VU Research Portal

Compensation for Commuting in Imperfect Urban Markets

van Ommeren, J.N.; Rietveld, P.

Link to publication in VU Research Portal

Compensation for commuting in imperfect urban markets[*]

Jos van Ommeren[1], Piet Rietveld[1]

[1] Free University, FEWEB, De Boelelaan 1105, NL-1081 HV Amsterdam, The Netherlands (e-mail: jommeren@feweb.vu.nl, prietveld@feweb.vu.nl)

Abstract. We develop an urban equilibrium job search model with employed and unemployed individuals where residential mobility of the unemployed is restricted. We assume a standard mono-centric model (firms are located in one location), but allow for imperfect labour markets. In contrast to models with perfect labour markets, the model predicts that the employed are only partially compensated for commuting costs in the form of wages. As a result, rent gradients are less steep than predicted by standard urban theories that assume perfectly competitive labour markets.

JEL classification: R12, R23, J64

Key words: Job matching, moving costs, rent gradient, spatial mismatch, urban economics

1 Introduction

Theories of urban residential location, which are based on perfect competitive labour and housing markets lead to predictions that are not always consistent with empirical evidence. We will focus here on two anomalies.

The first anomaly is that these theories predict that wages depend on workplace location, but not on residence location, ceteris paribus (Muth 1969). Zax (1991) argues that these theories implicitly assume that all workers at a particular work-

---

place share the same residence location. Wages, given workplace, are invariant to residence location, because residence location does not vary with workplace. However, this is not confirmed by empirical research: there is sufficient empirical evidence that at a particular workplace, workers' wages depend on residence location (for example, Zax 1991; Hazans 2004; see Kasper (1983) for a short review), suggesting the importance of labour market imperfections (e.g., bargaining power of firms) or housing market imperfections (e.g., moving costs). For evidence of the importance of labour market imperfections, see Manning (2003). Zax (1991) identifies the effect of commuting time on earnings for employees of one firm located in the CBD of one city (Detroit, Michigan, USA). The research design is unique, because in the empirical analysis the workplace locations are the same for all employees. Zax (1991) shows that employees' wages depend on the residence location, essentially rejecting the assumption of a perfectly competitive labour market, and suggests that firms have labour market power, which induces firms to compensate employees for their commuting costs. He finds also that males receive higher wages and more compensation than females. To the extent that males have more labour market power, his results suggest thus that workers with more bargaining power (e.g., due to less discrimination or more residential mobility) receive higher wages and more compensation for commuting costs.

Manning (2003) argues that the effect of residence location on wages, as for example identified by Zax (1991), may be a result of omitted variable bias related to the presence of unmeasured ability (see also Timothy and Wheaton 2001), since high-ability workers commute further and also earn higher wages. Kasper (1983) deals with this issue by employing another research design. The effect is estimated from a sample of one city (Glasgow, in the UK), where workers are randomly relocated within the city by a Housing Authority. The new location of residence was chosen by the Housing Authority, independent of location of workplace, and can be assumed to be exogenous. Kasper (1983) demonstrates that wages respond positively to increases in the commuting costs, also for those who do not change job, and that the increase is stronger for females than males (in contrast to the findings by Zax (1991)). Based on an analysis of cross-section data he finds similar results (although now the effect on the males' earnings is higher which is in line with Zax (1991)).

Another anomaly of the theories of urban residential location is that according to the standard theory, the price of housing services declines with the distance of residences from the centre to induce workers to choose locations at longer commuting distance (Muth, 1969). Because the theory assumes that workers do not receive compensation in the form of wages, workers receive full compensation in the housing market. However, this form of compensation, let alone full compensation, has to a large extent been empirically elusive (e.g., Dubin and Sung 1987; Ball 1973; Söderberg and Janssen 2001). Although there are a number of reasons why rent gradients are flatter than predicted by standard theory (e.g., variation in income levels, multiple employment centres), this paper aims to demonstrate that compensation in the housing market and in the labour market are mutually related by employing a model which shows that workers at a single employment centre (in the current paper, the Central Business District) are compensated by both wages and rents. So, we make standard assumptions on the spatial structure of the

economy, assume ex-ante identical individuals, but assume labour market imperfections. This model is essentially a variant of the urban equilibrium search model as introduced in the literature by Wasmer and Zenou (2002). Our model takes both labour market imperfections due to search frictions and imperfect residential mobility and wage bargaining between workers and employers into account. The model introduced in this paper differs from the model by Wasmer and Zenou (2002) that assumes perfect residential mobility and predicts that workers receive full compensation in the housing market, but no compensation in the form of wages. Our model is similar in spirit to the equilibrium search model recently introduced by Wasmer and Zenou (2006), with (small) positive relocation costs and exogenous wages.

In the labour economics literature, labour market imperfections are generally captured via search frictions (e.g., Pissarides 2000) or via efficiency wages, which are based on the assumption that monitoring of workers is costly, so workers tend to shirk (Shapiro and Stiglitz 1984). Firms may reduce shirking by offering wage surpluses, which are sufficiently high to make the threat of firing too costly for the workers. Recently, Zenou (2002) has introduced efficiency wages in a standard urban model with free residential mobility. In addition, he makes the additional assumption that effort level is a decreasing function of distance, because workers are more tired. He shows that wages are not a function of commuting distance, a similar result as Wasmer and Zenou (2000). In a similar vein, Zenou and Smith (1995) also demonstrate that in an urban context with free residential mobility and efficiency wage paying firms, firms do not pay wages to compensate for the actual commuting costs. So, evidence from the current literature indicates that labour market imperfections are insufficient to explain the positive relationship between wages and commuting distance in an urban market with free residential mobility.

