Measurement of $B \to X \gamma$ Decays and Determination of $|V_{td}/V_{ts}|$

(BABAR Collaboration)

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Using a sample of $383 \times 10^6 \bar{B}B$ events collected by the BABAR experiment, we measure sums of seven exclusive final states $B \to X_d \gamma$, where $X_d(X_s)$ is a nonstrange (strange) charmless hadronic system in the mass range 0.6–1.8 GeV/$c^2$. After correcting for unmeasured decay modes in this mass range, we obtain a branching fraction for $b \to d \gamma$ of $(7.2 \pm 2.7$ (stat) $\pm 2.3$ (syst)) $\times 10^{-6}$. Taking the ratio of $X_d$ to $X_s$ we find $\Gamma(b \to d \gamma)/\Gamma(b \to s \gamma) = 0.033 \pm 0.013$ (stat) $\pm 0.009$ (syst), from which we determine $|V_{td}/V_{ts}| = 0.177 \pm 0.043$.

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The decays $b \to d \gamma$ and $b \to s \gamma$ are flavor-changing neutral current processes. They are forbidden at tree level in the standard model (SM), but can occur via one-loop electroweak penguin diagrams involving the top quark. In the SM, the inclusive rate for $b \to d \gamma$ is suppressed compared to $b \to s \gamma$ by $|V_{td}/V_{ts}|^2$, where $V_{td}$ and $V_{ts}$ are Cabibbo-Kobayashi-Maskawa matrix elements. Measurements of $|V_{td}/V_{ts}|$ from $B \to (\rho, \omega) \gamma$ and $B \to K^* \gamma$ [1] have theoretical uncertainties of 7% from weak annihilation and hadronic form factors [2]. A measurement of the inclusive decay $b \to d \gamma$ relative to $b \to s \gamma$ could determine $|V_{td}/V_{ts}|$ with reduced theoretical uncertainties compared to the exclusive modes [3]. In theories beyond the SM [4], new particles may appear differently in the penguin loop diagrams for $b \to d \gamma$ and $b \to s \gamma$ compared to the box diagrams responsible for $B_d$ and $B_s$ mixing [5], leading to differences in $|V_{td}/V_{ts}|$.

This Letter presents the first measurements of $|V_{td}/V_{ts}|$ from $b \to d \gamma$ and $b \to s \gamma$ inclusive decays including the region above the $\rho/\omega$ resonances, with systematic uncertainties largely independent of those from the measurement provided by the exclusive reconstruction of the $B \to (\rho, \omega) \gamma$ and $B \to K^* \gamma$ decay channels.

We present measurements of the rare decays $B \to X_d \gamma$ using seven exclusive final states $B^0 \to \pi^+ \pi^- \gamma$, $B^+ \to \pi^+ \pi^0 \gamma$, $B^0 \to \pi^+ \pi^- \gamma$, $B^0 \to \pi^- \pi^0 \gamma$, $B^0 \to \pi^- \pi^+ \pi^- \gamma$, $B^+ \to \pi^+ \pi^0 \gamma$ and $B^+ \to \pi^+ \eta \gamma$ [6], in the hadronic mass range 0.6–1.0 GeV/$c^2$ (which contains the $\rho$ and $\omega$ resonances), and in the previously unmeasured region 1.0–1.8 GeV/$c^2$. We combine the results and correct for decay modes that are not reconstructed to obtain the inclusive branching fraction for $b \to d \gamma$ in the mass range 0.6–1.8 GeV/$c^2$. A parallel analysis of $B \to X_s \gamma$ using these modes with a $K^*$ replacing the first $\pi^+$ allows us to measure the ratio of inclusive rates $\Gamma(b \to d \gamma)/\Gamma(b \to s \gamma)$ in the same mass range.

This analysis uses $383 \times 10^6 \bar{B}B$ pairs collected at the $\Upsilon(4S)$ resonance with the BABAR detector [7] at the PEP-II B factory. The high-energy $\gamma$ is defined as an isolated energy cluster in the CsI(Tl) calorimeter, with a shape consistent with a single $\gamma$, and energy $1.15 < E_{\gamma} < 3.5$ GeV in the center-of-mass (c.m.) frame. We remove $\gamma$s forming a $\pi^0 (\eta)$ candidate with another $\gamma$ of energy greater than 30(250) MeV, if the two-photon invariant mass is in the range $105 < m_{\gamma\gamma} < 155$ MeV/$c^2$ ($500 < m_{\gamma\gamma} < 590$ MeV/$c^2$).

