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Mathesis Universalis from Leibniz to Husserl

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The idea of a *mathesis universalis* plays a prominent role in Edmund Husserl's *Formal and Transcendental Logic* (*FTL*).[1] It is clear that at this mature stage of his philosophy the idea he refers to with "*mathesis universalis*" is in large part due to Husserl's own development and cannot be straightforwardly derived from one single author or source anymore. In a historical respect, of course, the idea is most strongly associated with Leibniz, and indeed we see that Husserl does refer repeatedly to him in *FTL* when discussing the *mathesis universalis* (e.g. § 23b). However, Leibniz is not the only author that is relevant for the specific way in which Husserl fills out the notion of *mathesis universalis*, since he also repeatedly refers to more recent 19th century authors such as Bolzano and Lotze, as well as early modern authors such as Vieta and Descartes. Here, as elsewhere, we see Husserl's eclecticism at work.

Despite the attempt to situate Husserl's notion of *mathesis universalis* in a broader historical context, we certainly must acknowledge that one of the most important influences on late Husserl was early Husserl. Indeed, in a pivotal passage of *FTL* Husserl extensively quotes himself, i.e. his own *Prolegomena*, when introducing the core notion of a *Mannigfaltigkeit* (§ 28), stating that "I cannot improve on it".[2] Husserl even goes so far as to claim that we can find the first constitutional investigations of categorial objectivities as far back as his 1891 *Philosophy of Arithmetic* (*PA*). Husserl's early struggles in this "unripe" work[3] with the theories and problems addressed in this "*Erstlingswerk*"[4] then led him, according to his own testimony, to the fuller development in the *Prolegomena*, where the pure theory of manifolds is framed as a meta–theoretical enterprise, a

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1 The idea of a "*mathesis universalis*" in Leibniz sense is presented immediately after the preparatory considerations. See Husserl, Edmund (1974): Formale und transzendentale Logik. Versuch einer Kritik der logischen Vernunft. Mit ergänzenden Texten, Janssen, Paul (ed.), Husserliana XVII, Dordrecht, p. 53.

2 Ib., p. 79.

3 Husserl, Edmund (1956): Persönliche Aufzeichnungen, in: Philosophy and Phenomenological Research XVI/1956, p. 294.

4 This is Brentano's term, see Husserl, Edmund (1994): Briefwechsel. Schuhmann, Karl – Schuhmann, Elisabeth (eds.), Husserliana Dokumente III, Dordrecht, p. 6. Also see Ierna, Carlo (2015): A Letter from Edmund Husserl to Franz Brentano from 29 XII 1889, in: Husserl–Studies 31, 1/2015, p. 68.

theory of all possible forms of theory. This is the ultimate goal of a "theoretical science of theory in general".[5]

Husserl explicitly ties together the notions of pure theory of manifolds and *mathesis universalis*.[6] The *mathesis universalis* in this sense is formal, a priori and analytic. It is an analysis of the highest categories of meaning and their correlative categories of objects, the correlation of *Mannigfaltigkeitslehre* and *Mannigfaltigkeit*. In other words, in the mature stage of his thinking, Husserl identifies his "pure logic" from the *Prolegomena* as a *mathesis universalis* encompassing both the *kategoriale Bedeutungslehre* ("categorial theory of meaning") and the *formale Gegenstandslehre* ("formal theory of objects"), because the apriorical *Bedeutungsgeltung* ("validity pertaining to meaning") has a correlative equivalent in the apriorical *Gegenstandsgeltung* ("validity pertaining to objects").[7] Hence, it is not an unwarranted extension of formal mathematical methods to every other deductive domain, but a new foundation of the apriori validity of what can fall under the form of a manifold. At this highest level the logical analytic examines possible forms of theories as construed *a priori*.