The remainder of the paper is as follows. In section 2, the urban equilibrium model is introduced and analysed. Section 3 and 4 discuss extensions and compare the model with an equilibrium model with a different spatial structure. Section 5 concludes.

2 The basic job matching model

2.1 The job matching function

We assume a continuum of identical residences, which are homogeneously distributed over space.[1] Firms are located in one location: the Central Business District (the CBD hereafter).[2] The economy is linear and closed. Each residence is

---

[1] In a more general model, the size of the residence is optimally chosen (see, e.g., Fujita 1989). This does not affect the (qualitative) results of the paper.
[2] In our set up firms have an incentive to move away from the centre to economise on labour costs. We consider the spatial structure as given however. So, we implicitly assume that the agglomeration forces exceed the disagglomeration forces (Fujita and Krugman 1995). Fujita et al. (1997) relax the assumption of a given CBD and consider an endogenously determined secondary employment centre. In a companion paper to the current paper, we follow Marimon and Zilibotti (1999) by presuming a homogeneous distribution of firms, which do not have any incentive to move.

of fixed size normalised to 1 and inhabited by one individual, who is either unemployed or employed. The size of the city is equal to N, the number of individuals. The unemployed search for jobs, the employed do not search (for an equilibrium model which includes on-the-job search, see Mortensen 1994). The employed incur commuting costs t, which are proportional to distance d, so $t = \alpha d$. The commuting costs become known to the firm at the moment the unemployed job seeker and firm contact each other, but are also known to the firm if the employed moves residence. A firm consists of only one job, which is either filled or unfilled. In order to fill a job, firms post a vacancy. When a firm with a vacancy and an unemployed contact each other, the job will be filled. So, we assume that given a contact, it is advantageous for both firms and job seekers to form a match. The conditions under which this behaviour is optimal are derived later on.

Suppose there are L identical individuals in the labour force. We let u denote the unemployment rate and v denote the vacancy rate, defined as number of vacant jobs as a fraction of the labour force L. We assume the existence of a matching function that gives the number of matches between unemployed and firms per unit of time as a function of the number of unemployed uL looking for jobs and the number of firms looking for workers vL. The number of matches taking place per unit of time is given by $mL = m(uL, vL)$. The matching function is assumed increasing in both its arguments, concave, and has constant returns to scale. Empirical studies generally accept the assumption of an aggregate matching function with constant returns to scale, see Petrongolo and Pissarides (2001). The evidence from disaggregated empirical studies is not conclusive (e.g. Burda and Profit 1996).

Given the matching function, the probability for a vacancy to be filled per unit of time, denoted as q, is defined. Given the constant returns to scale assumption, it follows that:

$$q = \frac{m(uL, vL)}{vL} = m\left(\frac{u}{v}, 1\right) = m\left(\frac{1}{\theta}, 1\right), \tag{1}$$

where $\theta = v/u$. So, θ is a measure of labour market tightness, defined as the ratio of the vacancy to the unemployment rate. Thus, q, the rate at which vacancies become filled, depends negatively on the ratio of the vacancy to the unemployment rate, θ. We will assume that q goes to infinity, when θ approaches zero. Similarly, it can be seen that the rate at which unemployed become employed equals θq, where θq depends positively on θ (Pissarides 2000). Note that we assume that θq does not depend on the location of the unemployed. Later on, we will relax this assumption.

2.2 Employed and unemployed

An individual receives a wage w and incurs commuting costs t when employed, and receives unemployment benefits z when unemployed. To simplify notation, we assume that z equals zero. All individuals pay rent costs at the market price $R(t)$

to absentee landowners. Note that we write R as a function of commuting costs t, which is equivalent to writing it as a function of distance d. The commuting costs are exogenous to the worker. In contrast, the wage is endogenous. When firm and unemployed contact each other given the value of the commuting costs t, firm and unemployed will bargain about the wage w, so $w = w(t)$. Firms and workers also continuously bargain about the wage. Hence, if the employed moves residence and t changes, firms and employed individuals may agree on a different wage.[3] When the worker and firm do not agree on the wage, then the individual will remain / become unemployed and the firm and individual will continue their search. The worker will not keep the job forever. The job will be destroyed at rate λ and the worker will then become unemployed. The discount rate is denoted as r.

Urban economic theory assumes the absence of residential moving costs and assumes that individuals can choose their residence freely (Fujita 1989). Although the empirical literature on residential mobility has shown that moving costs are sufficiently high to deter residential mobility (see, among others, Boehm 1981), this assumption can usually be justified when focusing on the choice of the residence location. When choosing the optimal location, moving costs are discounted over long periods, so the optimal location is hardly affected by the presence of moving costs. This assumption is however problematic when individuals experience changes in their characteristics over time. For example, individuals may experience a change in income, number of children, or, as in the current paper, in their labour market position. The absence of moving costs implies that any change in an individual's characteristic would induce a residential move.

In the context of the labour market, the absence of moving costs implies that individuals move residence the moment they change their labour market status (from employment to unemployment or from unemployment to employment, e.g., Wasmer and Zenou 2002). This prediction is empirically implausible. For most individuals it is uneconomical to move residence the moment they become unemployed and to move again when they become re-employed at the same workplace location as before, because the costs of moving exceed the benefits of moving, viz. a temporary reduction in rent (see, also Wasmer and Zenou 2006). The expected duration of being unemployed is for most individuals only a couple of months. We impose therefore the assumption that over their lifetime, individuals are partially restricted to move.

