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A note on the worst case complexity for the capacitated vehicle routing problem

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A note on the worst case complexity for the Capacitated Vehicle Routing Problem

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Abstract

The best worst case complexity of $\mathcal{O}(n^2 2^n)$ to solve optimally the Traveling Salesman Problem is achieved by the Dynamic Programming algorithm of Held and Karp from 1962. This is exponentially better than an exhaustive enumeration of all $\mathcal{O}(n!)$ feasible solutions. For the Vehicle Routing Problem we were unable to find similar results in the literature. We present a Dynamic Programming algorithm for the Capacitated Vehicle Routing Problem based on the above mentioned algorithm. We also prove the complexity of this new algorithm to be exponentially better than exhaustive enumeration.

Keywords: vehicle routing problem, dynamic programming, complexity analysis

1. Introduction

For almost half a century the algorithm for solving the Traveling Salesman Problem (TSP) optimally with the best known worst-case complexity is Dynamic Programming (DP) proposed by Held and Karp [2]. This Dynamic Programming has a complexity of $\mathcal{O}(n^2 2^n)$. For the Capacitated Vehicle Routing Problem (CVRP) the best known worst-case complexity still seems to be an exhaustive enumeration of all possible solutions, which is $\mathcal{O}((n+v)!)$. Note that the worst case of branch and bound algorithms is the total enumeration of the solution space of the corresponding ILP formulation, often worse than the total enumeration of Feasible solutions. For the TSP all binary possibilities of n^2 variables is $\mathcal{O}(2^{n^2})$ which is much larger than $\mathcal{O}(n!)$. In this paper we propose an algorithm for the CVRP based on the Dynamic Programming algorithm of Held and Karp for the TSP using the Giant Tour Representation (GTR) for the VRP proposed bu Funke et al. [1] . First we will take a look at this Giant Tour Representation, then propose the Dynamic Programming for the CVRP and finally make an complexity analysis of this algorithm.

1.1. The Capacitated Vehicle Routing Problem

In the Capacitated Vehicle Routing problem n customers have to be visited by m vehicles. For each customer i we have a location r_i and a quantity q_i that has to be delivered to that customer ($i \in \{1, 2, \ldots, n\}$). For each vehicle j we have a start location s_j and a finish location f_j and a maximum capacity Q_j ($j \in \{1, 2, \ldots, v\}$). Note that the vehicle fleet can be heterogenous, different start, and finish locations and different capacities for each vehicle are allowed. Furthermore we have a complete directed graph $G(V, A)$ in which the vertices $V = \mathcal{R} \cup \mathcal{S} \cup \mathcal{F}$ are the locations $r_i \in \mathcal{R}$, $s_j \in \mathcal{S}$ and $f_j \in \mathcal{F}$, and the lengths of the arcs A represent the cost of traveling between these locations. The goal is to find the combination of routes with the lowest cost where every customer is visited exactly once by a single vehicle delivering the desired quantity and the total quantity delivered by vehicle j does not exceed capacity Q_j.

2. Giant Tour Representation

Funke et al. [1] introduce the Giant Tour Representation (GTR). With the representation all separate routes of a VRP solution are *glued* together as if it were one single route. Every finish node f_j in the graph G is connected with start node s_{j+1}, finally f_v is connected with s_1 creating a Hamiltonian path through graph G. In Figure 1 we present a solution to a VRP, the first vehicle, serving customers 1 and 4 starts and finishes in A, the second starts in A and finishes in B, and the third starts and finishes in B. Thus s_1, f_1 and s_2 all correspond to location A and f_2, s_3 and f_3 to location

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B, in Figure 2 we see the GTR corresponding to the same solution.