In this context Husserl keeps using the term "*mathesis*", in various combinations such as "pure *mathesis*", "formal *mathesis*", "logical *mathesis*", etc. We actually do not yet find such an extensive use of the term in 1900. In the entire *Prolegomena* there is just one paragraph that addresses the idea of a *mathesis universalis* (§ 60: "*Anknüpfungen an Leibniz*"). Here Leibniz is introduced as "*geistiger Vater der reinen Mannifaltigkeitslehre*" ("spiritual father of the pure theory of manifolds") and also elsewhere Husserl explicitly credits his studies of Leibniz as having paved the way for the breakthrough to anti–psychologism. But where and when did these "*Leibnizstudien*" take place and did they indeed have a noticeable effect on Husserl's development? As Husserl tells us in one of the fragments for a new introduction to the revised 1913 edition of the *Logical Investigations* (*LI*):

> The source of all my studies and the first source of my epistemological difficulties [*erkenntnistheoretischen Verlegenheiten*] lies in my first works on the philosophy of arithmetic and mathematics in general. Strictly scientifically and systematically, I wanted to begin from the ground up, i.e. with the concept of number and at first that of amount [*Anzahl*].[8]

Hence, we will likewise begin with Husserl's earliest writings and see if we can find these *Leibnizstudien* and their potential impact on the further development of his position. To begin with, we now know that there already was a Brentanist

[5] Husserl, Edmund (1975): Logische Untersuchungen (Erster Band: Prolegomena zur reinen Logik). Holenstein, Elmar (ed.), Husserliana XVIII, Den Haag, p. 249.
[6] Husserl, Edmund (2002): Logische Untersuchungen (Ergänzungsband: Erster Teil). Melle, Ullrich (ed.). Husserliana XX/1, Dordrecht, p. 285.
[7] Compare "Entwurf einer Vorrede. Erstes Fragment", ib., p. 291.
[8] Ib., p. 416.

philosophy of mathematics in place before Husserl began to write his early mathematical works,[9] which therefore contain an elaboration of issues already under discussion in the School of Brentano within a pre–existing framework. Specifically, we should bear in mind that one of the central tenets of the Brentanist philosophy of mathematics is that mathematics is analytic, deductive, and a priori:

> Hence mathematics is not an inductive, but a purely deductive, and in this sense, a priori science. Indeed, were it not, then there would be no science at all, neither deductive nor inductive. Because it is not induction that sanctions deduction, but deduction, and specifically mathematical deduction, that sanctions all rational scientific justified induction.[10]

Contrary to the case of mathematics, Brentano considered logic still as a practical procedure: the "art of judging" ("*Kunst des Urteils*"),[11] similar to the *Ars Iudicandi* in Leibniz's sense. Similarly, the project of designing a *characteristica universalis* would be an essentially empirical and practical task for descriptive psychology. Brentano remarked explicitly on his difference from Leibniz:

> We would operate in an essentially different manner as Leibniz and begin with the designation of each of our elementary concepts with a suitably constructed expression. Indeed, I am convinced that in the further development of psychology such signs will result.[12]

Here Brentano envisions the possibility that psychological analysis would ultimately lead to the discovery of a universal system of signs, a *characteristica universalis*. Contrary to Leibniz, the basic building blocks of all meaning would have to be discovered, not designed *a priori*. Symbolic logic or mathematical logic, as in the "algebra of logic" approach, would actually only cover a minor part, i.e. the one relating to judgments. The full project of descriptive psychology would provide a comprehensive symbolism that would allow an encompas-

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9 For a more extensive account of the Brentanist philosophy of mathematics and its influence on early Husserl see Ierna, Carlo (2011): Brentano and Mathematics, in: Revue Roumaine de Philosophie 55, 1/2011, pp. 149–167 and Ierna, Carlo (2017): The Brentanist Philosophy of Mathematics in Edmund Husserl's Early Works. In: Essays on Husserl's Logic and Philosophy of Mathematics. Centrone, Stefania (ed.), Synthese Library 384, Berlin, pp. 147–168.