We assume that employed individuals may move freely in each period following the standard assumptions of urban economic theory, whereas we assume that unemployed individuals do not move (so they will not bid for residences or can be

---

[3] At face value, this assumption seems unrealistic, because most workers do not explicitly renegotiate the wage after a residence relocation, but it captures in essence that the level of the commuting costs affects the bargaining position of the worker. For empirical evidence, see the study by Kasper (1983) which studies the change in wages of workers who move residence and where the residence occasion is randomly determined. For other indirect empirical evidence, we refer to the literature on job mobility, job search and commuting costs, which demonstrates that workers are more likely to leave their employer given larger commuting costs (see Manning 2003).

outbid).[4] Workers who become unemployed do not have the opportunity anymore to move residence until they become employed again (see, similarly, DeSalvo and Eeckhoudt 1982).[5] The location of an unemployed individual is therefore determined by the location chosen the moment just before the individual becomes unemployed.[6] Hence, effectively, employed workers bid for a residence location, realising that when they become unemployed, they will be locked in and may reside in a non-optimal location (until they become employed again).

We denote by $U(t)$ and $W(t)$ the expected (discounted) lifetime income of the unemployed and employed respectively. The lifetime income of the employed can be written as:

$$rW(t) = w(t) - t - R(t) + \lambda(U(t) - W(t)) + \max_{t'}(W(t') - W(t)). \qquad (2)$$

Interpretation of this Bellman equation is as follows. The lifetime income of the employed is equal to the flow of income in each period, which is equal to the net wage (the wage minus the commuting costs), minus the rent plus the expected change in lifetime income due to the probability of losing the job plus the expected change in lifetime income due to moving to another location. The expected employment duration equals $1/\lambda$. At the moment the individual becomes unemployed, the individual may not move residence (until he/she becomes re-employed).

We presume that search costs of the unemployed are absent (positive search costs do not change the results fundamentally). When unemployed, the lifetime income of the unemployed can be written as:

$$rU(t) = -R(t) + \theta q \left(\max_{t'} W(t') - U(t) \right). \qquad (3)$$

Interpretation of this Bellman equation is as follows: the unemployed pays rent equal to $R(t)$ and has per unit of time a probability θq of becoming employed. At the moment the unemployed becomes employed, (s)he may move to the location with a commuting costs of value t', which maximises lifetime income $W(t')$. The

---

[4] This assumption can be justified in a number of ways. For example, the unemployed are more restricted in the mortgage market, and are therefore often liquidity constrained, so they cannot bid for other residences. In addition, the assumption that the unemployed cannot be outbid can be justified on the grounds that contracts in the rental market are much longer than unemployment periods.

[5] In DeSalvo and Eeckhoudt (1982), the urban wage is exogenously given and does not explain why unemployment exists. Interestingly, an extension of their model may be used to obtain a relationship between wages and commuting, although the causation is the reverse and jobs are not identical. In the current paper, and this is essential, we assume identical jobs.

[6] This assumption may be contrasted to the assumption by Wasmer and Zenou (2006) that all individuals have to pay explicit residence costs when moving residence. They show that individuals choose an area within the city consistent with a certain residential mobility pattern. Given this set up, it may occur that mobile employed workers locate close to the CBD, whereas the mobile unemployed workers locate close to the edge of the city, and the immobile (employed and unemployed) workers live in between. The result is intuitive given low residential costs, as prevalent in the U.S. Given sizeable residence moving costs, prevalent for workers in most West European countries (e.g., in the Netherlands, average monetary moving costs are about 25,000 Euro for homeowners), their model predicts that all workers choose to be immobile ex post. Another difference is that we allow the wage to depend on the commuting costs.

expected unemployment duration equals $1/\theta q$. When the unemployed does not become employed, then (s)he will remain in the same location.

2.3 The bid rent function/optimal location

The moment the unemployed contacts an employer and decides to form a match and hence becomes employed, a new optimal residential location may be chosen to maximise lifetime income $W(t')$. Competition between the employed workers guarantees that all employed workers enjoy the same level of income, so $W(t') = W$, where W denotes the equilibrium lifetime income level. An employed worker may move to another residence location, only when outbidding other employed workers.[7] The optimal location of each worker is then determined by the maximum land rent that the employed worker is ready to pay, the so-called bid rent, to reach the equilibrium lifetime income level (e.g., Fujita 1989; Wasmer and Zenou, 2002). Given equation (2), the bid rent, denoted as Ψ, is equal to:

$$\Psi(t, W) = w(t) - t + \lambda U(t) - (r + \lambda)W. \tag{4}$$

Using equation (3), which defines the lifetime income of the unemployed, the bid rent can be conveniently rewritten as:

$$\Psi(t, W) = \frac{r + \theta q}{r + \theta q + \lambda}(w(t) - t) - rW. \tag{5}$$

Consequently, the bid rent function depends on the commuting costs t directly and indirectly (via wages). The marginal costs that a worker is ready to pay to be marginally closer to the CBD can be derived from the bid rent slope:

$$\frac{\partial \Psi(t, W)}{\partial t} = \frac{r + \theta q}{r + \theta q + \lambda}(w'(t) - 1) < 0. \tag{6}$$

The bid rent slope is negative (since $0 < w'(t) < 1$, which will be shown later on), and, importantly, less than one (in absolute value). It can easily be seen that the slope is equal to minus one, so workers receive full compensation in the housing market, when $\lambda = 0$ and $w'(t) = 0$. According to (6), employed individuals are not fully compensated for the commuting costs in the housing market, because we will demonstrate later on that workers are partially compensated in the labour market ($w'(t) > 0$). This makes sense given the assumption of the presence of labour market bargaining power. When workers have bargaining power, they will receive compensation for commuting costs in the labour market, and, as a result, will need less compensation in the housing market. Further, the employed worker takes into account the likelihood to become unemployed and to stay at the same location.

---

[7] Note that the unemployed do not relocate, so they do not bid or can be outbid.

When unemployed, the individual will not commute to the CBD, so the employed will bid less for a location closer to the CBD (implied by the factor $(r + \theta q)/(r + \theta q + \lambda) < 1$).

