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Spin asymmetries in jet-hyperon production at LHC

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Abstract

We consider polarized Λ hyperon production in proton–proton scattering, $pp \to (\Lambda^\uparrow \mathrm{jet})\,\mathrm{jet} X$, in the kinematical region of the LHC experiments, in particular the ALICE experiment. We present a new Λ polarization observable that arises from the Sivers effect in the fragmentation process. It can be large even at midrapidity and therefore, is of interest for high energy hadron collider experiments. Apart from its potential to shed light on the mechanisms behind the phenomenon of Λ polarization arising in unpolarized hadronic collisions, the proposed observable in principle also allows to test the possible color flow dependence of single spin asymmetries and the (non)universality of transverse momentum dependent fragmentation functions.

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1. Introduction

Since the observation of large transverse polarization of produced Λ hyperons in the inclusive reactions $pp \to \Lambda^\uparrow X$ [1] and $p\,Be \to \Lambda^\uparrow X$ [2] in the middle of the 1970's, there have been many experimental and theoretical investigations aimed at understanding this striking polarization phenomenon [3,4]. The polarization measurements of Λ hyperons produced in these inclusive reactions have been performed in fixed target experiments, and the data showed that the Λ polarizations are large only for large x_F. This poses a problem if one wants to investigate this observable further using high energy colliders such as RHIC, Tevatron or LHC, where the capabilities to measure Λ polarization are restricted to the midrapidity region, where x_F is very small. However, in this Letter we point out a way to by-pass this problem by making a less inclusive measurement: to select two-jet events and to measure the jet momenta K_j and $K_{j'}$ in addition to the momentum K_Λ and polarization S_Λ of the Λ that is part of either of the two jets. An asymmetry proportional to $\epsilon_{\mu\nu\alpha\beta} K_j^\mu K_{j'}^\nu K_\Lambda^\alpha S_\Lambda^\beta$ can then arise, which is neither power suppressed, nor needs to be zero[1] at vanishing x_F of the Λ. The observable is proportional to $(\mathbf{K}_j \times \mathbf{K}_\Lambda) \cdot \mathbf{S}_\Lambda$ in the center of mass frame of the two jets. Within a factorized description of the process under consideration, such an asymmetry can arise from spin and transverse momentum dependent fragmentation functions, which in turn can arise from two types of interactions. We will point out how the measurement of the suggested asymmetry in principle can allow for a differentiation between these two types of mechanisms. In this way also hadron collider experiments can contribute to our understanding of the underlying mechanisms that lead to Λ polarization, in addition to experiments with electron–positron and lepton–hadron collisions.

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[1] In $pp \to \Lambda^\uparrow X$ the Λ polarization needs to vanish at $x_F = 0$ due to symmetry reasons. In $pA \to \Lambda^\uparrow X$ nuclear effects could allow for a nonvanishing asymmetry at midrapidity. No such effects have been observed thus far.
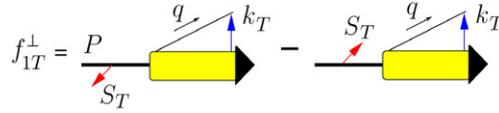
Fig. 1. Schematic depiction of the Sivers distribution function. The spin vector S_T points out of and into the page, respectively.
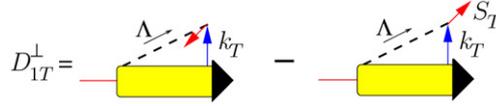

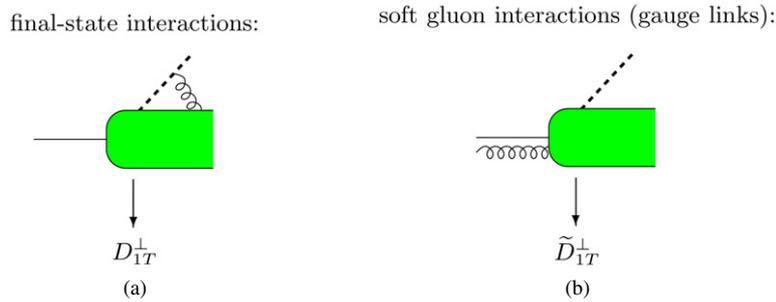Fig. 2. The Sivers or "polarizing" fragmentation function.

Fig. 3. Two possible mechanisms to generate single spin asymmetries in the fragmentation process: (a) through final-state interactions within the out-state composed of the outgoing hadron and the rest of the jet; (b) through soft gluonic interactions between the jet and the hard part.

It is well known that transverse momentum dependent distribution and fragmentation functions, nowadays commonly referred to as TMDs, can have a nontrivial spin dependence and that the so-called "T-odd" TMDs can lead to single spin asymmetries [5–8]. They are also often referred to as "naively T-odd", because the appearance of these functions does not imply a violation of time-reversal invariance. The Sivers distribution function $f_{1T}^\perp$, schematically depicted in Fig. 1, is the oldest example of such functions. It describes the difference between the momentum distributions of quarks inside protons transversely polarized in opposite directions. The Sivers effect was put forward [5,7] as a possible explanation for the large single spin asymmetries observed in $p^\uparrow p \to \pi X$ experiments [9]. Furthermore, it generates single spin asymmetries in semi-inclusive DIS [8,10], which have also been measured to be nonzero [11], and it results, e.g., in asymmetric di-jet correlations in $p^\uparrow p \to \mathrm{jet\,jet}\,X$ [12,13], which however are not yet visible in the data analyzed [14].

The fragmentation analogue of the Sivers distribution function is called $D_{1T}^\perp$ [15]. It describes the distribution of transversely polarized spin-1/2 hadrons, such as Λ's, inside the jet of a fragmenting *unpolarized* quark, cf. Fig. 2. For this reason it has been referred to as "polarizing fragmentation function" in Ref. [16]. It is an odd function of the transverse momentum of the observed hadron w.r.t. the quark direction, or equivalently, the jet direction. Despite the similarity between the definitions of $D_{1T}^\perp$ and $f_{1T}^\perp$, there are some important differences. Nonvanishing T-odd distribution functions require soft gluonic interactions between the target remnants and the active partons [10]. These interactions can be resummed into Wilson lines (gauge links), ensuring the gauge invariance of the operator definitions of the distribution functions [17–20]. On the other hand, there are two mechanisms to generate T-odd effects in the fragmentation process: through final-state interactions within the jet, e.g., between the observed outgoing hadron and the rest of the jet [6], or through soft gluonic interactions between the jet and the hard scattering part (sometimes also referred to as final-state interactions), see Figs. 3(a) and 3(b), respectively. As for the distribution functions, the latter interactions give rise to Wilson lines. Both effects can be expressed in terms of a TMD fragmentation function. The Λ polarization observable that is the subject of this Letter in principle allows for a differentiation between these two effects. There is considerable interest in this issue, as it could shed light on the color flow dependence of single spin asymmetries and the (non)universality of transverse momentum dependent fragmentation functions.

In recent years it has become apparent that T-odd TMDs enter in different ways in different processes, depending on the color flow in the partonic subprocesses.[2] The sign relation between the Sivers function appearing in semi-inclusive DIS and in Drell–Yan scattering was the first example of such process dependence [17,21]. More complicated relations were discussed soon afterwards for hadron production processes in hadronic collisions [22–25]. Process dependence may seem at odds with factorization [26], but since the color flow dependence can be explicitly taken into account through the determination of the Wilson line structure, the process dependence is explicitly calculable and hence, predictive power may be retained albeit in a less straightforward manner.

