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Empirical evidence of paradoxes of voting in Dutch elections

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Abstract. In this paper we analyze four national elections held in 1982, 1986, 1989 and 1994 in the Netherlands on the occurrence of the Condorcet paradox. In addition, we investigate these elections on the occurrence of three so-called majority-plurality paradoxes. The first paradox states that a party having a majority over another party may receive less seats. The second states that a Condorcet winner may not receive the largest number of seats and even may not receive a seat at all. The third says that the majority relation may be the reverse of the ranking of parties in terms of numbers of seats.

1. Introduction

The Condorcet paradox is concerned with voting situations in which no majority winner exists, in spite of the fact that voters have consistent preferences. The paradox was discovered in the eighteenth century by the French philosopher and mathematician Marquis de Condorcet (1743–1794). To illustrate the paradox, we present an example constructed by Condorcet (1789). The voting situation concerns sixty voters and three candidates called *Pierre, Paul* and *Jacques*. The distribution of the voters is as follows:

23 voters:	*Pierre*	*Paul*	*Jacques*
2 voters:	*Paul*	*Pierre*	*Jacques*
17 voters:	*Paul*	*Jacques*	*Pierre*
10 voters;	*Jacques*	*Pierre*	*Paul*
8 voters:	*Jacques*	*Paul*	*Pierre*

The voter preferences are supposed to be transitive. Thus a voter with for instance the preference (*Pierre, Paul, Jacques*) prefers *Pierre* to *Paul*, *Paul* to *Jacques* and, because of transitivity, *Pierre* to *Jacques*. According to this preference profile, *Pierre* has a 33 to 27 majority over *Paul*, *Jacques* has a 35 to 25 majority over *Pierre* and *Paul* has a 42 to 18 majority over *Jacques*.

Hence there is no candidate with a majority over every other candidate; there is no majority winner or Condorcet winner.

Condorcet also compared the majority rule with other voting rules, in particular the plurality rule. As is well known, the plurality rule selects the candidate with the highest number of first places in the voter preferences. Thus for the example above, this rule would have selected *Pierre* as the plurality winner, in spite of the fact that a majority of voters prefers *Jacques* to *Pierre*. Condorcet (1785, 1789) also discovered that a plurality winner and a Condorcet winner may differ. Moreover, he came to the general conclusion that different voting rules may yield different outcomes for the same voting situation. Though this insight seems trivial by now, it is in fact at the heart of a controversy between liberalism and populism (or, better, majoritarianism) in the theory of democracy. See, e.g., Riker (1982) and Tännsjö (1992).

Clearly, if the Condorcet paradox occurs, then either no winner can be selected when the majority rule is used or a minority candidate is selected in the case of another rule, for instance the plurality rule. Thus in the case of a paradox, the plurality rule selects a candidate that can be beaten by a majority. If instances of the paradox frequently occur, then either many decision deadlocks can be observed in the case of the majority rule or many minority candidates will be selected in the case of the plurality rule. Therefore, the question of how frequently the Condorcet paradox occurs is important.

In this paper we investigate four Dutch elections on the occurrence of the Condorcet paradox. The elections were held in 1982, 1986, 1989 and 1994. The party preferences of the voters in these elections are constructed by using data of the Dutch Parliamentary Election Studies. We also investigate the occurrence of other paradoxes related to the plurality rule and the majority rule. These so-called majority-plurality paradoxes are presented in Van Deemen (1993). The first paradox of this kind occurs when a party is preferred by a majority of the electorate to another party and yet receives less seats than that other party. The second paradox occurs when a Condorcet winner does not receive the largest number of seats. In fact, it is even theoretically possible in list systems of proportional representation like the one used in the Netherlands that the Condorcet winner receives no seats at all. Also this possibility will be investigated. Finally, the third majority-plurality paradox occurs when the ranking of parties in accordance with the vote or seat distribution is the reverse of the ranking of parties as obtained by the majority rule.

The paper is organized as follows. Section 2 shortly reviews existing research results concerning the relevance of the Condorcet paradox. In Section 3, the concept of election matrix is introduced and discussed. An election matrix is a suitable tool for the empirical representation of voter preferences in elections. Of course, we show how to detect a Condorcet paradox from an

election matrix. Section 4 presents the election matrices of the Dutch elections held in 1982, 1986, 1989 and 1994. Further we investigate the matrices on the occurrence of the Condorcet paradox and majority cycles. In the subsequent section, we investigate the data on the occurrence of majority-plurality paradoxes. The final section contains conclusions and discusses future research.