Charged particle tracks are reconstructed by means of a 5-layer silicon vertex detector and a 40-layer drift chamber coaxial with a 1.5 T magnetic field; a minimum laboratory momentum of 300 MeV/$c$ is required. To distinguish $\pi^+$s from $K^+$s we combine information from the detector of internally reflected Cherenkov light with specific ionisation energy loss measured in the tracking system. At a typical $\pi^+$ energy of 1 GeV, $\pi^+$ selection efficiency is $85\%$ with $K^+$ misidentification rate $3\%$. $K^+$s are selected by inverting the pion selection criteria. We reconstruct a $\pi^0(\eta)$ candidate with laboratory momentum greater than 300 MeV/$c$ from a pair of $\gamma$s, each with energy $>20$ MeV and satisfying $107 < m_{\gamma\gamma} < 145$ MeV/$c^2$ ($470 < m_{\gamma\gamma} < 620$ MeV/$c^2$). The $\pi^0(\eta)$ candidate, the high-energy $\gamma$ and the selected charged tracks are combined to form a $B$ meson candidate consistent with one of the decay modes. For a $B \to X_s \gamma$ decay one $K^*$ is required, with all other tracks required to be $\pi^+$s. For $B \to X_d \gamma$ decays, all tracks are required to be identified as $\pi^+$s. The charged particles are combined to form a common vertex for which the vertex fit probability is required to be greater than $2\%$.

The backgrounds encountered in this analysis arise mostly from continuum $e^+e^- \to q\bar{q}$ events, $q = (u, d, s, c)$, in which an energetic $\gamma$ comes from either initial state radiation or the decay of a $\pi^+(\eta)$ pair. We require $R_2 < 0.9$ and $|\cos\theta_T| < 0.8$, where $R_2$ is the ratio of the 2nd to 0th Fox-Wolfram moments [8], and $\theta_T$ is the angle between the $\gamma$ and the thrust axis of the rest of the event (ROE) in the c.m. frame. The ROE includes all the charged tracks and neutral energy in the calorimeter, excluding the $B$ candidate.

The quantity $\cos\theta_T$ and 12 other variables that distinguish signal from continuum events are combined in a neural network (NN). These include the ratio $R_2$, which is $R_2$ calculated in the frame recoiling against the $\gamma$ momentum, the $B$ meson production angle $\theta_{\gamma}^B$ in the c.m. frame with respect to the beam axis, and five Legendre polynomial moments of the ROE with respect to both the thrust axis of the ROE and the direction of the high-energy $\gamma$. Differences in lepton and kaon production between background and $B$ decays are exploited by including five flavor-tagging variables applied to the ROE [9]. We optimize the NN configuration for maximal discrimination between signal and background; this gives 50% signal efficiency and 0.5% misidentification of continuum, based on a Monte Carlo (MC) simulation.
We use the kinematic variables $\Delta E = E_B - E_{\text{beam}}$ and $m_{\text{ES}} = \sqrt{E_{\text{beam}}^2 - |\vec{p}_B^*|^2}$, where $E_B$ and $\vec{p}_B^*$ are the c.m. energy and momentum of the $B$ candidate, and $E_{\text{beam}}$ is the c.m. beam energy. Signal events should have a $\Delta E$ distribution centered at zero with a resolution $\sim 30$ MeV, and an $m_{\text{ES}}$ distribution centered at the $B$ meson mass with a resolution $\sim 3$ MeV/$c^2$. We retain candidates with $-0.3$ GeV < $\Delta E$ < 0.2 GeV and $m_{\text{ES}}$ > 5.22 GeV/$c^2$ to allow the combinatorial background yield to be extracted from a fit to the data. After all selection criteria are applied there are, on average, 1.75 candidates per event. In events with multiple candidates we select the one with the best $\pi^0(\eta)$ mass, or, where there is no $\pi^0(\eta)$, we select the candidate with the best vertex fit probability.