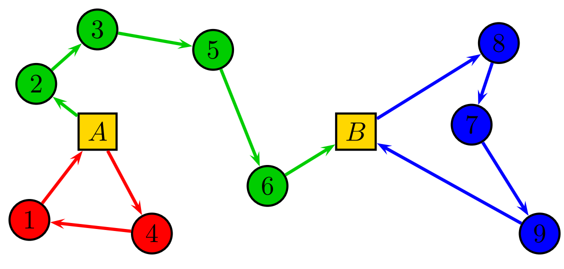

Figure 1: Example of a VRP solution with three vehicles

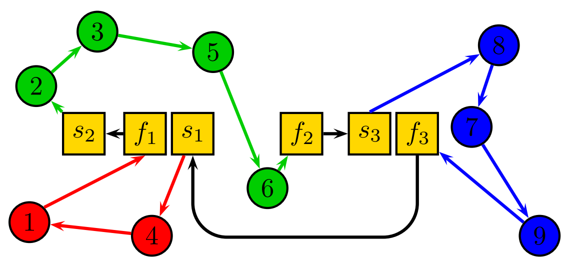

Figure 2: The Giant-tour representation of Figure 1

We can reduce the graph G by merging every node pair f_j and s_{j+1} to a single node d_j using the outgoing arcs of s_{j+1} and the incoming arcs of f_j, forming a new smaller graph $G'(V', A')$, with $V' = \mathcal{R} \cup \mathcal{D}$ where $\mathcal{D}$ consists of the merged nodes from $\mathcal{S} \cup \mathcal{F}$. Note that connecting the finish node of one trip with the start node of the next trip costs 0 anyhow. We can limit the arcs connecting d_j as the connecting arc (f_j, s_{j+1}) is the only other arc used for f_j and s_{j+1} and it is used in every solution represented in the form of a GTR.

3. Dynamic Programming

With the GTR every solution to a VRP becomes a hamiltonian path in G, where the optimal solution to the VRP is the shortest of such paths. This is exactly the same problem as solving the TSP for graph G. As we now have to solve a TSP we can use the Bellman equation of Held and Karp for the TSP. Which is formulated as follows.

$$\begin{cases} C(\{i\}, i) & = \quad c_{si} \\ C(S, i) & = \quad \min_{j \in S \setminus \{i\}} C(S \setminus \{i\}, j) + c_{ji} \end{cases}$$

Where $C(S, i)$ is the cost of the optimal path from starting node s through all nodes $S \subseteq V$ ending in node $i \in S$ (s not necessarily in S), and c_{ji} the cost to traverse the arc from j to i. Unfortunately this does not yet give us a valid algorithm for the CVRP because we need to ensure the feasibility of the solutions as some solutions of the TSP on graph G are infeasible for the CVRP. A solution has to represent a valid GTR and the quantity delivered by a single vehicle may not exceed the capacity of that vehicle. We can accomplish a correct GTR by only allowing subsets $S \subseteq V$ such that if $d_i \in S$ we have that $d_j \in S \; \forall_{j<i}$, ensuring the correct order of the vehicle start-finish nodes d_i. Note that for a set S the current vehicle in the GTR is uniquely determined, precisely the vehicle j such that $\min_{i=\{1,\ldots,v\}} d_i \notin S$. Preventing overloading of vehicles can be done by checking the residual capacity of the current vehicle and prohibiting the expansion to customers where a higher quantity has to be delivered.

However this can cause DP to find suboptimal solutions or not a solution at all, because DP may discard the partial path leading to the optimal solution during the minimum in the Bellman equation. This can happen when two partial solutions σ_1 and σ_2, with the same nodes S are compared, and σ_1 has a lower cost but σ_2 has more remaining capacity in the current vehicle. σ_2 may lead to the optimal solution while σ_1 may lead to a more expensive solution or may never lead to a feasible solution at all. To correct this we add the remaining capacity if the current vehicle to the Bellman equation, in a similar way as Malandraki et al.[3] incorporated the time dimension in their DP algorithm for the TSP with time windows. Now the Bellman equation becomes.

$$\begin{cases} C(\{i\}, i, Q_1 - q_i) & = \quad c_{d_v i} \\ C(S, i, q) & = \quad \min_{j \in S \setminus \{i\}, q' \geq q + q_i} \\ & \qquad C(S \setminus \{i\}, j, q') + c_{ji} \end{cases}$$

Where $C(S, i, q)$ represents the cost of the optimal path of the start node s_1 (incorporated in d_m through S ending in i having at least q residual capacity, and $q_i = -Q_j$ in case i represents the node that incorporates the start of vehicle j. This equation leads to Algorithm 3.1.

