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Passenger oriented railway disruption management by adapting timetables and rolling stock schedules ☆

Lucas P. Veelenturf [a,*], Leo G. Kroon [b,c,1], Gábor Maróti [c,d]

[a] School of Industrial Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
[b] Rotterdam School of Management, Erasmus University, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands
[c] Process quality and Innovation, Netherlands Railways, Utrecht, The Netherlands
[d] Faculty of Economics and Business Administration, VU University Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands

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In passenger railway operations, unforeseen events require railway operators to adjust their timetable and their resource schedules. The passengers will also adapt their routes to their destinations. When determining the new timetable and rolling stock schedule, the railway operator has to take passenger behavior into account. The operator should increase the capacity of trains for which the operator expects more demand than on a regular day. Furthermore, the operator could increase the frequency of the trains that serve stations with an additional demand.

This paper describes a real-time disruption management approach which integrates the rescheduling of the rolling stock and the timetable by taking the changed passenger demand into account. The timetable decisions are limited to additional stops of trains at stations at which they normally would not call. Several variants of the approach are suggested, with the difference in how to determine which additional stops should be executed.

Real-time rescheduling requires fast solutions. Therefore a heuristic approach is used. We demonstrate the performance of the several variants of our algorithm on realistic instances of Netherlands Railways, the major railway operator in the Netherlands.

1. Introduction

In passenger railway operations, unforeseen events (such as infrastructure malfunctions, accidents or rolling stock breakdowns) can make parts of the railway infrastructure temporarily unavailable. Then it is not possible to operate the timetable, rolling stock schedule and crew schedule as planned. Within minutes, or even better, seconds, a new timetable and new resource (rolling stock and crew) schedules must be available. In Cacchiani et al. (2014) an overview is given of recovering models and algorithms to solve these rescheduling steps. In this overview it becomes clear that, although the schedules are interdependent, most research focuses on rescheduling one of the schedules at a time. By the complexity of the rescheduling problems, there is not enough time to solve the integrated problem. In this paper we partly integrate the rescheduling of the rolling stock plan and the timetable. Our particular focus lies on passenger service, and we take passenger behavior explicitly into account.

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* Corresponding author.
E-mail addresses: l.p.veelenturf@tue.nl (L.P. Veelenturf), lkroon@rsm.nl (L.G. Kroon), g.maroti@vu.nl (G. Maróti).

[1] This paper is dedicated to the dear memory of Leo Kroon who passed away unexpectedly on the 14th of September, 2016.

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Current literature on integrated rescheduling of the timetable and the rolling stock schedule is scarce. Adenso-Díaz et al. (1999) and Cadarso et al. (2013) applied research on integrated rescheduling of the timetable and rolling stock on cases of the Spanish railway operator RENFE.

Like the main focus of this paper, (Cadarso et al., 2013) take the dynamics of the passenger behavior during a disruption into account. In the current paper, however, the fundaments of the approach are from Kroon et al. (2015). There, the focus is on improving passenger service by considering passenger behavior while rescheduling the rolling stock. They use an iterative procedure for rescheduling the rolling stock and evaluating the resulting passenger behavior which is inspired by the iterative framework of Dumas and Soumis (2008). Changing the timetable can also improve the passenger service. Therefore we extend the iterative procedure by allowing the timetable to be slightly adapted as well.

It is important to focus on the passenger service since a disruption does not only affect the timetable and the resource schedules, but also the passengers. However, it is difficult for railway operators without a seat reservation system to reschedule the passengers: the passengers choose their new travel plan by themselves.

By the changed passenger flows, the disruption causes changes in the demand for seats. Therefore, a rescheduling approach to handle a disruption must take the modified passenger flows into account dynamically, and not the passenger flows of a regular day. For example, since some passengers will take a detour, additional capacity on the detour routes is necessary. An increased capacity can be created by allocating more rolling stock or by timetable adjustments. The timetable adjustments can be achieved by inserting new timetable services or by introducing additional stops in existing services.

In this research we limit the timetable decisions to adapting the stopping patterns. More substantial timetabling measures, such as the insertion of additional services, would require non-trivial crew rescheduling efforts. In contrast, we assume that small delays caused by adapting the stopping patterns leave the crew schedule essentially intact.

Optimizing the stopping patterns form a key element in the approaches of Yue et al. (2016) and Jamili and Pourseyed Aghaee (2015) for medium to long term train timetabling. The time-dependent character of the passenger demand is explored by Niu and Zhou (2013) under oversaturated conditions and by Niu et al. (2015) under non-saturated conditions. To the best of our knowledge, our approach is novel in that we adjust the stopping patterns in a real-time rescheduling context, and we combine it with both with dynamic passenger flows, and with detailed rolling stock rescheduling.

Our focus lies on railway networks in which passengers have the free choice of taking trains, that is, they are not constrained by a seat reservation system. We consider rather large disruptions that lead to the cancellation of many services. Our method is best suited for disruptions where the traffic is not completely blocked; we hope to improve the service quality by applying careful changes to the stopping patterns besides rolling stock rescheduling measures.

Our algorithm is intended for network-level timetabling and rolling stock rescheduling. We consider a macroscopic view of the underlying network; the block occupation, signaling systems and the acceleration of the trains fall out of our scope. In particular, we assume that the very limited timetable adjustments of our algorithm result in a feasible timetable. For the sake of clarity we assume that the duration of the disruption is known upon its start. We note that the uncertain length of the disruption can be considered by embedding our models in a rolling horizon framework (see, e.g., Nielsen et al., 2012).

Delay management, which consists of deciding on whether or not trains have to wait for delayed connecting train services, is another problem in which slight timetable adaptations influence the passenger flows. Delay management is a hard problem on its own and thereby not considered in our approach. We refer the interested reader to Schachtebeck and Schöbel (2010), Kanai et al. (2011) and Dollevoet et al. (2012) for recent works on delay management approaches.

This paper is organized as follows. The framework of our rolling stock rescheduling approach is discussed in Section 2. We make use of an iterative procedure of which in Section 4 up to Section 5 the components are explained. A lower bound for the model is introduced in Section 6 and results of different variants of our approach, based on a scenario in the Netherlands are discussed in Section 7. Section 8 concludes this paper.

2. Problem description

2.1. Rolling stock and timetable rescheduling with dynamic passenger flows

We consider disruptions where passenger behavior has a large impact on the performance of the railway system if the timetable and rolling stock schedule are not changed. Passengers react to these disruptions by finding alternative routes to their destinations. However, the capacity on these alternative routes can be limited, resulting in overcrowded trains and thereby longer dwell times and delays.

Two ways to handle the increased demand on the alternative routes are to enlarge the capacity of the trains and to adapt the timetable. Adapting the capacity of the trains alone is not always enough. For example, it can be impossible to increase the capacity of a train by lack of time and/or reserve rolling stock or due to limited platform lengths. Therefore, timetable adaptations such as adding extra timetable services, rerouting timetable services or introducing extra stops in timetable services are worthwhile as well.