The signal yield in each $B$ decay category is determined from a two-dimensional unbinned maximum likelihood fit to the $(\Delta E, m_{\text{ES}})$ distributions of the sums of all seven final states. We consider the following contributions: signal, combinatorial backgrounds from continuum processes, $B \rightarrow X \pi^0/\eta$ decays, backgrounds from other $B$ decays, and cross-feed from misreconstructed signal $B \rightarrow X\gamma$ decays. The fit to the $B \rightarrow X_{d,\gamma}$ sample contains a component from misidentified $B \rightarrow X\gamma$ decays, but we neglect the small $B \rightarrow X_{d,\gamma}$ background in the $B \rightarrow X_{s,\gamma}$ sample. The $B$ background yields are determined from MC simulation, whereas the continuum background yield is free to vary in the fit.

Each background contribution is modeled by a probability density function (PDF) determined from MC events. Each signal PDF is the product of one-dimensional $m_{\text{ES}}$ and $\Delta E$ distributions determined from fits to the $B \rightarrow K^+\gamma$ data. For the signal cross-feed component, and the $B \rightarrow X_{d,\gamma}$ background in the $B \rightarrow X_{d,\gamma}$ fit, MC studies indicate that two-dimensional histogram PDFs are required to account for correlations that are not present in signal MC events. The contributions from $B \rightarrow X \pi^0/\eta$ are modeled by a Gaussian peak in each of $\Delta E$ and $m_{\text{ES}}$, where $\Delta E$ is displaced by $-80$ MeV due to the missing photon. The $B \rightarrow X_{s,\gamma}$ background in the $B \rightarrow X_{d,\gamma}$ sample also peaks, with $\Delta E$ displaced by $-50$ MeV due to $K^+$ misidentification. Continuum and other nonpeaking backgrounds are described by an ARGUS shape [10] in $m_{\text{ES}}$ and a second-order polynomial in $\Delta E$.

We perform separate fits for $B \rightarrow X_{d,\gamma}$ and $B \rightarrow X_{s,\gamma}$, in the two hadronic mass ranges. The signal and continuum yields, the continuum ARGUS shape parameter and the continuum polynomial shape parameters are allowed to vary. We scale the cross-feed contribution proportionally to the fitted signal yield, refit, and iterate until the fit converges. The fit projections for $B \rightarrow X_{s,\gamma}$ and $B \rightarrow X_{d,\gamma}$ are shown in Fig. 1.

The fit results are summarized in Table I. The reconstruction efficiency depends on the distribution of the signal yield among the final states. For $X_s$, we obtain this distribution from the data, but for $X_d$ this is not possible and so we use the phase space fragmentation model implemented in JETSET [11] for this purpose.

The branching fractions in Table II are obtained after correcting for missing final states. In the low mass region for both channels we assume that there are no nonresonant decays, an assumption consistent with our data in the $B \rightarrow X_{s,\gamma}$ channel. Our low mass $B \rightarrow X_{s,\gamma}$ measurement agrees with previous rate measurements for $B \rightarrow K^+\gamma$ [12], after accounting for the 50% of decays to neutral kaons. For the $X_d$ modes at low mass, the fraction of nonreconstructed $\rho$ and $\omega$ decays is small, and we find a branching fraction of

**FIG. 1 (color online).** Projections of the fits to data in the hadronic mass range 0.6–1.0 GeV/$c^2$ (a)–(d) and 1.0–1.8 GeV/$c^2$ (e)–(h). Projections of $\Delta E$ with 5.275 < $m_{\text{ES}}$ < 5.286 GeV/$c^2$ for (a),(e) $B \rightarrow X_{s,\gamma}$ and (c),(f) $B \rightarrow X_{d,\gamma}$, and of $m_{\text{ES}}$ with $-0.1 < \Delta E < 0.05$ GeV for (b),(g) $B \rightarrow X_{d,\gamma}$ and (d),(h) $B \rightarrow X_{s,\gamma}$. Data (points) are compared with the sum of all the fit contributions (solid curve) including the signal (dashed curve) and the $B \rightarrow X_{s,\gamma}$ contribution in the $B \rightarrow X_{d,\gamma}$ fit (dotted curve).