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[2] Although the possibility of such effects for T-even TMDs are not excluded, they will not be considered here.

For the specific example of the single spin asymmetry in $p^\uparrow p \to \text{jet jet} X$ the effects of color flow dependence have been taken into account [27], leading to a suppression w.r.t. the standard parton model expectation. For fragmentation functions the situation is complicated further by the fact that only one of the two mechanisms leading to nonzero T-odd fragmentation functions depends on the color flow structure [20]. However, a claim based on a model calculation [28] and on more general arguments [29] has been put forward that for T-odd fragmentation functions no color flow dependence arises at all. This makes an experimental test of this dependence, or of its absence, all the more interesting.

We propose to measure it in the process $pp \to (\Lambda^\uparrow \text{jet}) \text{jet} X$, where the brackets indicate that the Λ is part of one of the two observed jets which are almost back-to-back in the plane perpendicular to the beam axis and its polarization is measured. This measurement can be done for instance at LHC and RHIC, although—as we will point out—the situation at LHC is more straightforward thanks to the dominance of gluon–gluon scattering in the region of interest.

The Λ polarization observable we put forward here is similar to the Sivers effect observable in $p^\uparrow p \to \text{jet jet} X$ [12,13], but the Λ polarization observable does not depend on a non-collinearity of jets in the transverse plane. In other words, even if one were to assume the initial partons to be collinear to the parent hadrons, in which case the jets are exactly back-to-back in the plane perpendicular to the beam axis, an asymmetry can still arise. Therefore, it is expected that the Λ polarization observable is less sensitive to effects that would smear out the partonic $2 \to 2$ kinematics. This prompts us to ignore contributions that depend on nontrivial spin and transverse momentum dependent effects in the initial states (which would be proportional to $h_1^\perp H_1$ in the notation of Ref. [8]).

The observable turns out to be very sensitive to the unpolarized Λ fragmentation functions. The relative importance of contributing partonic subprocesses differs strongly depending on whether one uses for instance the fragmentation functions by De Florian et al. [30] (FSV) or by Albino et al. [31] (AKK). Therefore, even if no polarization effect is seen, the unpolarized data obtained by the proposed measurement can in any case lead to a considerable improvement in the determination of the unpolarized Λ fragmentation functions.

We will explain why it is necessary to measure both jet momenta and also how it differs from the process $pp \to \Lambda^\uparrow X$. We will start with a discussion of that process. Anselmino et al. [16] have attempted to describe the latter in terms of $D_{1T}^\perp$, but although a satisfactory description of the Λ polarization at larger values of the transverse momentum of the Λ could be obtained, considerable doubts about the applicability of the assumed factorization have arisen afterwards. A similar problem is unlikely to arise for the Λ polarization study in the process $pp \to (\Lambda^\uparrow \text{jet}) \text{jet} X$ at LHC and RHIC.

2. Λ polarization in $pp \to \Lambda^\uparrow X$

The Λ's measured in the process $pp \to \Lambda^\uparrow X$ [1] by the fixed target experiments have transverse momenta p_T w.r.t. the beam direction ranging up to 4 GeV/c. The Λ polarization transverse to the production plane first grows linearly with p_T, but around $p_T \approx 1$ GeV/c it levels off and remains approximately constant. For large p_T one expects the polarization to fall off as $1/p_T$, based on theoretical (collinear expansion) arguments [32]. The high energy collider experiments would be ideally suited to demonstrate this power-law fall-off behavior, if it were not for the small x_F values probed in those experiments.

The linearly rising behavior at very low p_T is well-described by recombination models employing spin-orbit couplings [4]. However, the plateau region for $p_T \sim 1$–4 GeV/c is less straightforward to incorporate in such soft physics models. Anselmino et al. [16] assumed that the p_T in this region may be considered a hard scale which then would allow for factorization of the cross section expression. However, since the scale is not too large either, instead of restricting to collinear factorization, transverse momentum dependence at the parton level was included. In this way the Λ polarization could be described in terms of the fragmentation function $D_{1T}^\perp$. Although a satisfactory description of the higher-p_T data was obtained in this way, a preliminary study of the unpolarized cross section within the same factorized approach resulted in considerable disagreement with the data, as mentioned in Ref. [33]. Apparently, at large x_F a p_T value of 3 or 4 GeV/c is not yet large enough to allow for a factorized description in the case of Λ production, as opposed to for instance pion production. Inclusion of intrinsic transverse momentum in the initial state parton distributions may lead to less discrepancy with the data, as studied for pion production [34], but remains to be done for Λ production.

Despite the doubts about whether $D_{1T}^\perp$ can be extracted from the existing fixed-target Λ polarization data, the collider experiments at RHIC and LHC are in a kinematic region that should allow for a factorized description of the cross section and thus can be considered safe. As mentioned, they do have the drawback that Λ measurements are generally restricted to small values of x_F, where the Λ polarization in $pp \to \Lambda^\uparrow X$ will be close to zero. Therefore, we next discuss a different process, where this drawback does not appear.

The main difference between the traditional process $pp \to \Lambda^\uparrow X$ and the one considered in this Letter, $pp \to (\Lambda^\uparrow \text{jet}) \text{jet} X$ comes from the way $D_{1T}^\perp$ enters in the asymmetry expressions. In the latter process it enters in an unintegrated way, i.e., the asymmetry is proportional to $D_{1T}^\perp(z, k_T^2)$, where z and k_T are observed. In the traditional process on the other hand, there is an integration over k_T, which implies that one essentially probes the first derivative of the partonic cross section (in case of collinear initial partons), which results in the aforementioned $1/p_T$ dependence. In $pp \to (\Lambda^\uparrow \text{jet}) \text{jet} X$ this $1/p_T$ factor does not arise.

3. Λ polarization in $pp \rightarrow (\Lambda^{\uparrow} \text{jet}) \text{jet} X$

We consider the process $p(P_1) + p(P_2) \rightarrow (\Lambda^{\uparrow}(K_\Lambda)\text{jet}(K_j)) + \text{jet}(K_{j'}) + X$. As indicated by the brackets, the Λ is part of the jet with observed momentum K_j, where the two jets are almost back-to-back in the plane perpendicular to the beam axis. The projections of the outgoing momenta in this plane will be denoted by $K_{\Lambda\perp}$, $K_{j\perp}$ and $K_{j'\perp}$. We will also use the scaled variables $x_{j\perp} = 2|K_{j\perp}|/\sqrt{s}$ and $x_{j'\perp} = 2|K_{j'\perp}|/\sqrt{s}$. At leading order the hadronic process is mediated by two-to-two partonic scattering $a(p_1)b(p_2) \rightarrow c(k)d(k')$. Since in the process under consideration both jet momenta are observed, the momenta of the outgoing partons are known, i.e., $k \equiv K_j$ and $k' \equiv K_{j'}$. For the parton momenta it is advantageous to make the Sudakov decompositions $p_i = x_i P_i + \sigma_i n_i + p_{iT}$ for incoming partons and $k = \frac{1}{z}K_\Lambda + \sigma n + k_T$ for the outgoing parton that will fragment into the hyperon. The n-vectors are lightlike four-vectors that have nonzero overlaps with the associated hadron momentum (i.e., $n_i \cdot P_i \neq 0$ and $n \cdot K_\Lambda \neq 0$). There is a certain degree of arbitrariness in the choice of these n-vectors, but it is required that σ_i and σ are suppressed by inverse powers of one of the hard scales in the process (such as $\sqrt{s}$ or $|K_{j\perp}|$). This requirement ensures in the first place that the cross section can be expressed in terms of TMDs, i.e., functions of only the momentum fractions and intrinsic transverse momenta.