2. The relevance of the Condorcet paradox

After a long period of slumbering existence, the interest in the Condorcet paradox was raised again by Arrow (1963) and Black (1958). Their rediscovery of the paradox has led to two important research fields in public choice. In the first, conditions are formulated that require voter preferences to satisfy certain regularities in such a way that the paradox is avoided. So the aim of this research is to find conditions that forbid the occurrence of the paradox.

The most-known condition in this respect is single-peakedness formulated by Black (1958). This condition requires that alternatives (or motions as Black called them) can be linearly ordered in such a way that the graph of each voter preference has one and only one peak. Black was able to prove that in case of single-peaked preferences the optimum of the median voter preference must be the majority winner when the number of voters is odd. This result is called the Median Voter Theorem. Further he showed that single-peakedness ensures the transitivity of majority decision-making in the case of odd numbers of voters and its quasi-transitivity in the case of even numbers of voters.

The second research field does not forbid the Condorcet paradox, but instead tries to find out how often it will occur in real decision-making situations. This research line can be split up into two sub-fields. The task in the first sub-field is to calculate the probability of the Condorcet paradox for several numbers of voters and alternatives. The second line aims at detecting empirically instances of the paradoxes. We discuss both sub-fields.

The probability approach was initiated, again, by Black (1958). He arrived at a probability of .0556 for the paradox in the case of three voters and three alternatives and given the fact that voter preferences are linear orderings (i.e., rankings without ties). He was not able to give further calculations but he conjectured that the probability of the paradox "increases rapidly with an increase in the number of motions" (Black 1958: 51). Since Black, several researchers have calculated the probabilities of the Condorcet paradox by means of computer simulations (Campbell and Tullock, 1965; Klahr, 1966) as well as by means of analytical expressions (DeMeyer and Plott, 1970; May, 1971; Niemi and Weisberg, 1968; Garman and Kamien, 1968; Gehrlein

Table 1. Probabilities of no Condorcet-winners for Dutch elections (1982–1994)

Election year	Number of parties	Probability of no Condorcet-winner
1982	13	.56869
1986	12	.54706
1989	9	.45453
1994	9	.45453

Source: Gehrlein (1983); Gehrlein & Fishburn (1976).

and Fishburn, 1976; Gehrlein, 1983). The general conclusion to be drawn from the calculations is that, indeed, the probabilities rapidly increase with an increasing number of voters but this increase is less than in the case of a growing number of alternatives.

To find the probabilities for the four Dutch elections under scrutiny, we suppose that the size of the electorate approaches infinity so that we can use the limit values for the several numbers of parties as calculated in the studies mentioned above. The probabilities for the elections are given in Table 1.

However, the results in this table should be interpreted with care. First, the calculations are based on the assumption of equally likely voter preferences. This assumption, which is called Impartial Culture, is highly implausible. Studies in which the probabilities of the paradox under alternative cultures or distribution assumptions are calculated, are scarce. In addition, the limit values under different cultures are still unknown.

The second reason to be careful is that most of the calculations start from the assumption that voter preferences are linear orderings, i.e., rankings without ties. This is not a realistic approach. As we shall see, we found many ties in the voter preferences in the four Dutch elections under scrutiny. Also see Niemi (1970). Recently, Jones, Radcliff, Taber, and Timpone (1995) calculated the probability of the paradox for weak orderings, thus for rankings with ties. However, they did not provide limit values for several numbers of alternatives so that their results cannot be used for national elections. Further they defined the paradox as the absence of a unique Condorcet winner. Now it is clear that in the case of weak orderings, two or more Condorcet winners may exist. In this work, we retain the definition of the paradox as the absence of a Condorcet winner and not as the absence of a *unique* one. So, situations with two or more Condorcet winners will not be considered as paradoxes.