**Table I.** Signal yield ($N_S$), average efficiency ($\epsilon$) and partial branching fraction ($\mathcal{B}$) for the measured decay modes. The first error is statistical, the second systematic.

<table>
<thead>
<tr>
<th>$M(X)/[\text{GeV}/c^2]$</th>
<th>$N_S$</th>
<th>$\epsilon$</th>
<th>$\mathcal{B}(\times 10^{-6})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6 &lt; $M(X_d)$ &lt; 1.0</td>
<td>1543 ± 46</td>
<td>8.5%</td>
<td>23.7 ± 0.7 ± 1.7</td>
</tr>
<tr>
<td>0.6 &lt; $M(X_d)$ &lt; 1.0</td>
<td>66 ± 26</td>
<td>7.0%</td>
<td>1.2 ± 0.5 ± 0.1</td>
</tr>
<tr>
<td>0.6 &lt; $M(X_d)$ &lt; 1.6</td>
<td>2279 ± 75</td>
<td>6.1%</td>
<td>48.7 ± 1.6 ± 4.1</td>
</tr>
<tr>
<td>0.6 &lt; $M(X_d)$ &lt; 1.6</td>
<td>107 ± 47</td>
<td>5.2%</td>
<td>2.7 ± 1.2 ± 0.4</td>
</tr>
</tbody>
</table>
(1.2 ± 0.5) × 10^−6, in agreement with previous measurements of \( \mathcal{B}(B \to (\rho, \omega)\gamma) \) [1]. In the high mass region for both channels, we correct for missing final states with ≥5 stable particles, or with multiple \( \pi^0 \)s, by using the fragmentation model described above. Alternative fragmentation models are used to estimate the associated uncertainty, as described below.

The sources of systematic uncertainty in the measurement of the branching fractions are listed in Table III. These include uncertainty in track reconstruction efficiency, \( \gamma \) and \( \pi^0/\eta \) reconstruction, the \( \pi^0/\eta \) veto, the NN selection, and the number of \( \bar{B}B \) pairs. The 2\% uncertainty in \( K^+/\pi^+ \) particle identification and the 20\% uncertainty in \( K^+ \) misidentification, which affects the fitted \( B \to X_s\gamma \) contribution to the \( B \to X_d\gamma \) fits, do not cancel in the ratio. The systematic errors associated with the variation of the fit PDFs also do not cancel because of the very different signal to background ratios in the two samples. We vary the signal PDF parameters within the range allowed by the fit to the \( B \to K^+\gamma \) data. The normalization of the signal cross-feed is varied by ±30\%, and the contribution of \( B \to X_s\gamma \) by ±100\%, in accordance with MC studies. The remaining peaking \( B \) backgrounds, including the \( B \to X_s\gamma \) contribution to the \( B \to X_d\gamma \) fits, are varied by ±20\%. We use simulated signal and background event samples to assign a systematic uncertainty due to possible bias in the fit method.

There is an additional systematic error on the efficiency due to the uncertainties in the measured fragmentation of the \( X_s \) hadronic system into the seven \( B \to X_s\gamma \) final states. The equivalent error for \( B \to X_d\gamma \) is obtained by comparing our fragmentation model for \( B \to X_d\gamma \) to the fragmentation observed for \( B \to X_s\gamma \) data. We assume that these errors are independent and so do not cancel in the ratio of branching fractions.

Table III also shows the systematic errors associated with corrections for the missing final states. There is no information from the data on the missing fraction of high multiplicity final states with ≥5 stable hadrons, or on the missing fraction of other final states with ≥1 \( \pi^0 \) or \( \eta \) mesons. We vary these fractions by ±50\% relative to their default phase space fragmentation values. Our choice of a ±50\% variation is motivated by studies of alternative MC signal models in which we replace half of the nonresonant width in the 1.0–1.8 GeV/c^2 mass range with a mix of \( X_d \) or \( X_s \) resonances. The missing fraction errors partially cancel in the ratio when the ±50\% variations are made in the same direction for \( B \to d\gamma \) and \( B \to s\gamma \).