By taking n to be a measurable four-vector in the process, e.g., $K_{j'}$ or one of the incoming hadron momenta P_i (or any linear fixed combination of them provided $K_\Lambda \cdot n$ is large), one ensures that the observed momentum fraction z and intrinsic transverse momentum squared k_T^2 are Lorentz invariant. Moreover, for any of these choices the same z and k_T^2 result, up to power corrections, which are neglected in our treatment. Below we will choose $n = K_{j'}$, for which $\sigma = (K_j \cdot K_\Lambda - z^{-1} M_\Lambda^2)/K_\Lambda \cdot K_{j'}$ is always small, of order of the hadronic scale squared divided by the partonic center of mass energy squared. The momentum fraction and intrinsic transverse momentum of the outgoing parton are then

$$z = \frac{K_\Lambda \cdot K_{j'}}{K_j \cdot K_{j'}} \quad \text{and} \quad k_T = K_j - z^{-1} K_\Lambda. \tag{1}$$

Note that k_T refers to the transverse component of the parton's momentum w.r.t. the Λ direction. The transverse component $K_{\Lambda T}$ of the Λ w.r.t. the jet direction is $K_{\Lambda T} = -z k_T = K_\Lambda - z K_j$ (note that the subscripts T of $K_{\Lambda T}$ and k_T refer to different frames [15]). The TMD fragmentation functions are commonly written with this vector as a variable.

For the cross section we take as a starting point the assumption that for each observed hadron separately the soft physics factorizes from the hard physics:

$$d\sigma_{pp \rightarrow (\Lambda^{\uparrow} j) j' X} = \frac{1}{2s} \frac{d^3 K_\Lambda}{(2\pi)^3 2E_\Lambda} \frac{d^3 K_j}{(2\pi)^3 2E_j} \frac{d^3 K_{j'}}{(2\pi)^3 2E_{j'}} 2(2\pi)^3 \int dx_1 \, d^2 p_{1T} \, dx_2 \, d^2 p_{2T} (2\pi)^4 \delta^4(p_1 + p_2 - K_j - K_{j'})$$

$$\times \sum_{a,\dots,d} \Phi_a(x_1, p_{1T}) \otimes \Phi_b(x_2, p_{2T}) \otimes \left| H^{ab \rightarrow cd}(p_1, p_2, K_j, K_{j'}) \right|^2 \otimes \Delta_c(z, k_T), \tag{2}$$

with $s \equiv (P_1 + P_2)^2 = E_{\text{cm tot}}^2$. The convolutions $\otimes$ indicate the appropriate Dirac and color traces for the partonic hard squared amplitude $|H|^2$. The momentum conservation delta function can furthermore be written as

$$\delta^4(p_1 + p_2 - K_j - K_{j'}) = \frac{2}{s} \delta\left(x_1 - \frac{1}{2}\left(x_{j\perp} e^{\eta_j} + x_{j'\perp} e^{\eta_{j'}}\right)\right) \delta\left(x_2 - \frac{1}{2}\left(x_{j\perp} e^{-\eta_j} + x_{j'\perp} e^{-\eta_{j'}}\right)\right)$$

$$\times \delta^2(p_{1T\perp} + p_{2T\perp} - K_{j\perp} - K_{j'\perp}), \tag{3}$$

where the $\eta_{j/j'} = -\ln \tan(\frac{1}{2}\theta_{j/j'})$ are the pseudorapidities of the outgoing jets and $\theta_{j/j'}$ denote polar angles in the c.o.m.-frame of the two incoming hadrons. The $p_{iT\perp}$ indicate the projections of the intrinsic transverse momenta p_{iT} of the incoming partons in the plane perpendicular to the beam axis. A factorization theorem as in Eq. (2) has not been proven rigorously for this process, but we believe that it is plausible to assume that if it exists, it will be of the schematic form of the expression in Eq. (2), possibly up to soft factors.

In principle there are several effects that contribute to the spin dependence of the cross section in Eq. (2), viz. the Boer–Mulders effect and the Sivers fragmentation effect. In the Boer–Mulders mechanism [8] the single spin asymmetry arises as a consequence of a correlation between the transverse momentum and transverse spin of a quark inside an unpolarized initial hadron. Hence, it relies on a k_T-effect in the initial state that will have to translate into an asymmetry in the final-state. Through the hard partonic scattering there can be k_T-smearing, leading to a dilution of the asymmetry in the final state. We will, therefore, assume that the Boer–Mulders effect is less important than the Sivers fragmentation effect. The latter is a k_T-effect that is directly in the final-state, such that there is no dilution from k_T-smearing in the partonic scattering. In addition, one may expect that the chiral-odd contributions ($\propto h_1^{\perp} H_1$) are generally smaller than the chiral-even ones ($\propto f_1 D_{1T}^{\perp}$), similar to what was found in Ref. [35] for the process $p^{\uparrow} p \rightarrow \gamma \text{jet} X$ using positivity bounds. Based on these considerations, we will simply neglect the intrinsic transverse momenta p_{iT} of the incoming particles, such that only the Sivers fragmentation mechanism contributes. As mentioned in the introduction, this effect is described by the TMD fragmentation function $D_{1T}^{\perp}(z, K_{\Lambda T}^2)$, which enters the expression in Eq. (2) through the parameterization of the

fragmentation correlator [15,36,37] (suppressing Wilson lines and with the convention $\epsilon^{0123} = +1$)

$$\Delta(z, k_T; K_\Lambda, S_\Lambda) = \sum_X \frac{1}{z} \int \frac{d(\xi \cdot K_\Lambda) \, d^2\xi_T}{(2\pi)^3} e^{-ik \cdot \xi} \langle 0|\psi(0)|\Lambda; X\rangle\langle\Lambda; X|\overline{\psi}(\xi)|0\rangle\rfloor_{\xi \cdot n = 0} \tag{4a}$$

$$= \left(D_1(z, K_{\Lambda T}^2) - D_{1T}^\perp(z, K_{\Lambda T}^2) \frac{\epsilon^{\mu\nu\rho\sigma} K_{\Lambda\mu} k_\nu S_{\Lambda\rho} n_\sigma}{M_\Lambda(K_\Lambda \cdot n)} \right) \slashed{K}_\Lambda + \text{other functions} \tag{4b}$$

$$= \left(D_1(z, K_{\Lambda T}^2) + D_{1T}^\perp(z, K_{\Lambda T}^2) \frac{(\hat{k} \times K_\Lambda) \cdot S_\Lambda}{z M_\Lambda} \right) \slashed{K}_\Lambda + \text{other functions}, \tag{4c}$$

in the case of quark fragmentation. Here k denotes the quark momentum, which equals the jet momentum K_j, and n is the Sudakov vector discussed before. The last line (4c) holds in any reference frame where n and $\hat{k}$ point in opposite directions. Sometimes the notation $\Delta^N D_{\Lambda\uparrow/a} = |K_{\Lambda T}| D_{1T}^{a\perp}/z M_\Lambda$ is used to indicate the Sivers fragmentation function [16].

