Are representations to be provided or generated in primary mathematics education? Effects on transfer

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Are representations to be provided or generated in primary mathematics education? Effects on transfer

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With regard to transfer, is it better to provide pupils with ready-made representations or is it more effective to scaffold pupils' thinking in the process of generating their own representations with the help of peers and under the guidance of a teacher in a process of guided co-construction? The sample comprises 10 classes and 239 Grade 5 primary school students, age 10–11 years. A pretest-posttest control group research design was used. In the experimental condition, pupils were taught to construct representations collaboratively as a tool in the learning of percentages and graphs. Children in the experimental condition outperformed control children on the posttest and transfer test. Both high- and low-achieving pupils profited from the intervention. This study shows that children who learn to design are in a better position to understand pictures, graphs, and models. They are more successful in solving new, complex mathematical problems.

Keywords: mathematics education; learning; transfer; collaborative learning; providing versus generating; models; primary school

Introduction

In most research that focuses on the use of strategies in learning and problem solving, ready-made representations or models provided by the teacher or a textbook are regarded as proper objects of study (Hattie, Biggs, & Purdie, 1996). Research findings support the idea that representations are important for the learning of science and mathematics (Ainsworth, 2006; Gilbert & Boulter, 2000; Greer, 1997; Lehtinen & Hannula, 2006; Mayer, 1989; Perkins & Unger, 1999). Moreover, learning to design representations can be expected to promote problem solving in relation to both familiar and new problems. When students are actively involved in the construction and collaborative evaluation of representations, they develop mathematical knowledge that enables them to generate new solution processes. This article reports on a research project aimed at describing and analyzing the effects of an experimental mathematics program that focuses on the construction and use of student representations in primary mathematics education.

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The starting point for this article constitutes one of the major questions in learning theory and strategy research: Are representations to be provided or generated? (Rosenshine, Meister, & Chapman, 1996). The outcomes of a number of studies into the design and use of representations (such as drawings, pictures, or diagrams) in mathematical problem solving show that self-constructed graphic representations do not always have the intended effects on students' performance in mathematics. De Bock, Verschaffel, Janssens, Van Dooren, and Claes (2003) mention two important factors. First, representations should be sufficiently detailed to enable students to use them in the process of problem solving. Second, the use of representations in problem solving appears to be efficient only when it is accompanied by active reflection.

In the present study, students were taught to design representations as a tool in the learning of percentages and graphs. In order to overcome the above-mentioned problem as described by Rosenshine et al. (1996), an experimental program was constructed and teachers were trained to assist students in generating representations in a process of guided co-construction. In the experiment, we compared the learning outcomes of students in the experimental program with control group learning outcomes, in which ready-made representations were provided. The experimental program was based on a large body of empirical and theoretical research that demonstrates the potential relevance of guided co-construction as a teaching-learning strategy in realistic mathematics learning situations (Ainsworth, 2006; Freudenthal, 1991; Keijzer & Terwel, 2003; Schwartz, 1995; Van Parreren, 1993).

The major subject to be discussed in the present article, however, relates to problem solving in relatively new domains and contexts. Will this kind of problem solving be more successful when students learn to co-construct representations compared to students who learn to work with ready-made representations? The guiding research question in this article is formulated as follows:

> With regard to transfer, what are the effects of an experimental primary mathematics program in which students participate as representation designers in a process of guided co-construction?

Our working hypothesis is that learning to design representations will have positive effects on the learning outcomes as measured by a mathematics transfer test. The basic idea underlying the hypothesis is that students will develop knowledge that enables them to generate new solution processes as a result of constructive involvement and dialogic inquiry (Wells, 1999). Following the Vygotskian perspective on learning (Vygotsky, 1978), we assume that design activities with collaborating peers will enable students to gain insight into mathematical structures. Representations that emerge in collaboration are *more abstract* as a consequence of the communicative need for a single representation that could bridge individual representations. In discussions toward consensus, students may reinvent represented domain structures. The capacity to solve new problems is determined by the extent to which the learner is aware of the underlying structural similarities between familiar and new mathematical problems (Ainsworth, 2006; Freudenthal, 1991; Lehtinen & Hannula, 2006; Schwartz, 1995). In designing representations, students learn how structuring processes develop, for example, through the use of graphs to represent geometric or topological relations. This knowledge enhances the capacity to generate new solution processes and to transform a representation according to changes in the

situation, facilitating the construction of solutions to relatively new and unfamiliar problems.

Although our theory is primarily based on European sources such as Vygotsky (1978), Katona (1940), Freudenthal (1991), Van Parreren (1966, 1993), and Van Oers (1998, 2001), it shows similarities with an important research tradition in the USA, in which meaningful understanding of the structure and the principles by which it works also plays a key role in theories of learning and transfer. This latter tradition includes a series of authors from Judd (1908) to Brown and Palincsar (1989) (see Lobato, 2006). Both research traditions criticized Thorndike's (1922) theory of "identical elements" and stressed the active role of the learner in the co-construction of underlying structure and the search for similarity between problems.