We are now ready to examine the equilibrium location of the individuals. The market rent is given by $R(t)$, and each employed individual takes it as an exogenous factor. Recall that W is the equilibrium lifetime income of the employed individual and $\Psi(t,W)$ is the maximum land rent costs an employed individual is willing to pay. Without loss of generality, we assume that the opportunity value of land is zero. This implies that t is an optimal location if and only if $R(t) = \Psi(t,W) \geq 0$. This implies that the urban land rent is 0 at the city fringe, which closes the model (because W is then determined).

Recall that we have assumed that unemployed do not bid (and cannot be out-bidded), because their residence location is determined in a previous period when employed. So where do the unemployed locate? Because the probability of entering and leaving unemployment is not related to space, it follows that in equilibrium, the spatial distribution of the unemployed in the city must be identical to the spatial distribution of the employed so the unemployed are randomly spread over space within the city fringe. This is consistent with our result later on derived in section 2.7 that the unemployed rate does not depend on space. One of the implications is that expost non-identical workers (employed and unemployed) mix over space. Hence, although individuals are identical ex ante (ex ante it is not known to individuals whether they will be employed or unemployed), the presence of moving costs causes them to mix over space (see also Wasmer and Zenou, 2006).

Given (2) and (3), it follows that $W - U(t) = (w(t) - t)/(r + \lambda + \theta q)$, so $U'(t) = (1 - w'(t))/(r + \lambda + \theta q) > 0$, since it is shown later on that $w'(t) < 1$. This result makes sense. When unemployed, workers living closer to the CBD are worse off, as they pay higher rents for which they are not compensated as they are locked in. This implies, as we will see later on, that while being employed, workers living closer to the CBD must have higher current incomes, because the lifetime income of the employed is (assumed to be) independent of space.

2.4 Job creation

Recall that all firms are located in one location. When opening a vacancy, firms do not know where the (next) job applicant who will fill the job is located. When firms and unemployed contact each other and form a match, the commuting costs of the worker are known to the firm. The value of a vacancy, V, can be written as:

$$rV = -c + q(J^e - V), \tag{7}$$

where c denotes the firms' hiring costs. Vacancies are filled at rate q and J^e denotes the expectation of the job's net worth. The job's net worth is unknown to the firm, because the residence location of the job applicant is unknown. However, the firm knows the distribution of commuting costs in the urban area, so the expected job's net worth is known. Firms do not influence directly the residence location of the worker.

The value of an occupied job is equal to the productivity level, denoted as p, minus the wage, $w(t)$, taking into account that with a rate equal to λ the job will be destroyed. Hence, the value of the filled job occupied by a worker who resides at t can be written as:

$$rJ(t) = p - w(t) + \lambda(V - J(t)), \quad \text{or, similarly, } J(t) = \frac{p - w(t) + \lambda V}{r + \lambda}. \quad (8)$$

In equilibrium, all profit opportunities from new jobs are assumed to be exploited, driving rents from vacant jobs to zero, so $V = 0$. Note that job creation occurs based on the *expected* value of the filled job. This equilibrium condition determines the supply of vacancies, implying that:

$$J^e = \frac{p - w^e}{(r + \lambda)} = \frac{c}{q}, \quad (9)$$

where w^e denotes the expectation of the wage paid to the employee. This equation states that the expected capitalised net return of the job is equal to the expected value of the firm's hiring cost. This condition is usually referred to as the job creation condition (Pissarides 2000).

2.5 Wage determination

We assume that individuals bargain about wages with firms conditional on a chosen residence location and therefore on the commuting costs. Unemployed are only allowed to bargain with the firm given a job contact (which arrives at a rate θq), whereas the employed continuously bargain with their employer. In equilibrium, job matches yield a local-monopoly surplus. In order to form the job match, the unemployed individual gives up $U(t)$ for W and the firm gives up V for $J(t)$. We assume that the total surplus, equal to the sum of the workers' surplus, $W - U(t)$, and the firms' surplus, $J(t) - V$, is shared according to the Nash solution to a bargaining problem.

When the unemployed bargain, we employ the following rule:

$$w(t) = \arg\max (W - U(t))^\beta (J(t) - V)^{1-\beta}, \quad (10)$$

where β is a measure of the workers' labour strength, and $U(t)$ and V can be interpreted as 'threat points'. β can also be interpreted as the workers' share of the total surplus. Let us now focus on the employed individuals. When on-the-job bargaining occurs, the employed individual threatens to give up W for $U(t)$ and the firm threatens to give up $J(t)$ for V. This implies that the above wage bargaining rule is identical for unemployed and employed workers (see also Pissarides 2000).

Hence, when bargaining while employed, U(t) is the threat point at the location of the employed individual.[8] We assume that $0 < \beta < 1$. The first-order equation satisfies:

$$W - U(t) = \frac{\beta}{1-\beta}(J(t) - V). \qquad (11)$$

In equilibrium $V = 0$, so that the wage can be written as (see Appendix 1):

$$w(t) = (1-\beta)t + \beta p + \beta c\theta + \frac{(1-\beta)\theta q(R(t) - R^e)}{r + \theta q}, \qquad (12)$$

where R^e denotes the average rent.

Equation (12) shows that the wage depends on commuting costs t, directly and indirectly via $R(t)$. Interpretation of the direct effect is as follows. Conditional on the commuting costs, firms and job seekers bargain about the wage. The higher the commuting costs, the smaller is the worker's surplus from the match, which is equal to $W - U(t)$, so the worker will ask (and receive) a higher wage to be compensated for the commuting costs. This direct effect of commuting costs on wages depends only on the bargaining power parameter β, and not on any other labour market variable. Further, equation (12) shows that the wage depends positively on the rent paid in the housing market (relative to the average rent). So, wages also compensate for the rent paid in the housing market.