As mentioned in the introduction, there is a complication specifically associated with TMD fragmentation functions. The Sivers fragmentation function of Eqs. (4b) and (4c) is actually a combination of the two functions in Fig. 3, arising from two different types of interactions. In the hadronic cross section the function $D_{1T}^\perp(z, K_{hT}^2)$ generated by the final-state interactions will appear convoluted with the usual partonic scattering cross sections. However, due to the dependence of the soft gluon interactions on the partonic subprocesses (in particular on its color flow structure), the TMD fragmentation function $\tilde{D}_{1T}^\perp(z, K_{hT}^2)$ generated by this effect will appear convoluted with modified partonic scattering cross sections. The contribution of the Sivers fragmentation mechanism to the hadronic cross section will thus have the form

$$\frac{d\hat{\sigma}_{ab \to cd}}{d\hat{t}} D_{1T}^{c\perp}(z, K_{hT}^2) + \frac{d\hat{\sigma}_{ab \to [c]d}}{d\hat{t}} \tilde{D}_{1T}^{c\perp}(z, K_{hT}^2), \tag{5}$$

where the $d\hat{\sigma}_{ab \to cd}$ are the usual partonic scattering cross sections and the $d\hat{\sigma}_{ab \to [c]d}$ are the modified partonic scattering cross sections (referred to as gluonic pole cross sections[3]) calculated in Refs. [13,24]. Therefore, the Sivers fragmentation functions in Eqs. (4b) and (4c) should be read as $D_{1T}^\perp \to D_{1T}^\perp + C_G^{[D]}\tilde{D}_{1T}^\perp$, where the $C_G^{[D]}$ are color fractions that depend on the particular term in the partonic squared amplitude H^*H which is given by the cut Feynman diagram D. These factors are perturbatively calculable and, once absorbed in the partonic scattering cross sections, lead to the gluonic pole cross sections. The gluonic pole cross sections, then, are gauge invariant weighted sums of cut Feynman diagrams D with the color fractions $C_G^{[D]}$ as weight factors.

After neglecting intrinsic transverse momentum effects for the incoming particles the Sivers fragmentation contribution to the cross section Eq. (2) *in the jet–jet c.o.m.-frame* becomes

$$\frac{E_\Lambda E_j E_{j'} \, d\sigma_{pp \to (\Lambda\uparrow j) j' X}}{d^3 K_\Lambda \, d^3 K_j \, d^3 K_{j'}} = \frac{1}{\pi} \delta^2(K_{j\perp} + K_{j'\perp}) \sum_{a,\ldots,d} x_1 f_1^a(x_1) x_2 f_1^b(x_2) \Biggl\{ \frac{d\hat{\sigma}_{ab \to cd}}{d\hat{t}} z D_1^c(z, (K_\Lambda - z K_j)^2) $$
$$+ \frac{S_\Lambda \cdot (\hat{K}_j \times K_\Lambda)}{z M_\Lambda} \left[\frac{d\hat{\sigma}_{ab \to cd}}{d\hat{t}} z D_{1T}^{c\perp}(z, (K_\Lambda - z K_j)^2) \right.$$
$$\left. + \frac{d\hat{\sigma}_{ab \to [c]d}}{d\hat{t}} z \tilde{D}_{1T}^{c\perp}(z, (K_\Lambda - z K_j)^2) \right] \Biggr\}, \tag{6}$$

where the momentum fractions x_1 and x_2 have been fixed by the first two delta functions on the r.h.s. of Eq. (3). The partonic scattering cross sections $d\hat{\sigma}_{ab \to cd}$ and the gluonic pole cross sections $d\hat{\sigma}_{ab \to [c]d}$ are functions of the Mandelstam variables $\hat{s} \equiv (p_1 + p_2)^2$, $\hat{t} \equiv (k - p_1)^2$ and $\hat{u} \equiv (k' - p_1)^2$ or, alternatively, of $\hat{s}$ and the variable y defined through

$$y \equiv -\frac{\hat{t}}{\hat{s}} = \frac{1}{e^{\eta_j - \eta_{j'}} + 1}, \quad \text{and} \quad -\frac{\hat{u}}{\hat{s}} = 1 - y. \tag{7}$$

Note that the expression of y in terms of η_j and $\eta_{j'}$ in Eq. (7) holds in the center of mass frame of the initial hadrons. Since we are ignoring masses w.r.t. $|K_{j\perp}|$, pseudorapidity and rapidity are the same and since differences of rapidities are invariant under boosts, Eq. (7) also holds in the jet–jet c.o.m.-frame. It should be emphasized that Eq. (6) does not hold in the c.o.m.-frame of the incoming hadrons.

Eq. (6) forms the central result of this Letter. Any nonzero measurement of the $S_\Lambda \cdot (\hat{K}_j \times K_\Lambda)$ asymmetry implies a nonzero Sivers effect in the fragmentation process and will thus be very interesting. It will imply nonzero $D_{1T}^\perp$ and/or $\tilde{D}_{1T}^\perp$. Due to the different partonic cross sections appearing in front of these two different Sivers fragmentation functions, measuring the y dependence of the asymmetry in principle offers the possibility to separate the two contributions, and to learn about their relative importance. This we will explore further in Section 4.

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[3] For T-odd gluon correlators in principle two distinct gluonic pole cross sections $d\hat{\sigma}^{(f)}$ and $d\hat{\sigma}^{(d)}$ appear. However, the latter cross section vanishes for the gluon–gluon scattering contribution [24], the only contribution considered in Section 4. Therefore, the function $\tilde{D}_{1T}^{g\perp}$ in this Letter refers to $\tilde{D}_{1T}^{g\perp(f)}$.
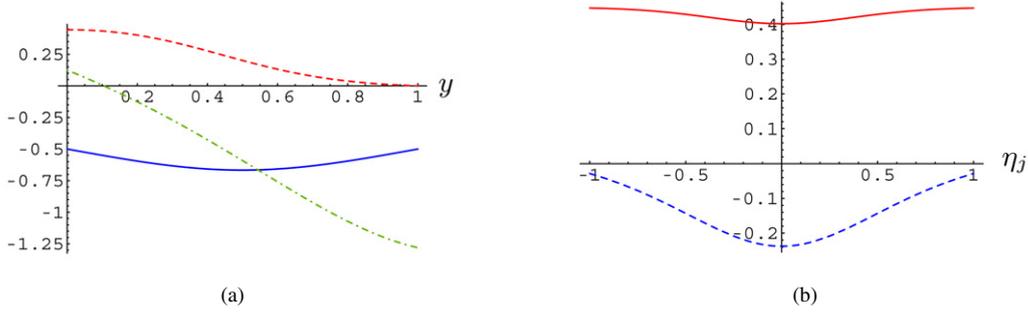
Fig. 4. (a) Solid line: $a_g(y)$; Dash-dotted line: $a_q(y)$; Dashed line: $b(y)$. (b) The combinations $b(y)+b(1-y)$ (solid line) and $b(y)a_q(y)+b(1-y)a_q(1-y)$ (dashed line) displayed as functions of η_j for $\eta_j = -\eta_{j'}$ events. All graphs are for $N = 3$.