Theoretical background

The concept of representation

Representations play a major role in problem solving (Anderson, 1995; Gilbert & Boulter, 2000; Schnotz & Bannert, 2003; Sternberg, 1994). A representation can be conceived of as a general term for symbolized versions of the external world, stored somewhere in the mind or materially expressed in a social context, for example, a classroom. This conception warrants use of the terms *mental representation* and *expressed representation.*

Representations refer to some form of knowledge about the world or the problem-as-experienced. Representations that retain much of the details of the original experience are termed perception-based representations. Their counterparts are meaning-based representations that represent the phenomenon or situation from a particular point of view and discard unimportant details (see Van Oers, 2001). Expressing and communicating a representation implies articulating what is significant. Consequently, it leads to a more abstract, meaning-based representation. Initially, there may be various individual representations, but after group discussions, shared representations as expressed by individual members can be sorted out or merged into one single, culturally accepted representation that is shared by the group.

In general, cognitive theories on thinking and problem solving appear to favor the term *representation*, whereas researchers in other domains prefer *model* and *modelling*. The latter terms are commonly used in domain-specific theories in science and mathematics education. In our view, the terms models and representations are, at least to a large extent, interchangeable; even more so when studies of teaching and learning in science or mathematics are involved (Gilbert & Boulter, 2000; Gravemeijer, Lehrer, Van Oers, & Verschaffel, 2003; Greer, 1997). If there is a difference between the two, it could probably be said that representation appears to be a more general, overarching concept from cognitive psychology – as also applied in multimedia learning –, while model is a more domain-specific term commonly used in science and mathematics education. In this light, we define a model as a specific structural form of representation. However, both terms refer to structural devices for referring to something else as perceived or conceptualized; they relate to human knowledge of the world. To put it differently, representations are important tools in attempts to understand some domain of knowledge (Gilbert & Boulter, 2000).

The importance of constructing representations for understanding

A survey of the educational literature provided us with several arguments to underpin the value of representations.

Although Perkins and Unger (1999) mentioned that learner-constructed representations are not always helpful because they are not necessarily adequate, they also warn that providing a model not always empowers the learner to perform in a flexible way. Previous research gave us some reason to believe that – under certain instructional conditions – self-constructed representations may be more productive in generating mathematical knowledge than ready-made, provided representations (Ainsworth, 2006; diSessa, 2004; Van Dijk, Van Oers, & Terwel, 2003).

The pros and cons mentioned in the literature could be conceived of as arguments for the idea that the appropriation of knowledge that enables students to generate relatively new solution processes depends both on the students' involvement in the design activity as well as on the features of the final, culturally accepted representation that emerge from collaborative designing in a process of guided co-construction. DiSessa (2002) also underlines the importance of representational adequacy. Student's representations are not necessarily good.

Like Freudenthal (1991), we assume that mathematical structuring ability might be more helpful than mastery of mathematical structures. We pointed out earlier that mastery of particular structures in itself is not enough to enhance problem solving (Perkins & Unger, 1999). But what happens if one involves students in a social activity such as learning to design representations and let them gradually (re)invent what was earlier invented by others? "If the learner is guided to reinvent all this, then valuable knowledge and abilities will more easily be learned, retained and transferred than if imposed" (Freudenthal, 1991, p. 49). Guided reinvention proceeds from rich to poor structures to make it more mathematical (Freudenthal, 1991). To clarify what more mathematical means, one may think of mathematical characteristics such as generality, certainty, exactness, and brevity. These are the characteristics of powerful, generative knowledge that enhances students' mathematical structuring (modeling) capacity. This knowledge can be built up by collaboratively engaging in activities such as generalizing, looking for analogies, classifying, reflecting, structuring, defining, symbolizing, and schematizing (Gravemeijer & Terwel, 2000). Several other authors (De Bock et al., 2003; Schwartz, 1995) particularly stress the role of active reflection in the process of designing mathematical representations as a condition for success in mathematical problem solving.

This issue has been pondered by many authors in the past. diSessa (diSessa, 2002, 2004; diSessa, Hammer, Sherin, & Kolpakowski, 1991) wondered, in addition, whether we could expect students to invent, in some reasonable sense, algebra, or decimal notation. His answer to that question was positive: A student can become "a designer of representational forms". In his studies, diSessa found that students were very well capable of inventing graphing. Although it seems prima facie impossible for students to reproduce in short order what took civilisation thousands of years to build, diSessa claims that asking students to design representations has several potential advantages (diSessa et al., 1991). First, it closes the gap between prior knowledge and the material they are involved with. Second, it provides opportunities for creative engagement and ownership of conceptually difficult material. And third, it enables students to exercise their meta-representational knowledge, which is expected to be of value in the creation of new representations. However, in spite of the advantages, diSessa noted "how rare it is to find instruction that trusts students

to create their own representations" (diSessa et al., 1991, p. 156). Although there have been positive changes in the past decades, we believe that today, diSessa's statement holds true for many classroom practices.

The concept of transfer

Transfer is a concept that does not remain constant across situations and theories of learning. As a consequence, references to the concept of transfer have little in common (Säljö, 2003). One general definition is: Transfer is about how what is learned in one situation affects or influences what the learner is capable of doing in another situation.

Some authors regard the concept as unnecessary because the concepts of *learning* and *transfer* cannot be distinguished. Others are far from optimistic about the possibility of transfer. These positions may lead to abandonment of the very concept.

There are also more moderate positions. Greeno, Smith, and Moore (1993) and Greeno (1997) interpreted transfer from their "situated cognition theory". Greeno et al. (1993) and Greeno (1997, 2006) reject the idea that a "situated cognition" perspective would necessarily imply that knowledge does not transfer between tasks. They propose that a term like *knowledge* should be banned because it attributes something like substance or content to the knower. Greeno prefers to use a process-evoking term such as *knowing* because it refers to patterns in the participation of a learner in interactions with others, like peers and teachers. In Greeno's view, learners acquire an activity in response to constraints and affordances of the learning situation. Transfer of an activity to a new situation involves a transformation process. Transfer may occur when the new situation implies similar constraints and affordances as the initial contexts that are perceived as such by the learner.