By adapting the timetable, the railway operator can influence the passenger flows: new alternative travel routes are provided and the demand for certain timetable services is changed. For example, a service can have an additional stop at a station to give passengers at that station an additional travel option with an earlier arrival time to their destination. At the same time, the demand is decreased for the next train stopping at that station and traveling in the same direction.

In this research we limit the timetable changes to adaptations of the *stopping patterns* of the timetable services. A stopping pattern indicates the stations where the train makes a stop. A stopping pattern can contain, next to the scheduled stops, also new stops at stations where the train did not have a scheduled stop. By the stopping pattern a timetable services is divided into *trips* representing a movement of a train between two consecutive planned stops.

The consequences of the timetable adaptations may not turn out to be advantageous for all passengers. For example, an additional stop in a timetable service will delay the service with a few minutes. As a consequence, passengers on board of the train will get an additional delay in favor of reducing the delay of the passengers at the station at which an additional stop is made. The small delay of the service can even lead to a large delay for the passengers if they miss their transfer at a later station. The railway operator has to find the right balance between the benefits and the disadvantages of the timetable adaptations.

2.2. A general mathematical framework

The performance of the disruption management process investigated in this paper arises from the interaction of three factors: (*i*) the timetable, (*ii*) the rolling stock schedule (seat capacity), and (*iii*) the passenger behavior. We propose a general framework for rescheduling the rolling stock and timetable by considering the passenger behavior.

Let $\mathcal{Z}$ denote the set of all possible timetables given the disruption, and let $\mathcal{X}_z$ denote the set of all feasible rolling stock schedules for timetable z. The function $f(x,z)$ returns the emerging passenger flows for timetable $z \in \mathcal{Z}$ and rolling stock schedule $x \in \mathcal{X}_z$. Note that the chosen timetable z and rolling stock schedule x uniquely determine the passenger flows y by the function f. This means that the only real decision variables are the rolling stock schedule x and the timetable z. Then the model read as follows.

$$\min c(x) + d(y) + e(z) \tag{1}$$
$$\text{subject to } z \in \mathcal{Z} \tag{2}$$
$$x \in \mathcal{X}_z \tag{3}$$
$$y = f(x,z) \tag{4}$$

The objective function consists of three terms. The function $c(x)$ gives the rolling stock rescheduling costs. The function $d(y)$ gives the service related costs of the passenger flows. The function $e(z)$ gives the timetable rescheduling costs. In our application, the highest priority is given to assigning at least one rolling stock unit to each timetable service, to prevent the service to be canceled by lack of rolling stock. Such cancellations will not only have a large negative influence on the passenger flows, but also make the crew schedule infeasible.

2.3. Iterative procedure

The optimization model (1)–(4) is very difficult to solve directly, mainly due to the structure of the passenger behavior function *f*. Recent research (such as Niu and Zhou, 2013; Niu et al., 2015; Wang et al., 2015) did address the problem of computing the passenger flows. However, the dependence of the flows on the timetable and on the rolling stock schedules does not appear to admit a tractable mathematical programming formulation for (1)–(4). In fact, we are not aware of any algorithmic framework that would be able to handle realistic instances of (1)–(4).

Therefore we propose an extension to the iterative heuristic of Dumas and Soumis (2008) and Kroon et al. (2015); the approach is sketched in Fig. 1.

The input of our algorithm consists of the original (i.e., undisrupted) timetable, the original rolling stock schedule as well as a list of timetable services that must be canceled as an immediate reaction to the disruption.

The algorithm starts with the rescheduling of the timetable. The removal of the inevitably canceled services due to the disruption gives the initially modified timetable. In each next iteration in the timetable rescheduling step, we evaluate which adaptations of the timetable could potentially improve the service quality. Each individual adaptation is a minor change, such as requiring a train to make an extra stop. Therefore we can assume that the just computed rolling stock schedule remains feasible. We describe several variants for finding the most promising timetable adaptation in Section 3.

Having decided on the timetable, the next iteration will start by launching a passenger simulation to evaluate the passenger flows. The simulation is based on the previous iteration's timetable and rolling stock schedule. Here the rolling stock schedule is only needed because it determines the capacities of the trains. We use the simulation model introduced by Kroon et al. (2015). The details of this simulation model are summarized in Section 4. Note that the first iteration assumes that each timetable service has rolling stock assigned with the same capacity as in the original schedule.

The passenger simulation pinpoints to the timetable services with rolling stock of insufficient capacity. The rolling stock rescheduling model computes a new schedule based on these findings, balancing it with other criteria, such as operational costs. For details we refer to Section 5.

Our method differs from the framework of Kroon et al. (2015) by adding the timetable adaptation step to the loop. Since the passenger flows can be heavily impacted both by a new rolling stock schedule and by an adapted timetable, we carry out passenger simulations after each of them.

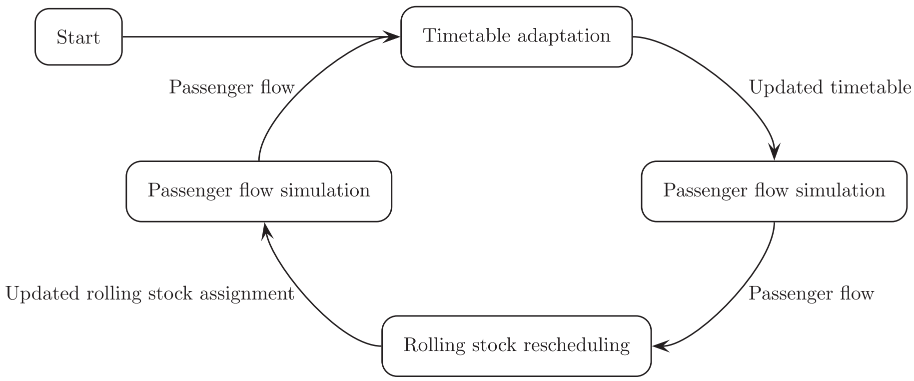

Fig. 1. Iterative procedure for solving the rolling stock rescheduling problem with dynamic passenger flows.

The iterative approach is purely heuristic; it does not necessarily converge, and has no optimality guarantee. Motivated by the limited time in real-life applications, we terminate our algorithm after a certain number of iterations, and we report the best solution found. In addition, we compute lower bounds, described in Section 6, in order to be able to judge the quality of the solution.

3. Timetable adaptations

The iterative procedure starts with updating the timetable. In the first iteration, the timetable is adapted by removing all services from the timetable which must be canceled as an immediate reaction to the disruption. In each next iteration we focus on timetable decisions in order to better facilitate the passenger flows. In this paper we limit the allowed timetable modifications to adding stops to timetable services.

In this research, adapting the stopping patterns means that trains may stop at stations where they normally just pass through. Making an additional stop results in new traveling options for some passengers but also in an increased travel time for others. Therefore it is necessary to make a trade off between the positive and negative effects of the changed stopping pattern. The objective of this research is to minimize the sum of the delays of all passengers. Therefore, we only allow timetable changes that do not increase the total delay of all passengers. We assume that an additional stop will delay all further trips of the timetable service by a fixed number of minutes and that those delays will not influence other train traffic.