10 Brentano, Franz: Megethologie, Ms. Meg 40025 f.: "*So ist denn die Mathematik nicht eine induktive, sondern eine rein deduktive und in diesem Sinne a priorische Wissenschaft. Und wäre sie es nicht, so würden weder deduktive noch induktive, und somit überhaupt keine Wissenschaft bestehen. Denn nicht die Induktion ist es, die der Deduktion, sondern die Deduktion und insbesondere Eine mathematische Deduktion ist es, die aller Vernünftige wissenschaftliche Berechtigten Induktion die Sanktion gibt.*"

11 Brentano, Franz (1884/85): Ms. Y 2, Die elementare Logik und die in ihr nötigen Reformen I. Vienna, 1884/85, p. 1.

12 Ib., p. 36. Compare Brentano, Franz: Ms. EL 72, Die elementare Logik und die in ihr nötigen Reformen. pp. 48 f.

sing codification of all mental phenomena: presentations, judgements and affects.

Hence, despite explicit and implicit differences between the preceding approaches in the School of Brentano, we can see that there is a broader context to Husserl's early works in the Brentanist philosophy of mathematics. On the more technical–mathematical side, Husserl engaged intensively with Schröder and Frege in the 1890s: his review of Schröder together with his "*Der Folgerungscalcul und die Inhaltslogik*" were Husserl's very first publications, and his reading of Frege's *Grundlagen* guided the elaboration of the 1887 Habilitation essay into the *PA*. Moreover, both Schröder and Frege had a similar interest in Leibniz' ideas of a *charactersitica universalis* and *mathesis universalis*. Both they and Husserl were at least partially influenced, as far as we know, by Trendelenburg's 1856 "*Über Leibnizens Entwurf einer allgemeinen Charakteristik*." ("On Leibniz' Project of a General Characteristic"), though they did not actually build on the specific details of Leibniz' system, since most of the relevant texts remained unpublished until Couturat's 1903 edition.

Hence, Trendelenburg was one of the few sources available to all three authors at the time. Trendelenburg extensively discusses the background of Leibniz' conceptions, taking into consideration Vieta (and his geometrical "*analysis speciosa*", 41), Raimundus Lullius (and his "*Ars Magna*" or "*Ars Universalis*", 41 f. as well as Leibniz' reception in *De Arte Combinatoria*), and of course Descartes (pp. 42–43). Trendelenburg underscores the idea of developing a *characteristica universalis* as a "*Universalsprache*" a universal language, explicitly mentioning the "cosmopolitical" aspects of the project, i.e. to unite people of all nations, stressing what is universal and common to all.[13] This universal language would allow us to order our thoughts in the same way as we order numbers, it would improve our capacity for correct judgment by being univocal, representing our thoughts in a clear and distinct way, so that mistakes become impossible. But already Trendelenburg sees the limits and dangers of the project, given that a *characteristica universalis* does not yet automatically give us a *calculus ratiocinator* (i.e. a reliable method to computationally "reason" with the signs). While the elementary components of a *charactersitica* might be self–evident, this self–evidence does not therefore automatically also attach to any of their compositions:

> Zusammengesetzte Begriffe bergen nicht selten Widersprüche in sich, welche nur dadurch entdeckt werden, daß die darin mit einander streitenden einfachen Begriffe hervortreten. […] Wenn ferner die Combinationsrechnung, wie es doch scheint, dem calculus ratiocinator zum Grunde gelegt werden soll, so kommt es darauf an, welches Princip aus der mit der Zahl der Elemente immer mehr und mehr wachsenden Zahl

13 Trendelenburg, Adolf (1856): Über Leibnizens Entwurf einer allgemeinen Charakteristik, in: Philosophischen Abhandlungen der Königlichen Akademie der Wissenschaften zu Berlin 2/1856, p. 58: "*Der Gedanke der Charakteristik ist nicht national, sondern, wenn man will, kosmopolitisch. In vielen Plänen geht Leibniz auf das Universale, auf das die Völker Verbindende hin.*"

> der Möglichkeiten die brauchbaren und unbrauchbaren Verbindungen ausscheide. Woher soll dies genommen werden?[14]

Neither Leibniz, nor Trendelenburg can give a satisfying answer here, which is probably why Husserl would dismiss Trendelenburg's take on Leibniz as "superficial" in the *Prolegomena*.[15] Given that Trendelenburg took the trouble of examining Leibniz' manuscripts in Hannover and included many primary materials in his 1856 appendix, this might seem somewhat unfair. Husserl, however, had actually done his own research on Leibniz early on: this is where we can find Husserl's own *Leibnizstudien*.