The second field in this research tradition is directed at finding empirical instances of the paradox. This research was initiated by Riker (1965), who found a paradox in the U.S. senate in 1911 and a paradox in the House of Representatives in 1956. It is useful here to make a distinction between voting in committees and voting in large elections. Since Riker's study, a number of other empirical instances of the paradox have been found in committee voting situations. Blydenburgh (1971) found two Condorcet paradoxes in the U.S. House of Representatives in 1932 and 1938. Jamison (1975), using experimental data about preferences in small groups, also detected a Condorcet paradox. More recently, Vergunst (1996) found a Condorcet paradox in a case treated in the Second Chamber of the Dutch parliament in 1994. In contrast to these results, instances of the paradox in large elections are difficult to detect. This is remarkable, since probability calculations indicate that the paradox should occur far more frequently in large elections than in comparatively small committees. So, Chamberlin, Cohen, and Coombs (1984) did not find a paradox in the case of elections for the American Psychological Association. Similarly, Niemi and Wright (1987) did not find a paradox in the Presidential Elections of 1980. Feld and Grofman (1992) investigated preference data of 36 elections held in professional organizations, unions and non-profit organizations. They did not find a Condorcet paradox either. Finally, Radcliff (1994) investigated president elections in the U.S. Neither he was able to detect the paradox. The only and so far unique exception is Niemi (1970), who discovered a paradox in a case of university elections. In this paper we search for Condorcet paradoxes in four national elections held in the Netherlands. Basic in our detection method is the concept of an election matrix.

3. Election matrices

Election matrices provide a powerful tool for analyzing election data. In particular, it is a convenient device for detecting paradoxes of voting. Let the number of voters who strictly prefer x_i to x_j be denoted by $N(x_i,Px_j)$ and let m be the number of alternatives. An election matrix is defined as a square matrix $E = ((e_{ij}))$, where $e_{ij} = N(x_iPx_j)$ for each i,j = 1,2,...,m and $i \neq j$, and where $e_{ii} = 0$ for i = 1,2,...,m. Thus each cell $((e_{ij}))$ in an election matrix contains the number of voters who strictly prefer alternative x_i to alternative x_j.

The matrix (1/n).E denotes the proportional election matrix. A cell $((x_i, x_j))$ in this matrix contains the proportion of voters strictly preferring x_i to x_j. Let E′ denote the transpose of E. Clearly, the matrix $S = E - E'$ is skew-symmetric, that is, $((s_{ij})) = ((-s_{ji}))$ for i, j = 1,2,...,m. The entries in S show the margins of the voters for an alternative over another alternative. If the margin is non-negative, then that alternative is preferred by a majority to the other

alternative. It is negative if the other alternative is strictly majority-preferred. Clearly, if a row contains only non-negative numbers, the alternative represented by the row is a Condorcet winner. Thus the matrix S provides an easy method for detecting Condorcet paradoxes: look for rows with only non-negative numbers. If there are none, the paradox has occurred.

4. Empirical evidence in four Dutch elections

In this paragraph we present and analyze the preference data concerning the four national elections of 1982, 1986, 1989 and 1994. First, the nature of the preference data will be explained. In order to illustrate our working procedure, we then present and analyze the election matrix of 1994 obtained from these data. Finally, the full majority relation for each of the elections of 1982, 1986, 1989 and 1994 is produced and analyzed. Of course, the data will be investigated on the occurrence of the Condorcet paradox.

The data are taken from the *Dutch parliamentary election study (NKO) of 1982, 1986, 1989 and 1994*. In the period before and after each election of the Dutch parliament, about 1500 respondents are interviewed about a wide range of issues. One of the many questions concerns the probability that a respondent will ever vote for a party. Each respondent can choose a point on a scale from 1 ("certainly never") through 10 ("some time certainly"). We will use the probability votes obtained in this way for the construction of the voters' party preferences. We assume that if a voter gives more points to party x than to party y, she strictly prefers x to y. Notation: $p_i(x) > p_i(y) \leftrightarrow xP_iy$, where $p_i(x)$ stands for the probability future vote score of respondent i for party x and where P stands for strict preference. Furthermore, $p(x) = p(y)$ for a voter means that this voter is indifferent with respect to parties x and y; notation: xIy. The voter preferences constructed in this way are then aggregated by the majority rule. According to this aggregation procedure, party x is majority-preferred to party y if the number of voters who strictly prefer x to y is at least as large as the number of voters who prefer y to x; notation: $xMy \leftrightarrow N(xP_iy) \geq N(yP_ix)$. In this case, we say that party x has a majority over party y. The binary relation M thus obtained over all pairs of parties is called the *majority relation*. Notice that according to this definition, indifference does not affect the majority relation between parties. In this context, a Condorcet winner is a party having a majority over every other party. In other words, x is a Condorcet winner if xMy for all parties y.