The (greedy) procedure adapts the stopping patterns as follows: (i) we collect a list of candidate timetable adaptations, (ii) we evaluate for each candidate the consequences, (iii) we apply the timetable adaptation with the best consequences.

Our approach requires a given list of candidate timetable adaptations in step (i). The dispatchers can give this as input to the approach. In the extreme case, every timetable service is allowed to make an additional stop at every station it passes.

The effect measured in step (ii) indicates how much the total delay of the passengers will change if only that single timetable adaptation will be applied. Therefore, in step (iii) we limit ourselves to allow only one timetable adaptation per iteration. If no candidate timetable adaptation reduces the total delay of the passengers, no timetable change is made.

For all candidate timetable adaptations, the consequences of applying the adaptation need to be computed. In Sections 3.1,2,3.4 we discuss several methods and approximations to compute these consequences. In Section 3.1 we discuss a method to compute the exact effect of the additional stop. The exact effect is computed by using a deterministic simulation for the passenger flows, but the computation can be time consuming. Therefore, we propose a faster algorithm to approximate the effects in Section 3.2. Furthermore, we introduce two heuristics in Sections 3.3 and 3.4 which are more transparent for use in practice.

In the different variants we evaluate the effects of different candidate timetable adaptations. In what follows, T is the current timetable, and $T^{\star}$ is a candidate timetable in which a timetable service i makes an additional stop at station b.

In this research we allow one single additional stop per iteration. We admit that the latter assumption limits the solution space. Our choice is pragmatic; it is motivated by the explosive growth of the number of possible combinations of additional stops.

3.1. Exact effect of an additional stop (EXACT)

In order to determine the exact consequences of an additional stop, we run the passenger flow simulation twice: first in T and then in $T^{\star}$. We compare the total passenger delay minutes of the two simulations; the difference shows the consequences of the candidate timetable adaptation. This approach measures the exact effect of the additional stop and is called *EXACT*.

A variant of *EXACT*, denoted by *EXACT*∗, will not use the current capacity of service i in the simulation but the capacity of the largest possible composition allowed for service i. By this change, we do not measure the exact effect of the extra stop but its *potential* effect. It is left for the rolling stock rescheduling phase to try to increase the capacity of service i.

3.2. Estimated effect of an additional stop (EST)

In this section we introduce an approach to *estimate* the effect of an additional stop. Both the positive and the negative effects of the additional stop are considered. We assume that the preferred path of all passengers is known, both in T and in $T^{\star}$. In this research, a path of a passenger refers to the sequence of train rides. The preferred paths arise from simple shortest path computations in our application (see Section 4).

Suppose that we decide to adopt $T^{\star}$ in which trip i makes an extra stop at a certain location. This decision is certainly beneficial for passengers who arrive earlier at their destination due to an additional stop at their origin or destination. Other passengers also profit from the delay caused by the additional stop, as it allows them to transfer to timetable service i; while this transfer is not possible in timetable T. All these passengers do not have i on their preferred path in T, but they do have it on their preferred path in $T^{\star}$.

The additional stop can lead to a negative effect for passengers who have i on their preferred path in T. The delay of i may force them to change their preferred path due to a missed connection; the new preferred path often has a later arrival time. Even if the preferred path remains unchanged, the passengers still can become delayed; this happens if i is the final part of their journey.

To estimate the effect of the additional stop we compare, for each passenger, the arrival time of the preferred paths in T and in $T^{\star}$. The sum of these differences is our estimation of the consequences of the additional stop. Note that this is an estimation since this method assumes that every passenger can take his preferred path which might not be true by the limited capacity of the rolling stock compositions.

The estimation of the additional stop's effect is governed by a parameter called *extra stop penalty* which expresses the duration of an additional stop in the computation of the preferred path. That is, the preferred path indicates the passenger's best (e.g., earliest arriving) journey to his/her destination under the assumption that the extra stop penalty is equal to trip i's arrival delay due its additional stop.

We want to point out that the extra stop penalty is used in the estimation only; the passenger flow simulation is always based on the actual duration of the additional stop. For example, if we assume the extra stop penalty to be smaller than the actual duration, we over-estimate the potential effect of the additional stop. We will investigate whether particular choices of the extra stop penalty would guide our algorithm to better solutions.

3.3. Rule of thumb I: Do not pass rejected passengers (PRACT1)

In case of insufficient capacities, passengers do not fit in a train. We say that these passengers are *rejected* by the train, and they have to wait for the next one. It is particularly frustrating for rejected passengers to see a train that passes their station without stopping. A simple rule of thumb in practice could be that it is not allowed to pass a station where rejected passengers are waiting. We will call this rule *PRACT1*.

To decide whether or not a timetable service i needs to make an additional stop at b we have to consider two situations.

The first considered situation is that service i needs to make an additional stop at b because at station a (the station before b) people traveling to b where rejected by the previous service h. The additional stop at station b let these rejected passengers travel from station a to b.

The second situation considered is that service i needs to make an additional stop at b because the previous service j stopping at b had rejected some passengers. Service i will make an additional stop at station b to let these rejected passengers enter the train. Note that in most cases service j refers to the same service as h, but it in principle it might also be a different service.

Since we allow one timetable adaptation per iteration we have to make a comparison on how effective the additional stop will be. To that end, we sum up the advantages for all passengers who were rejected to board service h in station a or service j in station b. For the passengers rejected to board h in station a the advantage is measured by the difference between the arrival time of i at station b and the arrival time of the first service from a to b after i. For the passengers rejected to board j in station b the advantage is estimated by the difference between the arrival time of i at the first station c after station b where both services h and i stop and the arrival time at c of the first service departing after i.

In this measurement we assume that all passengers are able to board i, so that i is assumed to have infinite capacity. We do not use the actual capacity; the rolling stock rescheduling phase will be responsible for increasing the capacity of i.

3.4. Rule of thumb II: including the negative effects (PRACT2)

The approach *PRACT1* only considers the positive effects of an additional stop. This results in an additional stop even if only one passenger may benefit from it. We propose another rule of thumb *PRACT2* in which we do include the negative effects, i.e., the delay for passengers traveling by service i. In this approach, the advantages for passengers are measured in the same way as approach *PRACT1*, and use a parameter for the inconvenience of a passenger who travels by i at the moment when i makes the additional stop at b. In our experiments this parameter is equal to the duration of the extra stop. That is, we neglect the fact that the inconvenience can be quite different, for example, by missing a transfer.

Note that practitioners need more information for *PRACT2* than for *PRACT1*. In *PRACT1*, the dispatchers only need to monitor whether there are services where some passengers did not fit in the train. In *PRACT2*, the dispatchers also need to know how many passengers did not fit in the train, and how many passengers are in the next service passing station b.