In the manuscript convolute K I 31 of the Husserl–Archives Leuven, originating in or around 1890,[16] we find extensive excerpts from works on logic and mathematics both from the 19th century as well as from the early modern period, in latin, french, and english. Husserl additionally made comments and notes on the history of the concept of number from antiquity to modern times,[17] remarking on the role of the Indian positional system, Arabic numerals, etc. as well as the attempts to systematize the basic notions of general arithmetic in those various approaches, including some notes on the concepts of magnitude and measurement as well.[18]

The excerpted authors in manuscript K I 31 include among others James Mill and John Stuart Mill, D'Alembert, Gergonne, Lipschitz, Boole, Sigwart, Cantor, Hankel, Chasles, but of course most importantly for our discussion here, Leibniz. Unsurprisingly, many excerpts from Leibniz here deal with the central topics of the *PA*, such as the relation between natural numbers and negatives, fractions, imaginary numbers, etc. that is to say, the progression towards so–called "impossible" numbers and how to deal with them.[19] For instance, the excerpts from Leibniz mention the problem of lifting restrictions on operations,[20] which then lead to the introduction of negatives, fractions, and "absurd" num-

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14 Ib., pp. 47, 54.

15 Husserl (1975) p. 224.

16 One set of notes at least is dated explicitly on July 1890: "*Entwickelung der Formalen Arithmetik* 5.VII.90", Husserl, Edmund: Ms. K I 31, p. 15a.

17 Ib., p. 10a: "How the concept of number and the algorithm of the theory of operations developed historically" ("*Wie entwickelten sich historisch die Zahlbegriffe und der Algorithmus der Operationslehre*").

18 See Ch. XII & XIII of the *PA* where Husserl praises the Indian positional system.

19 Husserl: K I 31, p. 21a: "impossible or imaginary quantities" ("*quantitates impossibiles seu imag.*[*inarias*]") and 21b "when we make the required changes in the given [example], the imaginary vanishes" ("*ubi facta debita mutatione in datis [exemplis], evanescent imaginariae*"). Compare Husserl: K I 31, pp. 3a–b, where Husserl gives an example of calculating with imaginary numbers as temporary assumptions, which disappear in the course of the calculation (see Husserl, Edmund (2005a): Vorlesung Über den Begriff der Zahl (WS 1889/90), Ierna, Carlo (ed.), in: The New Yearbook Vol. V/2005a, pp. 278–308).

20 Husserl: K I 31, p. 18a: "it is not permitted to subtract the greater from the smaller" ("*non licet subtrahere magis ab minore*").

bers.[21] This will be the central issue troubling Husserl at least up to 1894, which he then takes up again in the famous *Doppelvortrag*.

However, Leibniz is not the only early modern author excerpted in the manuscript, and also not the only author relevant to the idea of a *mathesis universalis*. Husserl likewise took notes from Wallis, and Newton, but also excerpts from Descartes' *Geometria*.[22] Notably, both from the French edition by Cousin, as well as the Latin by van Schooten.[23] Given that the Latin translation by van Schooten, in the edition quoted by Husserl, also contained several sets of comments, additions, and treatises by van Schooten and his students and collaborators, this points to a much broader context in which the idea of a *mathesis universalis* first developed and was applied, specifically in a work that served as a source for Leibniz himself. Husserl also excerpted some of these supplementary materials, which are decisively relevant for the development of his own early conception of *mathesis universalis*. These excerpts are about the notion of an "*algebra speciosa*", which is defined as the science of relations and proportions in general, going beyond its application to mere numbers in the "*algebra numerosa*".

