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Abelian monopoles and action density in $SU(2)$ gluodynamics on the lattice

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Abstract

We show that the extended Abelian magnetic monopoles in the Maximal Abelian projection of lattice $SU(2)$ gluodynamics are locally correlated with the magnetic and the electric parts of the $SU(2)$ action density. These correlations are observed in the confined and in the deconfined phases. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

The monopole confinement mechanism [1] in lattice gluodynamics seems to be confirmed by many numerical calculations [2]. Monopoles in the Maximal Abelian (MaA) projection [1,3] are condensed in the confinement phase of gluodynamics [4], their currents satisfy the classical equations of motion for the (dual) Abelian Higgs model [5] and the $SU(2)$ string tension is reproduced by the monopole currents [6]. The confining string connecting the static quark-antiquark pair is clearly seen [7]. The next problem to solve is to build the qualitative and quantitative model for this flux tube, or more generally the effective infrared Lagrangian for gluodynamics. The first steps in this direction are done already [8,9]. In brief the main results of the numerical study of the confinement problem are: the vacuum of gluodynamics behaves as the dual superconductor, the abelian monopoles playing the role of the Cooper

pairs and the confining string is an analogue of the Abrikosov-Nielsen-Olesen string.

On the other hand in the continuum theory the Abelian monopoles arise as singularities in the gauge transformations [10]. The definition of the Abelian monopoles is projection-dependent, monopoles defined in different projections are different in general ¹. Therefore it is not clear whether these monopoles are “physical” objects. The first argument in favour of the physical nature of the Abelian monopoles was given in Ref. [12]: it was found that the total action of $SU(2)$ fields is correlated with the total length of the monopole currents, so there exists a global correlation. Recently it was shown that the Abelian monopoles in the MaA projection are locally correlated with the non-Abelian action density [14].

¹ However, there exists a gauge invariant definition of the monopole current in any chosen Abelian projection, see Refs. [11].

Really it means that monopoles are the physical objects (not the artifacts of the singular gauge transformation), since by definition we call the object physical if it carries the action. In Ref. [15] the correlation of monopoles, ZZ_2 strings and the action density was discussed. The investigation of the correlations of monopoles, the topological density and the action density was performed in Refs. [13,16].

Thus monopoles are important dynamical variables for the confinement problem and the detailed study of their anatomy is interesting. At present we have no idea what is the general class of the gauge fields which generate the monopole currents in the MaA projection². But since the elementary monopoles carry nonabelian magnetic action [14] they are related with some nonabelian objects. The numerical study of the effective infrared Lagrangian of lattice gluodynamics shows [9] that to approach the continuum limit we have to consider also the extended (blocked) monopoles [18]. In the present publication we continue the study of correlations of the monopole currents and the action density started in Ref. [14]. We investigate the extended monopole currents, and also study the correlations of the electric part of the action with the monopole currents.

The couplings of the monopole action [9] obeys scaling, it means that these couplings do not depend separately on the monopole size in the lattice units and bare coupling, but only on the physical size of the monopole. This fact in turn means that the couplings lie on the renormalised trajectory, and we know the values of the coupling and the size of the monopoles in the continuum limit ($a \rightarrow 0$). The calculations presented in the present paper are done just for that sizes of monopoles and for that values of the bare coupling which correspond to the initial part ($2.2 < \beta < 2.5$) of the renormalised trajectory of Refs. [9]. In that sense our results correspond to the continuum limit.

There are two different types of extended monopoles (type-I and type-II monopoles) [18]. As we already discussed the type-II extended monopoles are important dynamical variables in lattice gluody-

namics [9,12]. The type-I extended monopoles play a non-trivial role for the dynamics of the phase transitions in electroweak theory [19] and in the $U(1)$ Abelian Higgs model [20].

The paper is organised as follows. In Section 2 we introduce two quantities η^E and η^M , which define the correlation of the magnetic and electric parts of the $SU(2)$ action with the (extended) monopole charge. In Section 3 we describe the results of numerical calculations. We discuss the results in Section 4.