4. Simulation of the passenger flows

Each time the timetable or rolling stock schedule is updated, a new simulation of the passenger flows is necessary. We use the passenger flow simulation algorithm of Kroon et al. (2015); in this section we give a brief summary of the main assumptions. It is important to mention that this is a deterministic simulation algorithm to calculate the emerging passenger flows. This means that, given a timetable and rolling stock schedule, there are uniquely defined resulting passenger flows. We emphasize that the approach is modular, which allows us to replace the simulation model by any other simulation model to model the passenger behavior.

4.1. Assumptions

To keep the simulation tractable, all passengers with the same characteristics (origin, destination and arrival time at the origin) are aggregated into *passenger groups*.

It is assumed that *passengers always know the most recent timetable*. This means that if the timetable is updated due to a disruption, they know which services are canceled, which services make additional stops, and which services are delayed. *Passengers do not know the future timetables*, so they cannot anticipate on cancellations, delays and additional stops before the disruption occurs. Furthermore, they do not know anything about whether or not they fit in the trains they would like to take.

Passengers want to reach their destination as early as possible. If several paths have the same earliest arrival time, the passengers prefer the path with the least transfers between trains. It is worthwhile to mention that in practice there is a more balanced trade off between transfers and travel time. It seems to be highly unrealistic that passengers are willing to transfer 2 times to save 1 minute of travel time.

Each passenger wants to reach his destination before a certain ***deadline***. If a passenger is not able to reach his destination before the deadline, it is assumed that the passenger gives up traveling by train. In this way it is modeled that passengers are not willing to wait endlessly.

If a train arrives, *first the passengers who want to leave the train get the option to do so*, then the passengers who wait at the platform and want to enter the train compete for the available capacity in the train.

It can happen that there is not enough capacity for all passengers. Then it is assumed that the number of passengers from each passenger group (i.e. passengers with the same destination) who actually board a train is proportional to the size of the group. Some of them have to stay behind. We say that these passengers are *rejected* by the train. The rejected passengers must find a new preferred path from their current location to their final destination.

4.2. Evaluating the passenger flows

In this research we evaluate the passenger flows by two criteria: (*i*) the delays which passengers face in comparison with their original expected arrival times and (*ii*) the number of passengers who gave up traveling by train since they were not able to reach their destination within their set deadline. In our experiments we try different ways to penalize delay minutes. In one setting the delay minutes are penalized uniformly and in another setting longer delays are penalized more, since one may argue that longer delays are worse than several small delays. For passengers who are not able to reach their destination within their deadline we penalize passengers leaving the system by the difference between the deadline and the expected arrival time of the intended traveling path. For each passenger, his delay or penalty for not reaching his destination within his deadline is called his *inconvenience*.

5. Rolling stock rescheduling

The rolling stock is rescheduled using the model described in Nielsen et al. (2012) which is an extension of Fioole et al. (2006). The basic decisions in the model are to assign a rolling stock composition to each trip such that as many of the passengers are accommodated. The difficulty arises from the fact that a composition consists of multiple combined rolling stock units.

During operations the operator can change the compositions by decoupling or coupling units in the front or the back of the train. These operations are called *shunting operations*. Shunting personnel must be arranged to facilitate these operations. Therefore, changing the shunting operations also includes new tasks for the shunting personnel, which is not preferred.

Since a composition can consist of different types of rolling stock units, the order in which they are combined within a composition matters (i.e. one could not decouple a unit in the middle).

The main objective of our approach is to prevent cancellations caused by lack of rolling stock. Therefore we first determine how many services need to be canceled due to lack of rolling stock. To do this we run the rolling stock rescheduling approach on the initially modified timetable with the single goal to find a feasible rolling stock schedule by minimizing the number of services without rolling stock. This means that we have only a penalty for services which do no get rolling stock assigned to them. All other penalties are set equal to 0. The result shows how many services need to be canceled inevitably by lack of rolling stock. In the rolling stock rescheduling steps we enforce the number of canceled services to be equal to this value to ensure that no more services than necessary are canceled. Still the rolling stock rescheduling approach has freedom to assign no rolling stock to a service.

For all remaining rolling stock rescheduling steps, the model has two objectives: it consists of a trade off between minimizing the rolling stock rescheduling costs and the inconvenience for the passengers. The rolling stock rescheduling costs express how much the rolling stock schedule is changed. For example one does not want to make too many new shunting operations, since these new shunting operations must be communicated (with a certain probability of miscommunication) and require personnel to perform them.

The inconvenience for passengers is based on the latest simulation run with the timetable and rolling stock schedule of the last iteration. Penalties are defined for assigning a certain composition to a trip. The penalties are determined by estimating the effect of the composition capacity on the total passenger inconvenience measured as discussed in Section 4.2.

We compute for each trip the average inconvenience of passengers who were not able to board the train. To do this, per passenger group the difference in inconvenience between passengers who were not able to board and passengers who were able to board is determined. Then, the weighted (based on group size) average of these differences is considered as the average inconvenience per passenger who is not able to board the train. In the objective function the number of seat shortages is multiplied by this average inconvenience per rejected passenger.

Kroon et al. (2015) report that the approach of updating the objective function could lead to cyclical behavior if the feedback from earlier iterations is ignored. We follow their described exponential smoothing procedure (which is based on Dumas and Soumis, 2008) to take feedback from earlier iterations into account as well. We use the setting which performed best in their case. This setting means that feedback from earlier iterations is weighted for 35%.

6. Lower bounds

The proposed approach does not guarantee to converge to an optimal solution. To consider the quality of our solutions, we check the gap between a lower bound and the value of our solution. Depending on the nature of the disruption, the lower bound on the rolling stock rescheduling costs will not differ that much from 0, but the lower bound on the passenger delays can be quite interesting. First we will discuss a lower bound which assumes operator control. Since it will be complex and time consuming to solve this lower bound, another lower bound is proposed which takes the positive effects of an additional stop into account.

6.1. Operator control

The rescheduling process discussed in Section 2 may result in a better outcome if the operator can directly influence the passengers' behavior by appropriately assigning them to the timetable services (rather than letting the passengers choose their routes). We call this situation *operator control*. In this section we describe an optimization model for operator control which is a relaxation of the model (1)–(4). We present this relaxation of the model to give an idea about its complexity, and thereby justifying our use of an iterative procedure. Since it is a computationally complex model we present another lower bound in Section 6.2 which will be used for our computational experiments.

We split all timetable services into trips $t \in \mathcal{T}$ representing a movement of a train between two consecutive planned stops. The main decision for the rolling stock schedule is to assign compositions to trips, where a composition consists of one or more combined rolling stock units. Let $\mathcal{G}_t$ be the set of all compositions g which can be assigned to trip t, and the capacity of composition g is denoted by Cap_g. Binary variables $x_{t,g}$ indicate whether composition g is used ($x_{t,g} = 1$) for trip t or not ($x_{t,g} = 0$).

For the timetable decisions every trip $t \in \mathcal{T}$ has a set $\mathcal{J}_t$ of possible stopping patterns for stops at the intermediate stations. Let $\mathcal{J} = \cup_{t\in\mathcal{T}}\mathcal{J}_t$ be set of all possible stopping patterns. Here a stopping pattern indicates a sequence of intermediate stations at which the train makes an additional stop. Binary variables z_j indicate whether stopping pattern j is used ($z_j = 1$) or not ($z_j = 0$).

