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Estimates of the social cost of carbon are the yardstick for climate policy targets. However, there is great uncertainty and we do not know how estimates have evolved over time. Here I present a meta-analysis of published estimates showing that the social cost of carbon has increased as knowledge about climate change accumulates. Correcting for inflation and emission year and controlling for the discount rate, kernel density decomposition reveals a non-stationary distribution. In the past 10 years, estimates of the social cost of carbon have increased from US\$9 per tCO_2 to US\$40 per tCO_2 for a high discount rate and from US\$122 per tCO_2 to US\$525 per tCO_2 for a low discount rate. This trend is statistically significant if sensitivity analyses are discounted and paper quality weighted. Actual carbon prices are below its estimated value almost everywhere and should therefore go up.

The social cost of carbon is a key indicator of the seriousness of climate change. However, we do not know whether its estimates have changed over time, whether we should raise our ambitions to reduce greenhouse gas emissions, and what we have learned since the first estimate was published in 1982 (ref. 1). There is broad agreement among scholars that greenhouse gas emissions should be taxed, but the uncertainty about the optimal level of that tax is very large. I estimate the probability distribution of published estimates of the social cost of carbon and how it changes over time. I develop and apply a non-parametric test for the stationarity of an entire probability distribution, and apply a range of other statistical tests to show that estimates of the social cost of carbon have changed. An upward trend can be discerned. Climate policy should be intensified.

The social cost of carbon is the damage done, at the margin, by emitting more carbon dioxide into the atmosphere. For an economy with no other distortions, if evaluated along the optimal emissions trajectory, the social cost of carbon equals the Pigou tax[2,3] that internalizes the externality and restores the economy to its Pareto optimum[4] where no one can be made better off without making someone else worse off. The social cost of carbon is then the optimal carbon price. It informs the desired intensity of climate policy.

Some have argued[5,6], although others disagree[7], that the debate on optimal climate policy is over since the Paris Agreement has set targets for international climate policy. Analysis should focus on the cheapest way of meeting these targets, and emissions should be priced on the basis of the 'shadow price of carbon', which is the scarcity value of the carbon budget[5]. The shadow price is different from the social cost of carbon—the current paper does not include estimated shadow prices—and their growth rates are also different. The shadow price understates the value of near-term climate policy[8]. However, the first stock-take of the commitments under the Paris Agreement[9] reveals that few countries plan to do what is needed to meet the agreed targets. The debate over the ultimate target of international climate policy, hence the debate on the social cost of carbon, is not over. Indeed, US President Biden reinstated the Interagency Working Group on the Social Cost of Greenhouse Gases to reassess the appropriate carbon price[10,11].

There is a large literature on the social cost of carbon spanning four decades[1,12]. The social cost of carbon depends on many things, including the total economic impact of climate change; potential tipping points; the scenarios for population, economy and emissions; changes in vulnerability and relative prices with development; the rate of degradation of carbon dioxide from the atmosphere; the rate and level of global warming; the discount rate; the distribution of impacts and inequity aversion; and the uncertainties about impacts and risk aversion. The estimates used here—5,905 estimates in 207

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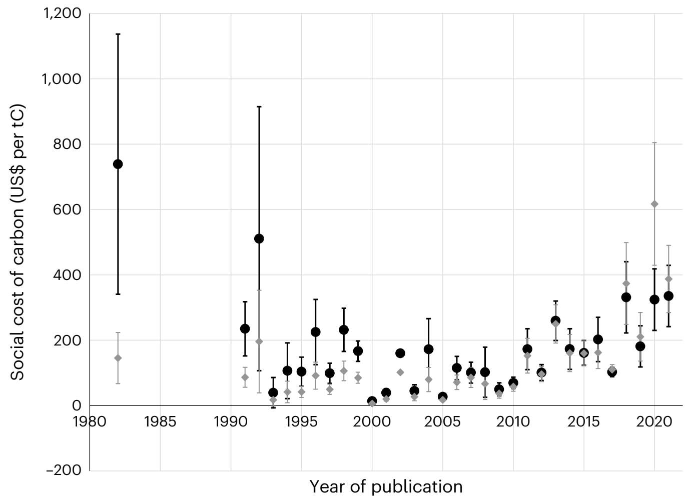

Fig. 1 | Average social cost of carbon by publication year. Grey diamonds are as reported, black dots are corrected for inflation and year of emission. Error bars are ± s.d. of the published estimates. Estimates are quality weighted and censored.

papers, published before 2022—make different assumptions about all these matters.