2. Correlations of monopoles with action densities

If the Abelian monopole carries the non-Abelian action, then the action density near the monopole current should be larger than the action density far from the monopole trajectory. One of the quantities which can show this effect is the relative excess of the mean action density in the region near the monopole current [14]. The total action can be divided into electric and magnetic parts. The relative excess of the magnetic (electric) action density is defined as:

$$\eta^{M(E)} = \frac{S_m^{M(E)} - S}{S}. \quad (1)$$

Here $S = \langle S_P \rangle \equiv \langle (1 - \frac{1}{2} \text{Tr} U_P) \rangle$ is the expectation value of the lattice plaquette action. The quantity S_m^M is the action averaged over the plaquettes closest to the monopole current $j_\nu(x)$. The definition of S_m^M is:

$$S_m^M = \left\langle \frac{1}{6} \sum_{P \in \partial C_\nu(x)} S_P \right\rangle, \quad (2)$$

where the summation is over the plaquettes P which are the faces of the cubes $C_\nu(x)$; a cube $C_\nu(x)$ is dual to the monopole current $j_\nu(x)$. For the static Abelian monopole $j_0(x) \neq 0$, $j_i(x) = 0$ ($i = 1,2,3$), and the boundaries of the cubes dual to the monopole current are formed by the space-like plaquettes $P_{i,j}$, $i,j = 1,2,3$. Therefore only the magnetic part of the $SU(2)$ action density, $\frac{1}{2} \text{Tr} F_{ij}^2$, contributes to S_m^M .

The quantity S^E in Eq. (1) is:

$$S_m^E = \left\langle \frac{1}{24} \sum_{P \in \mathcal{P}(C_\nu(x))} S_P \right\rangle, \quad (3)$$

² It is known [17] that instantons induce Abelian monopole currents in the Abelian gauge but it seems that they are not the only sources of Abelian monopoles.

where $\mathcal{P}(C_\nu(x))$ is the set of all plaquettes P which satisfy the following two conditions: all plaquettes P (i) have one, and only one, common link l_μ with the cube $C_\nu(x)$; (ii) they are lying in the planes, defined by the vectors $\hat{\mu}$ and $\hat{\nu}$. There are 24 such plaquettes corresponding to a cube $C_\nu(x)$. For the static monopole current these plaquettes lie in the planes $(0,i)$, $i = 1,2,3$; therefore only the electric part of $SU(2)$ action density, $\frac{1}{2}\text{Tr}F_{0i}^2$, contributes to the quantity S_m^E .

Thus, our definition of electric, S^E , and magnetic, S^M , parts corresponds to the electric and magnetic parts of the action density only for a static monopole. For non-static monopoles it is convenient to keep these notations.

In the naive continuum limit the expressions (2) and (3) and the plaquette action S have the following form:

$$S_m^M = \frac{1}{24} \langle \text{Tr} (n_\mu(x) \tilde{F}_{\mu\nu}(x))^2 \rangle, \quad (4)$$

$$S_m^E = \frac{1}{6} \langle \text{Tr} (n_\mu(x) F_{\mu\nu}(x))^2 \rangle, \quad (5)$$

$$S = \frac{1}{24} \langle \text{Tr} F_{\mu\nu}^2 \rangle, \quad (6)$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}$, and $n_\mu(x)$ is the unit vector in the direction of the current: $n_\mu(x) = j_\mu(x)/|j_\mu(x)|$ if $j_\mu(x) \neq 0$, and $n_\mu(x) = 0$ if $j_\mu(x) = 0$. For a static monopole Eqs. (4) and (5) give the normalised average of the chromomagnetic, $\frac{1}{3}(B_i^a)^2$, and chromoelectric, $\frac{1}{3}(E_i^a)^2$, action density at the point where the monopole is located. Eq. (6) gives the normalised total action density: $\langle S \rangle = \frac{1}{6} \langle (B_i^a)^2 + (E_i^a)^2 \rangle$.

3. Numerical results

Below we present the quantities η^M and η^E calculated on symmetric, 24^4 , and asymmetric, $24^3 \cdot 4$ lattices in standard $SU(2)$ lattice gluodynamics³. In all these cases, we find in the MaA projection [3]

³ To check the finite volume corrections we also performed calculations on the smaller lattices: 16^4 , 20^4 , $16^3 \cdot 4$ and $16^3 \cdot 4$. It occurs that the results obtained on these small lattices coincide within the statistical errors with the results obtained on 24^4 and $24^3 \cdot 4$ lattices

that the quantities $\eta^{M,E}$ are different from zero for all values of β . We also considered the F_{12} (diagonalization of the F_{12} lattice field strength tensor), Polyakov (diagonalization of the Polyakov line) and ‘‘random’’ Abelian gauges. The ‘‘random’’ Abelian gauge means no gauge fixing at all: we take a field configuration, apply a random gauge transformation and then treat the phases of the diagonal elements of the $SU(2)$ gauge field as the Abelian gauge field.