The "universality" is here specifically tied to the fact that we use general symbolizations, through letters, that can stand for absolutely anything, for objects in empty formal generality, for "*etwas überhaupt*" ("something in general" or "anything whatsoever"). The *mathesis universalis* is hence formulated here as the formal science of everything. Analytical geometry is simply a specific application of this science to the domain of space. However, in these texts, the *mathesis universalis* is quite explicitly described as a method and an art that is chosen for its immense practicality and the advantages of its universal applicability. The early modern discussion was not concerned with the ultimate justification of such applicability, while this will increasingly become the focus of Husserl's worries, in various stages. This issue is not only related to Husserl's problems with the expansion of the number domain, imaginary numbers, and the passage through the impossible, but also to his later concerns about the mathematization of the natural sciences. Yet, in 1890, Husserl's context is that of the re–elaboration of his *Habilitationsschrift* into his first book, the *PA*, and these excerpts were made within and due to this context. Hence, we see excerpts and comments about real and imaginary numbers, and that no quantity can correspond

[21] Ib., p. 19a: "these originate in the calculation" ("*oriuntur in calculo*").

[22] Ib., pp. 8a, 12a, and extensive excerpts from pp. 60a–64b from the French edition by Cousin, comments in Latin by Florimond de Beaune on "*algebra speciosa*", pp. 65a–66b, to the effect that it would apply to "everything that has any relation or proportion" ("*omne id, quod relationem quamdam habet aut proportionem*"), as Descartes refers in his treatise on method.

[23] Ib.,, p. 67a onwards, the "*universam Mathesin*" is mentioned on p. 68a, "*radices falsae*" ("false roots") on p. 68b and "*imaginarias*" on 69a: "no quantity corresponds to these" ("*nulla iis respondet quantitas*").

to the latter, which Husserl will work out only later, both in the manuscripts intended for the second volume of the *PA* and the *Doppelvortrag*. The excerpts from van Schooten's volume, however, are interesting, since Husserl also transcribed a passage from van Schooten's own treatise on the "*Principia Mathesis Universalis*".[24]

Nevertheless, it is clear that the manuscripts (perhaps together with other still unidentified ones) in convolute K I 31 are the result of the "Leibniz studies" that Husserl mentions in the *Prolegomena*. The numerous excerpts from Leibniz' various works reveal Husserl's deep engagement with the theme of the symbolical mathematical techniques that went under the various names of *Speciosa Generalis*, *Ars Characteristica*,[25] and the other names mentioned earlier, all referring to the algebraic technique capable of expressing "*omnes relationes quae sunt in rebus*" "all relations among things", and thereby gives "*auxilium ad ratiocinandum*", "support for reasoning", not only "*in memoriae usum ad retinendum cogitata*" "for better remembering our thoughts", but also "*ad vim mentis augendam*", "to empower our minds", "*ut incorporalia manu tangat*" so that it can "reach out and touch the abstract, incorporeal ideas".[26] In the excerpts we see Leibniz making explicit connections between the *mathesis universalis* and "mathematical logic" or "the logistics and the logic of the mathematicians",[27] where logic would encompass a theory of names, propositions, and inferences,[28] in parallel to the mathematical domain of quantities, truths about quantities (equations), and inferences (calculations),[29] as well as in both cases the appropriate methods for applying and performing them.

This tripartition does suggest at least a "family resemblance" to the tripartition of the tasks of pure logic in the *Prolegomena*. Moreover, Leibniz points out that there is a hierarchy between the most general and abstract levels of pure logic (as *mathesis universalis*) on the one hand, and the science dealing with quantity and numbers in general on the other (algebra), to which the sciences of concrete magnitudes and measurements are subordinated (arithmetic, geometry, mechanics, etc.) as specific instances.[30]

These intensive studies of Leibniz (and other early modern authors) around 1890 might justify or at least explain Husserl's pithy remark about the superficiality of Trendelenburg. Yet also Husserl's own very first reception and use of such conceptions turned out to be highly problematic as well due to its

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24 Ib., p. 71a.
25 Ib., p. 77a.
26 Ib., pp. 77a–b.
27 Ib., p. 80b.
28 Ibidem: "*In Logica autem sunt Nomines, Propositiones, Argumentationes*"
29 Ibidem: "*Idem est in Analysi Mathematica, ubi sunt quantitates, veritates de quantitatibus enunciatae (aequationes, majoritates, min.[oritates], analogiae, etc.), argumentationes, (nempe operationes calculi)*".
30 Ib., pp. 81b–88a.

superficiality. Husserl would struggle with these problems and confusions throughout the 1890s, both with respect to the symbolization of concepts through signs in a sign system as well as relating to the justification of operating with and applying these signs to various domains.