Table 2 contains the election matrix obtained from the individual scores collected in the Dutch national election in 1994. For example, the cell (D66, PvdA) in this matrix contains the number of 655 while the cell (PvdA, D66) contains the number of 580. This means that 655 respondents strictly prefer

Table 2. Majority scores for the 1994 elections in the Netherlands: $N(xP_iy)$

Party	D66	PvdA	CDA	VVD	Groen Links	RPF	SGP	GPV	CD
D66	0	655	818	753	988	995	1036	1026	1262
PvdA	580	0	715	767	905	914	946	946	1183
CDA	480	608	0	607	779	901	930	937	1151
VVD	471	540	594	0	783	847	883	885	1137
GroenLinks	208	300	508	505	0	723	743	743	996
RPF	152	191	99	160	240	0	114	94	545
SGP	136	194	89	154	237	107	0	95	573
GPV	146	191	92	153	238	74	79	0	562
CD	50	81	61	51	87	56	59	61	0

Source: Dutch Parliamentary Election Study (n = 1527).

D66 to PvdA and that 580 strictly prefer the PvdA. Since the total number of respondents is 1527, we conclude that 292 of the respondents are indifferent or have not given a future probability score for at least one of the two parties. (This means that the numbers of respondents are not the same for each pair.) The matrices for the elections of 1982, 1986, and 1989 are given in the Appendix.

The skew-symmetric election matrix for 1994 is the matrix difference between the election matrix and its transpose. It is given in Table 3. In this matrix, the first row contains only non-negative numbers. Therefore we conclude that the corresponding political party D66 is the Condorcet winner. Hence, the Condorcet paradox did not occur in this national election. Proceeding in the same way for the elections of 1982, 1986, and 1989, we arrive at the conclusion that no Condorcet paradox did occur in any of these elections. Notice that this does not mean that the concerned majority relations are acyclical. A Condorcet winner may beat any other party involved in a cycle, as long as this winner itself is not involved in a cycle. In other words, the existence of a Condorcet winner only implies that there is no top-cycle.

To detect majority cycles, we need the majority relation for each election. Such a relation can be constructed by means of the skew-symmetric election matrices by using the fact that a non-negative number in such a matrix implies the row party to have a majority over the column party. Proceeding in this way we arrive at the following majority relations for the concerned elections (see Table 4).

Table 3. Skew-symmetric election matrix 1994

Party	D66	PvdA	CDA	VVD	GRLNKS	RPF	SGP	GPV	CD
D66	0	75	338	282	780	843	900	880	1212
PvdA	–75	0	107	227	605	723	752	755	1101
CDA	–338	–107	0	13	271	802	841	845	1090
VVD	–282	–227	–13	0	278	687	729	732	1086
GRLNKS	–780	–605	–271	–278	0	483	506	505	909
RPF	–843	–723	–802	–687	–483	0	7	20	489
SGP	–900	–752	–841	–729	–506	–7	0	16	514
GPV	–880	–755	–845	–732	–505	–20	–16	0	501
CD	–1212	–1102	–1090	–1086	–909	–489	–514	–501	0

Table 4. Social rankings based on the majority rule and on the election results of the Dutch parliament, 1982-86-89-94

	1982		1986		1989		1994	
Ranking	MR	PR	MR	PR	MR	PR	MR	PR
1	CDA	PvdA	CDA	CDA	CDA	CDA	D66	PvdA
2	VVD	CDA	PvdA	PvdA	PvdA	PvdA	PvdA	CDA
3	PvdA	VVD	D66	VVD	D66	VVD	CDA	VVD
4	D66	D66	VVD	D66	VVD	D66	VVD	D66
5	DS70	PSP	PPR	SGP	GRLNKS	GRLNKS	GRLNKS	GRLNKS
6	PPR	SGP	PSP	PPR	SGP	SGP	RPF	CD
7	PSP	CPN	CPN	PSP	GPV	GPV	SGP	RPF
8	SGP	PPR	SGP	GPV	RPF	RPF	GPV	SGP
9	CPN	RPF	GPV	RPF	CD	CD	CD	GPV
10	GPV	CP	EVP	CPN				
11	RPF	GPV	RPF	CP				
12	CP	DS70	CP	EVP				
13	BP	BP						

MR: Majority Relation.
PR: Proportional Representation Ranking; ranking according to the proportions of votes in the elections.

Table 4 also gives the rankings of the parties as yielded by the Dutch system of proportional representation. In the next section, we will use this information for detecting the majority-plurality paradoxes mentioned in the introduction. In addition, we note that the majorities in all the four majority relations are strict.