To fix the MaA projection we use the overrelaxation algorithm of Ref. [21]. The number of gauge fixing iterations is determined by the following criterion [22]: the iterations are stopped when the matrix of the gauge transformation $\Omega(x)$ becomes close to the identity matrix: $\max_x \{1 - \frac{1}{2} \text{Tr} \Omega(x)\} \leq 10^{-6}$. We also check that a more accurate gauge fixing does not change our results. By performing a sufficient number of iterations between measurements we have made sure that the configurations on which we performed our measurements are statistically independent.

Fig. 1(a) shows the quantities $\eta^{M,E}$ in the MaA projection for the lattice 24^4 . The quantity η^M is 4–6 times larger than the quantity η^E for all considered values of $\beta = 4/g^2$. Thus the excess of the chromomagnetic action near the monopole position is larger than the excess of the chromoelectric action.

The correlations increase with increasing β . For small β monopoles are present almost everywhere, so the action averaged over the cubes containing monopoles differs very little from the action averaged over all cubes. The density of monopoles decreases with increasing β , thus the increase of the correlator as $\beta \rightarrow \infty$ means that at large β the Abelian monopoles disappear mainly in the regions with a small $SU(2)$ action density.

Note, that the monopole current j_μ is derived from the plaquettes ∂C_μ which contribute to S^M , thus the fact that $\eta^M \neq 0$ is rather natural. The plaquettes which contribute to S^E are not directly related to the monopole current and the fact that $\eta^E \neq 0$ probably means that there exist some structures in the vacuum of gluodynamics which generate monopole currents and carry electric and magnetic action.

Our numerical simulations show that the quantities $\eta^{E,M}$ for the F_{12} , Polyakov and random Abelian gauges coincide with each other within the numerical

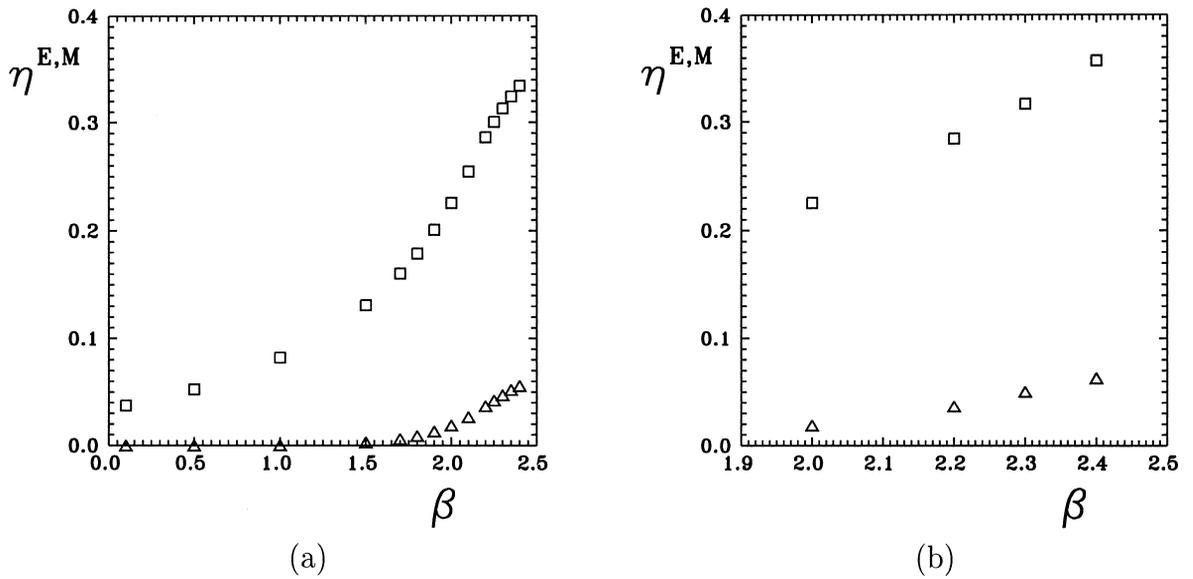


Fig. 1. (a) The quantities η^M (boxes) and η^E (triangles) versus β on the lattice 24^4 for the MaA projection. In all figures the error bars are much smaller than the sizes of the symbols used; (b) the same as in (a), but now for the lattice $24^3 \cdot 4$.