The estimates of the social cost of carbon are for carbon dioxide emitted in the recent past. The carbon tax should increase over time (until climate change has been mitigated to the point that its marginal impacts start to fall[13]). Some 94 papers estimate how fast, showing estimates of the social cost of carbon at two or more points in time, for a total of 1,974 estimates of the 'growth rate' of the social cost of carbon.

I apply meta-analysis to these estimates. Meta-analysis is not the only way to make the social cost of carbon more transparent. Sensitivity analysis[14] and model comparison[15] are also insightful. Decomposition of model updates[16,17] helps to understand the evolution of estimates, but only within the confines of a single model. Closed-form equations[18,19] give an exact relationship between input parameters and the social cost of carbon, but require rather restrictive and unrealistic assumptions. Meta-analysis, however, can show how the entire literature has evolved over time.

The mean and standard deviation of estimates of the social cost of carbon by year of publication are shown in Fig. 1. Estimates are shown with and without standardization. The social cost of carbon is expressed in 2010 US dollars per metric tonne of carbon (US$ per tC) for emissions in the year 2010. The literature uses nominal dollars and a variety of emission years (Extended Data Fig. 1). Particularly, later studies report the social cost of carbon in later dollars for later emission years. Average inflation is 2.9% over the period. The social cost of carbon grows by ~2.2% yr^{-1}; this is the average across the 94 studies that estimate its growth rate (Extended Data Fig. 2). Without correcting for emission year and inflation, the 'apparent' trend in the social cost of carbon equals 5.1% yr^{-1}. After correction, some of the early estimates are found to be the highest. Between 1993 and 2008, estimates went up and down without a discernible trend in either direction. Since around 2009, there appears to be an increase in the social cost of carbon, and 3 of the past 4 yr stand out. An upward trend is also clearly visible in the median and interquartile range per publication period (Extended Data Fig. 3). An earlier meta-analysis finds that the social cost of carbon has not increased over time[20] but a more recent one finds that it has[21]; the current paper uses more data and statistical methods that account for the skewness and kurtosis of the distribution of the observations. The mean for 2021 is higher than all but two other years; paired *t*-tests show that the 2021 mean is statistically significantly higher than all but four other years.

The range of estimates is large and has remained large over the years, perhaps even grown recently (Fig. 1). An assessment of the literature on the social cost of carbon should reflect that uncertainty, not just the first and second moment, but the entire probability distribution. I therefore use bespoke kernel methods to reflect the true uncertainties, including the uncertainties about parameter values, model structure, future scenarios and ethical parameters such as the rates of time preference, risk aversion and inequity aversion.

The uncertainty about the social cost of carbon is right-skewed[22] because the uncertainty about climate sensitivity is right-skewed[23], impact functions are nonlinear[24] and risk aversion emphasizes bad surprises over good ones[25]. This asymmetry is lost by adding and subtracting the standard error from the mean. The uncertainty about the social cost of carbon is also thick- or even fat-tailed[26]. There is considerably more probability mass outside the Gaussian confidence bounds. The above *t*-tests are overconfident. If the distribution of the social cost of carbon is right-skewed and fat-tailed, then recent estimates may not differ significantly from earlier estimates.

Kernel densities are a flexible alternative to parametric distribution functions[27]. Kernel densities have been used to visualize the uncertainty about the social cost of carbon[28]. I here add many more observations and decompose that uncertainty into discrete components, particularly publication periods, testing whether the components differ from one another. Simple kernel regression is helpful for specifying the relationship between two variables[29]. Kernel quantile regression can be used to show this relationship across the distribution[30]. However, these methods are not suitable if the explanatory variable is categorical, as is the case for assumed discount rates, authors or recorded years of publication. The method proposed here, kernel density decomposition, works for categorical data, shows both central tendency and spread, and does not make assumptions about functional form or the shape of the probability distribution. This method, while uncommon, is therefore best suited for the problem at hand.

