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# Teleworking and Congestion: A Dynamic Bottleneck Analysis

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# Teleworking and congestion:
# A dynamic bottleneck analysis

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**Abstract**

We analyze the welfare effects of part-day teleworking on road traffic congestion in the context of Vickrey's dynamic bottleneck model. Endogenous decisions to become equipped with a teleworking-enabling technology change the scheduling of arrival times at work for equipped drivers and, due to congestion externalities, affects travel costs of all drivers. We show that even costless teleworking might be marginally welfare reducing, after reaching the optimal penetration level, as an equipped driver imposes a higher travel externality on other equipped drivers than unequipped drivers do. We study various possible market configurations for the supply of the technology, and find that private monopolistic supply of the technology might yield a higher social welfare than perfectly competitive supply.

*JEL classification:* D62, O33, R41, R48

*Keywords*: traffic congestion, teleworking, bottleneck model

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## 1. Introduction

Road congestion is a challenging and persisting problem. Various policy measures have been proposed to tackle congestion, including investment in transport infrastructure and public transit, provision of traffic information to drivers, city zoning, road pricing, parking policies and flexible working hours. Since Pigou (1920), most economists regard marginal cost pricing of roads as the first-best solution to traffic jams; see, for example, the exposition in Small and Verhoef (2007). However, optimal tolling seems technically hard to implement in practice; and in part due to its redistributive effect, pricing suffers from low political acceptability that further hinders its wide implementation. There are only a few cities, the best-known being Singapore, Stockholm, and London, with road pricing schemes, usually in the form of a fixed or step cordon toll. The limited feasibility of the first-best policy motivates an ongoing search for alternatives.

Teleworking refers to out-of-office work arrangements, usually from home and sometimes with flexible time schedules, and is one of the suggested ways to reduce travel costs. Whole-day teleworking allows an individual to avoid commuting between the home and the workplace altogether, while part-day teleworking, supposedly, makes it easier to circumvent congestion by commuting during off-peak hours. Progress in information and telecommunication technologies (ICT), such as the availability of the remote access to secured databases, cloud computing, networks and a general advance of Internet technologies, expands both the intensive and the extensive margin of teleworking use. Moreover, governments stimulate teleworking use. For instance, in the USA, the Telework Enhancement Act of 2010 promotes teleworking among public servants (USA Government, 2010). Given the range of potential benefits on labor productivity, work-life balance, job matching, and given expected future technological progress, one may expect teleworking to be of increasing relevance in the future.[1]

Against this background, this paper will investigate the effects of part-day (morning) teleworking on congestion from economic perspective. Part-day teleworking is an empirically relevant phenomenon; for example, a recent UK survey shows that in 2007 part-day teleworking had a higher incidence rate among full-time employees, within a survey's reference week, than a whole-day teleworking, respectively 17.2 and 9.8 percent (Haddad, Lyons and Chatterjee, 2009).[2] In the context of this paper one might think of the employees performing some work tasks from home in the morning, and then coming to an office for the rest of the workday. This is an example of what has been dubbed "work fragmentation", which is not uncommon in reality (see, for instance, a survey by KPMG, 2011). The few papers that do model the impacts of teleworking on travel introduce a spatial dimension, to capture the long-term effects of whole-day teleworking on residential choice within a city; see, among others, Safirova (2002) and Rhee (2008). But time-of-day adjustments may also be relevant especially for part-day teleworking. To

[1] Increase of the teleworking incidence rate over time has occurred in the past. According to the Eurofound surveys (2005, 2010), an employees' self-reported EU average incidence rates of teleworking for at least one-quarter of their time were 4 and 7 percents in, respectively, 2000 and 2005.

[2] There is a large cross country, industry and day of the week variation in the incidence rates of teleworking, see, for example, Eurofound (2010) report on teleworking in the EU.

effectively incorporate this temporal aspect of part-day teleworking, we apply Vickrey's (1969) dynamic bottleneck model, a work-horse model in transportation economics, in which the drivers' scheduling decisions are endogenous.

We model the behavioral impacts of teleworking by assuming that access to the teleworking-enabling ICT raises utility that an individual derives from being at home at any given point in time. We therefore define a teleworking individual as a person who is equipped with a technology that allows her to perform various work tasks from home. An equipped individual values time spent at home higher than an unequipped one, and, as we show later, has an incentive to postpone the arrival time at work. The choice of whether to be equipped is determined within the model; thus our model may produce endogenous heterogeneity of drivers, if not everybody chooses to become equipped. We derive an inverse demand for the teleworking technology, and show that the marginal willingness to pay depends negatively on the number of teleworking people, due to the relatively large congestion externality equipped drivers impose on one another. We show that even costless technology might be marginally welfare reducing after teleworking reaches a certain optimal penetration level. We also study private market provision of the teleworking technology, both under perfect competition and monopoly, and define conditions when the social welfare is found to be higher under monopoly.

Our study fits into a wider literature that considers the potential impacts of ICT on congestion and social welfare. However, most of this literature focuses on the provision of traffic information to drivers; see for example, Arnott, de Palma and Lindsey (1996); de Palma and Lindsey (1998); and Emmerink, Verhoef, Nijkamp and Rietveld (1998a, 1998b). These studies consistently show that under an unpriced congestion externality, the marginal effect of information might sometimes be welfare decreasing. To the best of our knowledge, our study is the first to show this in the context of teleworking. Given the popularity of ICT-based solutions to traffic congestion, these results are important for practical policy making.

The paper is organized as follows. Section 2 introduces Vickrey's dynamic bottleneck model, and teleworking within that framework. Section 3 derives the marginal willingness to pay for and social benefits of teleworking. Section 4 considers private provision of the teleworking technology in markets of perfect competition and monopoly. We evaluate the relative efficiency of market outcomes, compared to the social optimum. Section 5 considers the impact of teleworking on travel in case first-best road tolling already addresses congestion externality. Section 6 summarizes the paper, highlights the main findings, and concludes with a list of possible extensions.

## 2. Teleworking within Vickrey's dynamic bottleneck model

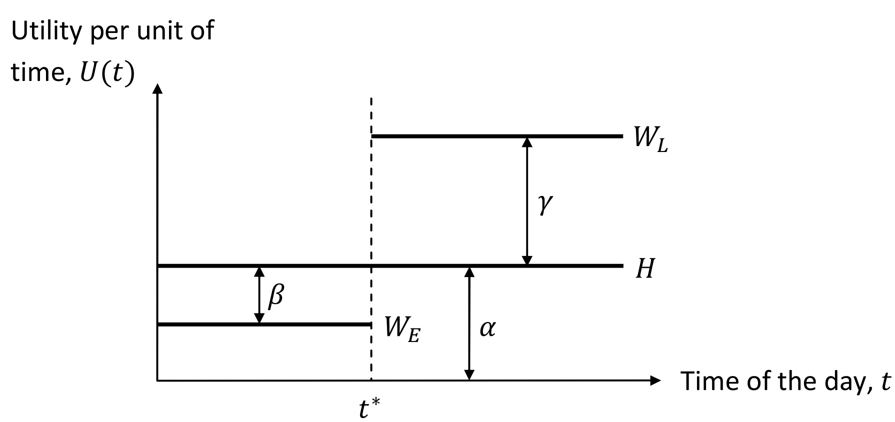
### 2.1. Basic model

Our analysis is cast in the framework of Vickrey's (1969) dynamic bottleneck model. The standard bottleneck model uses a conventional linearized scheduling model, attributed to Vickrey (1969) and

Small (1982), and now extensively used in the transportation economics literature. The model considers a traffic jam at a single bottleneck with a capacity $s$ on a road connecting one origin ("home") and one destination ("work"). $\bar{N}$ homogeneous atomistic drivers, with a common preferred arrival time at work $t^*$, commute (in the morning) from home to work. Time spent on travel (or travel delay) is entirely due to waiting in a "first-in first-out" traffic jam, which starts building up after the flow of drivers arriving to the bottleneck has first exceeded its capacity. The (free-flow) travel time is set to zero, without loss of generality in this context, implying that without a queue drivers depart from home, pass the bottleneck and arrive at work at the same moment. This stylized description of traffic congestion offers a framework for studying dynamic departure time decisions, and the dynamic evolving of traffic conditions over the peak within one analytical model that lends itself to closed-form solutions of optima and equilibria (e.g., Arnott, de Palma and Lindsey, 1993).