We take the spectral shape of the high-energy \( \gamma \) from Ref. [13] using the values \((m_{\gamma}, \mu_{\gamma}^2) = (4.65 \text{ GeV}/c^2, -0.52 \text{ GeV}^2)\) extracted from fits to \( B \to s\gamma \) and \( B \to c\ell\nu \) data [14]. We vary these shape parameters in a correlated way between \((m_{\gamma}, \mu_{\gamma}^2) = (4.60 \text{ GeV}/c^2, -0.60 \text{ GeV}^2)\) and \((m_{\gamma}, \mu_{\gamma}^2) = (4.70 \text{ GeV}/c^2, -0.45 \text{ GeV}^2)\). Systematic errors on the branching fractions result from these variations, but they are small and cancel in the ratio. The fraction of the spectrum in the mass range 0.6–1.8 GeV/c^2 is estimated to be \((51 ± 4)\% \) for \( B \to d\gamma \) and \((50 ± 4)\% \) for \( B \to s\gamma \). We do not extrapolate the ratio of branching fractions to \( M_{\gamma} > 1.8 \text{ GeV}/c^2 \), and so these errors, which mostly cancel in the ratio, are not included in Table III. If we make this correction, we obtain \( \mathcal{B}(b \to d\gamma) = (1.4 ± 0.5 ± 0.4 ± 0.1) \times 10^{-5} \) and \( \mathcal{B}(b \to s\gamma) = (4.3 ± 0.3 ± 0.7 ± 0.2) \times 10^{-4} \), where the first error is statistical, the second systematic and the third accounts for the uncertainty in extrapolating to the mass

<table>
<thead>
<tr>
<th>Systematic Error Source</th>
<th>( M(X_s) )</th>
<th>( M(X_d) )</th>
<th>( X_d/X_s )</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracking</td>
<td>1.7%</td>
<td>1.7%</td>
<td>1.7%</td>
<td>1.7%</td>
</tr>
<tr>
<td>High-energy photon</td>
<td>2.5%</td>
<td>2.5%</td>
<td>2.5%</td>
<td>2.5%</td>
</tr>
<tr>
<td>( \pi^0/\eta ) rec.</td>
<td>1.7%</td>
<td>1.7%</td>
<td>1.7%</td>
<td>1.7%</td>
</tr>
<tr>
<td>( \pi^0/\eta ) veto</td>
<td>1.0%</td>
<td>1.0%</td>
<td>1.0%</td>
<td>1.0%</td>
</tr>
<tr>
<td>( K/\pi ) identification</td>
<td>2.0%</td>
<td>2.0%</td>
<td>2.0%</td>
<td>2.0%</td>
</tr>
<tr>
<td>Neural network</td>
<td>5.0%</td>
<td>5.0%</td>
<td>5.0%</td>
<td>5.0%</td>
</tr>
<tr>
<td>( B\bar{B} ) pair counting</td>
<td>1.1%</td>
<td>1.1%</td>
<td>1.1%</td>
<td>1.1%</td>
</tr>
<tr>
<td>Fit PDFs</td>
<td>2.4%</td>
<td>3.6%</td>
<td>7.0%</td>
<td>8.3%</td>
</tr>
<tr>
<td>Backgrounds</td>
<td>0.3%</td>
<td>0.4%</td>
<td>2.4%</td>
<td>6.1%</td>
</tr>
<tr>
<td>Fit bias</td>
<td>0.4%</td>
<td>1.7%</td>
<td>0.4%</td>
<td>3.3%</td>
</tr>
<tr>
<td>Fragmentation</td>
<td>3.6%</td>
<td>7.7%</td>
<td>8.5%</td>
<td></td>
</tr>
<tr>
<td>Partial ( B )</td>
<td>7.0%</td>
<td>11.4%</td>
<td>10.0%</td>
<td>14.8%</td>
</tr>
<tr>
<td>Missing ≥5 body</td>
<td>5.6%</td>
<td>25.8%</td>
<td>21.0%</td>
<td></td>
</tr>
<tr>
<td>Other missing states</td>
<td>17.0%</td>
<td>23.8%</td>
<td>7.1%</td>
<td></td>
</tr>
<tr>
<td>Spectrum Model</td>
<td>1.8%</td>
<td>1.6%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total ( B )</td>
<td>7.0%</td>
<td>21.2%</td>
<td>10.0%</td>
<td>38.1%</td>
</tr>
</tbody>
</table>

TABLE II. Branching fractions \( \mathcal{B}(\times 10^{-6}) \) and their ratio in the two mass regions of \( M(X) \times [\text{GeV}/c^2] \), after correcting for missing final states. The first error is statistical and the second systematic.