Kernel density decomposition offers a valid basis for statistical tests. To test for changes over time, I split the sample into six periods demarcated by important events in the publication history of the social cost of carbon. These key events are the Second[31] and Third[32] Assessment Reports of the IPCC, the Stern Review[33], the Obama update on the social cost of carbon[34] and the 2018 Nobel Memorial Prize in Economic Sciences[35].

The kernel density is estimated with bespoke kernel functions, reflecting the deep and asymmetric uncertainty of the social cost of carbon. The data are weighted for quality—age, computational method, scenario, peer review, validity, novelty—but the results are largely robust to these weights. Furthermore, implausibly high estimates are censored or, in the appendix, winsorized. The decomposition of the kernel density is based on the fact that the weighted sum of probability densities is a probability density[36]. The statistical test is that for the equality of proportions[37] adjusted for finite sample size. Applied to different publication periods, this is a test for the stationarity of the distribution[38] of the social cost of carbon (see Methods for details and the Supplementary Information for sensitivity analyses).

The kernel density (Fig. 2, bottom) has the same shape as the histogram (Fig. 2, top): there is a little probability mass below zero, a pronounced mode and a thick right tail. Compared with the histogram, the kernel density is smooth and spread wider. Kernel mean and standard deviation are larger than their empirical counterparts (Table 1) because (1) I use the mode rather than the mean as the central estimate and (2) I assume a right-skewed kernel function.

Earlier studies exclude negative estimates. The kernel density decomposition shows a fatter right tail for recent years as shown by the contributions of estimates of the social cost of carbon published in a particular period to the overall kernel density and its quintiles (Supplementary Table 5). The null hypothesis that the quintile shares

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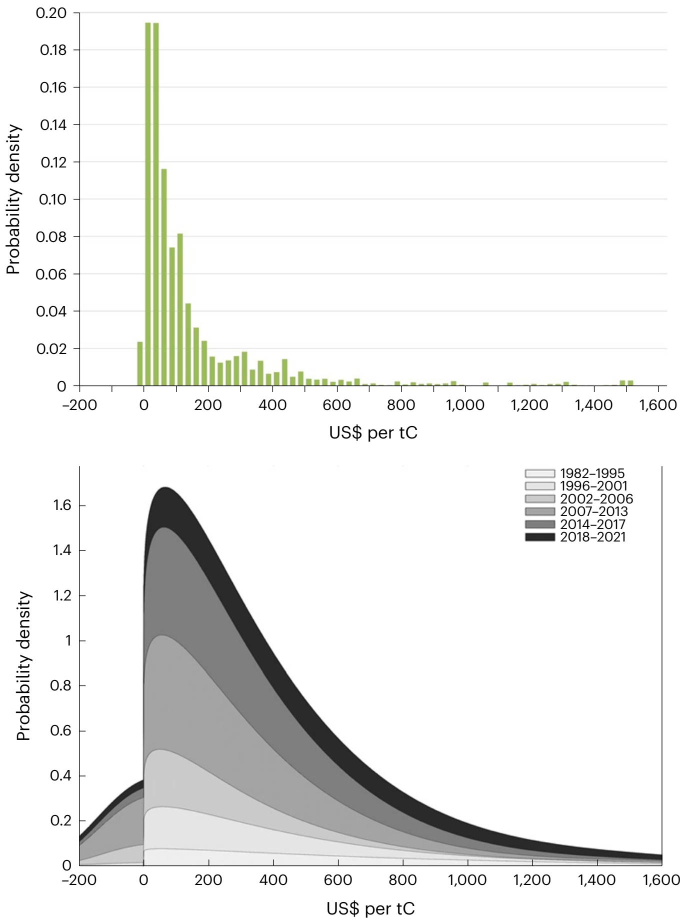

Fig. 2 | Histogram of the social cost of carbon. Top: composite kernel density of the social cost of carbon. Bottom: composition by publication period. Results are quality weighted and censored.