Vickrey (1973), and later Tseng and Verhoef (2008), proposed a somewhat more general specification of scheduling behavior and utility, which explicitly describes the underlying pattern of activities in terms of time-dependent utilities and opportunity costs. In this approach, a driver is confronted with time-variant utilities of being in various locations. This specification is useful for our purposes, as it allows us to incorporate the impact of teleworking technology in the bottleneck model in a structured way. In this approach, a driver derives utility from being either at home ($H(t)$ per unit of time), at work (W(t) per unit of time), or in a vehicle ($V(t)$, normalized at zero for convenience).[3] Tseng and Verhoef (2008) show that if (and only if) these functions are such that $H(t)$ is equal to some constant ($H$ henceforth) throughout the period considered, while $W(t)$ is piecewise constant with a certain upward jump at $t^*$, and $W(t) = W_E < H$ before $t^*$ and $W(t) = W_L > H$ after $t^*$, it is equivalent to the standard linear

**Figure 1.** Utility structure and opportunity costs for a driver in the conventional representation of Vickrey's bottleneck model




Based on Tseng and Verhoef (2008), Fig. 2.

[3] Strictly speaking, $H$, W and $V$ are Marshallian surpluses, which are the ratios of marginal utility of time spend at respectively, home, work and in a vehicle, over marginal utility of income. For brevity we refer to them as utilities.

scheduling model in which there is a time-invariant value of travel time ($\alpha$), a constant unit shadow price of schedule delay early ($\beta$), and one of schedule delay late ($\gamma$). The equivalence stems from the fact that the opportunity cost of being at work before $t^*$ is, then, $H - W_E = \beta$; the opportunity cost of not being at work after $t^*$ is $W_L - H = \gamma$; and the opportunity cost of being in a vehicle is $H - V = \alpha$. We plot this utility structure in Figure 1.

While various utility specifications for $H(t)$ and $W(t)$ are possible, we use the above linearized one primarily to stay as close as possible to the conventional linear scheduling model which assumes constancy of $\alpha$, $\beta$ and $\gamma$ (Small, 1982) and which has been applied in the bulk of the bottleneck model literature. It is therefore a natural choice, and we expect that insights on the desirability of different market structures for the supply of teleworking technology will not depend critically on this specific choice.

The single margin of behavior in this model is the arrival time at work, $t$, which a driver sets to maximize utility over the course of the morning that starts for every driver at the common time $t_S$ and finishes at $t_F$ (chosen such that the interval is wide enough to cover the entire peak). An individual's utility level is equal to an "ideal" utility level $\Gamma$, which a driver would reach over the course of the morning, had she both departed and arrived at the preferred arrival time $t^*$, minus the generalized travel cost she actually incurs. The latter consists of travel time and schedule delay costs, and will be denoted $P(t)$, where $t$ denotes the arrival time at work. A driver incurs schedule delay cost when the time of her arrival at work is not $t^*$. Each minute of arriving at work either early or late has a value of, respectively, $\beta$ and $\gamma$. In turn, each minute of travel delay, $T(t)$, has a value of time $\alpha$.

Thus, the utility level of a driver arriving to work at time $t$ is:

$$(1) \qquad U(t) = \Gamma - P(t) = \alpha(t^* - t_S) + (\alpha + \gamma)(t_F - t^*) - \alpha T(t) - \begin{cases} \beta(t^* - t) & | \; t \leq t^* \\ \gamma(t - t^*) & | \; t > t^* \end{cases}$$

Note that the first two terms are constants, and the final two (time-depended) terms correspond to the conventional generalized cost of travel.

In a dynamic equilibrium, the utility levels of all (homogeneous) drivers must be equal; i.e., no one is able to adjust her arrival time and consequently gain in utility. The very first driver arrives at work at time $t_q$ and faces no travel time costs as she freely passes the bottleneck. But she incurs schedule delay costs from being early at work; thus, her generalized cost is $\beta\left(t^* - t_q\right)$. Likewise the very last driver who arrives at time $t_{q'}$ incurs the cost of arriving late at work; the generalized travel cost is then $\gamma\left(\, t_{q'} - t^*\right)$. Because the ideal utility level $\Gamma$ is identical across drivers, the equilibrium condition implies equality of generalized travel costs. Given that the duration of the peak period is $\frac{\bar{N}}{s} = t_q - \; t_{q'}$, traffic starts and ends at:

(2) $$t_q = t^* - \frac{\gamma}{\beta+\gamma}\frac{\bar{N}}{s}$$

(3) $$t_{q'} = t^* + \frac{\beta}{\beta+\gamma}\frac{\bar{N}}{s}$$

The generalized travel costs are then:

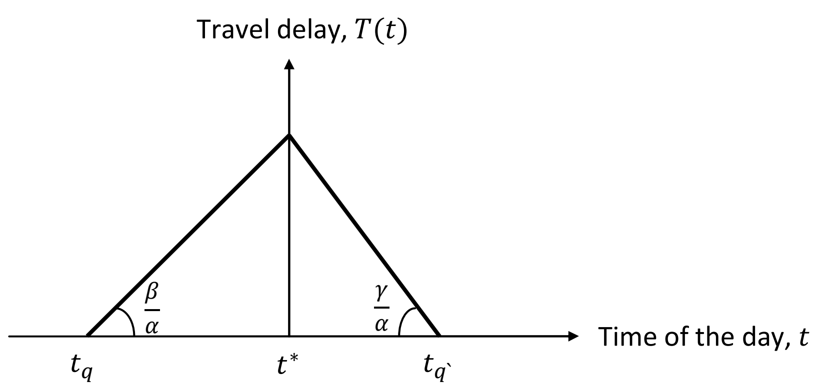
(4) $$P(t) = \frac{\beta\gamma}{\beta+\gamma}\frac{\bar{N}}{s}$$

(see also Arnott, de Palma and Lindsey, 1993).

The driver who arrives at work at time $t^*$, only incurs travel time cost $\alpha T(t^*)$. In equilibrium, she has the same generalized travel cost as the first driver; hence, $T(t^*) = \frac{\beta\gamma}{\beta+\gamma}\frac{\bar{N}}{s}\frac{1}{\alpha}$.

The dynamic bottleneck model highlights an important equilibrating mechanism affecting behavior in traffic congestion: the trade-off that drivers make between schedule costs of arriving at an inconvenient time, versus the travel delay cost of waiting in the queue. In equilibrium, when generalized costs are identical across drivers, for those who arrive before $t^*$ each additional one minute arrived closer to $t^*$ decreases the schedule delay cost by $\beta$, but must increase travel time by $\frac{\beta}{\alpha}$ to keep generalized cost constant over time. In the same fashion, for arrivals later than $t^*$ the travel delay decreases with $\frac{\gamma}{\alpha}$ by arrival time to keep generalized cost constant. Figure 2 shows the equilibrium combinations of arrival times and travel delays. The slopes of the triangle naturally depends on the parameters $\alpha$, $\beta$, $\gamma$; while the width depends on $\frac{\bar{N}}{s}$, which determines the duration of the peak (the time interval between $t_q$ and $t_{q'}$). As the generalized costs are constant over time, one may interpret the graph as an isocost function. As there are no arrival times with a generalized cost level below the equilibrium level, the equilibrium in Vickrey's dynamic bottleneck model is a Nash equilibrium.

**Figure 2.** Isocost function of the homogeneous drivers



2.2 Introduction of teleworking technology

The framework shown in Figure 1 helps us to make a structured and well-motivated assumption on how the availability of teleworking technology would affect the value of time components $\alpha$, $\beta$ and $\gamma$. Sticking to a piecewise constant utility structure, and assuming that the technology would affect the utility of being at home (not of being at work or in the vehicle), the natural assumption to make is that it shifts $H$ upwards; and does so by some constant to maintain the qualitative pattern displayed in Figure 1. This means that, in terms of the conventional scheduling formulation, adaptation of teleworking technology will lead to equally large increases in $\alpha$ and $\beta$, and a decrease in $\gamma$ that is equally large in absolute size. Intuitively, an individual equipped with the teleworking technology would put a higher value on time spent at home before $t^*$, as being at home results in higher utility due to the possibility of teleworking. At the same time, for an equipped driver, who arrives at work after $t^*$, a trip does not cause as much cost as for an unequipped one, as a teleworker partly "compensates" the disutility of a delayed arrival at work with working from home.

A constant shift in $H$ due to teleworking technology implies that an equipped driver starts gaining higher utility from being at home right after the beginning of the morning, at $t_S$. That might represent that an equipped driver works during the entire morning, or, alternatively, that due to the availability of technology an individual is able to reschedule other activities (not modeled explicitly) in such a way that the utility derived from being at home before the start of teleworking rises.

Of course, other assumptions could be made on how teleworking would affect the individual's utility function. We believe our assumption captures the most relevant aspect of the issue, in the simplest possible utility specification. Specifically, only with a constant upward shift of $H$ would the individual, both before and after being equipped, have a utility function that can be characterized by three constant shadow prices $\alpha$, $\beta$ and $\gamma$.

We thus assume the technology raises the unit value of staying at home by a constant $\Delta < \gamma$. The latter constraint assures that also those drivers who are equipped with the teleworking technology still find it worthwhile to be at work at time $t \geq t^*$. Note from Figure 1 that our specification leaves the preferred arrival time $t^*$ unchanged. This is in fact a welcome feature, because it secures that any shift of the peak period that results from adaptation of technology by drivers (which we do find) can be ascribed solely to the impact of changes in $H$, $\alpha$, $\beta$ and $\gamma$; and not to a change of $t^*$.