errors. For all studied values of the coupling constant β the values of the quantities $\eta^{M,E}$ calculated in these gauges are more than 10 times smaller than

those for the MaA gauge. This fact probably indicates that the Abelian monopoles in the F_{12} and Polyakov Abelian projections carries much less in-

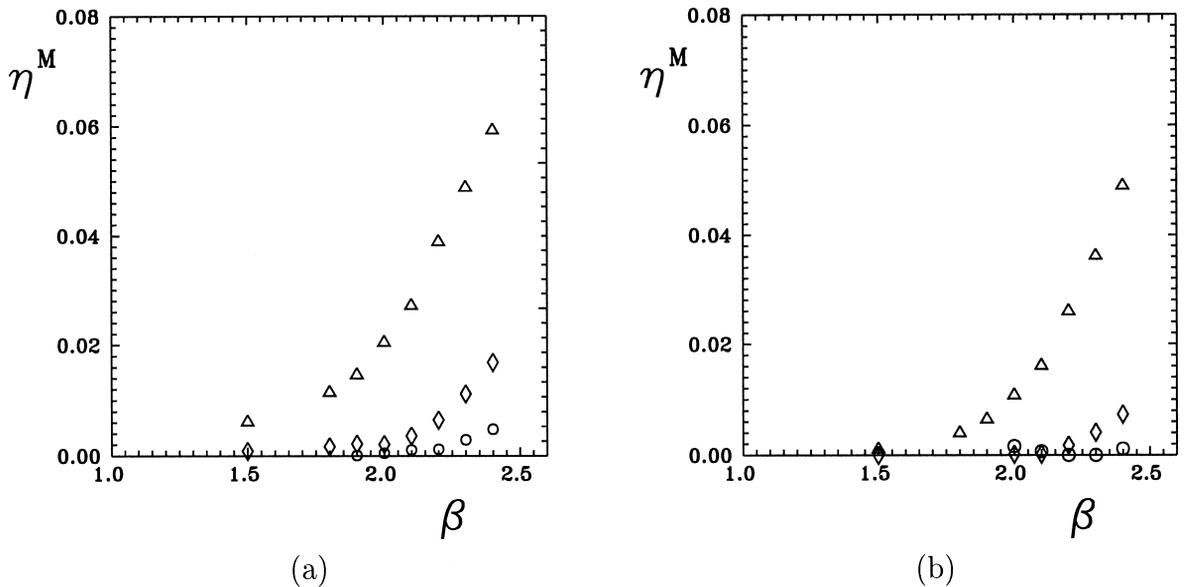


Fig. 2. (a) The correlator η^M in the MaA gauge for a 24^4 lattice versus β for type-II extended monopoles of sizes 2 (triangles), 3 (diamonds) and 4 (circles); (b) The same as in (a), but now for type-I monopoles.

formation about the properties of the non-Abelian vacuum than the Abelian monopole in the MaA projection⁴.

The finite-temperature analysis of the correlators $\eta^{M,E}$ is performed on an asymmetric lattice. We found that at finite temperature the correlators in the MaA projection turned out to be much larger than the correlations in the F_{12} , Polyakov and random gauges. We show the quantities η^M and η^E in the MaA gauge in Fig. 1(b). These calculations are performed on a $24^3 \cdot 4$ lattice. It is seen that the confinement–deconfinement phase transition (which occurs at $\beta = \beta_c = 2.3$) has no observable influence on the behaviour of the correlators $\eta^{M,E}$.

The correlation between electric and magnetic action in the vicinity of Abelian monopoles is small. We measured the correlation of the product of the electric and the magnetic action with the monopole currents. We find that the correlator

$$\eta^{EM} = \frac{\langle S^E(x) S^M(x) \rangle}{\langle S^E(x) \rangle \langle S^M(x) \rangle} - 1 \quad (7)$$

vanishes within the statistical error for the studied region of the bare coupling β on the lattice 24^4 . This occurs not only when the averages are taken over the full lattice, but also if only the cubes associated with monopoles are included in the average. This result is independent of the gauge fixing condition. Therefore the magnetic and electric fluctuations around the Abelian monopole in the MaA gauge are independent.