Lobato's "actor-oriented viewpoint" describes how individuals generate their own similarities between problems by making generalizations based on affordances and constraints (Lobato, 2003, 2006).

The importance of situational "affordances" for the occurrence of transfer is also acknowledged in the work of the Dutch psychologist Van Parreren (1966). As a follower of Kurt Lewin's (1943) activity theory, Van Parreren speaks of the "valence" of situations. He preferred the term "equivalent situations", while stressing the role of the characteristics and intentions of the learner in recognizing situations as equivalent in relation to actions to be accomplished. Thus, whether or not a situation is perceived as equivalent depends on the acknowledgement by the learner of the action possibilities of the given situation. Second, transfer does not occur automatically but is always connected to learner efforts. In Van Parreren's view, transfer has to be actively constructed by the learner (see also Van Oers, 1998, 2001).

We are now in a position to clarify our position in this debate and add a few conclusions. Following Van Parreren (1966), Greeno (1997, 2006), Sfard (1998), and Marton (2006), we believe that there is no reason to abandon the concept of transfer. What a student has attained in previous experiences may influence what the learner is doing in new situations thanks to perceived equivalencies. This always requires an effort from the learner in order that the differences and similarities between structures and principles in various situations may be understood. For instance, a mathematician recognizes differences and similarities in mathematical structures and principles in real-life situations thanks to what Freudenthal (1991) called "mathematization". This is a reflective and constructive activity to make things more "mathematical". Designing

representations in a process of guided co-construction may promote these kinds of understanding, which in turn may facilitate transfer.

Conditions that facilitate transfer

In their studies with elementary school pupils, Brown and Palincsar (1989) specified the conditions for transfer still further. They showed that transfer of old knowledge to new problems can take place when (a) learners are shown how problems resemble each other; (b) when learners' attention is directed to the underlying goal structure of comparable problems; (c) when the learners are familiar with the problem domains; (d) when examples are accompanied by rules, particularly when the latter are formulated by the learners themselves; and (e) when learning takes place in a social context, whereby justifications, principles, and explanations are socially fostered, generated, and contrasted. When these conditions are taken into account, transfer can take place.

In line with the ideas and research findings mentioned, we assumed that the self-(re)construction of flexible schematic problem-solving models contributes to understanding the underlying goal structure of comparable problems and consequently to transfer. To illustrate this, we present an example of such a basic structure that is related to the mathematical topics of fractions and percentages. In learning mathematics, students encounter various underlying concepts and structures such as the notion of number and the part–whole relation. In learning fractions, students become familiar with the part–whole structure. In realistic mathematics education (RME), this structure is not imposed on children. Rather, it emerges in guided collaborative work on problems in real-life situations (Streefland, 1990). At the time when percentages are introduced, the same underlying part–whole structure forms the basis for solving real-life problems. Also, the structure here is not imposed but reinvented or reconstructed by the students. Understanding of the underlying structure can be promoted by engaging in activities such as generalizing, looking for analogies, classifying, reflecting, structuring, defining, symbolizing, and schematizing. Student-generated representations such as drawings, schemes, graphs, diagrams, and so forth, can be used as tools in these activities, for example, in transforming a percentage-representation into a fraction and vice versa. These activities are part of the human activity called "mathematization" (Freudenthal, 1991; Keijzer & Terwel, 2003; Keijzer, Van Galen, & Oosterwaal, 2004).

In line with Lehtinen and Hannula (2006), we believe that transfer theories emphasizing abstract knowledge and situational approaches are not incompatible but could be included in the same framework. Abstract representations seem to be a necessary but not sufficient precondition for transfer. Therefore, the teacher should mediate between the abstract representation and the characteristics of the problem situation by engaging students in the process of mathematization (see the foregoing definition of mathematization and also our description of the program characteristics in the methods section).

Representations that emerge from collaboration may show a higher level of abstraction as a result of the need for bridging and consensus in discussions on an individual's representations (Ainsworth, 2006; Schwartz, 1995). These co-constructed representations need to evolve in a particular, well-specified direction. Therefore, teachers play an important role in guiding their students towards culturally accepted representations. Having learned how to construct mathematically

adequate and more abstract representations presumably places the students in a better position for solving new and even complex problems.

Methods

Participants

This field experiment, comprising an experimental group and a control group, involved 8 schools, 10 classes, 10 teachers, and 238 Grade 5 students (age 10–11 years). A pretest-posttest control group design was used. Of the students, 117 were assigned to the "providing" condition: Teacher-made representations were provided to the students while the latter were learning the percentage concept. This providing approach is the way in which regular education takes place in most primary schools in The Netherlands. The experimental ("co-constructing") group, consisting of 121 students, was exposed to the same mathematical content, but here the emphasis was on "guided co-construction" by students and teacher. This co-constructive learning approach can be characterized as a form of teaching in which the students participate as representation designers in a mathematical context, jointly constructing mathematical representations for the solution of complex problems. The schools participating in the experiment were situated in, or near, two cities in the center of The Netherlands. Experimental and control schools were either middle-class schools or schools with high proportions of ethnic minority children. Both types of schools were equally represented in the two conditions.