## 3. Marginal willingness to pay for and externalities of teleworking technology

### 3.1 Marginal willingness to pay for teleworking technology

In this section we derive the marginal willingness to pay for acquiring teleworking technology. We will show that this willingness to pay depends on the aggregate level of technology penetration: if fewer drivers are equipped with the technology, an individual driver is willing to pay more for it.

The marginal willingness to pay for teleworking technology can be determined as the difference between the utility that a driver reaches over the course of the morning if one is equipped with the technology, minus the utility when being unequipped. As follows from Equation (1), changes in opportunity costs $\alpha$, $\beta$ and $\gamma$, affect an individual's utility via a change in the ideal utility, $\Gamma$, and in the generalized travel costs that one incurs, $P$. A driver equipped with teleworking technology has a higher ideal utility than an unequipped driver, because between $t_S$ and $t^*$ a higher utility of being at home is enjoyed.[4] This increase in ideal utility is, of course, identical for all teleworkers, and does not depend on one's arrival time at work.

Both equipped and unequipped drivers choose their arrival time at work $t$ to minimize generalized travel costs. With different time values $\alpha$, $\beta$ and $\gamma$, the slopes of isocost functions as shown in Figure 2 may differ between drivers as well. Therefore, a driver who adopts the technology may have an incentive to change the arrival time at work, in order to minimize generalized travel costs under the new time values.

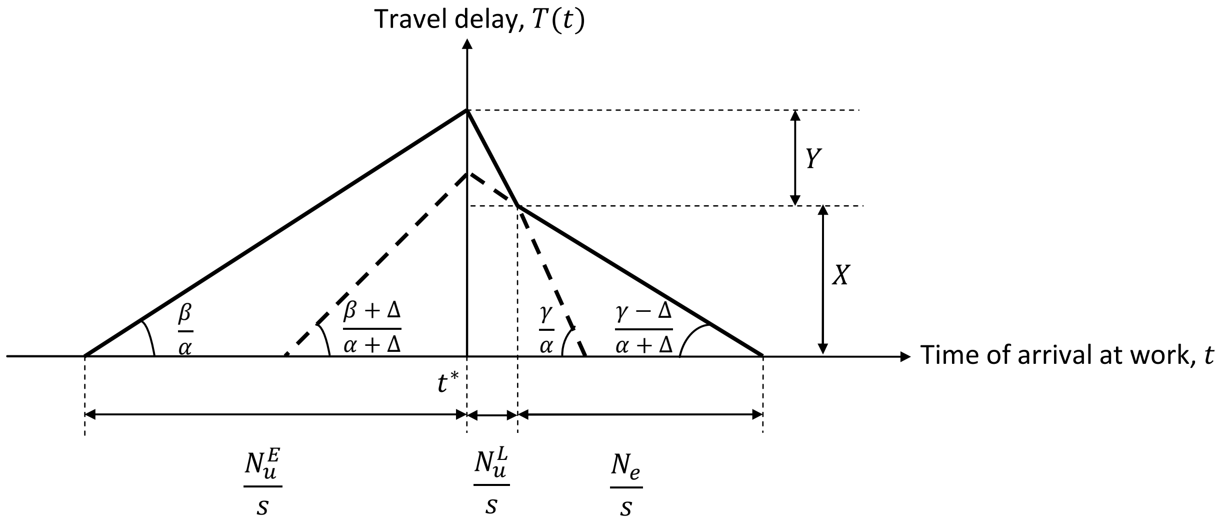
Dynamic equilibrium requires that for both groups of travelers (if both are greater than zero in size) the generalized travel costs are equal at moments when arrivals occur, and not lower at other times. We will see shortly that this will involve temporal separation of travelers if both types exist.

To see why this occurs, first note that the upper envelope of the isocost functions, for both groups, corresponds to the equilibrium pattern of travel delays. Let $N_e$ be the number of equipped drivers, $N_u^L$ and $N_u^E$ are the numbers of unequipped drivers arriving at work, respectively, after ("late") and before ("early") $t^*$. For a fixed overall number of drivers, $\bar{N}$, the duration of the peak period will be $\frac{\bar{N}}{s} = \frac{N_e}{s} + \frac{N_u^L}{s} + \frac{N_u^E}{s}$. The timing of the beginning of the peak, however, is endogenous.

To determine equilibrium level of generalized cost, we have to distinguish between cases with relatively low numbers of equipped drivers ($0 \leq N_e \leq N_e^{\#}$), and high numbers ($N_e^{\#} < N_e \leq \bar{N}$), where $N_e^{\#}$ is defined later. With low numbers, equipped drivers will arrive only after $t^*$. Figure 3 illustrates this type of equilibrium, and shows the equilibrium isocost functions for both types of drivers. The isocost lines of unequipped drivers have slopes $\frac{\beta}{\alpha}$ and $\frac{\gamma}{\alpha}$; those of equipped drivers have slopes $\frac{\beta+\Delta}{\alpha+\Delta}$ and $\frac{\gamma-\Delta}{\alpha+\Delta}$. The slope for equipped drivers is, therefore, steeper for early arrivals and flatter for late ones. It is very easy to prove that the first single (atomistic) driver who gets equipped prefers to be the last traveler passing the bottleneck. Given the equilibrium isocost line for unequipped drivers, which gives the equilibrium pattern of travel times for this group, the last arrival time brings the single equipped driver at the lowest achievable isocost line. This driver spends the peak at home, benefits from teleworking, and then travels to work incurring generalized travel costs of $\frac{\beta(\gamma-\Delta)}{\beta+\gamma}\frac{\bar{N}}{s}$. As more drivers become equipped, their equilibrium isocost function will shift upwards to accommodate the increasing number $N_e$. At the same time, the equilibrium isocost of the unequipped group moves downward as there is decreasing demand for early arrivals. The different slopes of isocost lines in Figure 3 thus induce a temporal separation of travelers where equipped drivers arrive later than unequipped ones.

[4] The difference in ideal utility levels of equipped drivers $\Gamma_e$ and unequipped ones $\Gamma_u$ is $\Gamma_e - \Gamma_u = \Delta(t^* - t_S) \geq 0$, which is greater than zero whenever $t^* > t_S$ and $\Delta > 0$, as we assume.

**Figure 3.** Isocost functions of the heterogeneous drivers, if equipped drivers arrive at work late




To calculate equilibrium cost levels as a function of $N_e$, we indicate in Figure 3 the travel delay that the first arrived equipped driver incurs as $X$, while $Y$ gives the absolute difference between $X$ and the travel delay of a driver who arrives at work at time $t^*$. We can write $X$ and $Y$ as follows:

(5) $$X + Y = \frac{\beta}{\alpha}\frac{N_u^E}{s} = \frac{\beta}{\alpha}\frac{\bar{N}-N_e-N_u^L}{s}$$

(6) $$X = \frac{\gamma-\Delta}{\alpha+\Delta}\frac{N_e}{s}$$

(7) $$Y = \frac{\gamma}{\alpha}\frac{N_u^L}{s}$$

These equalities can easily be verified in Figure 3. After substituting (6) and (7) into (5), we, can express $\frac{N_u^L}{s}$ (i.e., the duration of the interval where unequipped drivers arrive after $t^*$) and $\frac{N_u^E}{s}$ (i.e., the duration of the peak before $t^*$) as a function of $N_e$:

(8) $$\frac{N_u^L}{s} = \frac{\beta}{\beta+\gamma}\frac{\bar{N}}{s} - \frac{(\alpha+\Delta)\beta+\alpha(\gamma-\Delta)}{(\alpha+\Delta)(\beta+\gamma)}\frac{N_e}{s}$$

(9) $$\frac{N_u^E}{s} = \frac{\gamma}{\beta+\gamma}\frac{\bar{N}}{s} - \frac{\Delta(\alpha+\gamma)}{(\alpha+\Delta)(\beta+\gamma)}\frac{N_e}{s}$$

Note that (8) is non-negative if $N_e$ is below the level that defines the threshold value $N_e^{\#}$:

$$(10)\qquad N_e^{\#} = \frac{\beta(\alpha+\Delta)}{\beta(\alpha+\Delta)+\alpha(\gamma-\Delta)}\overline{N}$$

Multiplying (9) by $\beta$, we get the generalized travel costs of unequipped drivers as a function of $N_e$ (for $0 \leq N_e \leq N_e^{\#}$):

$$(11)\qquad P_u = \frac{N_u^E}{s}\beta = \frac{\gamma\beta}{\beta+\gamma}\frac{\overline{N}}{s} - \frac{\Delta\beta(\alpha+\gamma)}{(\alpha+\Delta)(\beta+\gamma)}\frac{N_e}{s}$$