Table 1 | Empirical and kernel average (standard deviation) of estimates of the social cost of carbon (US$ per tC) by PRTP

PRTP (%)	Empirical	Kernel
3	43	72
	(54)	(66)
2	238	714
	(492)	(342)
1	155	342
	(318)	(401)
0	407	342
	(539)	(401)
all	179	458
	(382)	(499)

Estimates are censored and quality weighted.

are the same as the overall shares is rejected; $\chi^2_{20} = 12.77$ is larger than the critical value at 1%.

This analysis only considers time. The discount rate used to estimate the social cost of carbon has varied over time (Extended Data Fig. 4). Particularly, the once-popular pure rate of time preference (PRTP) of 3.0% has been largely replaced by 1.5%. This would increase estimates of the social cost of carbon (Table 1 and Extended Data Fig. 6). Note

Table 2 | Test for the equality of quintiles between selected publication periods and the whole record, for the whole sample and for selected PRTPs

	Test statistic	*P* value	10%	5%	1%
All	14.45	0.81	5.09	6.15	8.51
PRTP = 0%	1.15	1.00	2.60	3.35	5.02
PRTP = 1%	7.26	1.00	3.56	4.50	5.87
PRTP = 2%	16.24	0.70	2.61	3.47	4.94
PRTP = 3%	13.86	0.84	1.86	2.76	4.75

The three rightmost columns are bootstrapped critical values. For illustration, the third column shows the incorrect *P* value of the asymptotic test.

that economists' use of a different discount rate does not mean that the discount rate itself has changed[39,40].

I therefore repeat the analysis for the four PRTPs for which there are observations in every time period: 0%, 1%, 2% and 3% (Table 2). Papers that used other PRTPs, other discount rates or other discount factors were omitted (Supplementary Table 1). Conditional on the PRTP, the Equality of Proportions test finds statistically significant differences between the publication periods, except for the lowest discount rate. The Kolmogorov-Smirnov test finds differences at a finer resolution but not for quintiles (Supplementary Table 10). Note that these tests do not reveal 'how' things have changed, only that they have. The social cost of carbon has increased (Fig. 1).

Nine more analyses are included in the Supplementary Information. These analyses are less appropriate as they assume normality of error terms, avoiding the asymmetry and thick tail that are a feature of the social cost of carbon, making upward trends harder to detect. Weighted linear regression shows that the PRTP is highly significant and the year of publication is non-significant; either result is independent of the weights used (Supplementary Table 15). Higher-quality estimates show less unexplained variation (Extended Data Fig. 3), as in the kernel decomposition. Binning the utility discount rate leads to the same results, except that the weakly significant time trend for paper weights becomes non-significant. Dropping the PRTP, the time trend is still non-significant. Replacing the linear time trend with a flexible time trend, the effect of publication year is non-significant regardless of weights (Extended Data Fig. 5). Using quantile regression, the PRTP is significant for almost all quantiles and weights; the year of publication for almost none. However, the social cost of carbon appears to increase over time if estimates are weighted for quality, and attention is restricted to the central parts of the distribution (Supplementary Table 15). Replacing the PRTP with time preference dummies, a significant time trend appears for quality-weighted estimates in the lower half of the distribution. Dropping the PRTP, a trend appears in the 30th percentile and in the median and 90th percentile if quality weights are used. This cannot be explained by the quality weights themselves, as these 'fall' slightly over time because a shrinking share of estimates of the social cost of carbon relies on new estimates of the total economic impact of climate change[41]. Rather, as shown above, quality-weighted estimates show less variation in the social cost of carbon, so that the standard error of estimated coefficients is lower and their significance higher. Replacing the time trend with dummies for the six publication periods, for both mean regression and quantile regression, a trend appears when estimates are quality weighted; this trend is more significant in the lower quantiles (Supplementary Table 16).

Altogether, the upward trend in Fig. 1 is primarily because analysts have used lower discount rates but also partly because higher-quality studies have tended to report increasing estimates of the social cost of carbon. The bespoke kernel decomposition method proposed in this paper supports this conclusion much more strongly than standard methods of statistical analysis[20,21].

