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In spite of the fact that the main claim of our paper (Withagen et al. [2003]) remains valid, its proof included incorrect components and needs amendment. In this note we briefly indicate the essence of the corrections needed. A new full version of the paper has appeared in Asheim [2007, Chapter 12] and it can be downloaded from http://folk.uio.no/gasheim/WAB-Nc03.pdf.

An important step in the proof was, and still is, to characterize the solution to the minimum resource use problem. This is the problem of finding the program that minimizes total resource use needed to achieve the maximin consumption, given at the outset. In the original paper it was claimed that minimum resource use coincides with the resource stock initially available to the economy. Our proof borrowed an idea used by Cairns and Yang [2000] entailing that, since a uniform decrease of the consumption rate allows for a finite decrease of the resource

stock, a uniform increase of the consumption rate would require a finite increase of the resource stock. This being so implies that the maximin path must exhaust the resource because otherwise consumption can be increased uniformly, contradicting that the path under consideration is maximin. However, as such, the argument is not valid since it makes an implicit assumption on the curvature of the production function. We now propose an alternative, which can broadly be sketched as follows.

We assume that the raw material is important in production (Mitra [1978, p. 121]), meaning that its production elasticity is uniformly bounded away from zero: there exists $\alpha > 0$ such that $f_r(k, r)r/f(k, r) \geq \alpha$ for all $k > 0$ and $r > 0$, where k and r denote capital and raw material, respectively, and f is the production function. Suppose we are on the maximin path where consumption c is at least as large as $c^* > 0$, with $\inf_t c(t) = c^*$. Assume, for now, that capital is nondecreasing $\left(i(t) = \dot{k}(t) = f(k(t), r(t)) - c(t) \geq 0\right)$. Then $f(k(t), r(t)) \geq c^*$. Moreover,

$$\int_0^\infty \frac{1}{f_r(k(t), r(t))}\, dt = \int_0^\infty \frac{r(t)}{f_r(k(t), r(t))r(t)}\, dt$$

$$\leq \int_0^\infty \frac{1}{\alpha f(k(t), r(t))}\, r(t)\, dt$$

$$\leq \int_0^\infty \frac{1}{\alpha c^*}\, r(t)\, dt = \frac{1}{\alpha c^*} \int_0^\infty r(t)\, dt < \infty.$$

Then, we observe that

$$\int_0^\infty \frac{1}{f_r(k(t), r(t))}\, dt$$

is the marginal resource cost of a uniform increment to consumption, given the initial capital stock k_0. It remains to be shown that resource use minimization requires a nondecreasing capital stock. Intuitively this is clear, but a formal proof is provided in the full-length version. The idea behind the proof given there is that it is inefficient in terms of raw material use to have a decrease of capital that has to be compensated by an increase later on.

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