Husserl began with the aim of founding arithmetic on the concept of number, through an analysis that was both philosophical and psychological. He progressively distanced himself from his former teachers Weierstrass and Brentano and came to the conclusion that the justification of calculations with imaginary concepts could only be due to the system of axioms, specifically that it had to be definite. Husserl elaborated this account in the years 1890 – 1900. The *Prolegomena* and the *LI* are then "The breakthrough of phenomenology" which was "connected to investigations that the Author [Husserl] was concerned with earlier and for many years, investigations that at first regarded the philosophy of arithmetic and pure mathematics in general".[31] Husserl himself remarks repeatedly on the continuity between the *PA* and the *LI*, and indeed we saw that he retrospectively places the source of his epistemological struggles in his early philosophy of mathematics.[32] Already in his very first publications we see that Husserl makes a fundamental distinction between the calculus of logic and the logic of calculus. In his 1891 review of Schröder Husserl's position is clearly delineated: we need to investigate the justification of knowledge obtained through formal deductive methods, because the mechanical "calculus of logic" is not the conceptual "logic of calculus".

> Furthermore, all highly developed deductive disciplines apply symbolic methods to derive truths, they calculate with various algorithms. But is calculating a [form of] deducing? In no way. Calculating is blind symbol manipulation according to mechanically reproduced rules of conversion and transposition of the signs of the respective algorithm. [...] The whole procedure spares us and surrogates for [*erspart und ersetzt*] a manifold of pure deductions, but it is itself not one.[33]

This is the crux of the matter: mathematics and logic which are regarded as the strictest of sciences, which claim to be able to operate with truths to yield other truths do not actually perform conceptual deductions. Instead formal disciplines operate blindly and mechanically with symbols that substitute for the actual concepts. So the status of these disciplines hinges on the justification of the application of the algorithm to the signs: is the mechanical symbol manipulation warranted? If it is indeed true that "calculating is not deducing, but an external surrogate of deduction",[34] then we must look into the process that provides us with these surrogates. Signs substitute for concepts and blindly executed al-

31 Husserl (1975) (Ergänzungsband: Erster Teil), p. 294.
32 Ib., p. 416.
33 Husserl, Edmund (1979): Aufsätze und Rezensionen (1890–1910). Rang, Bernard (ed.), Husserliana XXII, Den Haag/Boston/London, p. 7.
34 Ib., p. 8: "*Rechnen ist aber kein Folgern, sondern ein äußerliches Surrogat des Folgerns*".

gorithms substitute for insightful cognitive processes. How do we obtain the signs and algorithms?

> In the exposition of calculus one can clearly proceed in two ways: Either we can set it up as mere technique, as a sign game so to speak, regardless of any applications. [...] Or we proceed from the outset from a specific domain of application, deriving the primitive formulae from the nature of its concepts, and from there the entire calculus.[35]

Indeed, in his review of Schröder and the related article Husserl argues that *Umfangslogik* (extension–logic) and *Inhaltslogik* (content–logic) operate with the same formal, algorithmic methods. The content of the concepts upon which calculus is based appears to be irrelevant, because the algorithm is the same, whether one uses extension–logic or content–logic. Hence, we see that Husserl separates the logical technique of calculation (*Logikkalkül*) from the logical foundations of calculation (*Logik des Kalküls*). The *Kunstlehre* is just a technique, but does not carry in itself the justification for the application of this technique to its various domains (such as arithmetic). A real logic of calculus must supply a justification (*Berechtigung*) for its application.