The results are surprising indeed. The majority relations are all transitive; none of them contains a cycle. So not only top cycles are absent, but any majority cycle whatsoever. What is the reason of "so much stability" (Tullock, 1981)? The research line on domain conditions as briefly discussed above may provide an answer. It is possible that there is a underlying pattern that restricts the voter preferences in such a way that the paradox cannot occur. As we have seen, the best-known condition in this respect is Black's condition of single-peaked preferences. We apply this to the Dutch situation in the concerned elections.

Remember that voter preferences are single-peaked if there is a linear ordering of parties such that, in passing from one party to the next in this ordering, each voter preference shows only one top (or a plateau in the case of indifferent preferences at the top). In the four Dutch elections under scrutiny we could not find such a linear ordering. First we tried out, of course, the traditional left-right dimension in Dutch politics. For example, using data of the National Parliamentary Election Studies we arrived at the following left-right ranking of the parties for 1994:

GroenLinks – PvdA – D66 – CDA – VVD – SGP – GPV – RPF – CD

See also Vergunst (1995). For this linear ordering we found that only 35.1% of the voter preferences were single-peaked. The other possible orderings of parties did not work either, not only for this case but for every case we studied.

Our conclusions concerning the observed stability cased by preference regularities are preliminary. As the above results show, single-peakedness of voter preferences in Dutch elections is not a very likely cause. However, other domain restrictions like Sen's (1966) value restriction may be operational. In order to be more conclusive, much more empirical research on preference patterns and domain restrictions is needed.

5. Majority-plurality paradoxes in the four elections

The main goal of this paper is to analyze Dutch elections on the occurrence of the Condorcet paradox. In order to detect the paradoxes and, as a by-product, possible majority cycles, the majority relation for each election under scrutiny was reconstructed (see Table 4). These majority relations now will be used

Table 5. Comparing rankings by majority rule to rankings by the proportional system in the Netherlands (1982–1994)

Election year	Kendall's tau
1982	.6410
1986	.7576
1989	.9444
1994	.6667

Source: Rankings Table 4.

for detecting three possible majority-plurality paradoxes as discussed in the introduction.

The three paradoxes are:

1. *The More-Preferred-Less-Seats Paradox*, which states that a party having a majority over another party may receive less seats;
2. *The Condorcet-Winner-Turns-Loser Paradox*, which states that a Condorcet winner need not receive the largest number of seats and even may not receive a seat at all, and;
3. *The Majority-Reversal Paradox*, which states that the majority relation for an election may be the reversal of the ranking of the parties in correspondence with their number of seats as assigned by the system of proportional representation.

It can be shown that each of these paradoxes may occur in the Dutch system of proportional representation (Van Deemen, 1993). However, do they occur in reality? Table 4 contains, besides the majority relations in the left part of each column (called MR), the rankings in terms of the number of seats as actually assigned to the parties by the system of proportional representation. See the right part of each column (called PR) in Table 4. A first look at this table immediately shows the differences between the two rankings MR and PR. For the elections of 1982, 1986 and 1994, the More-Preferred-Less-Seats Paradox occurs abundantly. It also occurs in 1989, but to a less extent. In that year, there was only one pair of parties with reversed positions, namely VVD and D66.

To gain more insight into the coherence of both relations for each election, we calculated Kendall's tau (Table 5). Clearly, the smaller Kendall's tau, the more majority-plurality reversals can be observed. We conclude that the election of 1982 contains the most reversals.

The Condorcet-Winner-Turns-Loser Paradox occurred twice. In 1994, D66 was the majority winner, but PvdA received the largest number of seats. Also CDA and VVD received more seats than D66, in spite of the fact that D66 had a majority over both parties. The second case occurred in 1982 when CDA received more seats than the majority winner PvdA. We did not observe the fact that a Condorcet winner may not receive a seat at all.

The Majority-Reversal Paradox did not occur in any of the four elections. Although the rankings based on the majority rule can be quite different from the rankings based on the system of proportional representation, there is no election year in which the majority ranking is completely reversed.

6. Conclusions

Voting in accordance with the majority rule is often considered as a necessary condition for real democracy (Dahl, 1989). The major problem of the majority rule, however, is the possibility of cycles that prevent the existence of a Condorcet winner. If this is the case, we speak of the Condorcet paradox. In this paper we tried to find instances of the Condorcet paradox in four national elections held in 1982, 1986, 1989 and 1994 in the Netherlands. We did not find the paradox. The majority relation for every election appeared to be fully transitive.