If one were to study instead the process $p(P_1)+p(P_2) \to (\Lambda^\uparrow(K_\Lambda)\text{jet}(K_j))+X$ without constructing $K_{j'}$, then one does not have the full partonic kinematic information and one will be probing the combination

$$D_{1T}^{c\perp}(z, K_{hT}^2) + \frac{d\hat{\sigma}_{ab\to[c]d}/d\hat{t}}{d\hat{\sigma}_{ab\to cd}/d\hat{t}} \tilde{D}_{1T}^{c\perp}(z, K_{hT}^2), \tag{8}$$

averaged over y. In addition, referring to a correlation $\mathbf{S}_\Lambda \cdot (\hat{\mathbf{K}}_j \times \mathbf{K}_\Lambda)$ in the jet–jet c.o.m.-frame, involving three-vectors (corresponding to the choice $n \propto K_{j'}$) would not make sense. One could for instance choose $n \propto P_1$ and consider this correlation in terms of three-vectors in any frame in which $\hat{\mathbf{P}}_1$ and $\hat{\mathbf{K}}_j$ point in opposite directions. In the remainder of this article we will focus on the case where $K_{j'}$ is measured and present results in the jet–jet c.o.m.-frame.

4. Phenomenology

One can decompose the hadronic cross section in a spin-independent and a spin-dependent part:

$$\frac{d\sigma}{d^3K_\Lambda d^3K_j d^3K_{j'}} = \frac{d\sigma_U}{d^3K_\Lambda d^3K_j d^3K_{j'}} + \frac{\mathbf{S}_\Lambda \cdot (\hat{\mathbf{K}}_j \times \mathbf{K}_\Lambda)}{zM_\Lambda} \frac{d\sigma_T}{d^3K_\Lambda d^3K_j d^3K_{j'}}. \tag{9}$$

Comparing to Eq. (6) it follows that within our approximations the ratio of the spin-dependent and spin-independent cross sections is given by

$$\frac{d\sigma_T}{d\sigma_U} = \frac{\sum_{a,\dots,d} x_1 f_1^a(x_1) x_2 f_1^b(x_2) \{d\hat{\sigma}_{ab\to cd} D_{1T}^{c\perp}(z, (K_\Lambda - zK_j)^2) + d\hat{\sigma}_{ab\to[c]d} \tilde{D}_{1T}^{c\perp}(z, (K_\Lambda - zK_j)^2)\}}{\sum_{a,\dots,d} x_1 f_1^a(x_1) x_2 f_1^b(x_2) d\hat{\sigma}_{ab\to cd} D_1^c(z, (K_\Lambda - zK_j)^2)}. \tag{10}$$

This ratio appears in the single spin asymmetry

$$\text{SSA} = \frac{d\sigma(+\mathbf{S}_\Lambda) - d\sigma(-\mathbf{S}_\Lambda)}{d\sigma(+\mathbf{S}_\Lambda) + d\sigma(-\mathbf{S}_\Lambda)} = \frac{\mathbf{S}_\Lambda \cdot (\hat{\mathbf{K}}_j \times \mathbf{K}_\Lambda)}{zM_\Lambda} \frac{d\sigma_T}{d\sigma_U}. \tag{11}$$

The relevant partonic hard functions for our study are [24]

$$\frac{d\hat{\sigma}_{gg\to gg}}{d\hat{t}} = \frac{\pi\alpha_s^2}{\hat{s}^2} \frac{2N^2}{N^2-1} \frac{(\hat{s}^4+\hat{t}^4+\hat{u}^4)(\hat{s}^2+\hat{t}^2+\hat{u}^2)}{2\hat{s}^2\hat{t}^2\hat{u}^2}, \qquad \frac{d\hat{\sigma}_{gg\to[g]g}}{d\hat{t}} = -\frac{\pi\alpha_s^2}{\hat{s}^2} \frac{2N^2}{N^2-1} \frac{(\hat{s}^2+\hat{t}^2+\hat{u}^2)^2}{4\hat{t}^2\hat{u}^2}, \tag{12a}$$

$$\frac{d\hat{\sigma}_{qg\to qg}}{d\hat{t}} = -\frac{\pi\alpha_s^2}{\hat{s}^2} \frac{\hat{s}^2+\hat{u}^2}{2\hat{s}\hat{u}} \left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} - \frac{1}{N^2}\right), \qquad \frac{d\hat{\sigma}_{qg\to[q]g}}{d\hat{t}} = \frac{\pi\alpha_s^2}{\hat{s}^2} \frac{\hat{s}^2+\hat{u}^2}{2\hat{s}\hat{u}} \left(\frac{\hat{s}^2}{\hat{t}^2} - \frac{N^2+1}{N^2-1}\left(\frac{\hat{u}^2}{\hat{t}^2} - \frac{1}{N^2}\right)\right). \tag{12b}$$

All cross sections are given for massless partons. The cross sections with the initial or final state particles interchanged are obtained from these by a $\hat{t} \leftrightarrow \hat{u}$ substitution. We also define the ratios (displayed in Fig. 4(a))

$$a_g(y) = \frac{d\hat{\sigma}_{gg\to[g]g}}{d\hat{\sigma}_{gg\to gg}} = -\frac{\hat{s}^2}{2} \frac{\hat{s}^2+\hat{t}^2+\hat{u}^2}{\hat{s}^4+\hat{t}^4+\hat{u}^4} = -\frac{1}{2} \frac{1+y^2+(1-y)^2}{1+y^4+(1-y)^4}, \tag{13a}$$

$$a_q(y) = \frac{d\hat{\sigma}_{qg\to[q]g}}{d\hat{\sigma}_{qg\to qg}} = \frac{N^2+1}{N^2-1} - \frac{2N^2}{N^2-1} \frac{\hat{s}^2}{\hat{s}^2+\hat{u}^2-\frac{1}{N^2}\hat{t}^2} = \frac{N^2+1}{N^2-1} - \frac{2N^2}{N^2-1} \frac{1}{1+(1-y)^2-\frac{1}{N^2}y^2}, \tag{13b}$$