Husserl remained ambivalent about the *PA:*[36] while positively quoting it in his later works, he also privately remarked in 1906 that it was "unripe, naïve, and almost childish," that he was "already beyond it" when it was published.[37] In February 1890, before it was even published, Husserl went so far as to claim that it was mistaken in its most fundamental assumption.[38] This is contextualized and clarified by his letter to Brentano from two months earlier:

> I had great difficulties with the full understanding of the logical character of the system of signs of the *arithmetica universalis* with its negative and imaginary, rational and irrational numbers. The matter is not so simple that everything could be completely settled with the concept of amount and the theory of improper presenting.[39]

The solution to this problem is spread over various writings, starting in 1890. In chronological order, step by step we can assemble all the necessary elements. First of all, in his lecture on the concept of number from January 1890 Husserl pointed out that:

> The fundamental error we made was to overlook the fact that all presentations of number that we have at all beyond about 5, are not given to us as real ones, but only as symbolic ones. This fact, which determines the whole character, sense and purpose of arithmetic, has been generally over looked by logicians and mathematicians, and this is to a considerable extent why they could not arrive at a real understanding of this science. And the goal of our preceding deliberations was to vividly portray to them

35 Ib., pp. 29 f.

36 For a more extensive discussion, on which this section is partially based, see Ierna (2017).

37 Husserl (1956) p. 294.

38 Husserl (1994) Vol. I, p. 158.

39 Ierna (2015) p. 71.

> that an understanding of this science based on the proper concepts of number is impossible.[40]

On the basis of Brentano, who on his turn credits Leibniz,[41] Husserl makes a significant distinction between proper and improper presentations and problematizes this precisely in the context of his early philosophy of mathematics. It is this distinction that led to the division of the *PA* into two parts, and it is this problem that occasioned his writing of the lengthy appendix on semiotics "On the Logic of Signs" ("*Zur Logik der Zeichen*").[42]

> "I. The Proper Concepts of Multiplicity, Unity and Amount. II. The Symbolic Concepts of Amount and the Logical Sources of the Arithmetic of Amounts. III. The General Arithmetic of Amount. IV. The Arithmetical Algorithm in Other Domains. V. Concluding Remarks. Appendix: The Investigations into Semiotics".[43]

After having discussed the proper and improper concepts of number and gone into the symbolic methods of calculation in the last chapter of the first volume, Husserl intended to bring his account to a higher level of abstraction by discussing general arithmetic. This is a theme that pervades the first volume already: general arithmetic, the *arithmetica universalis*. The arithmetical algorithm can be developed by starting in a specific domain, but we can abstract from it and see that this arithmetic, applied to the domain in question is just an instantiation of a more general structure. Husserl begins with the arithmetic of amounts (i.e. cardinals), sees that its methods are formal and hence "agnostic" with respect to the domain of application, and then purports to extend arithmetic to new domains. Indeed, Husserl already in his letter to Stumpf from February 1890 tells us that the "algorithm is the same everywhere" and moreover that "already the analysis of the ordinal number led me to this": we cannot "deduce" other kinds of numbers from amounts, i.e. cardinals, by any "tricks" of "improper presenting".[44] This analysis of ordinals is the "*Arithmetik der Reihen*":[45]

> They [all number forms developed up to now] can be conceived independently from each other, and even, when they are not, each area requires a special arithmetic for its-

[40] Husserl (2005a) p. 297.

[41] Brentano: EL 72, p. 268.

[42] For a detailed discussion of this text, including useful references to other secondary sources, see Byrne, Thomas (2017): Husserl's Early Semiotics and Number Signs: Philosophy of Arithmetic through the Lens of "On the Logic of Signs (Semiotic)", in: The Journal of the British Society for Phenomenology 48, 4/2017, pp.287–303.

[43] Husserl: Ms. K VI 2, p. 18.

[44] Husserl (1994) Vol. I, pp. 158 ff.

[45] Husserl, Edmund (1983): Studien zur Arithmetik und Geometrie. Strohmeyer, Ingeborg (ed.), Husserliana XXI, Den Haag/Boston/Lancaster, pp. 154–214. Also see Ierna, Carlo (2005): The Beginnings of Husserl's Philosophy (Part 1: From Über den Begriff der Zahl to Philosophie der Arithmetik), in: The New Yearbook for Phenomenology and Phenomenological Philosophy Vol. V/2005, pp. 47–48, n. 170.