The Grade 5 classes were randomly assigned to the control condition or the experimental condition. There were no "drop-outs" (schools, teachers, or classes) during the study.

Program characteristics

In this study, the elements known in the literature to enhance transfer were applied in an intervention aimed at learning how to solve mathematical problems derived from real-life situations. Students were assisted in their attempts to design problem situation models in such a way that the models can be processed mathematically. Students in the experimental program were stimulated to:

(1) work in a productive and active manner on open problems in real-life situations;
(2) contextualize and recontextualize the concepts, structures, and strategies in different contexts;
(3) share in class-wide and small-group discussions on how they dealt with the tasks;
(4) learn from each other by asking for and providing explanations, and to reflect on their representations and problem-solving methods;
(5) learn different methods of constructing representations through teacher demonstrations;
(6) work from informal representations in a well-specified direction to make them more mathematical.

The mathematical content was the same in the experimental and control conditions. The main difference concerned the activity of modeling. In the control

group, a restricted range of representations was *provided* by the teacher and the textbook, while students in the experimental condition were stimulated to *design* their own representations.

Although both conditions partially emphasized the same educational principles, some significant differences were created: In the control group, "working in a productive and active manner" (see Principle 1) meant that children individually worked with, and drew ready-made representations, whereas children in the "generating condition" learned to design and develop their own representations. Instructional principle 2 (contextualize) in the experimental group meant that self-designed models were adapted and applied in various contexts, whereas students in the "providing condition" merely applied provided models in different situations. Instructional principles 2 and 3 in the control group meant that students discussed the way they worked by means of representations provided to them, whereas students in the generating condition discussed their own representations and improved on them. Instructional principles 4, 5, and 6 are especially directed to the design process and thus specified for the experimental students. The research design and the outcomes of this intervention program will be discussed in the section on research design and the results section.

Procedures

The experiment started with separate workshops for each of the conditions. The program and the teacher manual were explained, and materials were discussed. The teachers who participated in the experiment joined one of the two workshops. In the autumn, all teachers started the program in the same week and ended the program 3 weeks later.

The intervention consisted of a 1-hr lesson every day for (almost) 3 weeks. It was composed of 13 lessons: an orientation lesson on representations and their functions and 12 lessons on percentages and graphs. A particular version of the program was constructed for each condition. The versions differed according to the way students learned to work with representations. In the experimental condition, the students learned to design and improve their own representations as a tool for solving percentages problems. In the control condition, the students learned to apply ready-made representations and graphs provided by the teacher. They did not learn to design representations themselves but to use them insightful. All teachers were visited at least twice during the course of the investigation, to give them the opportunity to ask questions about the material. The visits enabled the researcher to monitor the course of the lessons, the validity of the conditions, and the integrity of the intervention. In addition, the teachers completed a short questionnaire after each lesson.

For control of the experimental and control conditions, as well as the measurement of the outcomes, we administered a national standardized pretest and two specially designed tests: a posttest and a transfertest, which will be described in the next section.

Instruments and analyses

Both before and after the intervention, student perceptions of their learning environment were measured as checks on the implementation of the experimental

and control program. An adapted version of the PERCIA–questionnaire (PERception of the Curriculum In Action) was used (Herfs, Mertens, Perrenet, & Terwel, 1991, pp. 173–179). In the present study, the focus was on a newly designed "Model Construction" scale which measures the extent to which students perceive themselves as participants in model construction. One example of a scale item is the following: "Our teacher stimulates us to design our own representations". Students could answer on a 5-point scale ranging from: *this almost never happens* to this *almost always happens*. The alpha coefficients from the pre- and postmeasurements on the perception scale of "Model Construction" are .63 and .74, respectively.

We used a national standardized pretest, a specially designed posttest, and a separate transfer test. The pretest, posttest, and transfer tests were the same for both groups. All three tests proved to be reliable with alpha coefficients of .90, .83, and .76, respectively.

The pretest (covariate) was intended to check students' mathematical knowledge and skills before commencement of the intervention (150 items). The content of the standardized pretest was in accordance with the part of the national math curriculum covered by this particular age group. To a certain extent, this curriculum resembles the principles of Realistic Mathematics Education (RME) of the Freudenthal Institute in The Netherlands (Gravemeijer & Terwel, 2000; Keijzer & Terwel, 2003). Subjects include calculation, measuring, proportions, geometry, and so forth, which may all be regarded as prerequisites for the subjects in the next 2 years of the primary school, including fractions, percentages, and graphs.

Subsequently, the learning outcomes were measured by a posttest (27 items) with situation and domain problems directly related to those used during the lessons.

The transfer test (17 items) was administered last. The transfer items were relatively new problems involving situations and domains that were not directly related to the ones used during the lessons. However, the test was constructed in such a way that both students from the experimental group and students from the control group could be expected to possess the required knowledge to solve the problems presented. All items basically required knowledge of percentages and graphs embedded in real-life problems that involved proportions and relations in various domains. The transfer test consisted of partly open and partly closed tasks, based on new problems that required application of previously learned knowledge in new ways. For example, students were asked to work with permilleages, after a short textual introduction. The principles behind permilleages strongly resemble the principles underlying percentages, but to solve permilleage problems correctly, students are required to reconsider their problem-solving methods.

Figure 1 shows some examples of students' work on the permilleage task in the transfer test. Some children made the connection between their existing knowledge of percentages and representations and used their representations with some adaptations, as can be seen in Figure 1. In the transfer test, the students were introduced to the permilleage concept only by the following text: "Now you've learned that percentage means 'something out of a hundred'. Something similar to percentage is 'per mille'. That means, 'something out of a thousand'." The assignment was formulated as follows, "Can you show in a drawing what 750 permille means?"