Table 3 | The empirical and kernel means of the social cost of carbon (US$ per tC) by publication period and PRTP

PRTP (%)	1982–1995	1996–2001	2002–2006	2007–2013	2014–2017	2018–2022
empirical						
any	227	157	102	138	153	286
3	22	66	26	25	58	106
2	40	46	55	126	177	994
1	410	92	69	155	101	266
0	260	1,005	316	198	702	917
kernel						
any	847	392	247	286	362	618
3	26	128	34	30	63	146
2	44	60	74	195	257	1,219
1	686	110	99	340	254	293
0	573	1,097	532	378	979	1,436

Estimates are quality weighted and censored.

The analysis is limited to year of publication and utility discount rate. In the Supplementary Information, author is added. The social cost of carbon is influenced by many other assumptions, including emission scenario, climate sensitivity, damage function, risk aversion and inequity aversion. Unfortunately, documentation is uneven. Considering additional explanatory variables would rapidly reduce the number of observations.

Earlier claims of an increase in the social cost of carbon[17,42–44] are confirmed. There is an apparent upward trend because (1) estimates are reported for later years, (2) there is price inflation and (3) later analyses tend to use lower discount rates. Correcting for these factors and properly accounting for the asymmetric heavy-tailed uncertainty, there is indeed a statistically significant time trend in published estimates of the social cost of carbon.

Between earlier periods, estimates of the social cost of carbon went up and down (Table 3). In the past 10 yr, however, there has been a steady increase. For a PRTP of 3%, 2% or 0%, the kernel estimates more than quadrupled in 2018–2022 compared with 2007–2013—an increase from US$33 per tC to US$146 per tC (3% PRTP), from US$195 per tC to US$1,315 per tC (2% PRTP) and from US$446 per tC to US$1,925 per tC (0% PRTP). For all estimates, regardless of the discount rate used, the increase is smaller, while estimates decrease for a 1% PRTP. The empirical means show much the same pattern but have increased slightly faster than the kernel means. Overall, economists have found higher estimates of the social cost of carbon in more recent times.

Above, the growth rate of the social cost of carbon is mentioned in passing to make comparable estimates for different years of emission. As investment in greenhouse gas emission reduction is driven by the expectation of future climate policy, the growth rate of the social cost of carbon is as important as the social cost of carbon itself. The central estimate is that the social cost of carbon becomes 2.2% larger every year—a factor of 3 over a 50-yr period. The uncertainty about the growth rate is discussed in the Supplementary Information.

Research continuously refines our knowledge and updates our estimates, sometimes upwards, sometimes downwards. There are still many things left to research[45] and things that have been studied but are not yet reflected in our estimates of the social cost of carbon[46–48]. Researchers will continue to disagree on how best to express the ethical dimensions of the social cost of carbon. Moreover, the social cost of carbon reflects the impact of future climate change and the future will remain uncertain.

The implication of the meta-analysis presented here is that the literature on the social cost of carbon does justify an upward revision of carbon prices and emission reduction targets. Furthermore, the literature, summarized in Table 1, suggests that in most of the world, the price of carbon is too low. This is partly because other kinds of climate policies suppress the price of emission permits or reduce the need for a carbon tax. Almost 80% of greenhouse gas emissions is not priced at all[49]. Only the European Union Emissions Trading System, now with permit prices around US$250 per tC, is in the ballpark of published estimates of the social cost of carbon[50]. There is often a gap between the announced emissions targets and the policies supposed to achieve those targets[9]. Besides raising the social cost of carbon, the 'recommended' carbon price, policymakers should focus on raising the 'actual' price of carbon.

Online content

Any methods, additional references, Nature Portfolio reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41558-023-01680-x.

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Methods

Kernel density decomposition

A 'kernel density' is defined as

$$f(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right) \quad (1)$$

where x_i are a series of observations, h is the bandwidth and K is the kernel function. The kernel function is conventionally assumed to be a (1) non-negative (2) symmetric function that (3) integrates to one, with (4) zero mean and (5) finite variance[27]. That is, any standardized symmetric probability density function can serve as a kernel function. The Normal density is a common choice.