This result is in line with previous research on cycles in large elections. For some reason or another, cycles in large elections are scarce. Now it is well-known from theoretical studies that regularities in voter preferences may prevent the occurrence of the paradoxes. Single-peaked preferences constitute the most clear example of this. However, we did not find evidence for single-peakedness in the four elections investigated by us. But of course, other preference regularities we did not investigate yet, may be operational. In order to reveal the real causes of the absence of Condorcet paradoxes, much empirical research has to be done.

We also tried to detect instances of majority-plurality paradoxes. Two of the three paradoxes formulated by us actually did occur. First, we found many instances of the paradox that a party having a majority over another party received less seats than that other party. A good indication for this paradox is Kendall's tau. Especially the elections of 1982 and 1994 are characterized by a small Kendall's tau and hence by a high number of reversals. Further, we found two instances of the paradox that a Condorcet winner does not receive the largest number of seats. In the election of 1994, three parties received more seats than the Condorcet winner and in the election of 1982 only one. We did not find instances of the paradox that the majority relation over the

parties is the complete reverse of the ranking of the parties as yielded by proportional representation.

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Table App. 1. Majority scores for the 1982 elections in the Netherlands: $N(xP_iy)$

Party	CDA	VVD	PvdA	D66	DS70	PPR	PSP	SGP	CPN	GPV	RPF	BP	CP
CDA	0	486	583	600	634	675	702	727	760	743	733	778	788
VVD	351	0	513	528	596	607	633	656	690	672	659	715	720
PvdA	460	473	0	480	544	551	597	615	665	636	634	661	671
D66	320	303	364	0	498	508	581	593	637	614	599	648	660
DS70	110	105	194	140	0	251	303	306	346	329	326	376	397
PPR	232	239	116	180	242	0	276	342	348	354	356	384	408
PSP	189	191	85	144	192	115	0	249	254	272	267	299	321
SGP	44	78	130	100	106	143	161	0	195	108	123	196	228
CPN	154	156	50	117	153	79	93	194	0	209	207	222	226
GPV	44	74	129	95	108	138	152	77	191	0	92	171	218
RPF	47	73	119	88	102	125	139	83	171	76	0	161	194
BP	20	27	62	40	44	73	75	50	103	46	55	0	110
CP	21	18	37	35	35	45	45	43	53	45	40	40	0

Source: Dutch Parliamentary Election Panel Study 1981–86 (n = 1206).

Table App. 2. Majority scores for the 1986 elections in the Netherlands $N(xP_iy)$

Party	CDA	PvdA	D66	VVD	PPR	PSP	CPN	SGP	GPV	EVP	RPF	CP
CDA	0	645	624	668	762	800	844	855	860	848	846	944
PvdA	575	0	536	672	727	760	805	756	769	794	772	875
D66	456	472	0	565	706	757	825	804	815	830	821	930
VVD	286	471	403	0	590	622	661	634	642	653	646	759
PPR	301	168	162	355	0	349	445	445	467	477	474	588
PSP	234	120	154	283	152	0	329	314	333	334	334	467
CPN	212	90	122	254	109	146	0	265	281	275	367	367
SGP	56	174	109	121	179	177	247	0	133	201	148	328
GPV	53	171	106	113	163	163	224	77	0	164	100	314
EVP	85	114	86	145	121	133	206	163	155	0	160	330
RPF	53	141	86	100	132	137	196	77	55	139	0	285
CP	11	28	17	14	20	21	25	23	21	23	21	0

Source: Dutch Parliamentary Election Study 1986 (n = 1356).

Table App. 3. Majority scores for the 1989 elections in the Netherlands: $N(xP_iy)$

Party	CDA	PvdA	D66	VVD	GRLNKS	SGP	GPV	RPF	CD
CDA	0	728	698	807	889	990	989	975	1113
PvdA	637	0	613	763	845	925	931	931	1074
D66	581	573	0	737	873	1002	1006	1008	1146
VVD	335	472	435	0	646	761	759	762	912
GRLNKS	365	226	229	498	0	646	646	651	806
SGP	90	187	152	186	214	0	154	181	443
GPV	88	204	154	195	226	151	0	149	443
RPF	80	180	137	169	201	125	72	0	408
CD	43	48	47	43	67	61	62	62	0

Source: Dutch Parliamentary Elections Study 1989 (n = 1506).