Other items from the transfer test required students to use their previously learned knowledge in more open, complex situations and to reflect on the applicability of this knowledge to the new situation and to new subject areas, for example, geometry, biology, or science.
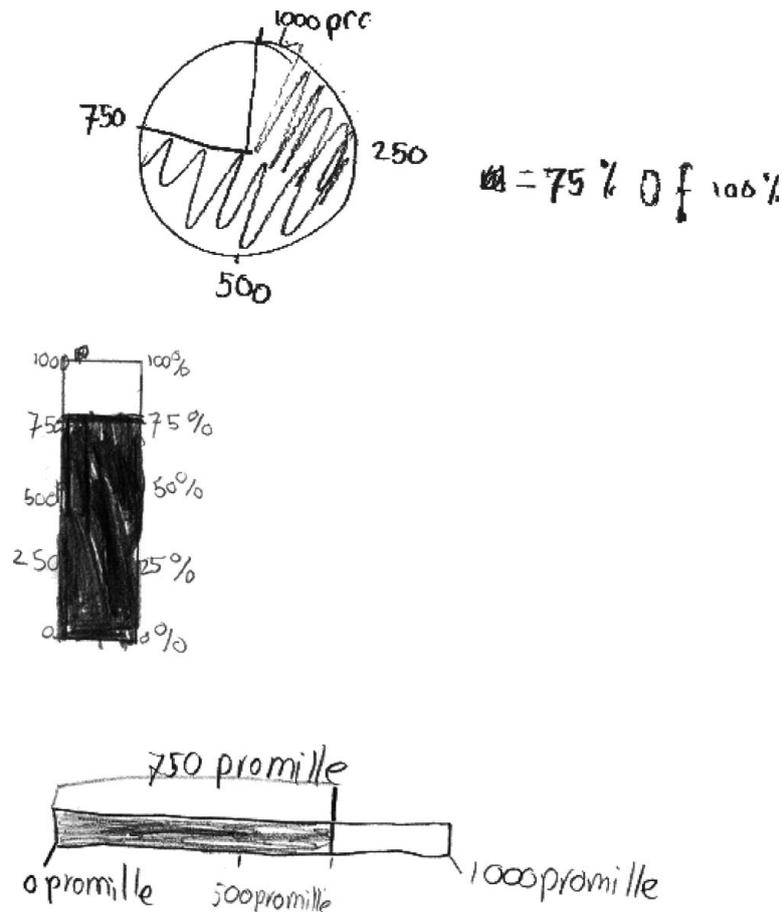
Figure 1.   Examples of permilleage worked-out problems (students' work).

With the aid of these types of item, the test was constructed to give information about (a) what the students had learned about percentages and graphs and (b) their ability to apply these learning outcomes in new ways and to generalize them in solutions to complex problems and to new domains and situations. The problems were designed in such a way that students from both programs were presumed to be able to find solutions, either by using their ready-made representations or self-constructed representations, or a combination of these.

A subseries of tasks in the transfer test was accompanied by a request to show the model used to solve the task. One of these assignments (test items) was about the circulation of water. After a short verbal description of the global water cycle, including elements such as the oceans, the sun, evaporation, condensation, clouds, rain, mountains, rivers, sea, et cetera, students were asked: "Your friend in Groningen has never heard of this cycle in nature. Could you write a letter and make a model (drawing) to explain to your friend how it works?" We present three alternative solutions to this problem, produced by Nienke (see Figure 2), Daniel (Figure 3), and Vanessa (Figure 4), respectively.

Both Nienke and Daniel are students in the experimental group. To give an example of the differences in representation from students of both conditions, we present Vanessa's control group model in Figure 4.

In the next part of the assignment, students were asked the following question: "A few days later you received a letter from your friend. She wrote, 'Thanks for your letter. If I understand you correctly, could I say that not a single drop of water is
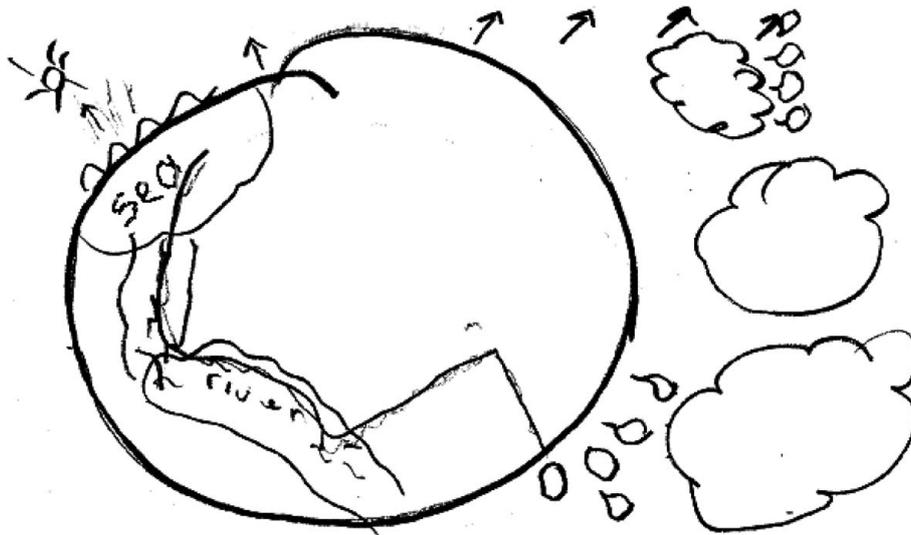
Figure 2. Naturalistic, detailed process design (Nienke's model).