Conventions are just that. As long as the kernel function is non-negative (the assumption of non-negativity is relaxed for bias reduction[51]) and integrates to one, an appropriately weighted sum of kernel functions is non-negative and integrates to one—such a sum is a probability density function[36,52,53].

The kernel density is thus defined as the sum of probabilities (see equation (1)). It is a vote-counting procedure[54] where the votes are uncertain. This interpretation fits the nature of the data. Estimates of the social cost of carbon are neither 'data' in the conventional sense of the word, nor can integrated assessment models be seen as 'data generating processes'. Besides, I use the population of estimates rather than a sample. There is therefore no Frequentist interpretation of the proposed method. There is no Bayesian interpretation either. While we might take the kernel function to express degrees of belief, a Bayesian procedure would take the first estimates[1] as previous and later estimates as likelihoods, 'multiplying' rather than 'adding' probability densities. Because some of the estimates are mutually exclusive with other estimates, a Bayesian interpretation is problematic. In this interpretation, dependence between studies and estimates is not an issue: the repetition of a previous estimate raises confidence in that estimate.

A kernel density can also be seen as a mixture density[55,56]. This reinterpretation opens a route to decomposition. We can construct the kernel density of any subset of x_i. The weighted sum of the kernel densities of all subsets is a kernel density.

With the right weights and bandwidths, the weighted sum of the kernel densities of subsets of the data is 'identical' to the kernel density of the whole data set. To see this, partition the observations into m subsets of length m_j with $\sum_j m_j = n$, as $x_1, \dots x_{m_1}, x_{m_1+1}, \dots, x_{m_1+m_2}, x_{m_1+m_2+1}, \dots, x_n$ Then

$$f(x) = \sum_{j=1}^{m} \frac{m_j}{n}\frac{1}{m_j h}\sum_{i=\sum_{k=1}^{j-1} m_k+1}^{\sum_{k=1}^{j} m_k} K\left(\frac{x - x_i}{h}\right) =: \sum_{j=1}^{m}\frac{m_j}{n} f_j(x) \quad (2)$$

In the middle expression, the inner sum is a sum of kernel functions. The sum is over a subsample of the data. The outer sum is over all subsamples. As the weights and bandwidths are the same, the resulting kernel density is identical to equation (1). Moreover, as shown by the rightmost expression, each of the components f_j of the composite kernel density f is itself a kernel density. This is the kernel density for a subsample of the data.

Kernel decomposition works with any set of weights that add to one and with any kernel function or bandwidth for the subsets:

$$f(x) = \sum_{j=1}^{m} \frac{w_j}{m_j h_j}\sum_{i=\sum_{k=1}^{j-1} m_k+1}^{\sum_{k=1}^{j} m_k} K_j\left(\frac{x - x_i}{h_j}\right) =: \sum_{j=1}^{m} w_j f_j(x; h_j) \quad (3)$$

In this case, the composite kernel density is not the same as the kernel density fitted to the complete data set. It is hard to argue for different kernel functions K_j for different subsets of the data, but different subsets of the data would have different spreads and hence bandwidths h_j. I did not do this here, instead I used the same bandwidth for every subsample.

Inference

Equation (3) holds that the kernel density $f(x)$ is composed of m kernel densities $f_j(x)$ with weight m_j/n. For each interval $\underline{x} < x < \bar{x}$, I tested whether the shares of the component densities equalled their weight, using the Equality of Proportions test[37] but using the bootstrapped distribution rather than the asymptotic χ^2 distribution proposed by Pearson. Suppose, for example, that a component density makes up 17% of the overall density. Then, the null hypothesis is that the left tail, central part and right tail of the component density also make up 17% of the left tail, central part and right tail of the overall density.

Let intervals correspond to p percentiles of the composite distribution. The test statistic is

$$\chi^2_{(m-1)(p-1)} = \frac{n}{p}\sum_{k=0}^{p}\sum_{j=1}^{m}\frac{\left(\int_{P_k}^{P_{k+1}} f_j(x)\mathrm{d}x - \frac{m_j}{n}\right)^2}{\frac{m_j}{n}} \quad (4)$$

The test only works if there are two or more components, $m \geq 2$. If not, there would be nothing to compare. The distribution needs to be split in two quantiles or more, $p \geq 2$, because each component density adds up to its weight m_j/n by construction. Again, there would be nothing to compare with fewer than two quantiles.