A passenger $p \in \mathcal{P}$ should take a path from its origin to its destination within his/her proposed deadline, where a path itself is a sequence of rides on trains between two stations. Let K^p be the set of all paths that passenger $p \in \mathcal{P}$ could take and let $K^p_t \subset K^p$ be all paths including (part of) trip t which passenger p could take. Note that the paths in K^p and K^p_t can be based on every possible stopping pattern. Let L_j be the set of paths that are incompatible with stopping pattern j. The binary variable y^p_k is 1 if passenger p picks path k and 0 otherwise. The parameter d^p_k indicates the associated cost (delay) of passenger p taking path k.

Then, in case of operator control, the model of (1)–(4) can be relaxed by:

$$\min \quad c(x) + \sum_{p \in \mathcal{P}} \sum_{k \in K^p} y_k^p d_k^p + e(z) \tag{5}$$

$$s.t. x \in \overline{\mathcal{X}}_z \tag{6}$$

$$\sum_{j \in \mathcal{J}_t} z_j = 1 \quad \forall t \in \mathcal{T} \tag{7}$$

$$\sum_{k \in K^p} y_k^p = 1 \quad \forall p \in \mathcal{P} \tag{8}$$

$$\sum_{k \in K^p \cap L^j} y_k^p + z_j \leqslant 1 \quad \forall p \in \mathcal{P}, \text{ and } \forall j \in \mathcal{J} \tag{9}$$

$$\sum_{p \in \mathcal{P}} \sum_{k \in K_t^p} y_k^p \leqslant \sum_{g \in \mathcal{G}_t} x_{t,g} Cap_g \quad \forall t \in \mathcal{T} \tag{10}$$

$$y_k^p \in \{0,1\} \quad \forall p \in \mathcal{P} \text{ and } \forall k \in K^p \tag{11}$$

$$z_j \in \{0,1\} \quad \forall j \in \mathcal{J} \tag{12}$$

The objective function (5) is to minimize the total costs of the rolling stock rescheduling, the passenger flows (sum of delays) and the timetable rescheduling. Constraints (6) compactly summarize the constraints on the underlying rolling stock rescheduling problem (see e.g. Nielsen et al., 2012). These rolling stock decisions are influenced by the chosen timetable z since there are some minimum process times required in the rolling stock schedule. So, if some trips take longer than planned certain processes can become infeasible. Constraints (7) determine that for every trip exactly one stopping pattern must be selected. Every passenger must pick exactly one path, which is modeled by Constraints (8). Constraints (9) ensure that only matching paths and stopping patterns can be chosen. The chosen paths by the passengers should also match with the available capacity in the trains which is modeled by Constraints (10).

6.2. Lower bound for the computational experiments

The main idea for our lower bounds is the following observation. Consider a hypothetical timetable where (i) the arrival times are equal to those in the original timetable; (ii) each potential additional stop is made; and (iii) a transfer between two trips is possible whenever this transfer is possible in at least one of the feasible timetables. Moreover, suppose that each train has an unlimited capacity.

By carrying out a passenger simulation in this hypothetical timetable, we obtain a lower bound of the passengers delays of *any* timetable subject to *any* rolling stock schedule. Indeed, the hypothetical timetable admits all passenger journeys that can be realized in at least one feasible timetable, and passengers are always able to board the trains. Therefore the simulation in the hypothetical timetable simultaneously yields a lower bound on the delay of each individual passenger group.

Such a hypothetical timetable can be constructed by defining the event times as follows. Each train's arrival time is equal to its original arrival time. For a train's departure time, consider its departure times in all feasible timetables, and take the latest one of them. Recall that feasible timetables do shift the departure and arrival times as a consequence of the additional stops.

7. Computational results

We tested the proposed approach on instances based on cases of Netherlands Railways (NS) which is the major railway operator in the Netherlands. In these instances, a disruption, due to some blocked switches, caused that fewer timetable services than normally can be operated on certain tracks.

7.1. Detailed case description

The instances take the busiest part of the Dutch railway network into account which is represented in Fig. 2. The original passenger flows are constructed conform a regular weekday of Netherlands Railways, which resulted in 15,064 passenger groups with a total of about 450,000 passengers.

In almost all parts of the network we have four Intercity services per hour in each direction. An *Intercity service* is a train which only stops at larger stations. All Intercity services in this network are considered, furthermore the *regional services*, that stop at every station, between The Hague (Gv) and Utrecht (Ut) are also considered.

For the rolling stock rescheduling, four types of rolling stock are available; two types for regional services and two types for intercity services. The regional rolling stock types can be coupled together, which leads to 5 possible compositions, and the Intercity rolling stock types can also be coupled together in 10 different compositions.

In Fig. 2 the dotted line represents the disrupted area. On a normal day each hour 4 Intercity services and 4 regional services run in each direction over the tracks of the disrupted area. We constructed two instances with a disruption in the rush

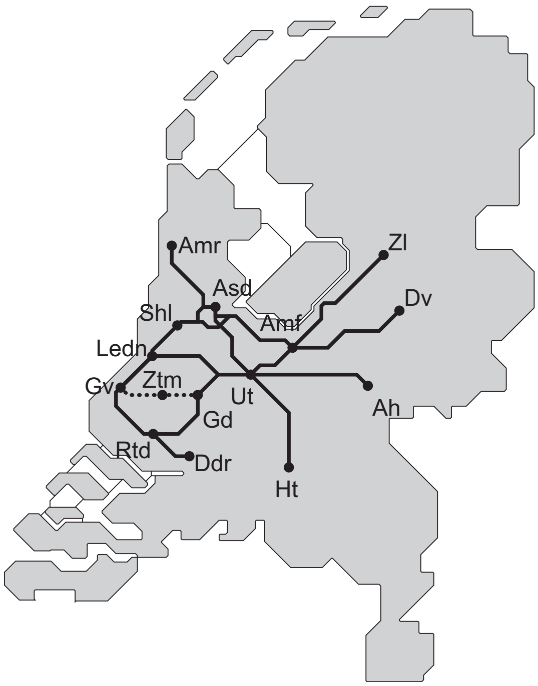

Fig. 2. Part of the Dutch railway network.

hours between 7:00 A.M. and 10:00 A.M. In the first instance (*ZTM1*), 2 regional services per hour per direction are canceled. In the second instance (*ZTM2*) also 2 Intercity services per hour per direction are canceled. This means that in instance *ZTM1* in each direction 6 services per hour are still operated between Gouda (Gd) and The Hague (Gv), and only 4 services per hour in each direction in instance *ZTM2*.

During the disruption all Intercity trains which pass regional station Zoetermeer (Ztm), may perform an additional stop at this station. This means that there are 24 candidate timetable adaptations in instance *ZTM1* and 12 candidate timetable solutions in instance *ZTM2*.