Figure 3. Naturalistic, detailed stages design (Daniel's model).
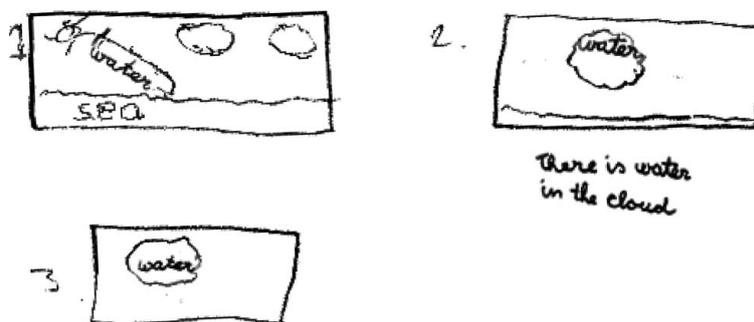
Figure 4. Global stages design (Vanessa's model).

lost?"' The assignment continues with the following question: "Did your friend understand your explanation? How do you know this?" Daniel's answer is: "Yes, not a single drop is lost because the water comes from the sea and goes back to the sea." Vanessa's answer is: "Yes, because I showed what happens in my drawing."

Comparing Models 2 and 3 (experimental students) on the one hand and Model 4 (control student) on the other, there are differences in creativity and originality. It appears that students in the control condition were less flexible in their choice and applications of the ready-made representations provided, while the experimental students presented representations which clearly demonstrated that they were self-constructed products. This demonstrates a kind of "authorship" which may facilitate generalization to novel situations. In addition, the representations designed by the experimental students showed a higher degree of explanatory and communicative value. The same holds true for the verbal explanations given by the students from both conditions.

The representations were scored by an observer and the outcomes compared with the scores of a second rater for 30 students. Inter-observer agreement was determined by Cohen's Kappa (.69). Since resources of time and money were limited, second observers were not feasible for all cases. The observer scored the representations in a blind procedure. She did not know the name of the student, nor the class or intervention the student had attended. A special identification number was attached to each individual test, which was later combined with the original respondent number.

The observer scored all representations on four criteria inspired by the work of diSessa (2002). The four criteria were intended to represent criteria that appear to be important for a qualitatively good model: a solid structure, clarity, accuracy, and completeness. In our view, a good model constitutes a representation that is accurate, neatly structured, conveniently arranged, and complete. Each individual student representation was scored according to these four criteria. Scores on each of the criteria varied from a minimum of 0 to a maximum of 4 points. After each individual representation had been scored, the representation scores were added to the scores on the other transfer test items. The alpha for the total, 17 items on the transfertest, was .76. The results of the total test (17 items) were processed and analyzed.

Results

Classroom processes

Students were closely monitored in the learning process. Monitoring focused on the construction processes that occurred in work on tasks in real-life contexts. Video tapings uncovered several differences in approaches in the two conditions. The control students often struggled to work with representations that were not self-designed and wasted a great deal of time on it. By contrast, students in the generating condition used the same amount of time to think and talk about self-constructed representations and did not seem to find it difficult to apply these self-constructed representations to their solutions.

In some cases, the representations were strongly context bound. However, in time, the representations sometimes came to resemble the formalized representations that were used in the providing class, such as bar models and pie charts. Since the generating students invented these representations themselves in co-operation with others, it was less difficult for them to apply representations to new and unfamiliar situations. The role of the teacher in our experiment was different from the teacher's activity in the traditional classroom, where the teacher mainly

passes on knowledge. In our experimental condition, the teacher's role consisted in being a critical participant in the designing process, by providing counterevidence and/or conventional representations. This meant that teachers were initiating and guiding discussions, asking critical questions, reformulating aspects of students' mathematical contributions, and creating opportunities to reflect on the students' own representations and those of others (including the conventional representations introduced by the teacher). After class-wide discussions, the teacher made no suggestions as to what representation should be used in a particular task. He expected that those who saw the advantages of a particular representation would use it on their own initiative. This way, pupils were given time to develop their use of representations at their own pace or adopt versions of other students' representations.

Before and after the experiment, all students completed the perception questionnaire (PERCIA-scale) on Model Construction. Prior to the experiment, no differences between the experimental and control group were found on Model Construction, which was expected. After the experiment, students were asked to complete the same perception questionnaire. The difference between the mean scores of both groups was significantly in favor of the experimental group. A regression analysis also showed a significant effect of the intervention on the extent to which students perceived themselves as participants in Model Construction over and above the variance already explained by the first measurement on the perception scale. Thus there was clear evidence, also from the students' own perceptions of their learning environment, that our intervention was implemented in classroom processes along the crucial Model Construction variable parameter. Extensive descriptions from observations and video recordings of classroom processes had been made earlier in our studies (Van Dijk, Van Oers, & Terwel, 2003; Van Dijk, Van Oers, Terwel, & Van den Eeden, 2003).

Outcomes

In order to determine the intervention effects, one-way ANOVA and multiple regression analyses were used. Table 1 presents the characteristics of the distribution of the pretest, posttest, and transfer test. In the trajectory from pretest and posttest to transfer test, the experimental group proved to be more successful than the control group. Table 1 shows that the mean scores of the experimental group on

Table 1. Characteristics of the distribution of the standardized pretest, the posttest, and the transfer test for all students in the two conditions (*N*-Students = 238, *N*-Classes = 10).