This shows that the generalized travel costs of unequipped drivers is decreasing when the share of equipped drivers rises. Thus, equipped drivers in some sense impose a positive external effect on unequipped drivers. More precisely, they impose a smaller external cost on unequipped drivers than unequipped drivers do themselves. The underlying reason is that the groups have different preferences for arrival time adjustments, where equipped drivers have a less strong demand for early arrivals. The utility of unequipped drivers is the ideal utility minus the generalized travel costs:

$$(12)\qquad U_u = \Gamma_u - P_u = \alpha(t^* - t_S) + (\alpha+\gamma)(t_F - t^*) - \frac{\gamma\beta}{\beta+\gamma}\frac{\overline{N}}{s} + \frac{\Delta\beta(\alpha+\gamma)}{(\alpha+\Delta)(\beta+\gamma)}\frac{N_e}{s}$$

Next, the generalized travel costs of equipped drivers amount to

$$(13)\qquad P_e = \left(\frac{N_u^L}{s} + \frac{N_e}{s}\right)(\gamma-\Delta) = \frac{\beta(\gamma-\Delta)}{\beta+\gamma}\frac{\overline{N}}{s} + \frac{\Delta(\gamma-\Delta)(\alpha+\gamma)}{(\alpha+\Delta)(\beta+\gamma)}\frac{N_e}{s}$$

The higher the number of equipped drivers, the larger their travel costs. Equipped drivers impose a negative marginal externality on their own group, that exceeds the negative externality from unequipped drivers. Both terms in (13) are positive, where the second one represents the additional impact of an equipped driver on $P_e$ above the one of unequipped drivers. The utility of equipped drivers is therefore:

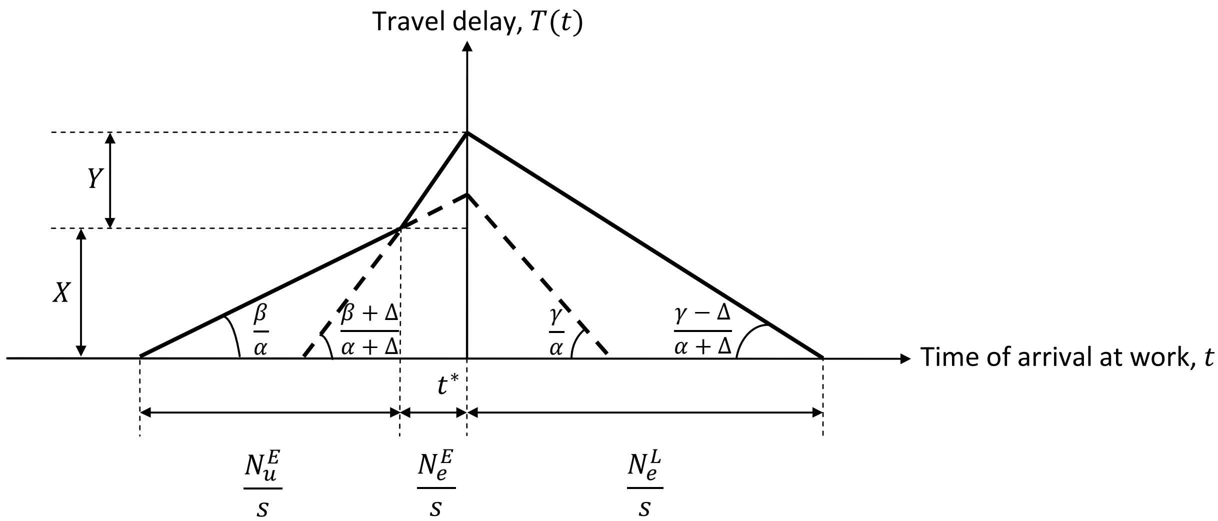
$$(14)\qquad U_e = \Gamma_e - P_e = (\alpha+\Delta)(t^* - t_S) + (\alpha+\gamma)(t_F - t^*) - \frac{\beta(\gamma-\Delta)}{\beta+\gamma}\frac{\overline{N}}{s} - \frac{\Delta(\gamma-\Delta)(\alpha+\gamma)}{(\alpha+\Delta)(\beta+\gamma)}\frac{N_e}{s}$$

By subtracting (12) from (14) we find the marginal willingness to pay ($MWTP$) for teleworking technology as a function of the number of adopters $N_e$, when their number is "low", $0 \leq N_e \leq N_e^{\#}$:

$$(15)\qquad MWTP_L = U_e - U_u = \Gamma_e - P_e - \Gamma_u + P_u = \Delta(t^* - t_S) + \frac{\beta\Delta}{\beta+\gamma}\frac{\overline{N}}{s} - \frac{\Delta(\beta+\gamma-\Delta)(\alpha+\gamma)}{(\alpha+\Delta)(\beta+\gamma)}\frac{N_e}{s}$$

$MWTP_L$ is downward sloping with respect to $N_e$, so if more drivers are equipped, an additional individual driver is willing to pay less for it. That is true for both the marginal unequipped driver who becomes equipped, but also for the already equipped drivers, who have the same benefit of being equipped as the marginal equipped driver. Hence, the total benefit for the equipped drivers collectively is $MWTP_L N_e$.

**Figure 4.** Isocost functions of the heterogeneous drivers, if equipped drivers arrive at work late and early




The logic of the derivation of $MWTP$ stays the same if the number of teleworkers becomes larger than $N_e^{\#}$, but the difference is that the temporal separation will take place before $t^*$. As Figure 4 shows, the change in generalized travel costs for drivers is then determined by the relative differences of the isocost slopes of early arrivals, not the later ones.

Let $N_e^E$ be the number of equipped drivers arriving early, and $N_e^L$ the number arriving late, so that $N_e = N_e^E + N_e^L$. For the present case $N_e^{\#} < N_e \le \bar{N}$, we can express $X$ and $Y$, as shown in Figure 4, as:

(16) $$X + Y = \frac{\gamma-\Delta}{\alpha+\Delta}\frac{N_e^L}{s} = \frac{\gamma-\Delta}{\alpha+\Delta}\frac{\bar{N}-N_u^E-N_e^E}{s}$$

(17) $$X = \frac{\beta}{\alpha}\frac{N_u^E}{s} = \frac{\beta}{\alpha}\frac{\bar{N}-N_e^E-N_e^L}{s}$$

(18) $$Y = \frac{\beta+\Delta}{\alpha+\Delta}\frac{N_e^E}{s}$$

After substituting (17) and (18) into (16), we derive $\frac{N_e^E}{s}$ (i.e., the duration of the interval when equipped drivers arrive early) and $\frac{N_e^L}{s}$ (i.e., the duration of the peak after $t^*$):

(19) $$\frac{N_e^E}{s} = -\frac{(\alpha+\Delta)\beta}{\alpha(\beta+\gamma)}\frac{\bar{N}}{s} + \frac{(\alpha+\Delta)\beta+\alpha(\gamma-\Delta)}{\alpha(\beta+\gamma)}\frac{N_e}{s}$$

(20) $$\frac{N_e^L}{s} = \frac{(\alpha+\Delta)\beta}{\alpha(\beta+\gamma)}\frac{\bar{N}}{s} + \frac{(\alpha-\beta)\Delta}{\alpha(\beta+\gamma)}\frac{N_e}{s}$$

The generalized cost for equipped drivers, and the implied utility, become:

(21) $$P_e = \frac{N_e^L}{s}(\gamma-\Delta) = \frac{(\gamma-\Delta)(\alpha+\Delta)\beta}{\alpha(\beta+\gamma)}\frac{\bar{N}}{s} + \frac{(\alpha-\beta)\Delta(\gamma-\Delta)}{\alpha(\beta+\gamma)}\frac{N_e}{s}$$

and

(22) $$U_e = \Gamma_e - P_e = (\alpha+\Delta)(t^* - t_S) + (\alpha+\gamma)(t_F - t^*) - \frac{(\gamma-\Delta)(\alpha+\Delta)\beta}{\alpha(\beta+\gamma)}\frac{\bar{N}}{s} - \frac{(\alpha-\beta)\Delta(\gamma-\Delta)}{\alpha(\beta+\gamma)}\frac{N_e}{s}$$

For an unequipped driver we find:

(23) $$P_u = \left(\frac{\bar{N}}{s} - \frac{N_e^L}{s}\right)\beta = \left(\frac{\bar{N}}{s} - \frac{(\alpha+\Delta)\beta}{\alpha(\beta+\gamma)}\frac{\bar{N}}{s} - \frac{(\alpha-\beta)\Delta}{\alpha(\beta+\gamma)}\frac{N_e}{s}\right)\beta$$

and

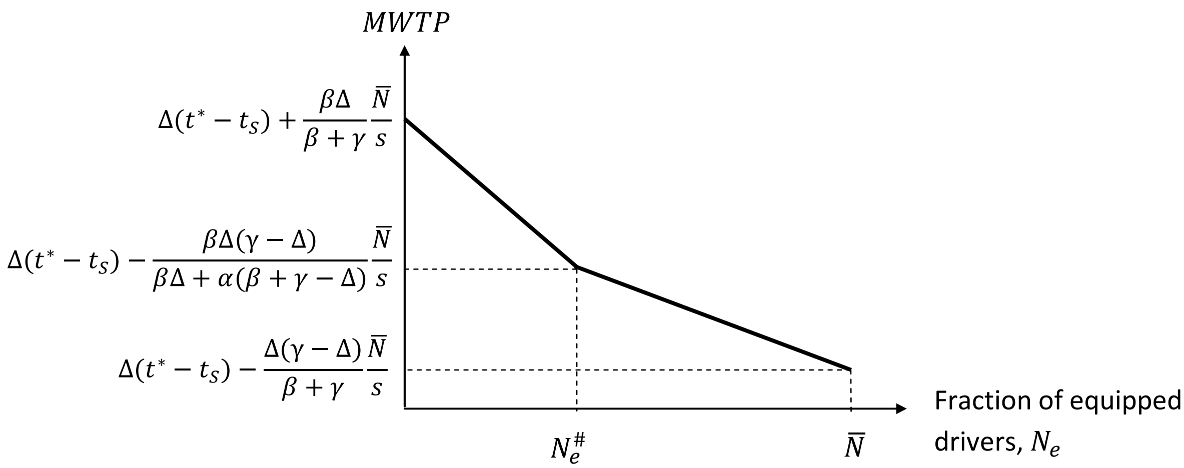
(24) $$U_u = \Gamma_u - P_u = \alpha(t^* - t_S) + (\alpha+\gamma)(t_F - t^*) - \left(\frac{\bar{N}}{s} - \frac{(\alpha+\Delta)\beta}{\alpha(\beta+\gamma)}\frac{\bar{N}}{s} - \frac{(\alpha-\beta)\Delta}{\alpha(\beta+\gamma)}\frac{N_e}{s}\right)\beta$$

By subtracting (24) from (22) we find the $MWTP$ when $N_e$ is "high" or $N_e^{\#} < N_e \leq \bar{N}$:

(25) $$MWTP_H = U_e - U_u = \Gamma_e - P_e - \Gamma_u + P_u = \Delta(t^* - t_S) - \frac{\beta\Delta(\beta-\alpha+\gamma-\Delta))}{\alpha(\beta+\gamma)}\frac{\bar{N}}{s} - \frac{\Delta(\beta+\gamma-\Delta)(\alpha-\beta)}{\alpha(\beta+\gamma)}\frac{N_e}{s}$$

The slope of $MWTP$ in the range $N_e^{\#} < N_e \leq \bar{N}$, implied by (25), is flatter than in the range $0 \leq N_e \leq N_e^{\#}$ (see equation (15)), resulting in a kink in $MWTP$ function at $N_e^{\#}$. Equations (15) and (25) imply the same $MWTP$ value at $N_e^{\#}$. Figure 5 illustrates the marginal willingness to pay function as given by equations (15) and (25). Although the $MWTP$ is declining, also the very last driver to become equipped has a positive willingness to pay, because we set the start of the day $t_S$ before the first driver arrives at work when teleworking possibility is not available (i.e., $t^* - t_S \geq \frac{\gamma}{\beta+\gamma}\frac{\overline{N}}{s}$). Under this constraint, the $MWTP$ value at $\overline{N}$, as shown in Figure 5, is positive, since then $MWTP > \frac{\Delta^2}{\beta+\gamma}\frac{\overline{N}}{s}$. A positive $MWTP$ also for the last driver to become equipped is consistent with the notion that even when not changing departure time, this driver has benefited at a rate Δ over the time spent home between the start of the day and the moment of departing.

**Figure 5.** Marginal willingness to pay for teleworking technology




3.2 Marginal social benefits function

All drivers' generalized travel costs change when an additional unequipped driver becomes equipped. More precisely, with each additional driver becoming equipped, the already equipped drivers will incur higher costs, while unequipped drivers' costs decrease; see equations (11), (13) for the "low" levels of $N_e$, and (21) and (23) for the "high" ones. Changes in costs for other drivers are external to the driver who becomes equipped. The (private) marginal willingness to pay function therefore does not correspond to the marginal social benefits function, as the latter includes externalities.

To calculate the socially optimal level of teleworking, we derive the marginal social benefit function of teleworking technology ($MSB$). With zero marginal cost, the adaptation of the technology by an additional driver is socially desirable as long as $MSB$ is non-negative; otherwise the number of teleworkers is socially excessive.

To determine the $MSB$, the entire population can be subdivided into three groups: the single (atomistic) driver who becomes equipped, the drivers already equipped ($N_e$), and those unequipped ($\overline{N} - N_e$). $MSB$ is then equal to the sum of marginal willingness to pay $MWTP$ of the driver who is becoming equipped, minus the marginal external cost for all equipped drivers, plus the marginal external benefits[5] of all unequipped drivers. The marginal external benefits of all unequipped drivers is the derivative of their generalized travel costs with respect to number of equipped drivers, multiplied by the number of unequipped drivers; and similarly for equipped drivers.

One can also determine the $MSB$ by taking the derivative of the total generalized travel costs of all drivers jointly with respect to the number of equipped drivers. $MSB$ is then minus the resulting derivative plus the increase in ideal utility for a single driver who is getting equipped. It has been verified that the two approaches lead to the same result.

Let us first consider the case where the number of teleworkers is "low", i.e., $0 \leq N_e \leq N_e^{\#}$. The marginal benefits of all unequipped drivers is then:

(26) $$MSB_u = -\frac{dP_u}{dN_e}(\bar{N} - N_e) = \frac{\Delta\beta(\alpha+\gamma)(\bar{N}-N_e)}{(\alpha+\Delta)(\beta+\gamma)s}$$

Likewise, the overall marginal costs of already equipped drivers is:

(27) $$MSB_e = -\frac{dP_e}{dN_e}N_e = -\frac{\Delta(\gamma-\Delta)(\alpha+\gamma)}{(\alpha+\Delta)(\beta+\gamma)}\frac{N_e}{s}$$

The marginal social benefit function, conditional on "low" number of equipped drivers, $0 \leq N_e \leq N_e^{\#}$, is then the sum of (15), (26) and (27), which gives:

(28) $$MSB_L = \Delta(t^* - t_S) + \frac{\beta\Delta(2\alpha+\Delta+\gamma)}{(\alpha+\Delta)(\beta+\gamma)}\frac{\bar{N}}{s} - 2\frac{\Delta(\beta+\gamma-\Delta)(\alpha+\gamma)}{(\alpha+\Delta)(\beta+\gamma)}\frac{N_e}{s}$$

In the same fashion, but now using cost functions for "high" levels of penetration, we can define $MSB$ in the range $N_e^{\#} < N_e \leq \bar{N}$. This results in:

(29) $$MSB_H = \Delta(t^* - t_S) - \frac{\beta\Delta(2\beta-2\alpha+\gamma-\Delta)}{\alpha(\beta+\gamma)}\frac{\bar{N}}{s} - 2\frac{\Delta(\beta+\gamma-\Delta)(\alpha-\beta)}{\alpha(\beta+\gamma)}\frac{N_e}{s}$$

Note that $MSB_L$ and $MSB_H$ have different slopes and intercepts, and overall $MSB$ is discontinuous at $N_e^{\#}$. This discontinuity stems from the differences in external effects that drivers impose upon one another in early arrivals compared to later ones.

[5] For brevity we refer to the decrease in marginal external cost for unequipped drivers as if it were a marginal external benefit.

Comparing $MSB$ as given in equations (28) and (29) to $MWTP$, in equations (15) and (25), we can establish that the slope of the $MSB$ function is twice as steep as the slope of the $MWTP$.