<table>
<thead>
<tr>
<th>( M(X) )</th>
<th>( \mathcal{B}(b \to d\gamma) )</th>
<th>( \mathcal{B}(b \to s\gamma) )</th>
<th>( \mathcal{B}(b \to d\gamma)/\mathcal{B}(b \to s\gamma) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6–1.0</td>
<td>1.2 ± 0.5 ± 0.1</td>
<td>47 ± 1 ± 3</td>
<td>0.026 ± 0.011 ± 0.002</td>
</tr>
<tr>
<td>1.0–1.8</td>
<td>6.0 ± 2.6 ± 2.3</td>
<td>168 ± 14 ± 33</td>
<td>0.036 ± 0.015 ± 0.009</td>
</tr>
<tr>
<td>0.6–1.8</td>
<td>7.2 ± 2.7 ± 2.3</td>
<td>215 ± 14 ± 33</td>
<td>0.033 ± 0.013 ± 0.009</td>
</tr>
</tbody>
</table>
range. The result for \( B \to X_s\gamma \) is consistent with the measured inclusive \( b \to s\gamma \) branching fraction of \( (3.55 \pm 0.24) \times 10^{-4} \) [12].

We convert the ratio of partial widths from the full mass range \( 0.6-1.8 \) GeV/c\(^2\), \( \Gamma(b \to d_s) / \Gamma(b \to s\gamma) = 0.033 \pm 0.013 \pm 0.009 \), into a value for \( |V_{td}|^2/|V_{ts}|^2 \) using Table I and Eq. (26) of Ref. [3]. We obtain \( |V_{td}|^2/|V_{ts}|^2 = 0.177 \pm 0.043 \pm 0.001 \), where the first error is experimental, including systematic errors, and the second error is from theory. The theory error includes uncertainties in the CKM parameters \( \rho \) and \( \eta \), and on \( 1/m_c^2 \) and \( 1/m_b^2 \) corrections, but includes no uncertainty for the restriction to the region below \( 1.8 \) GeV/c\(^2\).

As a check, we use the low mass region to determine \( |V_{td}|^2/|V_{ts}|^2 \) using predictions for exclusive \( B \to (\rho, \omega)\gamma \) and \( B \to K^*\gamma \) from [2]. We find \( |V_{td}|^2/|V_{ts}|^2 = 0.214 \pm 0.046 \pm 0.028 \) where the errors are as before. This is in good agreement with previously published results [1].

In summary, we have made the first measurement of \( B \to X_d\gamma \) decays in the hadronic mass range up to \( 1.8 \) GeV/c\(^2\), and have extracted \( |V_{td}|^2/|V_{ts}|^2 \) from an inclusive model with small theoretical uncertainties. These results are consistent with the measurements of \( |V_{td}|^2/|V_{ts}|^2 \) from the exclusive decays \( B \to (\rho, \omega)\gamma \) [1], and with \( B_d/B_s \) oscillations [5]. Future studies applying this method to larger data sets could provide a substantial improvement in the determination of this quantity via radiative \( B \) meson decays. This offers the possibility that new physics effects could be revealed by the comparison of this determination with that from \( B_d/B_s \) oscillations. A measurement of the \( CP \)-violating parameters for inclusive \( b \to d\gamma \) may also be possible.

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[6] Charge conjugate states are implied throughout this paper.
[10] The ARGUS function is defined as: \( P(x) = \frac{1}{\sqrt{2}} \left[ 1 + \left( \frac{x}{m_c^2} \right)^2 \right] \exp\left( -\frac{x}{m_c^2} \right) \), H. Albrecht et al. (ARGUS Collaboration), Phys. Lett. B 185, 218 (1987).