	M	*SD*	Min	Max
Control program *N*-students = 117				
Pretest	136.07	22.02	49	189
Posttest	43.55	14.76	10	73
Transfer test	30.96	10.18	7.50	59.50
Experimental program *N*-students = 121				
Pretest	137.87	17.80	95	181
Posttest	49.40	13.76	17	75
Transfer test	37.38	9.94	10	60.25

the posttest and the transfer test exceed the control group's mean scores by 6 percentage points.

The effect sizes as defined by Cohen (1988) were calculated. As expressed in the posttest and the transfer test, the "total" effect sizes are .40 and .63, respectively, which are moderate effects. However, if we estimate the "net" effect size for the transfer test – by taking into account the effect of the intervention already expressed in the posttest and correcting for scale differences between both scales – the effect size as expressed in the transfer test shrinks to .23. This net effect size as expressed in the transfer test (over and above the effect size of the posttest) can be regarded as a small effect.

After presenting the improvement on the transfer test as a whole, we will give an impression of the improvement at item level. For reliability reasons, we could not conduct analyses on subsets of the 17 transfer items. However, it appeared that students in the experimental program were ahead on the more complex transfer items, like permilleages tasks (see Figure 1). This task, in which students are supposed to translate their knowledge of the concept of percentages into a model in order to handle a concept as permilleages, is considered a more complex, transfer problem, since students have to transform part of their available knowledge for the correct solution of such problems.

Students of the experimental group score higher at all complex problems, though with different magnitudes. Especially the difference in mean scores on the permilleages task is remarkable. While the students of the control group score a mean of 1.2 points (indicating that most of the students tried to solve the task with some model but that they were not very successful), the students of the experimental group score a mean of 2.7 points, which is twice as much (and indicates that most students received a score of 2 or 3 on a scale of 4).

Analysis

In a one-way analysis of variance, no significant differences on the pretest scores were found (pretest: $F (1,237) = .484$, $p = .487$). However, a one-way analysis of variance between the conditions on the posttest and the transfer test resulted in significant differences in favor of the experimental group, $F (1.237) = 10.01$, $p = .002$) and $F (1, 237) = 24.33$, $p = .000$), respectively. The conclusion is therefore warranted that there is a generally positive effect of the experimental program both on learning results as measured by the posttest and the transfer test. This result is in line with the hypothesis of the study.

We then examined the effects of the variables "pretest", "posttest", and "intervention" on the transfer test outcomes by conducting a multiple linear regression analysis. The pretest, posttest, and intervention variables were subsequently included in the equation by means of a stepwise regression analysis. No interaction effects were found. The outcomes are presented in Table 2 and the coefficients in Table 3.

Table 2 shows that the pretest explains 46% of the variance in the transfer test. The Model 2 posttest explains 9% of the variance in the transfer test. Finally, in Model 3, the intervention explains 4% over and above the variance in the transfer test already explained by the pretest and the posttest. Thus, in this study we were able to explain 59% of the total transfer test variance.

Table 2.   Summary of multiple linear regression analysis for variables predicting the scores on the dependent variable transfer test ($N = 238$).

	Model R	R Square	R^2change	Fchange	Sig. R^2change
1	.681 (a)	.464	.464	204.2	.000
2	.744 (b)	.554	.090	47.4	.000
3	.770 (c)	.593	.039	22.3	.000

a Predictors: (Constant) Pretest.
b Predictors: (Constant) Pretest, Posttest.
c Predictors: (Constant) Pretest, Posttest, Intervention.

Table 3.   Coefficients regression analysis in Table 2 (dependent variable transfer test, $N = 238$).

	Model Variables	B	SE	Beta	Sig.
1	(Constant)	− 14.379	3.444		.000
	Pretest	.355	.025	.681	.000
2	(Constant)	− 3.561	3.519	.313	
	Pretest	.159	.036	.306	.000
	Posttest	.344	.050	.480	.000
3	(Constant)	− 7.137	3.453		.040
	Pretest	.191	.036	.367	.000
	Posttest	.280	.050	.391	.000
	Intervention	4.264	.902	.205	.000

Conclusions and discussion

Before proceeding to the conclusion, we need to say something about the limitations of this study. Although we presented data about the implementation of the programs in both the experimental and control group by using *student perceptions* from all 238 students in both conditions, we also mentioned that, compared to systematic observations, perceptions are limited as measures of implementation. In addition, we were unable to provide detailed descriptions of the interaction and learning processes in the classroom. Extensive, qualitative descriptions of these processes, as well as samples of curriculum materials, were presented in earlier research (Van Dijk, Van Oers, & Terwel, 2003; Van Dijk, Van Oers, Terwel, & Van Eeden, 2003).

A second limitation concerns our definition of transfer. We do not pretend to have covered all forms of transfer. Our measure of transfer as determined by the transfer test took place within the context of the classroom for a specific age group and is limited to certain real-life situations, domains, and concepts.

A third limitation refers to the effects on transfer in the long run. There was no long-term retention test in this project, thus this question remains open and needs to be included in further research.

In this article, we addressed the question of transfer from both a theoretical and an empirical point of view. Our research question was the following:

> With regard to transfer, what are the effects of an experimental primary mathematics program in which students participate as representation designers in a process of guided co-construction?