Data availability

All data can be found in GitHub. Source data are provided with this paper.

Code availability

All code can be found in GitHub.

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Author contributions

This is a single-authored paper. There are no ghostwriters and no research assistants.

Competing interests

The author declares no competing interests.

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Additional information

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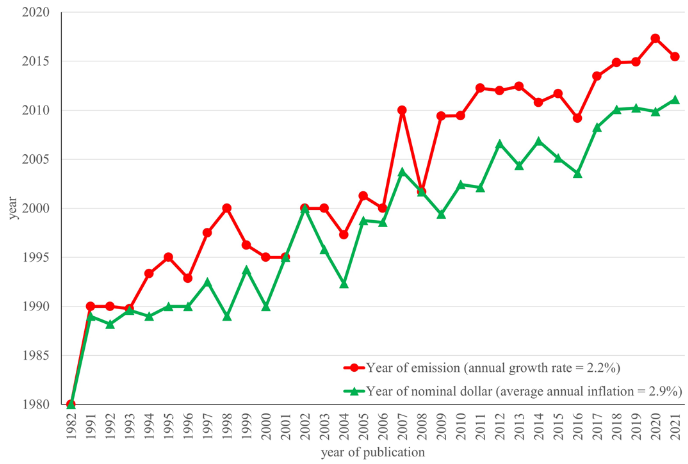

Extended Data Fig. 1 | Year of emission and year of nominal dollar by year of publication. Estimates are weighted such that every published paper counts equally.

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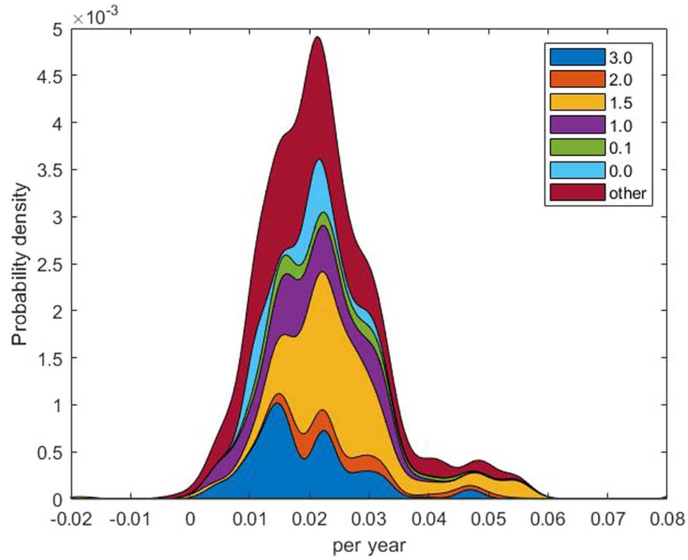

Extended Data Fig. 2 | Composite kernel density of the growth rate of the social cost of carbon and its composition by discount rate.

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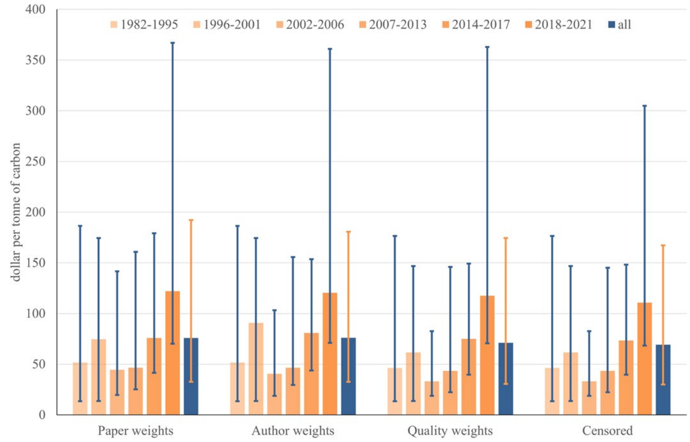

Extended Data Fig. 3 | Empirical median and interquartile range of the social cost of carbon for six subperiods and the whole sample, and for four alternative weights. Sample sizes and further statistics are in Supplementary Tables S1 and S2.