In particular, let $TSC$ be the total social costs, $\varphi = \frac{dP_u}{dN_e}$, and $\mu$ and $\theta$ are constants to be defined below. Following equations (11), (13), (21), (23) we can write the difference in generalized travel costs as a function of the number of equipped drivers $N_e$: $P_u - P_e = \mu\bar{N} - \theta N_e$, with

$$\mu = \begin{cases} \frac{\beta\Delta}{(\beta+\gamma)s}, & | \ 0 \leq N_e \leq N_e^{\#} \\ \frac{\beta\Delta(\alpha-\beta-\gamma+\Delta)}{\alpha(\beta+\gamma)s}, & | \ N_e^{\#} < N_e \leq \bar{N} \end{cases}$$

$$\theta = \begin{cases} \frac{\Delta(\alpha+\gamma)(\beta-\Delta+\gamma)}{(\alpha+\Delta)(\beta+\gamma)s}, & | \ 0 \leq N_e \leq N_e^{\#} \\ \frac{\Delta(\alpha-\beta)(\beta-\Delta+\gamma)}{\alpha(\beta+\gamma)s}, & | \ N_e^{\#} < N_e \leq \bar{N} \end{cases}.$$

For both “low” and “high” ranges of the technology penetration we can write $MWTP$ as:

$$MWTP = U_e - U_u = (\Gamma_e - P_e) - (\Gamma_u - P_u) = \Gamma_e - \Gamma_u + P_u - P_e = \Delta(t^* - t_S) + \mu\bar{N} - \theta N_e.$$

$TSC$ is the sum of the costs of equipped and unequipped drivers:

$$TSC = N_u P_u + N_e P_e = (\bar{N} - N_e)P_u + N_e P_e = \bar{N}P_u + N_e(P_e - P_u) = \bar{N}P_u + N_e(\theta N_e - \mu\bar{N})$$

Note that by definition $MSB = -\frac{dTSC}{dN_e} + (\Gamma_e - \Gamma_u)$; that leads to:

$$-MSB = \frac{dTSC}{dN_e} - (\Gamma_e - \Gamma_u) = \bar{N}\frac{dP_u}{dN_e} + 2\theta N_e - \mu\bar{N} - \Delta(t^* - t_S) = \varphi\bar{N} + 2\theta N_e - \mu\bar{N} - \Delta(t^* - t_S)$$

From equations (11) and (23) we see that $\varphi$ is a (negative) constant which does not depend on $N_e$, hence:

$$MSB = \Delta(t^* - t_S) + (\mu - \varphi)\bar{N} - 2\theta N_e$$

## 4. Supply of teleworking technology

### 4.1 Perfect competition

In this section we examine the pricing strategies of private (profit maximizing) and public (welfare maximizing) firms that could supply the teleworking technology to drivers. In particular, we are interested in the relative efficiency of private market outcomes, compared to the social optimum. The profit maximizing price is, for a given market structure, determined by the marginal willingness to pay and marginal revenue on the one hand, and marginal costs on the other. At the same time, to reach the social optimum, a public provider should set a price that assures the equality of marginal social costs and marginal social benefits.

We assume that the marginal (social) cost of technology provision is zero. Besides simplifying the analysis, this assumption strengthens our finding that unrestricted supply of the teleworking technology might be marginally socially detrimental. The essential outcomes are not likely to change with the introduction of positive marginal costs. First, we will consider a market with perfect competition, where congestion is the single market friction. Then we introduce another market friction in combination to congestion: the existence of market power by a monopolist.

Under perfect competition the price will be equal to zero marginal costs, so that all drivers will be equipped with teleworking technology: $N_e^{PC} = \bar{N}$. The reason is that the $MWTP$, following the discussion in the previous section, is always positive (see Figure 5). The total social benefits under perfect competition ($TSB^{PC}$) is then the integral of marginal social benefits, given by equations (28) and (29):

$$(30) \qquad TSB^{PC} = \int_0^{N_e^{\#}} MSB_L \, dN_e + \int_{N_e^{\#}}^{\bar{N}} MSB_H \, dN_e$$

In contrast, a public firm sets the price and corresponding level of technology penetration, $N_e^{FB}$, such that the total social benefits are maximized:

$$(31) \qquad TSB^{FB} = \max_{\bar{N} \geq N_e^{FB}} \left( \int_0^{N_e^{\#}} MSB_L \, dN_e + \int_{N_e^{\#}}^{N_e^{FB}} MSB_H \, dN_e \right)$$

Indeed, as used in (31), it is easy to prove that, under zero marginal cost, the first-best level of technology penetration, $N_e^{FB}$, is always in the range $\bar{N} \geq N_e^{FB} > N_e^{\#}$, implying that some equipped drivers arrive before $t^*$.[6] If the corner solution $N_e^{FB} = \bar{N}$ holds, a competitive market provides the optimal outcome. Otherwise, $TSB^{PC} < TSB^{FB}$, meaning that taxation is required to achieve higher social welfare, by bringing down the number of equipped drivers from $\bar{N}$ to $N_e^{FB}$.[7]

We first compare the outcomes of perfect competition and first-best. Some results are cumbersome to present algebraically, so in this section we present results graphically. In this model numerical analysis

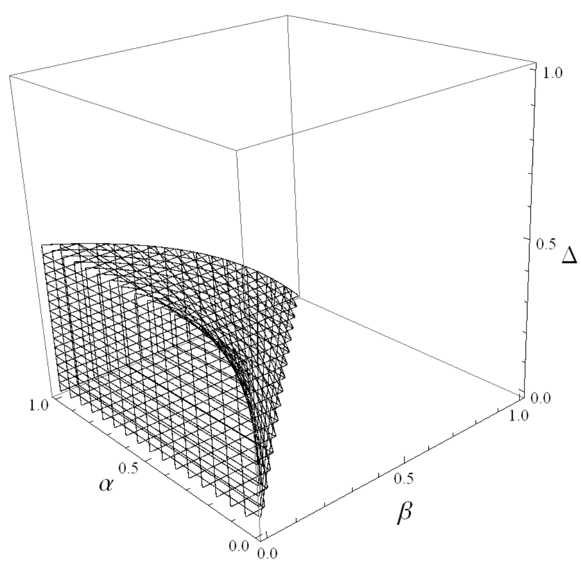
[6] $MSB$is a discontinuous function which might cross marginal cost line of zero in two points: in the low and high ranges of penetration $N_e$. We compared the two integrals of $MSB$, one where the upper limit corresponds to the point of intersection in the low range, and one in the high range. The latter integral is always larger than the former. This also holds when $MSB$ crosses the horizontal line only in the low range, while in the high range it ends up in the corner solution.

[7] When marginal costs are positive and large enough, it might be possible that perfect competition will supply less than the optimal number of drivers, and then a subsidy is appropriate. From equations of $MWTP$ (15) and $MSB$ (28) it follows that if marginal costs are larger than $\Delta(t^* - t_S) - \frac{\beta\Delta(\gamma-\Delta)\bar{N}}{(\alpha+\Delta)(\beta+\gamma)s}$, then the competitive technology penetration level falls short of social optimal one, $N_e^{PC} < N_e^{FB}$.

can in fact be rather exhaustive, because all functions that are necessary for the analysis are dependent only on four parameters $\alpha$, $\beta$, $\gamma$ and $\Delta$. Without loss of generality, we normalize opportunity cost of being late at workplace as $\gamma = 1$. The model restricts the effect from teleworking to $\Delta \in (0, \gamma)$. Most of the empirical literature suggests the relationship $\gamma > \alpha > \beta$ (e.g., Small, 1982). The relevant parameter space might then be shown as a cube with the edges $\alpha$, $\beta$ and $\Delta$, each of a length 1 (if desired, one could easily relax the constraint to allow both $\alpha$ and $\beta$ to be larger than $\gamma$). Without loss of generality, we normalize the overall number of drivers $\bar{N}$ to 100, and set road capacity $s$ to 1, so that the duration of the peak is 100. This does not affect the results of interest. In subsequent computations we define the beginning of the day $t_S$ as the arrival time of the first driver when no teleworking is available (see equation (2)). We can safely do this, because an introduction of teleworking shifts the arrival window to later times. The reallocation of $t_S$ to an earlier stage increases the ideal utility $\Gamma_e$, as the time during which drivers are able to gain benefits from teleworking expands. But the increase takes place over a period where no one travels under any equilibrium, and we like to keep this "benefit" as small as possible. Changes in the end time $t_F$ do not affect the comparative performance of equilibria with and without teleworking, as this involves times of the day where only $W(t)$ matters for overall welfare, and this is not affected by the adaptation of the technology.

Figure 6 shows the domain of parameters values which make the perfect competition outcome of full penetration of the teleworking technology socially less desirable than the first-best outcome. The combinations of $\alpha$, $\beta$ and $\Delta$ within the meshed body are those for which $TSB^{PC} < TSB^{FB}$. The domain outside the meshed body in Figure 6, in so far as it complies with the restriction $\alpha > \beta$, corresponds to the values where the competitive market generates the first-best outcome.

**Figure 6.** Parameter combinations that correspond to an above-optimal level of teleworking penetration under perfect competition, $\gamma = 1$



To determine the conditions under which perfect competition could lead to an above-optimal penetration level, first observe that when the last unequipped driver gets equipped, she gains $\frac{\Delta^2}{\beta+\gamma}\frac{\bar{N}}{s}$ in terms of private benefits (from equation (25)) and, at the same time, imposes a negative externality $\frac{\Delta(\alpha-\beta)(\gamma-\Delta)}{\alpha(\beta+\gamma)}\frac{\bar{N}}{s}$ on all other drivers (derived as $\frac{dP_e}{dN_e}\bar{N}$, where $P_e$ is from equation (21)). From the inequality $\frac{\Delta^2}{\beta+\gamma}\frac{\bar{N}}{s} \geq \frac{\Delta(\alpha-\beta)(\gamma-\Delta)}{\alpha(\beta+\gamma)}\frac{\bar{N}}{s}$ we can then derive the condition for perfect competition to yield the first-best welfare gain (again, when the marginal costs are zero):

(32) $\Delta \geq \frac{\alpha-\beta}{2\alpha-\beta}$

given that $\gamma = 1$. When $\beta \to 0$, this condition becomes $\Delta > 0.5$. If $\beta \to \alpha$, then it becomes $\Delta > 0$. The intuition behind this pattern is as follows.