> elf. The fact that this arithmetic possesses the same formal rules everywhere, i.e., that the algorithm is identical, that is a fact in itself.[46]

One cannot alchemically transmute the cardinals into ordinals or imaginaries.[47] During the crucial year of 1890 Husserl seems to have been significantly inspired by his reading of Leibniz, given how closely his position seems to match what we quoted from the excerpts above ("*evanescent imaginariae*"):

> Finally, I noticed that through the calculation itself and through its rules, as defined for those fictional numbers, the impossible vanishes and a correct equation remains.[48]

A central insight here is that if the rules are right, the result will be right ("*die Rechnung bleibt richtig, wenn sie regelrecht ist*"), whatever (proper or improper, cardinal or ordinal) it is applied to. The universality of the algorithm justifies the application, not the foundation on a specific kind of number.

This is also Husserl's position in last phases of the *PA*, and the second volume would have shown that it is not the proper concept of amount that is fundamental, but the algorithm: the *arithmetica universalis*. General arithmetic should be need considered as "a general theory of operations" i.e. actually as part of a more encompassing logic as an "art of signs",[49] in a nearly complete parallel to the hierarchy pointed out in the excerpts from Leibniz we saw above.

Between the *PA* and the *Prolegomena*, we see the same approach in Husserl's 1896 logic lectures. Here, like in 1891, what makes the *arithmetica numerosa* as well as the *arithmetica universalis* work, is still the parallelism between signs and concepts,[50] and signs serve still the role that Brentano, Trendelenburg, and Leibniz identified for them: to avoid the ambiguities of natural language.[51] Beginning around 1895 Husserl puts more emphasis on the justification of the application of the signs and their rules. Husserl acknowledges that descriptive psychology cannot give any foundational insight into such a justification, similarly to Trendelenburg's observation that the *characteristica* alone does not straightforwardly yield a *calculus*. In 1895, however, Husserl begins in the same way as his treatise on ordinals, remarking that we can develop the algorithm starting

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46 Husserl (1983) p. 175.

47 Ib., pp. 42 f. This account is familiar from his own later retrospective self–analysis: the issue of calculating with seemingly contradictory concepts (e.g. $\sqrt{-1}$) led him to the position of the 1901 *Doppelvortrag* on the "Passage through the Imaginary." See Schuhmann, Elisabeth, and Schuhmann, Karl (2001): Husserls Manuskripte zu seinem Göttinger Doppelvortrag von 1901, in: Husserl Studies 17/2001, pp. 87–123 and Ierna, Carlo (2011): Der Durchgang durch das Unmögliche. An Unpublished Manuscript from the Husserl–Archives, in: Husserl–Studies 27, 3/2011, pp. 217–226.

48 Husserl (1994) Vol. I, p. 160.

49 Ib.,p. 161.

50 Husserl, Edmund (2001): Logik Vorlesung 1896. Schuhmann, Elisabeth (ed.), Husserliana Materialienbände I, Dordrecht, pp. 247–248, and in 1895 explicitly also between actual and mechanical deduction, see Ib., pp. 311 f.

51 Ib., p. 252.

from either cardinals or ordinals and then apply it to all others, but then also adds that we could develop the system straightaway *in abstracto*.[52]

This additional possibility, of defining formal numbers directly, then seems to lead to the theory of manifolds as we know it from the *Prolegomena*. The *arithmetica universalis* can be purely analytical only if we construct the concept in reflection on the form of the rules.[53] In order to provide a deductive, analytic, a priori foundation of mathematics (as in the Brentanist philosophy of mathematics), Husserl had to go beyond his initial framework to a larger more encompassing account, which subsumes it without invalidating it. Based on the excerpts found in K I 31, Leibniz clearly played an important role in various steps of this development. I hope to have shown here, however briefly and superficially, that there is a remarkable continuity in Husserl's concerns from his early beginnings up to the latest phases of his thinking. A continuity of concerns, not necessarily of terminology and theory, but also a continuity in inspiration, as we saw in particular here, with respect to the idea of a *mathesis universalis* in Leibniz.

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[52] Ib., pp. 314 f.

[53] Ib., p. 315.

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