7.2. Parameter settings

The objective function consists of the timetable adaptation costs, of the rolling stock rescheduling costs, and of costs for the passenger delays. For the timetable adaptations we do not consider any penalties other than that we assume that an additional stop will delay a train by 3 minutes. The omission of a purely timetable-specific objective term is due to lacking data and due to our goal to keep the discussion of results more clear and focused.

The rolling stock rescheduling costs are given in Table 1. Most important is that the rolling stock schedule should not change too much from the original plan, since changed plans require communication between the dispatchers and the personnel, and a failure in this communication is easily made. Therefore we introduce costs for having other shunting operations than planned. Changing the shunting operations also includes new tasks for the shunting personnel, which is not preferred. We consider the carriage kilometers as least important.

The passenger service costs consist of the passenger delay minutes as discussed in Section 4.2, where the penalties for passengers who left the system because of not reaching their end station within their deadline are also measured in delay minutes. The deadline is the same for all passengers belonging to the same group; the deadline is based on the assumption that passengers are accepting an increase in travel time up to 50% of the planned travel time plus 90 minutes.

We limit our computational tests to at most 15 iterations. To solve the composition model of the rolling stock rescheduling we used CPLEX 12.5. The test instances are run on a laptop with a Intel(R) Core(TM) i7-3517U 1.9/2.4 Ghz and 4.0 GB RAM.

7.3. Results

This section provides the results of the two test instances. For the timetable rescheduling part we have different approaches to decide which Intercity services should make an additional stop. We compare the effect of the different approaches on the final solution. We also compare our approach (which includes the option to adapt the timetable) with the method of Kroon et al. (2015) (which does not have an option to adapt the timetable) referred to as (*NO_STOP*). In the

Table 1
Rolling stock rescheduling costs.

Type of costs	value
New shunting operation	500
Changed shunting operation	500
Canceled shunting operation	100
Off balances at the end of the day, per unit	200
Seat shortage per seat per kilometer	0.1
Carriage Kilometers	0.0001

approach (*EST*) we estimated in the timetable rescheduling step the effect of an additional stop. Within this estimation we discussed that we could assume different lengths of the additional stops. This assumed length of the additional stop is also called the extra stop penalty. In our experiments we used 0, 1, 2 and 3 minutes for the extra stop penalty. Note that an extra stop penalty of 0 minutes means that it is assumed that nobody faces negative effects of the additional stop. Furthermore, note that the realized timetable adaptation always includes a 3 minute delay caused by the additional stop.

In Table 2 we provide the best result found in the iterative procedure for each of the variants of the approach. Note that the iterative procedure does not necessarily converge to an optimal solution and thus the best solution can be found at any iteration. Therefore, we included the number of the iteration where the best solution was found. For each approach the *lower bound* as discussed in Section 6.2 is given. Furthermore, the table contains the value of the *objective* function which consist of the sum of the *rolling stock rescheduling costs* (by considering the parameters in Table 1) and the passenger inconvenience (measured in *passenger delay minutes* as discussed in Section 4.2). In the table fractional values for the total passenger delay minutes are rounded to whole minutes. We also report how many *extra stops* are included in the timetable of the best result. The *computation time* is measured in seconds and reports the computation time over all 15 iterations, and not just the computation time up to the moment the best solution is found. The latter would not be fair, since beforehand it is not known at which iteration the best solution will be found.

The rolling stock rescheduling costs indicate that the shunting processes need 6 essential changes. Although we cannot judge the practical difficulty of these shunting changes, we still believe that practitioners would find these rolling costs rescheduling costs very acceptable in a network as large and as complex as our test instances.

7.4. Performance

In the approach *NO_STOP* based on Kroon et al. (2015), timetable adaptations were not allowed. Our results show that allowing the stopping patterns to be adapted can reduce the passenger delays significantly by about 25–35%.

From Table 2 we can deduce that the approach *EXACT* led in both cases to the lowest passenger delay minutes and the lowest rolling stock rescheduling costs. *EXACT*∗, the variant of the approach *EXACT*, reaches the same solutions, but it takes more iterations to get there. The estimation approach works well as long as we overestimate the positive effects of the additional stop by having a lower extra stop penalty (0 or 1 minute) than the realized delay (3 minutes).

Table 2
Results.

Solution Method	Lower bound	Objective	Passenger delay minutes	Rolling stock rescheduling costs	Extra stops	Iteration of best solution	Computation time (s)
Instance *ZTM1*							
(EXACT)	33,526	55,927	55,874	53	4	4	563
(EXACT∗)	33,526	55,927	55,874	53	4	9	573
(EST 0min)	33,526	57,372	56,818	554	3	3	349
(EST 1min)	33,526	58,857	58,304	554	4	4	352
(EST 2min)	33,526	57,372	57,318	53	5	9	352
(EST 3min)	33,526	91,534	90,980	554	0	1	291
(PRACT1)	33,526	64,304	64,251	53	3	4	282
(PRACT2)	33,526	64,304	64,251	53	3	4	307
(NO_STOP)	49,626	91,534	90,980	554	–	1	235
Instance *ZTM2*							
(EXACT)	110,848	139,588	136,527	3061	3	3	456
(EXACT∗)	110,848	139,588	136,527	3061	3	8	427
(EST 0min)	110,848	139,630	136,368	3262	3	4	352
(EST 1min)	110,848	139,630	136,368	3262	3	4	315
(EST 2min)	110,848	162,228	159,167	3061	2	6	351
(EST 3min)	110,848	177,123	173,861	3262	0	1	291
(PRACT1)	110,848	152,062	149,000	3061	4	4	320
(PRACT2)	110,848	166,356	163,295	3061	3	7	326
(NO_STOP)	120,373	177,123	173,861	3262	–	1	249

The performance of the approach *EST 0min* is surprising. It underestimates the negative effects and overestimates the positive effects of the additional stop but it is still able to reach solutions which do not differ much from the solutions reached by the approach *EXACT*. In deciding on which train should make an additional stop, the approach *EST 0min* assumes that an additional stop does not cause any delay and thereby no one faces negative effects of the additional stop. In every iteration an additional stop is introduced (by assuming that every additional stop has at least some positive effect).

On the other hand, the bad performance of the approach *EST 3min* is also surprising. Especially since in this approach the duration of the additional stop in the estimation approach matches the realized duration of an additional stop. However, this approach finds it never worthwhile to make an additional stop. Since this approach does not consider the capacities of the trains, it does not take rejected passengers into account. The *EST* approaches, thereby underestimate the positive effect the additional stop could have for rejected passengers. It seems that in *EST 0min* and *EST 1min* this underestimation is balanced by the overestimation of the other positive effects, but in the *EST 3min* approach the underestimation is not corrected by another overestimation.

The rules of thumb approaches *PRACT1* and *PRACT2* are outperformed by our exact approach *EXACT* and by our estimation approaches *EST 0min* and *EST 1min*. This shows that our more complex approaches are able to come to better solutions.