$$b(y) = \frac{d\hat{\sigma}_{qg\to qg}}{d\hat{\sigma}_{gg\to gg}} = -\frac{N^2-1}{2N^2} \frac{\hat{s}\hat{u}(\hat{s}^2+\hat{u}^2)(\hat{s}^2+\hat{u}^2-\frac{1}{N^2}\hat{t}^2)}{(\hat{s}^4+\hat{t}^4+\hat{u}^4)(\hat{s}^2+\hat{t}^2+\hat{u}^2)} = -\frac{N^2-1}{2N^2} \frac{(1-y)(1+(1-y)^2)(1+(1-y)^2-\frac{1}{N^2}y^2)}{(1+y^4+(1-y)^4)(1+y^2+(1-y)^2)}. \tag{13c}$$
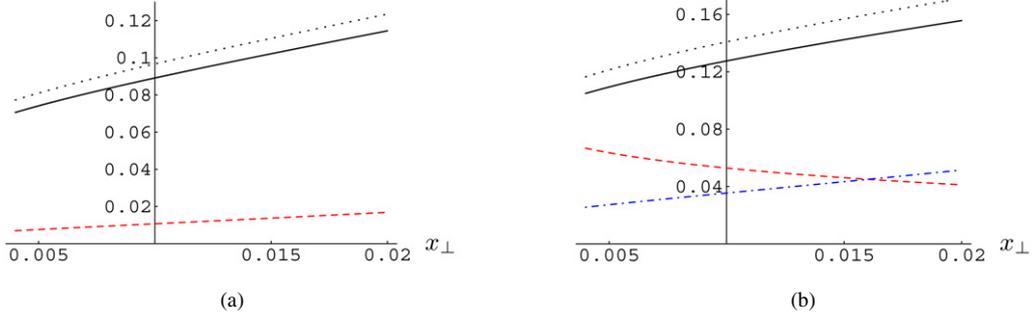
Fig. 5. (a) The partonic cross sections multiplied by the unpolarized collinear distribution functions, integrated over $-1 \leqslant \eta_{j/j'} \leqslant 1$, normalized to the corresponding quantity for gluon–gluon scattering (numerator and denominator integrated separately). Solid line: $qg \to qg/gg \to gg$, dashed line: $qq' \to qq'/gg \to gg$. For the unpolarized distribution functions we have used the CTEQ5L distribution functions evaluated at 10 GeV. For comparison $qg \to qg/gg \to gg$ at 20 GeV was also included (dotted line). We have approximated $x_{j\perp} \approx x_{j'\perp} \equiv x_\perp$. (b) As in (a), but now with $qq' \to qq'/qg \to qg$ (solid line), $gg \to q\bar{q}/qg \to qg$ (dashed line) and $qq \to qq/qg \to qg$ (dash-dotted line). Over the entire kinematical region considered here all other partonic channels are dominated by those shown in the figure. For comparison $qq' \to qq'/gg \to gg$ at 20 GeV was also included (dotted line).

In Fig. 5 we show the expressions $x_1 f_1^a(x_1) x_2 f_1^b(x_2) d\hat{\sigma}_{ab \to cd}$ appearing in the denominator of Eq. (10), integrated over the pseudorapidity ranges $-1 \leqslant \eta_{j,j'} \leqslant +1$, normalized to the corresponding quantity for gluon–gluon scattering (Fig. 5(a)) and quark-gluon scattering (Fig. 5(b)). The pseudorapidity integration range is inspired by the η coverage of the ALICE detector at LHC, which has the particle identification capabilities that allow to measure Λ's with transverse momenta p_T of several GeV/c. We have taken $\sqrt{s} = 14$ TeV as is relevant to the LHC situation and for the unpolarized distribution functions we have used the CTEQ5L parametrizations evaluated at the scale 10 GeV. The figures do not change significantly with the scale in this region (see Fig. 5). What we infer from the figure is that the combination $x_1 f_1^a(x_1) x_2 f_1^b(x_2) d\hat{\sigma}_{ab \to cd}$ which determines the inclusive cross section is dominated by the gluon–gluon ($gg \to gg$) scattering process in this particular kinematic region. The quark–gluon ($qg \to qg$) scattering contribution is on the order of 10% of the ($gg \to gg$) contribution at perpendicular scales $x_\perp$ typical for LHC, if we consider jets with transverse momenta between 30 and 100 GeV/c. Similarly, the other partonic contributions are of the order of 10% (or less) of the $qg \to qg$ contribution (Fig. 5(b)).

Whether or not the unpolarized Λ production cross section $d\sigma_U$ can also be approximated by the gluon–gluon scattering contribution, depends heavily on the Λ fragmentation functions employed. Several such functions have been fitted to experimental data [30,31,38,39], unfortunately with vastly different results for the ratios D_1^g/D_1^q for the various quark and antiquark flavors. If one restricts to the subprocesses $gg \to gg$ and $qg \to qg$, then the unpolarized cross section $d\sigma_U$ contains

$$d\sigma_U \sim x_1 f_1^g(x_1) x_2 f_1^g(x_2) d\hat{\sigma}_{gg \to gg} D_1^g(z) \{1 + \varepsilon(\eta_1, \eta_2)\}, \tag{14}$$

with

$$\varepsilon \equiv \frac{\sum_q x_1 f_1^q(x_1) x_2 f_1^g(x_2) [d\hat{\sigma}_{qg \to qg} D_1^q(z) + d\hat{\sigma}_{qg \to gq} D_1^g(z)]}{x_1 f_1^g(x_1) x_2 f_1^g(x_2) d\hat{\sigma}_{gg \to gg} D_1^g(z)}$$

$$+ \frac{\sum_q x_1 f_1^g(x_1) x_2 f_1^q(x_2) [d\hat{\sigma}_{gq \to qg} D_1^q(z) + d\hat{\sigma}_{gq \to gq} D_1^g(z)]}{x_1 f_1^g(x_1) x_2 f_1^g(x_2) d\hat{\sigma}_{gg \to gg} D_1^g(z)} \tag{15a}$$

$$= b(y) \frac{\sum_q f_1^q(x_1) D_1^q(z)}{f_1^g(x_1) D_1^g(z)} + b(1-y) \frac{\sum_q f_1^q(x_1)}{f_1^g(x_1)} + b(y) \frac{\sum_q f_1^q(x_2)}{f_1^g(x_2)} + b(1-y) \frac{\sum_q f_1^q(x_2) D_1^q(z)}{f_1^g(x_2) D_1^g(z)}. \tag{15b}$$

The function ε is quite sensitive to the choice of unpolarized fragmentation functions, as demonstrated in Fig. 6, which shows a comparison using the functions by De Florian et al. [30] (FSV) and by Albino et al. [31] (AKK), for a factorization scale of 10 GeV.

For the AKK set the assumption of gluon–gluon scattering dominance may be good to 30%, but for the FSV set one can reach this conclusion only for small z values. At ALICE Λ's with transverse momentum of several GeV are expected to be measured in sufficiently large quantities. A typical z value could be 0.1 for a 5 GeV Λ within a 50 GeV jet. Therefore, we expect the midrapidity data to be restricted to small z values at LHC and the assumption of gluon–gluon scattering dominance to be valid despite the large variation in Λ fragmentation function sets.

A future investigation of Λ production at LHC or RHIC should, therefore, first focus on describing the unpolarized cross section correctly, especially regarding the y-dependence, before the y-dependence of the polarization effect is studied. Therefore, even if no polarization effect is seen, the unpolarized data obtained by the proposed measurement can in any case lead to a considerable improvement in the determination of the unpolarized Λ fragmentation functions.
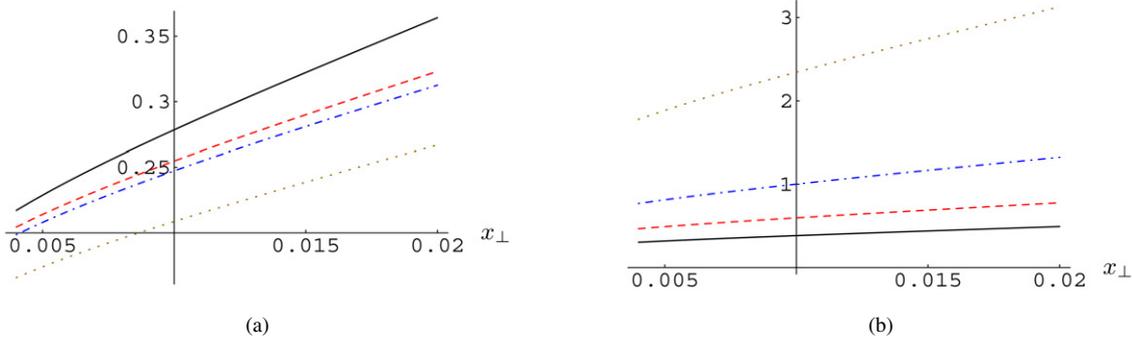
Fig. 6. Expression in Eq. (15a) integrated over $-1 \leqslant \eta_{j/j'} \leqslant 1$ (numerator and denominator integrated separately) with (a) AKK and (b) FSV Λ-fragmentation functions, for $z = 0.1$ (solid line), $z = 0.3$ (dashed line), $z = 0.5$ (dash-dotted line) and $z = 0.7$ (dotted line). Distribution and fragmentation functions are evaluated at 10 GeV.