4. Complexity

In Algorithm 3.1 the first four loops in the main part of the algorithm loop over all states $C(S, i, q) \neq \infty$. Then these states are expanded to at most $n + 1$ nodes. It is not possible to get $C(S, i, q) \neq \infty$ for all combinations of S, i and q, even if we only allow S which can represent a feasible GTR solution. Especially not for all q it is possible to have $C(S, i, q) \neq \infty$, a good example is that for $C(\{i\}, i, q)$ we only have a single value of q that gives a finite value $(Q_1 - q_i)$. For all sets S we have $S \subseteq V'$ and $|V'| = n + v$ so we have at most

Algorithm 3.1 Dynamic Programming for the CVRP

Input: A graph $G'(V', A')$
Assume: $C(S, i, q) = \infty \ \forall_{S,i,q}$
Output: The cost optimal solution to the CVRP

```
for all i ∈ R do
  C({i}, i, Q_1 − q_i) = c_{d_v i}

C({d_1}, d_1, Q_2) = c_{d_v d_1}                      // only if empty vehicles are allowed

for l = 1 to |V'| do                                // for all sequence lengths
  for all S ⊆ V' : |S| = l and d_i ∈ S if d_j ∈ S with i < j do
    Let j be the current vehicle.                   // min_{i={1,...,v}} d_i ∉ S
    for all q = 0 to Q_j do
      for all i ∈ S do
        if C(S, i, q) ≠ ∞ then
          for all k ∈ (R ∪ {d_j}) \ S do
            if k = d_j then
              C(S ∪ {d_j}, d_j, Q_j) = min{C(S ∪ {d_j}, d_j, Q_j), C(S, i, q) + c_{i d_j}}
            else if q ≥ q_i then
              C(S ∪ {k}, k, q − q_i) = min{C(S ∪ {k}, k, q − q_i), C(S, i, q) + c_{ik}}

C = ∞
for all q = 0 to Q_v do
  C = min{C, C(V', d_v, q)}
return C
```

2^{n+v} subsets, however we can limit this by eliminating all subsets for which no valid GTR can be constructed. For every set S for which a valid GTR exists we have a value g such that $d_1, \ldots, d_g \in S$ and $d_{g+1}, \ldots, d_v \notin S$. For a fixed g and $l = |S|$ we have $\binom{n}{l-g}$ ($g \leq l$ and $l - g \leq n$) sets S for which a valid GTR can be constructed. If we sum this over all g and l we get $\sum_{g=0}^{v} \sum_{l=g}^{n+g} \binom{n}{l-g} = (v+1)2^n$ possible sets $S \subseteq V'$ for which a valid GTR can be constructed. We have $|V'| = n + v$ possibilities for i and q can take the values $0, \ldots, Q$ where $Q = \max_{j=1,\ldots,v} Q_j$ is the maximum vehicle capacity. With these elements we get a total complexity of dynamic programming for the CVRP of

$$\mathcal{O}\left((Q+1)(v+1)(n+1)^2 2^n\right).$$

This complexity $\mathcal{O}\left(Qvn^2 2^n\right)$ is for every bounded Q exponentially less than brute-force with a complexity of $\mathcal{O}((v+n)!)$.

5. Conclusion

In this paper we presented an algorithm to solve the CVRP to optimality based on the Dynamic Programming formulation of Held and Karp for the TSP. This algorithm has a worse case complexity of $\mathcal{O}(Qvn^2 2^n)$, which is exponentially better complexity than exhaustive enumeration with complexity $\mathcal{O}((n+v)!)$, for every bounded value of Q, which represents the maximum capacity of the vehicles involved. Furthermore this algorithm accommodates vehicle fleets with heterogeneous capacities different start and finish locations and asymmetric traveling costs. The Capacitated Vehicle Routing Problem in its most general form.

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