td>
</tr>
<tr>
<td>( (0.04) )</td>
<td>( (0.02) )</td>
</tr>
<tr>
<td>( \beta_4 )</td>
<td>( -0.07*** )</td>
</tr>
<tr>
<td>( (0.02) )</td>
<td>( (0.02) )</td>
</tr>
<tr>
<td>( \gamma_4 )</td>
<td>( -0.37*** )</td>
</tr>
<tr>
<td><strong>Taylor rule</strong></td>
<td></td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>( 0.56*** )</td>
</tr>
<tr>
<td>( (0.04) )</td>
<td>( (0.02) )</td>
</tr>
<tr>
<td>( \beta_5 )</td>
<td>( 1.85*** )</td>
</tr>
<tr>
<td>( (0.35) )</td>
<td>( (0.27) )</td>
</tr>
<tr>
<td>( \beta_6 )</td>
<td>( 0.75*** )</td>
</tr>
<tr>
<td>( (0.08) )</td>
<td>( (0.06) )</td>
</tr>
<tr>
<td>( \gamma_5 )</td>
<td>( -0.51*** )</td>
</tr>
</tbody>
</table>

**Note:** Outcomes of GMM estimation of system of Equations (1), (1a), 2, (2a), 3, (3a) with data for the euro area. Sample period 1995–2018q3 (quarterly observations). The model is estimated by heteroskedasticity and autocorrelation consistent GMM (HAC), applying pre-whitening to soak up the correlation in the moment conditions. Standard errors in parentheses, where *** indicates significance at the 1% level, ** at the 5% level and * at the 10% level.