Equation (12) also shows that the wage is increasing in the productivity level and the average hiring costs per unemployed ($c\theta$ is equal to the hiring costs times the number of vacancies divided by the number of unemployed and can be interpreted as the average hiring costs per unemployed). Finally, note that the current interpretation of equation (12) is partial, because $R(t)$, but also θ, R^e and q are endogenous variables.

The wage equation can also be written as (see Appendix 2):

$$w(t) = (1-\beta)t + \beta p + \beta c\theta - \frac{\beta(1-\beta)\theta q(t - t^e)}{r + \beta\theta q + \lambda}, \qquad (13)$$

where t^e denotes the expected commuting costs. Given the assumption of a linear economy and the distribution of the employed over space, the expected commuting costs is determined. Given a random distribution of employed individuals over space within the city fringe, $t^e = \frac{1}{2}\alpha N$, where N is the number of individuals in the city. So, keeping the expected commuting costs constant, it follows that:

---

[8] In case that the employed individual moves from t to t', the same bargaining rule is defined at t', so $w(t') = \arg\max(W - U(t'))^\beta(J((t') - V)^{1-\beta}$ which is identical to (10). Hence, although the employed worker is indifferent with respect to the residence location (because W does not depend on t), note that the threat point of the unemployed U(t) is location specific.

$$w'(t) = \frac{(1-\beta)(r+\lambda)}{r+\beta\theta q+\lambda}. \tag{14}$$

It follows that $0 < w'(t) < 1$ (because $0 < \beta < 1$ and $\beta\theta q > 0$). Note that the right-hand side of (14) does not depend on t. The commuting costs compensation in the form of wages is positive, but partial, due to the residential lock-in effect of moving costs. Note that all employed workers are equally well off, so in equilibrium, wages do not compensate for commuting, but compensate for the difference between the commuting costs and the housing rent. In a housing market without moving restrictions, the difference is zero, but given moving restrictions the difference is positive. Effectively, the employed take into account that being further from the CBD increases commuting costs, which are not fully compensated by lower rents (this will be made more explicit later on). Hence, workers with a longer commute will bargain for a higher wage. This can be contributed to the residential lock-in effect of the moving restrictions, which induces them later on while being unemployed to reside in non-optimal locations.

In most labour markets the job finding rate θq is larger than the discount r and job-quitting rate λ. Suppose a labour market with expected job durations of 6–7 years, and expected unemployment durations of 6 months, so λ equals 0.15 and θq equals 2. The annual discount rate r is usually assumed to be around 0.15. Pissarides (2000) argues that a reasonable value of β is 0.5 (so workers and firms have equal bargaining power), implying that the marginal compensation for commuting costs in the form of wage is 0.12. Thus, the model predicts that wage compensation for commuting is sizeable, and should be empirically relevant for the labour market.

The wage compensation for commuting depends on the chosen values of the parameters. In a market where workers have less (more) bargaining power, the marginal compensation in the form of wages is higher (lower). For example, when $\beta = 0.25$, marginal compensation is 0.28; when $\beta = 0.75$, marginal compensation is 0.07. This makes sense. Workers with more bargaining power will receive higher wages (a larger share of the surplus), so firms will be less 'willing' to compensate for the commuting costs. Consequently, according to the current bargaining model, workers who belong to groups which are disadvantaged in the labour market (for example, females, blacks), who have plausibly less labour market power, will receive lower wages and will receive more compensation for the commuting costs. This result is consistent with the panel data analysis of Kasper (1983), but inconsistent with the cross-section analysis of Kasper (1983) and Zax (1991).

The marginal compensation for commuting costs depends negatively on the job-finding rate of the unemployed (given the discount and quitting rate), because the job-finding rate determines for how long unemployed workers are locked in by the residential moving costs. For example, keeping the assumption that $\beta = 0.5$, but the job finding rate is 1 (so the expected unemployment duration is one year), implies that the marginal compensation in the form of wages is 0.19. Hence, the less rapidly the unemployed find work, the higher is the compensation.

2.6 The bid rent slope

We have established that workers are partially compensated for the commuting costs in the labour market. The consequences for the housing market can easily be derived. The bid rent slope equation can be rewritten as:

$$\frac{\partial \Psi(t, W)}{\partial t} = \frac{r + \theta q}{r + \theta q + \lambda}(w'(t) - 1) = -\beta \frac{r + \theta q}{r + \lambda + \beta \theta q} > -1. \qquad (15)$$

Hence, the worker receives marginal compensation for the commuting costs in the housing market, which is less than one (in absolute value). Presuming again that β equals 0.5 (0.25), marginal compensation in the form of rent is 0.83 (0.67), which is clearly less than one. The rent gradient is therefore less steep than predicted by standard urban economics theory.[9] In conclusion, the urban equilibrium job search model predicts that workers located in the CBD receive compensation for commuting costs both in the housing and labour market. Given reasonable values of the labour market parameters, the compensation in terms of rent tends to be larger than in terms of wages.

For employed workers, the total compensation flow for commuting is typically less than 100 percent. When the commuting costs increase by one unit, then an employed worker's flow income changes by $-1 + w'(t) - R'(t) = \frac{\lambda}{r + \theta q + \lambda}(w'(t) - 1) = \frac{-\beta \lambda}{r + \beta \theta q + \lambda}$.

The change in flow income is non-positive in general, and goes to zero for extreme values of some of the parameters. In particular, the change becomes zero when (i) $\lambda = 0$, (ii) $\theta q \to \infty$, or (iii) $\beta = 0$. The cases (i) and (ii) imply that workers are fully compensated when the search frictions vanish (we will see later on that then unemployment disappears) as would happen in a frictionless competitive labour market. Case (iii) states that when workers have no bargaining power, and are therefore paid their reservation income, wages need to fully compensate for commuting costs, because less than full compensation would drive net income (wages minus commuting costs) below their reservation income.