To illustrate the main idea regarding the y dependence of the asymmetry, we will simply assume that the unpolarized cross section is known sufficiently well and that gluon–gluon scattering dominates, and consider two distinct scenarios: 1) $D_{1T}^{g\perp} \neq 0$; 2) $D_{1T}^{g\perp} = 0$ or more generally, $D_{1T}^{g\perp} \ll D_{1T}^{q\perp}$.

In scenario 1 ($D_{1T}^{g\perp} \neq 0$) one finds from Eq. (10):

$$\frac{d\sigma_T}{d\sigma_U} \approx \frac{x_1 f_1^g(x_1) x_2 f_1^g(x_2) d\hat{\sigma}_{gg \to gg} D_{1T}^{g\perp}(z, K_{\Lambda T}^2) + x_1 f_1^g(x_1) x_2 f_1^g(x_2) d\hat{\sigma}_{gg \to [g]g} \tilde{D}_{1T}^{g\perp}(z, K_{\Lambda T}^2)}{x_1 f_1^g(x_1) x_2 f_1^g(x_2) d\hat{\sigma}_{gg \to gg} D_1^g(z, K_{\Lambda T}^2)}$$

$$= \frac{D_{1T}^{g\perp}(z, K_{\Lambda T}^2)}{D_1^g(z, K_{\Lambda T}^2)} + a_g(y) \frac{\tilde{D}_{1T}^{g\perp}(z, K_{\Lambda T}^2)}{D_1^g(z, K_{\Lambda T}^2)}. \tag{16}$$

If there would be no Wilson line or color flow dependence in the fragmentation process in the way as has been claimed in Refs. [28,29], $\tilde{D}_{1T}^{g\perp}$ would arise with the usual partonic cross section. In that case there is no need to consider two separate functions to begin with and one should not find any y dependence at all. Observing the y dependence $a_g(y)$ (which is rather weak unfortunately) would be a signal for color flow dependence of the T-odd effects in the fragmentation process.

In scenario 2 ($D_{1T}^{g\perp} \ll D_{1T}^{q\perp}$) one finds on the other hand:

$$\frac{d\sigma_T}{d\sigma_U} \approx \frac{\sum_q (x_1 f_1^q(x_1) x_2 f_1^g(x_2) d\hat{\sigma}_{qg \to qg} D_{1T}^{q\perp}(z, K_{\Lambda T}^2) + x_1 f_1^q(x_1) x_2 f_1^g(x_2) d\hat{\sigma}_{qg \to [q]g} \tilde{D}_{1T}^{q\perp}(z, K_{\Lambda T}^2))}{x_1 f_1^g(x_1) x_2 f_1^g(x_2) d\hat{\sigma}_{gg \to gg} D_1^g(z, K_{\Lambda T}^2)}$$

$$+ \frac{\sum_q (x_1 f_1^g(x_1) x_2 f_1^q(x_2) d\hat{\sigma}_{gq \to qg} D_{1T}^{q\perp}(z, K_{\Lambda T}^2) + x_1 f_1^g(x_1) x_2 f_1^q(x_2) d\hat{\sigma}_{gq \to [q]g} \tilde{D}_{1T}^{q\perp}(z, K_{\Lambda T}^2))}{x_1 f_1^g(x_1) x_2 f_1^g(x_2) d\hat{\sigma}_{gg \to gg} D_1^g(z, K_{\Lambda T}^2)}$$

$$= b(y) \frac{\sum_q f_1^q(x_1) D_{1T}^{q\perp}(z, K_{\Lambda T}^2)}{f_1^g(x_1) D_1^g(z, K_{\Lambda T}^2)} + b(y) a_q(y) \frac{\sum_q f_1^q(x_1) \tilde{D}_{1T}^{q\perp}(z, K_{\Lambda T}^2)}{f_1^g(x_1) D_1^g(z, K_{\Lambda T}^2)}$$

$$+ b(1-y) \frac{\sum_q f_1^q(x_2) D_{1T}^{q\perp}(z, K_{\Lambda T}^2)}{f_1^g(x_2) D_1^g(z, K_{\Lambda T}^2)} + b(1-y) a_q(1-y) \frac{\sum_q f_1^q(x_2) \tilde{D}_{1T}^{q\perp}(z, K_{\Lambda T}^2)}{f_1^g(x_2) D_1^g(z, K_{\Lambda T}^2)}. \tag{17}$$

This expression has a rather involved y dependence. It is probably too difficult to disentangle the separate contributions. Therefore, we suggest a further step. Since the momentum fractions x_1 and x_2 are fixed by the jet momenta, one can select those events where they are equal, which corresponds to $\eta_{j'} = -\eta_j$. In that case the expression simplifies

$$\frac{d\sigma_T}{d\sigma_U} \approx \frac{\sum_q f_1^q(x_1)\{[b(y)+b(1-y)] D_{1T}^{q\perp}(z, K_{\Lambda T}^2) + [b(y) a_q(y) + b(1-y) a_q(1-y)] \tilde{D}_{1T}^{q\perp}(z, K_{\Lambda T}^2)\}}{f_1^g(x_1) D_1^g(z, K_{\Lambda T}^2)}, \tag{18}$$

where now $y = (e^{2\eta_j}+1)^{-1}$ and $x_1 = x_2 = x_\perp/2\sqrt{y(1-y)}$. The two different combinations of y-dependent terms are depicted in Fig. 4(b) as a function of η_j. One can see that one term in Eq. (18) varies more strongly with η_j than the other term. This may possibly allow a discrimination of the two effects.

5. Discussion and conclusions

In this Letter we have proposed a measurement of Λ polarization in the process $pp \to (\Lambda^\uparrow \mathrm{jet})\mathrm{jet}X$, where the Λ is part of one of the two observed jets which are almost back-to-back in the plane perpendicular to the beam axis. Unlike the traditional

measurement in the process $pp \to \Lambda^\uparrow X$, this new observable need not vanish at midrapidity. This makes it of interest for high energy collider experiments, such as RHIC, Tevatron and LHC, as they typically can detect Λ's at midrapidity. We have studied the asymmetry for the ALICE experiment specifically, due to the fact that ALICE has good PID capabilities that allow detection of Λ's of momenta of several GeV/c. The experimental situation at LHC furthermore leads to the dominance of gluon–gluon and gluon–quark scattering, allowing for a simplification of the expressions to good approximation.