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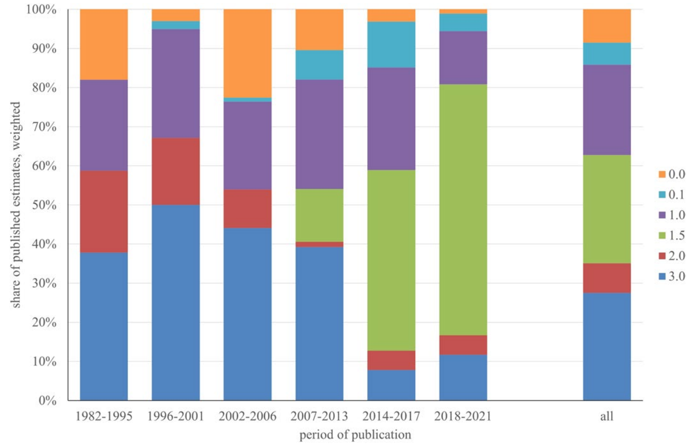

Extended Data Fig. 4 | The pure rate of time preference used to estimate the social cost of carbon by publication period. Estimates are weighted such that every published paper counts equally.

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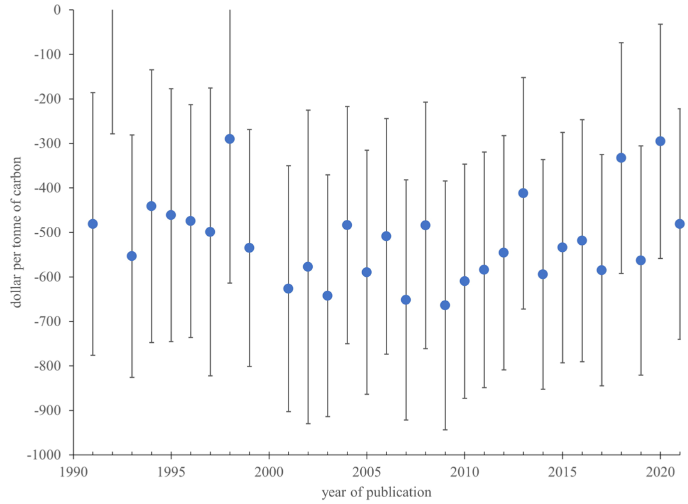

Extended Data Fig. 5 | Year fixed effects from a regression of the social cost of carbon on the pure rate of time preference, using quality weights. The base year is 1982; dots denoted the estimated coefficients; error bars denote the 67% confidence interval.

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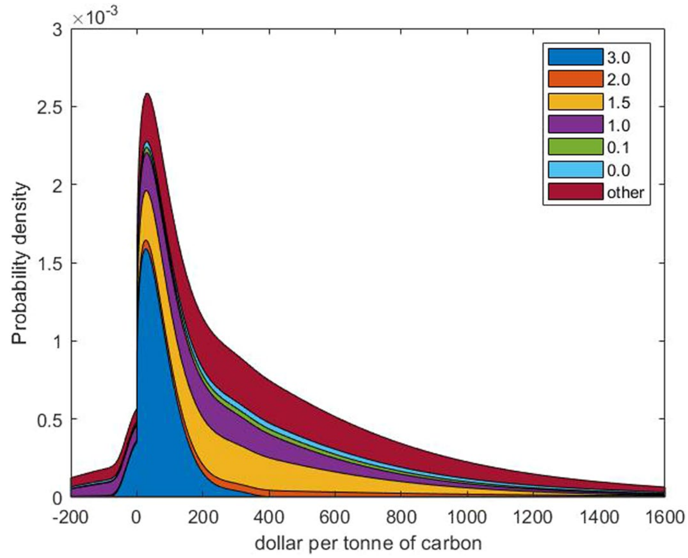

Extended Data Fig. 6 | Composite kernel density of the social cost of carbon and its composition by the pure rate of time preference.