We also study the correlations of extended monopoles [18] with the electric and magnetic action. There are two types of extended monopoles [18]: type I corresponds to the plaquettes of size $l \times l$; type II uses all 1×1 plaquettes that tile the faces of an l^3 sized cube associated with a monopole current. We measured the correlations of the magnetic and the electric action densities with the ex-

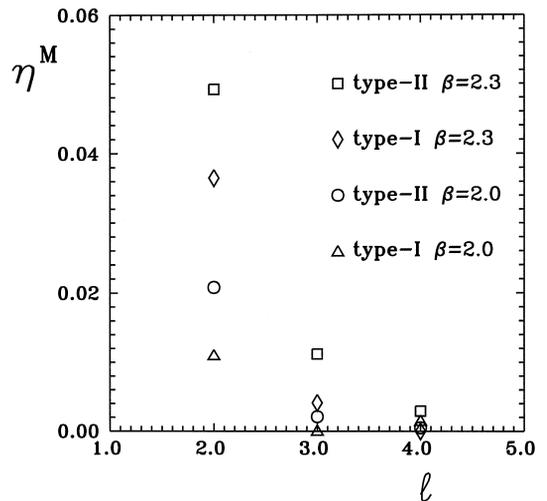


Fig. 3. The correlator η^M plotted as a function of the linear dimension l of the extended monopoles, the lattice size is 24^4 .

tended monopoles of sizes $l = 2, 3$ and 4 . It turns out that for the whole range of the bare coupling β studied, the quantity η^M for type-II monopoles is larger than that for the type-I monopoles. In Fig. 2(a,b) we show the dependence of the quantity η^M on β for type-II monopoles and type-I monopoles. In order to show a similarity between different types of monopoles we plot the quantity η^M for the type-I and type-II monopoles versus linear size of the extended monopoles l (Fig. 3). The figure clearly shows that the larger the size of the monopole the smaller the correlation η^M is. This fact is not unexpected since with increasing monopole size the part of the lattice which belongs to a monopole gets larger and therefore the averaged action associated with the monopole gets closer to the total averaged action. If the correlations η^M and η^E are physical quantities then they should depend on the physical monopole size, $b = la$, where a is the physical lattice spacing. We are planning to study this dependence in our next publication.

4. Discussion and conclusions

We discussed the local correlations of the electric and magnetic parts of the $SU(2)$ action with Abelian

⁴ In Ref. [16] the correlator η^M is studied under the smoothing procedure. It was found that for elementary monopoles in different gauges this correlator is of the same order. The smoothing procedure removes short-range fluctuations, therefore the result of Ref. [16] probably indicates that the small correlation of the monopoles with the action density in, say, the Polyakov gauge, is due to ultraviolet vacuum fluctuations.

monopoles in various Abelian projections. We have shown that monopoles in the Maximal Abelian projection are correlated with the electric and magnetic parts of the action density at zero and at finite temperature. The same result is obtained also for type-I and type-II extended monopoles. The correlators $\eta^{M,E}$ for the type-II monopoles are always larger than the correlators for the type-I monopoles. Thus, for the description of the vacuum of $SU(2)$ gluodynamics the type-II monopoles are more suitable variables than the type-I monopoles, in agreement with Refs. [9,12].

The correlation of the monopoles with the electric part of the action density is smaller than the correlation with the magnetic part of the action density. The correlations of the Abelian monopole with both parts of the $SU(2)$ action density in the Polyakov, F_{12} and random gauges are of the same order; all of them are much smaller than the correlations in the MaA gauge.

We note here that the existence of the correlation of the electric and the magnetic action densities with the Abelian monopoles can be understood from the fact that the Abelian monopoles are correlated with the topological charge density [13,23,24]. Indeed this correlation means that the monopole currents are accompanied by a non-zero density of the topological charge. This charge is non-zero if and only if both the electric and magnetic action densities are non-zero.

We conclude that the Abelian monopoles in the Maximal Abelian projection are physical objects which carry both magnetic and electric parts of the $SU(2)$ action density.

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