7.5. Iterative behavior

In Figs. 3–8 we show for six of the variants how the solution of the case *ZTM1* changes over the iterations. The black dot indicates the first solution and by arrows we indicate how the solution evolves. On the horizontal axis we have the rolling stock rescheduling costs and on the vertical axis we have the passenger delays.

Mind that each iteration gives rise to two data points: one after the timetabling step and another one after the rolling stock rescheduling step. With the rolling stock rescheduling both the rolling stock rescheduling costs and the passenger delays can change. However a timetable adaptation only influences the passenger delays, and therefore, a vertical drop or increase in the figure can generally be associated with a timetable adaptation.

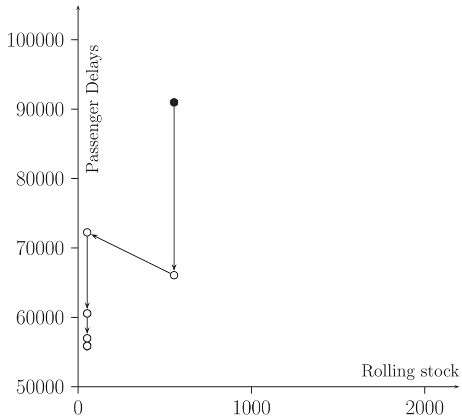

Fig. 3. *EXACT* case *ZTM1*.

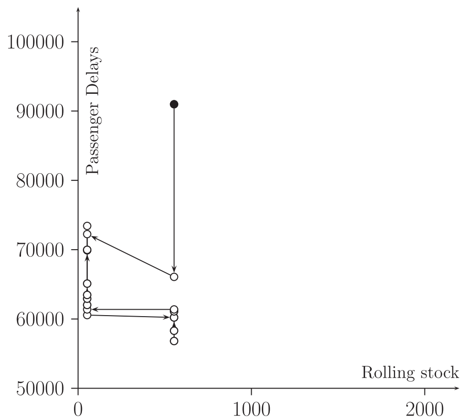

Fig. 4. *EST 0min* case *ZTM1*

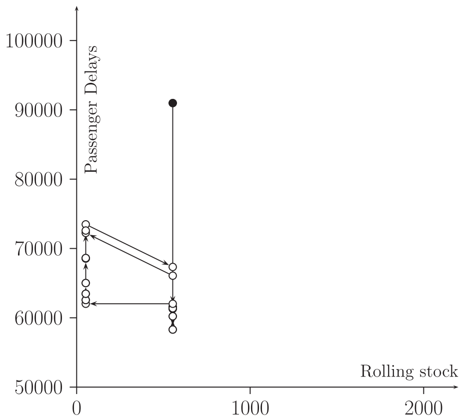

Fig. 5. *EST 1min* case *ZTM1*

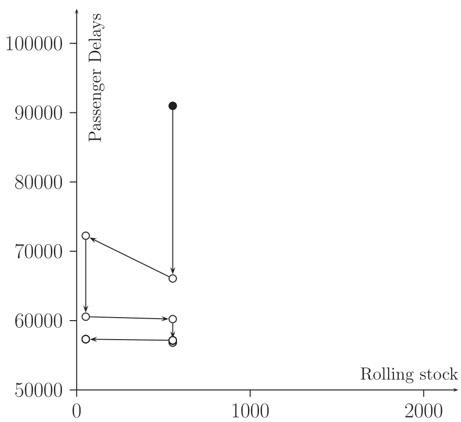

Fig. 6. *EST 2min* case *ZTM1*

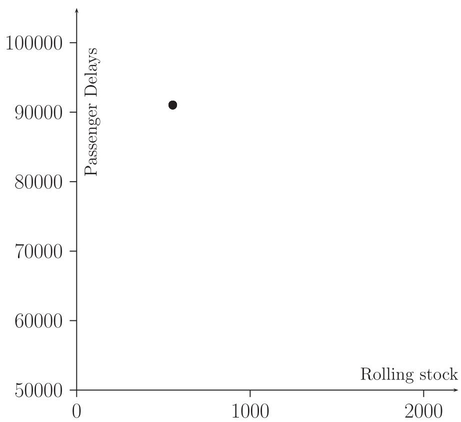

Fig. 7. *EST 3min* case *ZTM1*

The *NO_STOP* approach did not show any iterative behavior (i.e. the solution did not change). The solution found with the *NO_STOP* approach is the first dot in all of Figs. 3–8. As can be seen in Table 2, the approaches *EXACT* and *EXACT*∗ perform quite similarly, and the same holds for the approaches *PRACT1* and *PRACT2*. Therefore only one of each pair is provided is the figures.

The *EXACT* approach has a quite clear converging path to its best solution by decreasing passenger delays and rolling stock rescheduling costs. The solution of the approaches *EST 0min* and *EST 1min* first goes to solutions with low passenger delays

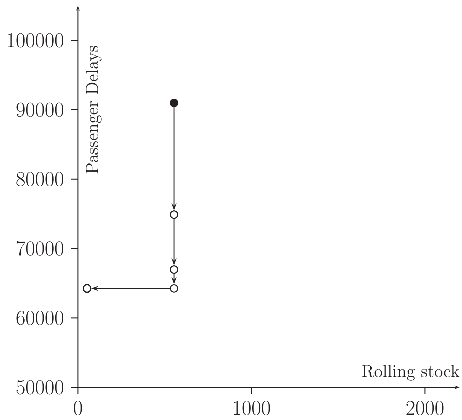

Fig. 8. *PRACT1* case *ZTM1*

and low rolling stock rescheduling costs, but from a certain moment, the passenger delays are increasing again. The approaches *EST 2min* and *PRACT1* converge like the *EXACT* approach to their best solution, but especially the solution of *PRACT1* does not come close to the solution of *EXACT*. The approach *EST 3min* has (just like the *NO_STOP* approach without extra stop possibilities) in every iteration the same solution. This means that the approach did not judge any of the timetable adaptations to be profitable in the first iteration; the rolling stock rescheduling could not find an improvement, either, thus the approach got stuck at this solution.

In Figs. 9 and 10 the iterative behavior of our best performing variants (*EXACT* and *EST 0min*) on case *ZTM2* are given. All variants did not show converging behavior for this case. The figures demonstrate that the *EXACT* approach explores a smaller region of solutions. The approach *EST 0min* first goes to solutions with large passenger delays, then gets to solutions with low passenger delays and in the end it goes again into the direction of solutions with high passenger delays.

7.6. Computation time

Our approach added a module which adapts the timetable to the iterative procedure of the approach *NO_STOP* of Kroon et al. (2015). This means that we assume that by the additional computations our approach cannot be faster than the *NO_STOP* approach.

If we use the *EXACT* approach, an instance is solved in about double the time of the *NO_STOP* approach. The other variants of the approach solve the instances faster (within 6 minutes).

The first rolling stock rescheduling step, to determine the number of timetable services without rolling stock, is carried out in about 80 s. Then, next rolling stock rescheduling steps take 4–5 s per iteration. The timetable rescheduling phase takes 8–15 s per iteration within the *EXACT* approach, since multiple simulations must be carried out. The computation time of the timetable rescheduling phase drops to 1–6 s per iteration for the estimation approaches *EST* and to less than 1 s for the approaches *PRACT1* and *PRACT2*.