The reason why perfect competition might not lead to the first-best outcome, is that there is a difference between external costs that unequipped drivers impose on others, and the external costs from equipped drivers. That difference is small when $\beta \to \alpha$; and at the limit, when $\beta = \alpha$, it disappears completely. This happens because the slopes of isocost lines of early arrivals of both equipped and unequipped drivers become identical: $\frac{\beta}{\alpha} = \frac{\beta+\Delta}{\alpha+\Delta} = 1$. This mean that both groups trade off travel time and schedule delay costs identically, that the groups are not separated in time, and thus impose the same external effects on each other (see Figure 4). An individual decision to become equipped then does not imply a change in the individual's external cost, so that as long as the individual herself benefits from doing so, also the net social welfare gain is positive. Even a small positive Δ is then enough to make full penetration of teleworking socially beneficial. However, when the difference in slopes of isocost lines between equipped and unequipped becomes larger ($\alpha$ and $\beta$ diverge), implying larger differences in imposed negative externalities, a bigger gain Δ is required to "compensate" for larger net external costs imposed, and to make full penetration also socially optimal. This explains the shape of the body in Figure 6.

### 4.2 Private monopoly

A private monopolistic provider is assumed to set the profit maximizing price. The profit of the monopolist ($\Pi^M$), given zero marginal costs, and ignoring fixed costs is the maximized integral of the marginal revenue ($MR$) function, which itself directly follows from the $MWTP$ functions (15) and (25). For both ranges of levels of technology penetration, high and low, $MR$ is twice as steep as the $MWTP$. The monopolist's profit is:

$$(33) \qquad \Pi^M = \begin{cases} \int_0^{N_e^M} MR_L \, dN_e & \Big| 0 \le N_e^M \le N_e^\# \\ \int_0^{N_e^\#} MR_L \, dN_e + \int_{N_e^\#}^{N_e^M} MR_H \, dN_e & \Big| N_e^\# < N_e^M \le \overline{N} \end{cases}$$

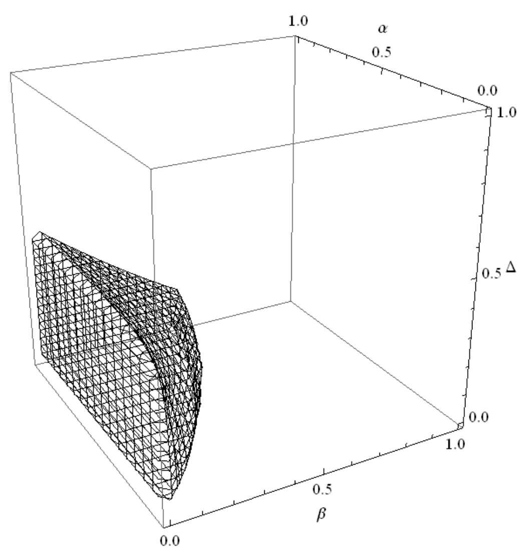
where the number of equipped drivers under monopoly is $N_e^M = \underset{N_e}{argmax}\, \Pi^M$. The corresponding total social benefits ($TSB^M$) is then the integral of marginal social benefits (equations (28) and (29)), with $N_e^M$ as the upper limit:

$$(34) \qquad TSB^M = \begin{cases} \int_0^{N_e^M} MSB_L \, dN_e & \Big| 0 \le N_e^M \le N_e^\# \\ \int_0^{N_e^\#} MSB_L \, dN_e + \int_{N_e^\#}^{N_e^M} MSB_H \, dN_e & \Big| N_e^\# < N_e^M \le \overline{N} \end{cases}$$

Because $MR$ and $MSB$ are generally not equal, the monopolist matches the first-best outcome only when achieving full penetration; i.e., when $N_e^M = \overline{N}$. This requires $MR$ to be high enough, for instance, when $\Delta$ is large.

$MSB$ always exceeds $MR$ (as follows from discussion in Section 3); thus, the private monopolist will never supply more than optimal number of drivers with teleworking technology. One underlying reason is that the private provider internalizes the negative external effects that its customers impose upon one another. The positive externalities of teleworking to unequipped drivers are, however, left outside the pricing rule, implying that the profit-maximizing price exceeds the welfare-maximizing price for reasons other than the classical demand-related mark-up. We can summarize our findings on the level of penetration under different market forms as follows: $N_e^M \le N_e^{FB} \le N_e^{PC}$.

**Figure 7.** Parameter combinations that correspond to the larger total social welfare under monopoly than under perfect competition, $\gamma = 1$



Next, in Figure 7 we compare the welfare outcomes of perfect competition versus the private monopoly. The combination of $\alpha$, $\beta$ and $\Delta$ within the meshed body are those for which $TSB^{PC} < TSB^{M}$, so monopoly produces a higher social welfare than perfect competition does. The parameter space outside the meshed body, insofar as $\alpha > \beta$, corresponds to the values where either perfect competition outperforms monopoly, or where both yield the same outcome in terms of social welfare.

For a sizable parameter space, a monopoly market leads to a higher social welfare than perfect competition. The body of Figure 7 lies entirely within that of Figure 6. That is: total social benefits under monopoly can be larger than under perfect competition only if perfect competition itself is not the first-best outcome. If the strict inequality $N_e^M < N_e^{FB} < N_e^{PC}$ holds, the monopoly level of penetration might be "closer" (in terms of welfare) to the first-best level than perfect competition, as the latter always produces $\overline{N}$. Figure 7 shows that perfect competition is particularly "harmful" in terms of oversupply when the differences between external effects of unequipped on equipped vs. equipped on themselves is large; i.e., when $\beta$ diverges from $\alpha$. Not shown explicitly in Figure 7 is the subset of parameter values where the monopolist prefers to be in the low range of penetration; i.e. $0 \leq N_e^M \leq N_e^{\#}$. That area touches the one shown in Figure 7, and is located in the bottom part (low $\Delta$), in the corner with high $\alpha$ and $\beta$ (but $\alpha > \beta$). There, the resulting $TSB^{M}$ is so low, that oversupply of perfect competition is socially preferable.

## 5. Teleworking with the first-best road toll

We have now established how, in the presence of congestion, the use of teleworking technology by equipped drivers causes externalities for others. A consequence is that it may not be optimal to supply the technology at marginal cost; zero, in our case. The second-best distortion that is responsible for this, is the unpriced congestion at the bottleneck. An interesting question is whether the externality in consumption of the technology, and hence the optimal deviation from marginal cost pricing, vanishes when congestion at the bottleneck is optimally priced.

A central result in the literature on Vickrey's dynamic bottleneck model is that waiting time, and thus travel time cost, is a pure social loss which can be fully eliminated (Vickrey, 1969; Arnott, de Palma and Lindsey, 1993). Time-dependent road pricing can achieve this. The social optimum is achieved by levying a first-best time-dependent road toll that exactly equals the travel time costs in the no-toll equilibrium, at each moment of arrival. Thus, instead of waiting in the queue, drivers pay a toll and incur no waiting time. With homogeneous drivers, the generalized travel price thus remains unchanged, compared to the no-toll case considered earlier. But from the social viewpoint, a toll is not a cost component, but a welfare neutral monetary transfer from road users to government. The welfare gain from first-best pricing is therefore equal to the total toll revenues, and therewith to the total savings in travel time cost. For more in-depth discussion of the model with pricing we refer to, among others, Arnott, de Palma and Lindsey (1993).

**Figure 8.** Isoprice function of the homogeneous drivers

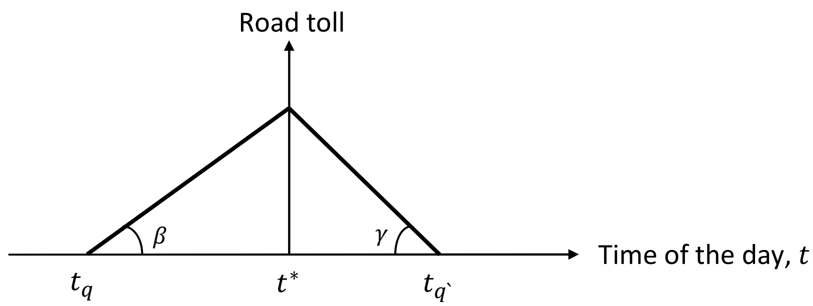



Figure 8 shows the optimal toll schedule for homogeneous unequipped drivers. The schedule depends entirely on the parameters $\beta$, $\gamma$, $s$ and $\bar{N}$. Note that the peak starts at the same time as in the no-toll case, because the very first and the very last driver in both regimes incur schedule delay costs only, which should be equalized in equilibrium. The generalized price of travel is therefore also equal to that in the no-toll equilibrium. The toll schedule has slopes $\beta$ and $-\gamma$ to keep the price constant over time without a queue, and reaches its maximum at the preferred arrival time.