Observation of a nonzero $\boldsymbol{S}_\Lambda \cdot (\hat{\boldsymbol{K}}_j \times \boldsymbol{K}_\Lambda)$ asymmetry in the jet–jet c.o.m. frame indicates a nonzero Sivers effect in the fragmentation process. The y dependence of this observable in principle could allow for a study of the relative importance of the two types of interactions that could exhibit such a Sivers effect. It allows for a study of possible color-flow dependence of the asymmetry and of the (non)universality of the Sivers fragmentation functions, issues currently of much interest and that cannot be addressed in $pp \to \Lambda^\uparrow X$. But even if no asymmetry is observed, the unpolarized data that is obtained from the polarization measurement would help to constrain the unpolarized Λ fragmentation functions, which are currently not well-determined. High energy collider experiments such as to be performed at LHC would be very helpful in this respect, as the question of factorization of the unpolarized cross section would not pose a problem.

The study of color flow dependence we suggest here can be done without an actual extraction of $D_{1T}^\perp$, namely by just studying the dependence of the observable on particular kinematic variables. This should prove useful even if a trustworthy extraction of the Sivers fragmentation functions themselves turns out to be too difficult.

Currently no knowledge on the magnitude of the various Sivers fragmentation functions is available, therefore, no predictions can be given for the actual size of the asymmetry. But the y dependence of the various contributing terms could be derived. They were discussed for two different scenarios: one in which the gluon Sivers fragmentation function is important to include and one in which it is irrelevant compared to the quark function.

The proposed measurement allows for a determination of the z and k_T^2 dependence of the Sivers fragmentation functions. For completeness, we mention that there are other processes from which this information can be extracted. This can be done in electron–positron annihilation experiments via the process $e^+e^- \to (\Lambda^\uparrow \mathrm{jet})X$ in a straightforward manner [40]. It allows to probe the combination $D_{1T}^\perp + \tilde{D}_{1T}^\perp$. It can also be done via neutral or charged current semi-inclusive DIS: $\ell p \to \ell'(\Lambda^\uparrow \mathrm{jet})X$, where the combination $D_{1T}^\perp - \tilde{D}_{1T}^\perp$ is probed. The comparison to the functions from $pp \to (\Lambda^\uparrow \mathrm{jet})\mathrm{jet}X$ would allow for another test of the (non)universality of the Sivers fragmentation functions.

Acknowledgements

References

[1] A. Lesnik, et al., Phys. Rev. Lett. 35 (1975) 770.
[2] G. Bunce, et al., Phys. Rev. Lett. 36 (1976) 1113.
[3] For a review of experimental data, see, e.g., K. Heller, in: C.W. de Jager, et al. (Eds.), The Proceedings of Spin'96, World Scientific, 1997;
    Or see A.D. Panagiotou, Int. J. Mod. Phys. A 5 (1990) 1197.
[4] For a review of theoretical models, see, e.g., J. Felix, Mod. Phys. Lett. A 14 (1999) 827.
[5] D.W. Sivers, Phys. Rev. D 41 (1990) 83;
    D.W. Sivers, Phys. Rev. D 43 (1991) 261.
[6] J.C. Collins, Nucl. Phys. B 396 (1993) 161.
[7] M. Anselmino, M. Boglione, F. Murgia, Phys. Lett. B 362 (1995) 164.
[8] D. Boer, P.J. Mulders, Phys. Rev. D 57 (1998) 5780.
[9] D.L. Adams, et al., FNAL-E704 Collaboration, Phys. Lett. B 264 (1991) 462.
[10] S.J. Brodsky, D.S. Hwang, I. Schmidt, Phys. Lett. B 530 (2002) 99.
[11] A. Airapetian, et al., HERMES Collaboration, Phys. Rev. Lett. 94 (2005) 012002.
[12] D. Boer, W. Vogelsang, Phys. Rev. D 69 (2004) 094025.
[13] A. Bacchetta, C.J. Bomhof, P.J. Mulders, F. Pijlman, Phys. Rev. D 72 (2005) 034030.
[14] B.I. Abelev, et al., STAR Collaboration, arXiv: 0705.4629 [hep-ph].
[15] P.J. Mulders, R.D. Tangerman, Nucl. Phys. B 461 (1996) 197;
    P.J. Mulders, R.D. Tangerman, Nucl. Phys. B 484 (1997) 538, Erratum.
[16] M. Anselmino, D. Boer, U. D'Alesio, F. Murgia, Phys. Rev. D 63 (2001) 054029.
[17] J.C. Collins, Phys. Lett. B 536 (2002) 43.
[18] X. Ji, F. Yuan, Phys. Lett. B 543 (2002) 66.
[19] A.V. Belitsky, X. Ji, F. Yuan, Nucl. Phys. B 656 (2003) 165.
[20] D. Boer, P.J. Mulders, F. Pijlman, Nucl. Phys. B 667 (2003) 201.
[21] S.J. Brodsky, D.S. Hwang, I. Schmidt, Nucl. Phys. B 642 (2002) 344.
[22] C.J. Bomhof, P.J. Mulders, F. Pijlman, Phys. Lett. B 596 (2004) 277.

[23] C.J. Bomhof, P.J. Mulders, F. Pijlman, Eur. Phys. J. C 47 (2006) 147.
[24] C.J. Bomhof, P.J. Mulders, JHEP 0702 (2007) 029.
[25] P.G. Ratcliffe, O.V. Teryaev, hep-ph/0703293.
[26] J. Collins, J.W. Qiu, Phys. Rev. D 75 (2007) 114014.
[27] C.J. Bomhof, P.J. Mulders, W. Vogelsang, F. Yuan, Phys. Rev. D 75 (2007) 074019.
[28] A. Metz, Phys. Lett. B 549 (2002) 139.
[29] J.C. Collins, A. Metz, Phys. Rev. Lett. 93 (2004) 252001.
[30] D. de Florian, M. Stratmann, W. Vogelsang, Phys. Rev. D 57 (1998) 5811.
[31] S. Albino, B.A. Kniehl, G. Kramer, Nucl. Phys. B 734 (2006) 50.
[32] G.L. Kane, J. Pumplin, W. Repko, Phys. Rev. Lett. 41 (1978) 1689.
[33] M. Anselmino, D. Boer, U. D'Alesio, F. Murgia, Phys. Rev. D 65 (2002) 114014.
[34] M. Anselmino, M. Boglione, U. D'Alesio, E. Leader, F. Murgia, Phys. Rev. D 71 (2005) 014002.
[35] A. Bacchetta, C. Bomhof, U. D'Alesio, P.J. Mulders, F. Murgia, hep-ph/0703153.
[36] A. Bacchetta, U. D'Alesio, M. Diehl, C.A. Miller, Phys. Rev. D 70 (2004) 117504.
[37] A. Bacchetta, et al., JHEP 0702 (2007) 093.
[38] D. Indumathi, H.S. Mani, A. Rastogi, Phys. Rev. D 58 (1998) 094014.
[39] C. Boros, J.T. Londergan, A.W. Thomas, Phys. Rev. D 62 (2000) 014021.
[40] D. Boer, R. Jakob, P.J. Mulders, Nucl. Phys. B 504 (1997) 345.