Presuming again that β equals 0.5, total compensation flow for commuting equals 0.94, whereas for $\beta = 0.25$ and $\beta = 0.75$, total compensation flow is almost the same (0.95 respectively 0.94). So, roughly 5 percent of compensation for the location does not show up in the income flow of workers, but in the income flow of unemployed workers via lower rents in the housing market. Hence, employed workers who live further away from their workplace are partially compensated in the future when they are unemployed (by paying lower rents than other unemployed workers who live closer to the workplace). We believe that this finding is relevant, because it demonstrates that empirical studies that universally analyse

---

[9] In case that unemployed and employed individuals may move freely, then the rent gradient is equal to −1 in the area where the employed live, but may differ from −1 in the area where the unemployed live (Wasmer and Zenou 2002). Because the employed cover about 95 percent of the total urban area (given an unemployment rate of 5 percent), the rent gradient is for most of the area equal to −1, and for a small part close to −1.

workers' income flows as a function of commuting costs focus merely on the location of employed workers.

2.7 Equilibrium

In the steady state, the proportion of individuals who enter unemployment, $\lambda(1 - u)$, must be equal to the proportion who would leave unemployment, θqu. So, the unemployment rate does not vary over space and can be written as:

$$u = \frac{\lambda}{\lambda + \theta q}. \tag{16}$$

The expected wage, w^e, can be written (using equation (12)) as:

$$w^e = (1 - \beta)t^e + \beta p + \beta c\theta, \tag{17}$$

where, as before, $t^e = 1/2\alpha N$, which is determined by the size of the city N.

As can be seen above, the partial effect of expected commuting costs on the wage is positive (keeping labour market tightness constant). In equilibrium however, an increase in the expected commuting costs decreases labour market tightness (and increases unemployment; the effect on vacancies is undetermined). This can be demonstrated by incorporating the expected wage equation (17) into the job creation condition (9):

$$(1 - \beta)(p - t^e) - \frac{(r + \lambda)c}{q} = \beta c\theta. \tag{18}$$

Equation (18) can be solved uniquely for θ given t^e. Given θ, the equilibrium unemployment rate u is determined (see equation (12)). So, the full equilibrium has been defined. The overall effect of higher expected commuting costs on the expected wage can be easily demonstrated. The wage curve is an increasing function of labour market tightness, whereas the job creation curve implies a negative relationship between the wage and labour market tightness. The job creation curve does not depend on the expected commuting costs (see (9)), whereas the expected wage curve shifts up where the job entry costs increase (see (17)). Consequently, the overall effect of higher expected commuting costs is an increase in the expected wage. The negative effect on labour market tightness and therefore the positive effect on unemployment follow from the same figure (the effect on the vacancy rate can be shown to be ambiguous).[10] Finally, note that if

---

[10] We have assumed that each contact between firm and unemployed generates a job match (see section 2.1). This puts a limit on the maximum commuting costs and therefore on the size of the urban area. The maximum commuting costs acceptable to the unemployed job seeker, denoted as t^m, are defined by the condition that $W - U(t^m) = 0$. When t exceeds t^m, the job seeker will reject the match. This condition implies that $J(t^m) = 0$, so $w(t^m) = p$ (see equation (11) and (8)) and therefore $t^m = (1 - 1/2\beta)^{-1}(p - \beta c\theta/(1 - \beta))$. Hence, $1/2 N \le \alpha t^m$.

$\lambda = 0$ or $\theta q = 0$, then $u = 0$, so the search frictions disappear and if $\beta = 1$, then $\theta = 0$ (see (18)), so $u = 1$ (because employers do not open vacancies).

Recall that we have assumed that employed workers can move freely, but unemployed workers are immobile. The model implies that unemployed workers who become employed may move residence, but have no reason to do so. Hence, our model outcomes are consistent with ex-post immobile employed workers, but also with mobile employed workers, so the employed's residential mobility rate is not determined and is therefore consistent with any empirical residential mobility pattern. This should be contrasted to the results by Wasmer and Zenou (2002), who impose free mobility of unemployed workers and obtain the result that unemployed and employed workers change residence when they change labour market status (from unemployment to employment or vice versa). Our results are more in line with Wasmer and Zenou (2006), who show that a proportion of the population of the employed and unemployed workers does not move residence given a change in labour market status.

3 Search and spatial mismatch

We will extend the model now by allowing for spatial search. The main reason for such an extension is that employing equations (8) and (11), and noting that w depends positively on t, it can be seen that $W - U(t)$ and $J(t)$ are both a negative function of t and $U(t)$ is a positive function of t. As noted above, the latter occurs because the unemployed residing further away from the CBD pay lower rents. The implication is that unemployed job seekers located further from the CBD are better off and have an incentive to search less intensively than those located closer to the CBD. To examine this issue, we extend the model by assuming the presence of search costs, σ, which vary with search intensity, s. The search intensity is optimally chosen by the unemployed, and the unemployed matching probability is assumed to be an increasing concave function of search intensity, so $q' > 0$ and $q'' \leq 0$ (for a similar approach, see Wasmer and Zenou 2002). A justification of the latter assumptions can, for example, be found in the work of Pissarides (2000). See for similar assumptions, Smith and Zenou (2003) or Wasmer and Zenou (2006).