On theoretical grounds, we hypothesized that the strategy of learning to design representations would be more effective than providing students with ready-made representations. The core of our hypothesis is that students in the previous designing process gained valuable understanding of structures and principles in the mathematical domain, which enables them to generate new solution processes. We, therefore, as a consequence of students' enhanced problem-solving capacities, expected higher scores in transfer problem solving. In order to test this expectation, we created different learning situations for the experimental and control groups, which, by hypothesis, affected the developing generative knowledge in mathematics differently. In a previously published study (Van Dijk, Van Oers, Terwel, & Van den Eeden, 2003), we found, as measured by a mathematical achievement test, significant differences in mathematical problem solving in more familiar contexts. But what can be said about problem solving in new situations and domains? In the present study, we found significant differences between the two conditions. Given our initial hypothesis and the analyses presented, we draw the conclusions given immediately below.

First, taking the ANOVA and the multiple regression analysis into account, we conclude that, compared to the outcomes of the control group, the intervention has a significant and practically relevant effect on learning results. The outcomes of this study clearly show the expected learning results. After controlling for initial pretest differences, and additional control for differences on the mathematical posttest, the program explained 4% of the variance in transfer test outcomes. The total effect size on this test was .63, which indicates a moderate intervention effect. However, the posttest effect is included in this figure. Therefore the posttest effect – which is intermediate – should be taken into account. The remaining net effect size expressed in the transfer test is .23, which corresponds to the 4% variance explained over and above the variance already explained in the posttest.

Second, we found a relatively large effect of domain-specific preknowledge on the transfer test score. Preknowledge in mathematics, as measured by a standardized pretest in mathematics, explained 46% of the transfer test score variance. Over and above the variance already explained by the pretest, the posttest in mathematical achievement explained 9% of the transfer test score variance. These outcomes are in line with Ausubel's well-known statement about the influence of preknowledge on learning: "The most important single factor influencing learning is what the learner already knows" (Ausubel, 1968, p. 18). It is also in line with the literature about transfer, which indicates the domain-specific character of transfer. Although this additional finding requires us to exercise caution in our expectations about the effects of instructional programs on learning and (especially) transfer, we may conclude that students who learn to collaboratively design representations score better on transfer tests than students who learned to work with ready-made models provided by the teacher. In explaining our findings in light of our hypothesis, the important question is: Why did we find precisely those effects? Was it because the experimental students simply learned more of the mathematical content and consequently were able to bring more mathematical knowledge to the transfer test situation? Or were experimental students more effective in applying what they learned to new situations? As presented in Table 2, our analysis allows us to give a clear answer: Students in the experimental condition learned more and consequently could bring more to the final posttest, but if we control for this by including the scores on the first mathematics posttest in the analysis, there remains a significant effect of our

experimental program on the separate transfer test. It is precisely the effect we were looking for. We have good reason to believe that it comes from increased insight as a result of involvement in the process of constructing representations. The question is: What is the precise nature of this insight?

For the experimental students, the model has added value. It originates from the fact that their model was constructed by themselves, albeit under the guidance of the teacher in a well-specified direction toward a culturally accepted representation. The added value consists in greater insight into the underlying abstract structure of a problem that comes from the act of "constructing". In constructing representations, students are focused on the structure of problems rather than simply following the procedural steps involved in the solution of one specific problem. This focus on structure rather than on rule-following is helpful in generating mathematical knowledge, especially when the learner's attention is directed to the underlying structure of classes of comparable problems.

Designing a model forces the designer to reflect on the basic structure of an object and on the core elements in their mutual relations. To participate in the construction and deconstruction of an object or a model enables the person to see the same object or model in a different light. Models can be used for the same purpose and situation from which they originate, but they can also be transformed for use in new situations.

Learning to design representations helps students bring more model-based knowledge to the structure of mathematical problems. Presumably, it also makes them keener on transforming previous knowledge for use in new applications. Students who learn to design representations in co-construction choose to make representations that are more in harmony with their competence level and their daily-life experiences (Van Oers, 1998). At the same time, they have to negotiate on the various representations that emerge from the process of collaboration in order to construct a more abstract consensus model; that is, a model capable of bridging the single models created by individual students. If properly guided by the teacher, the co-construction of representations proceeds – according to Freudenthal (1991) – from "rich to poor", from the particular to the general. This process is a largely student-driven activity but guided by the teacher in a well-specified direction. The resulting, more abstract mathematical model is more adequate and allows for the solution of unfamiliar problems. It helps students to better understand the domain covered by the model. Learning how to construct adequate representations places students in a better position to solve new problems for which no ready-made representations are either provided or available.

As described earlier, we see our "guided co-construction approach" as a way of learning how to organize phenomena – physical, social, and mathematical – and how to reinvent mathematical structures such as the number line and part–whole relations, rather than "mastering" the steps of a formal procedure in mathematical problem solving. Used as a tool, a representation can be moved to new places and be applied more successfully once students understand the structure and the principles by which it works. This may develop a sense of knowledge ownership, which makes students feel free to transform this knowledge as the situation requires. The present study clearly shows that children who learn to design are in a better position to understand pictures, graphs, schemes, models, or similar intellectual tools and are more successful in solving new, complex mathematical problems on relations and proportions. In short, the process of designing representations in collaboration is a

way of *reinventing* the more *abstract mathematical structures and principles*. The process of reinvention, in turn, enhances problem solving in new situations. It follows that increased attention to the process of model designing may be useful in classroom practice.

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