Figure 8 represents the isoprice function, and thus resembles Figure 2 for the no-toll case, the only difference being the slopes. These are $\beta$ and $-\gamma$ with first-best pricing, and $\frac{\beta}{\alpha}$ and $-\frac{\gamma}{\alpha}$ in the no-toll equilibrium. This difference only reflects the different units used in the vertical dimension (money in Figure 8 versus time in Figure 2).

It is now straightforward to repeat the entire analysis from the section 3, under the new conditions of optimal road pricing. The slopes of the isoprice curves for early arrivals and for the late ones are, respectively, $\beta$ and $\gamma$ for unequipped drivers, and $\beta + \Delta$ and $\gamma - \Delta$ for equipped ones. The isoprice slope is flatter for equipped drivers for late arrivals and steeper for early ones. Thus, a temporal separation in arrival times of equipped and unequipped drivers occurs in qualitatively the same manner as in the no-toll case: unequipped travelers go first, equipped ones go last.

Furthermore, w can calculate $MWTP$ and $MSB$ in the same way as described in, respectively, subsections 3.1 and 3.2. It is easy to prove that, because the order of arrival at work is the same as in no-toll case, the revenues from the toll collection are exactly 50% of the combined generalized travel costs that all drivers incur (as is true in the conventional bottleneck model with homogeneous users). Thus, the total social costs are cut by half. The $MWTP$ and $MSB$ functions are now identical:

(35) $$MWTP = MSB = \Delta(t^* - t_S) + \frac{\beta\Delta}{\beta+\gamma}\frac{\bar{N}}{s} - \frac{\Delta(\beta+\gamma-\Delta)}{\beta+\gamma}\frac{N_e}{s}$$

The equality of $MWTP$ and $MSB$ implies that the teleworking technology becomes a standard good when congestion externality is cleared by the first-best road toll. That is, no policy interventions are required to bring $MSB$ equal to the marginal costs under perfect competition. And under monopoly, the regular overpricing due to market power occurs.

Finally, note that for both low ($0 \leq N_e \leq N_e^{\#}$), and high numbers ($N_e^{\#} < N_e \leq \bar{N}$) of equipped drivers, the functions are the same, i.e., there is now no kink in $MWTP$, and no discontinuity in the $MSB$. The reason why the kink in $MWTP$ disappears is that equipped drivers impose an identical unpriced net externality, namely zero under first-best pricing, whether they arrive after or before $t^*$. It was the difference in unpriced externalities imposed by early versus late equipped drivers that caused the kink for the no-toll case, and this difference now no longer exists with optimal pricing.

## 6. Summary and conclusions

We investigated the welfare effects from teleworking becoming available for a congested bottleneck, using Vickrey's (1969) dynamic bottleneck model. Teleworking was modeled as an increase in the utility that a person derives from being at home, sufficiently small to keep commuting worthwhile. We derive the marginal willingness to pay for teleworking as the difference in utility that a driver gains when being equipped, compared to being unequipped. Getting the possibility of teleworking creates differences in the utility parameters of otherwise homogeneous drivers, and therefore affects their dynamic travel behavior. As the marginal external costs differ between equipped and unequipped drivers, the decision to become equipped influences travel costs of all other drivers. We derive generalized travel costs for both equipped and unequipped drivers, and the total social benefits of teleworking as a function of the number of equipped drivers. The optimal level of technology penetration is then such that the marginal social benefit is equal to the marginal social cost. We compare the relative efficiency of private market outcomes, under monopoly and perfect competition, to the social optimum. Finally, we examine the effect of teleworking on travel costs when the congestion externality is internalized using the time-dependent first-best road toll.

We find that even costless teleworking might have an adverse marginal effect on social welfare, when a certain level of technology penetration is reached, due to the negative externality it creates. The very first unequipped driver who becomes equipped prefers to be the only one teleworking, as equipped drivers impose higher external travel cost on one another than unequipped impose on them. The more people are teleworking, therefore, the lower the benefits of teleworking for each individual equipped driver. The remaining unequipped drivers enjoy positive effects of teleworking: the negative externality of equipped drivers on unequipped ones is lower than what unequipped drivers impose up on one another. Although full penetration of costless teleworking is always socially more beneficial than no teleworking at all, there exists an optimal degree of driver heterogeneity. An increase in the number of equipped drivers above that level lowers social welfare.

Our results show that a private monopolistic supplier of a teleworking technology might yield a higher social welfare than perfect competition does. Under perfect competition, with zero marginal cost, all drivers are teleworking, as the endogenous marginal willingness to pay for teleworking is always positive. A full penetration might, however, be socially excessive. At the same time, a monopolist charges a mark-up, while taking into account the negative effects its customers impose one each other, ignoring the positive effects on unequipped drivers. The level of penetration under monopoly could consequently be below the optimal level. We identified the conditions under which the monopoly outcome is “closer” to optimal, from a social welfare viewpoint, than that of perfect competition.

A policy that would eliminate the congestion externality would also get rid of the changes in externalities resulting from purchase of the technology. Time-dependent first-best road toll achieves this, and makes teleworking technology a conventional good, which does not require any policy interventions.

To the best of our knowledge, this paper is the first to model the effect of (part-day) teleworking on generalized travel costs. We have used a conventional linearized scheduling model, as considered by Vickrey (1969), Small (1982), Arnott, de Palma and Lindsey (1993), but we assumed it stems from preferences of being at home and being at work in a way as described by Vickrey (1973) and, later, by Tseng and Verhoef (2008). We found this framework suits well for the analysis and yields interesting insights.

There is ample scope for further research on the effects of teleworking on travel within the considered framework. Among the possible extensions are a consideration of initial driver heterogeneity; variation in teleworking technology; and more complex road networks allowing for an explicit consideration of spatial and network effects, in addition to the temporal dimension considered here. It might also be interesting to incorporate other effects of teleworking besides those on travel costs, such as effects on productivity, work-life balance, etc., to get the full picture of the overall effect of teleworking on welfare.

**References**

Arnott, R., A. de Palma, and R. Lindsey, "A Structural Model of Peak-Period Congestion: A Traffic Bottleneck with Elastic Demand," *American Economic Review* 83 (1993), 161-179.

——, ——, and ——, "Information and Usage of Free-Access Congestible Facilities with Stochastic Capacity and Demand," *International Economic Review* 37 (1996), 181-203.

De Palma, A., and R. Lindsey, "Information and Usage of Congestible Facilities under Different Pricing Regimes," *Canadian Journal of Economics* 31 (1998), 666-692.

Emmerink, R.H.M., E.T. Verhoef, P. Nijkamp, and P. Rietveld, "Information policy in road transport with elastic demand: Some welfare economic considerations," *European Economic Review* 42 (1998a), 71-95.

——, ——, ----, and ----, "Information Effects in Transport with Stochastic Capacity and Uncertainty Costs," *International Economic Review* 39 (1998b), 89-110.

Eurofound, *Third European Survey on Working Conditions 2000* (Dublin, 2005).

——, *Teleworking in the European Union* (Dublin, 2010).

Haddad, H., G. Lyons, and K. Chatterjee, "An examination of determinants influencing the desire for and frequency of part-day and whole-day homeworking," *Journal of Transport Geography* 17 (2009), 124-133.

KPMG, *Work fragmentation: How much is too much?* (2011).

Pigou, A.C., *Wealth and Welfare* (London: Macmillan, 1920).

Rhee, H.-J., "Home-based telecommuting and commuting behavior," *Journal of Urban Economics* 63 (2008), 198-216.

Safirova, E., "Telecommuting, traffic congestion, and agglomeration: A general equilibrium model," *Journal of Urban Economics* 52 (2002), 26-52.

Small, K.A., "The scheduling of consumer activities: work trips," *American Economic Review* 72 (1982), 467-479.

——, and E.T. Verhoef, *The Economics of Urban Transportation* (London: Routledge, 2007).

Tseng, Y.-Y., and E.T. Verhoef, "Value of time by time of day: A stated-preference study," *Transportation Research Part B: Methodology* 42 (2008), 607-618.

USA Government, *Telework Enhancement Act* (2010).

Vickrey, W.S., "Congestion theory and transport investment," *American Economic Review (Papers and Proceedings)* 59 (1969), 251-260.

——, "Pricing, metering, and efficiently using urban transportation facilities," *Highway Research Record* 476, 36-48.