We assume that the cost function σ is a convex function of s. Search intensity is chosen by the unemployed worker to maximise the lifetime utility $U(t, s)$, where $U(t, s)$ can be written as follows:

$$rU(t, s) = -\sigma(s) - R(t) + \theta q(s)\left(\max_{t'} W(t') - U(t, s)\right). \tag{19}$$

Note that (19) extends (3), allowing for search costs σ and variation in search intensity. The worker chooses the intensity of search to maximise $U(t, s)$. The optimal s^* satisfies:

$$-\sigma'(s^*) + \theta q'(s^*)\left(\max_{t'} W(t') - U(t, s^*)\right) = 0. \tag{20}$$

It follows that the optimally chosen s^* is a negative function of t (since $\partial U/\partial t > 0$, $\sigma'' > 0$ and $q'' = 0$. Hence, the unemployed residing further from the CBD will search less intensively[11] and the expected unemployment duration (equal to $1/\theta q(s^*(t))$) will be shorter for those individuals located closer to the CBD. For a similar result, see Smith and Zenou (2003), who assume free mobility or Wasmer and Zenou (2006).

In the steady state, the proportion of individuals at t who enter unemployment $\lambda(1 - u(t))$, must be equal to the proportion at t who leave unemployment, $\theta q(s^*(t))u(t)$. So, the unemployment rate at t can be written as:

$$u(t) = \frac{\lambda}{\lambda + \theta q(s^*(t))}. \tag{21}$$

It follows that the unemployment rate is lower for individuals residing closer to the CBD. The reason is that in equilibrium unemployed individuals who reside closer to the CBD pay higher rents, and search therefore more intensively for jobs. This result provides an explanation for the positive unemployment gradient found in some countries, for example, in the work of Vipond (1980, 1984).

Wasmer and Zenou (2000) point out that in the case that the unemployed also have to travel to the CBD, then the unemployed may reside close to jobs when these costs are relatively high. Given this assumption, it can be easily demonstrated that our paper would imply a negative unemployment gradient as reported for most cities in the United States. Hence, our results should be interpreted more generally as to confirm previous theoretical and empirical results in the mismatch literature which show that bad job access worsens employment opportunities, because the unemployed residing further away from employment centres will search less intensively for jobs (Holzer 1991; Smith and Zenou 2003).

4 Spatial structure and compensation

In the empirical urban economics literature, the extent to which workers are compensated for commuting costs has extensively been investigated (starting with Muth 1969). Far less attention has been given to the implications of urban structure for compensation.

Above we have presumed that all firms are located in the CBD. Let us therefore compare results derived above with the results of a similar labour market model but where a continuum of identical firms are homogeneously distributed over space. So spatial variations in rents are absent, which characterises a non-urban area (see Van Ommeren and Rietveld 2005). In this case, the wage equation, similar to (12), can be written as: $w(t) = (1 - \beta)t + \beta p + \beta c\theta$. So, the marginal compensation for commuting costs in the form of wages in non-urban areas is equal to $1 - \beta$, which exceeds the marginal compensation in urban areas (see (14)).

---

[11] A similar result has been obtained by Myers and Philips (1979) in a partial equilibrium model and by Wasmer and Zenou (2000) and Wasmer and Zenou (2006).

This is intuitive, because in urban areas, workers are also compensated by rents. This finding appears to be consistent with empirical studies. For example, it has been reported that firms located in large cities tend to reimburse less than those located elsewhere. In the United Kingdom, firms located in London are 50 percent less likely than firms located outside London to offer a contribution to individual commuting expenses at the moment of recruitment (RCI 2001).

5 Conclusion

We set out to analyse an urban equilibrium model presuming imperfect labour markets (bargaining power, search behaviour) and allowing for residential moving costs aiming to explain the empirical observation that for workers at a particular workplace (in the current model, the CBD), wages depend on residence location and therefore on commuting costs (Zax 1991). Labour market imperfections (for example, search costs) and bargaining between workers and employers play an essential role in the model. In contrast to models that exclude (labour) market imperfections, but in line with the empirical literature, workers are partially compensated for the incurred commuting costs in the labour market. We demonstrate that labour market power determines the extent to which workers can shift the burden of commuting expenses onto their employers, but, maybe surprisingly, the model predicts that workers belonging to groups which have more labour market power will receive higher wages but less compensation for commuting costs (since due to the higher wages, the firm's surplus is less).

   One of the implications of the model is that due to residential moving costs and a single employment centre, rent gradients are less steep than predicted by standard urban theories in line with a range of empirical studies (Dubin and Sung 1987). Using a theoretical urban economics model with multiple employment centres, Crane (1996) and Turnbull (1998) both obtained a similar result when presuming future job site uncertainty and the presence of residential moving costs that constrain households to live in the same place. In these models, the residential location is based not only on where the current job is located, but also on the expectation of where future jobs will be located. Although our model is quite different from the models of Crane (1996) and Turnbull (1998), in particular, we assume wage bargaining and only one employment centre, these models share the assumption of imperfect residential mobility and labour market imperfections. This seems to imply that the functioning of the labour market may contribute to the explanation why rent gradients are less steep than predicted by standard urban theories. More generally, this suggests that the introduction of residential moving costs combined with labour market imperfections, may be fundamental to our understanding of the relationship between urban labour and housing markets and the implications for commuting compensation. Another implication of the model is that employed individuals total commuting compensation flow derived from the labour and housing market is less than 100 percent, because they expect to be compensated for their location later on in the housing market when unemployed.

   To conclude, we call for a better understanding of the effects of residential moving costs on wages, unemployment and rents in urban markets. Our study

should be seen as a first step towards this goal. More realistic models, for example urban models that include on-the-job-search, endogenous firm location and congestion, or alternative models with different types of labour market imperfections (e.g., efficiency wage paying firms), may be needed to confirm our results.