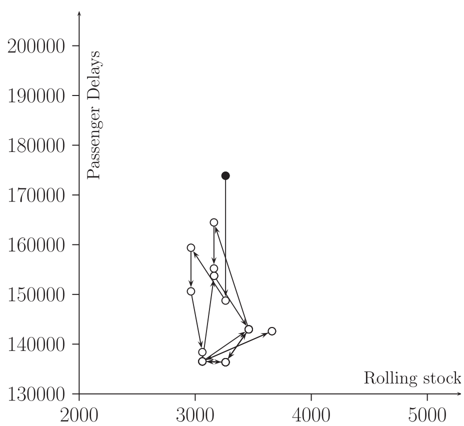

Fig. 9. *EXACT* case *ZTM2*

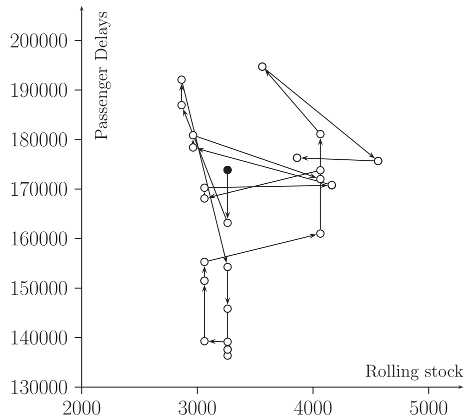

Fig. 10. *EST 0min* case *ZTM2*

Table 3
Results with costs of the passenger flows times 10.

Solution Method	Lower bound	Objective	Passenger delay minutes	Rolling stock rescheduling costs	Extra stops	Iteration of best solution	Computation time (s)
Instance *ZTM1*							
(EXACT)	335,260	558,793	55,874	53	4	7	568
(EXACT*)	335,260	558,793	55,874	53	4	14	541
(EST 0min)	335,260	568,737	56,818	554	3	3	346
(EST 1min)	335,260	583,589	58,304	554	4	4	351
(EST 2min)	335,260	568,737	56,818	554	3	6	350
(EST 3min)	335,260	910,355	90,980	554	0	1	363
(PRACT1)	335,260	642,566	64,251	53	3	7	297
(PRACT2)	335,260	642,566	64,251	53	3	7	296
(NO_STOP)	496,260	910,355	90,980	554	-	1	210
Instance *ZTM2*							
(EXACT)	1,108,480	1,363,686	135,962	4064	3	11	431
(EXACT*)	1,108,480	1,363,686	135,962	4064	3	5	430
(EST 0min)	1,108,480	1,375,949	137,189	4064	4	4	346
(EST 1min)	1,108,480	1,366,944	136,368	3262	3	4	298
(EST 2min)	1,108,480	1,593,181	158,992	3262	2	4	331
(EST 3min)	1,108,480	1,741,873	173,861	3262	0	1	359
(PRACT1)	1,108,480	1,473,060	146,900	4064	2	2	313
(PRACT2)	1,108,480	1,628,519	162,446	4064	3	12	293
(NO_STOP)	1,203,730	1,741,873	173,861	3262	–	1	237

7.7. Additional tests

To see what happens with the solutions if we put more weight on the passenger delays, we run all approaches also with an objective in which the cost of the passenger delays is multiplied by 10. The results are presented in Table 3. The results and the performance are similar to the results in Table 2. We observe that the rolling stock rescheduling costs barely increase in spite a 10-fold decrease of their relative importance. Mind that the difference amounts the costs of just one or two essential changes in the underlying shunting processes. This indicates that our cost structure controls rolling stock rescheduling costs rather tightly even though the magnitude of the passenger delay minutes is much larger that the magnitude of the rolling stock rescheduling costs.

In a third test we experiment on how the approaches behave if we give additional penalties to longer delays. In these tests we penalize delays between 15 and 30 minutes with an additional 5 minutes delay and delays longer than 30 minutes with an additional 10 minutes delay. Again one can see from the results in Table 4 that these additional penalties do not influence the solutions. In more than half of the cases, the best solution found is the same as in the situation without these additional penalties (as presented in Table 2). For the other cases the differences were not large.

These two additional tests show that our approach is not sensitive to changes in the evaluation of the passenger inconvenience.

Table 4
Results with 5 minutes additional penalty for delays larger than 15 minutes and 10 minutes additional penalty for delays larger than 30 minutes.

Solution Method	Lower bound	Objective	Passenger delay minutes	Rolling stock rescheduling costs	Extra stops	Iteration of best solution	Computation time (s)
Instance *ZTM1*							
(EXACT)	36,171	59,342	55,874	53	4	8	572
(EXACT*)	36,171	59,342	55,874	53	4	8	563
(EST 0min)	36,171	60,777	56,818	554	3	3	341
(EST 1min)	36,171	62,297	58,304	554	4	4	348
(EST 2min)	36,171	60,777	56,818	554	3	3	361
(EST 3min)	36,171	94,969	90,980	554	0	1	360
(PRACT1)	36,171	67,684	64,251	53	3	8	290
(PRACT2)	36,171	67,683	64,251	53	3	8	273
(NO_STOP)	52,741	94,969	90,980	554	–	1	214
Instance *ZTM2*							
(EXACT)	114,988	144,513	136,527	3061	3	5	417
(EXACT*)	114,988	144,513	136,527	3061	3	5	492
(EST 0min)	114,988	144,540	136,368	3262	3	4	312
(EST 1min)	114,988	144,540	136,368	3262	3	4	331
(EST 2min)	114,988	167,383	159,167	3061	2	5	344
(EST 3min)	114,988	182,073	173,861	3262	0	1	339
(PRACT1)	114,988	155,026	146,930	3061	4	4	324
(PRACT2)	114,988	167,639	159,563	3061	2	5	309
(NO_STOP)	124,813	182,073	173,861	3262	–	1	201

8. Conclusions and further research

In this paper we proposed a disruption management approach which integrates the rescheduling of rolling stock and the adaptation of stopping patterns with the aim of improving passenger service.

Computational tests are performed on realistic large-scale instances of the Dutch railway network. The two tested instances show that allowing the timetable to be adapted can reduce the total delay of passengers by more than 20% without increasing the rolling stock rescheduling costs. We proposed several variants of the approach, with the difference lying in the way of how the timetable changes are evaluated. These variants lead to different results and different computation times, but the results per variant are not quite sensitive to the exact cost parameter settings.

Our solution approach does not necessarily converge to an optimal solution. The lower bounds indicate that the gap between the solution and the lower bound is decreased by allowing stopping pattern adaptations. However the gap is still significant, we believe that this is caused by the weak lower bound. This is a topic for future research.

In our future research we will incorporate other timetable decisions as well, for example rerouting of timetable services. Furthermore we want to proceed with integrating delay management decisions into the model, which will be quite challenging since the delay management approach is already a difficult problem to solve on its own.

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