Appendix 1 The wage equation

Equations (11) and (8) imply that:

$$W - U^e + U^e - U(t) = \frac{\beta}{1-\beta} \frac{p - w(t)}{r + \lambda}, \tag{A1}$$

where U^e denotes the average lifetime income of an unemployed job seeker (the average is taken over space). The lifetime income of an unemployed job seeker can be calculated combining (3) and (11), taking expectations using (9).

$$rU^e = -R^e + \frac{\beta \theta c}{1 - \beta}, \tag{A2}$$

where R^e denotes the average (or expected) rent. From equation (3) it follows that:

$$U(t) - U^e = -\frac{R(t) - R^e}{r + \theta q}. \tag{A3}$$

Equation (2) can be rewritten as:

$$W - U^e = \frac{w(t) - t - R(t) + \lambda (U(t) - U^e) - rU^e}{r + \lambda}. \tag{A4}$$

Substituting (A2), (A3) and (A4) into (A1), reveals that:

$$\frac{w(t) - t - \dfrac{\beta \theta c}{1 - \beta} - \lambda \dfrac{R(t) - R^e}{r + \theta q}}{r + \lambda} + \frac{R - R^e}{r + \theta q} = \frac{\beta}{1 - \beta} p - w(t). \tag{A5}$$

Reordering gives us wage equation (12).

Appendix 2 Rewriting the wage equation

$$R(t) - R^e = \frac{r + \theta q}{r + \theta q + \lambda} (w(t) - w^e - (t - t^e)) = -\frac{r + \theta q}{r + \beta \theta q + \lambda} \beta (t - t^e), \tag{B1}$$

since $w(t) - w^e - (t - t^e) = ((1 - \beta) - 1)(t - t^e) + (1 - \beta)\theta q(R(t) - R^e)/(r + \theta q)$. Substituting (B1) into (12) gives (13).

References

Ball, MJ (1973) Recent empirical work on the determinants of relative house prices. *Urban Studies* 10: 213–233

Boehm TP (1981) Tenure choice and expected mobility: a synthesis. *Journal of Urban Economics* 10: 375–389

Burda MC, Profit S (1996) Matching across space: evidence on mobility in the Czech Republic. *Labour Economics* 3: 255–278

Crane R (1996) The influence of uncertain job location on urban form and the journey to work. *Journal of Urban Economics* 39: 342–356

DeSalvo JS, Eeckhoudt L (1982) Household behavior under income uncertainty in a monocentric urban area. *Journal of Urban Economics* 11: 98–111

Dubin RA, Sung CH (1987) Spatial variation in the price of housing: rent gradients in non-monocentric cities. *Urban Studies* 24: 193–204

Fujita M (1989) *Urban economic theory*, Cambridge University Press. Cambridge, UK

Fujita M, Krugman P (1995) When is the economy monocentric? Van Thünen and Chamberlin unified. *Regional Science and Urban Economics* 25: 505–528

Fujita M, Thisse JF, Zenou Y (1997) On the endogenous formation of secondary employment centers in a city. *Journal of Urban Economics* 41: 337–357

Hazans M (2004) Does commuting reduce wage disparities? *Growth and Change* 35: 360–390

Holzer HJ (1991) The spatial mismatch hypothesis: what has the evidence shown? *Urban Studies* 28: 105–122

Kasper H (1983) Toward estimating the incidence of journey-to-work costs. *Urban Studies* 20: 197–208

Manning A (2003) The real thin theory: monopsony in modern labour markets. *Labour Economics* 10: 105–131

Marimon R, Zilibotti F (1999) Unemployment versus mismatch of talents: Reconsidering unemployment benefits. *Economic Journal* 109: 266–291

Mortensen DT (1994) The cyclical behaviour of job and worker flows. *Journal of Economic Dynamics and Control* 18: 1121–1142

Muth RF (1969) *Cities and housing: the spatial pattern of urban residential land use*. The University of Chicago Press Chicago IL

Myers SL, Philips KL (1979) Housing segregation and black employment: another look at the ghetto dispersal strategy. *American Economic Review* 69: 248–302

Petrongolo B, Pissarides CA (2001) Looking into the black box: a survey of the matching function. *Journal of Economics Literature* 49: 390–431

Pissarides CA (2000) *Equilibrium Unemployment Theory*. MIT Press, second edition, Cambridge

RCI (2001) Recruitment Confidence Index, December 2001. Cranfield School of Management, United Kingdom

Shapiro C, Stiglitz J (1984) Equilibrium unemployment as a worker discipline device. *American Economic Review* 74: 433–444

Smith TE, Zenou Y (2003) Spatial mismatch, search effort and urban spatial structure. *Journal of Urban Economics* 54: 129–156

Söderberg B, Janssen C (2001) Estimating distance gradients for apartment properties. *Urban Studies* 38: 61–79

Timothy D, Wheaton WC (2001) Intra-urban wage variation, employment location, and commuting times. *Journal of Urban Economics* 50: 338–366

Turnbull G (1998) Housing prices and the residential land use under job site uncertainty. *Journal of Housing Economics* 7: 1–20

Van Ommeren JN, Rietveld P (2005) The commuting time paradox. *Journal of Urban Economics* 58: 437–454

Vipond J (1980) Intra-urban unemployment differentials in Sydney, 1971. *Urban Studies* 11: 39–46

Vipond J (1984) The intra-urban unemployment gradient: the influence of location on unemployment. *Urban Studies* 21: 377–388

Wasmer E, Zenou Y (2000) Space, search and efficiency. IZA, 181

Wasmer E, Zenou Y (2002) Does city structure affect job search and welfare? *Journal of Urban Economics* 51: 515–541

Wasmer E, Zenou Y (2006) Equilibrium search unemployment with explicit spatial frictions. *Labour Economics, forthcoming*

Zax JS (1991) Compensation for commutes in labor and housing markets. *Journal of Urban Economics* 30: 192–207

Zenou Y (2002) How do firms redline workers? *Journal of Urban Economics* 52: 391–408

Zenou Y, Smith T (1995) Efficiency wages, involuntary unemployment and urban spatial structure. *Regional Science and Urban Economics* 25: 547–573