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On the foundations of competitive search equilibrium with and without market makers ☆

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Abstract

The literature offers two interpretations of competitive search equilibrium, one based on a Nash approach and the other on a market-maker approach. When each buyer visits only one seller, the two approaches are equivalent. However, when each buyer visits multiple sellers, this equivalence can break down. We present a model in which every buyer visits 2 sellers. A buyer who trades with one seller receives a value of s, while a buyer who trades with 2 sellers receives value 1. Letting s vary from 0 (perfect complements) to 1 (perfect substitutes) we characterize the competitive search equilibrium under the two interpretations. We show that for low values of s, the Nash and market-maker competitive search equilibria coincide, but the common equilibrium is inefficient. For intermediate values of s, the two equilibria again coincide and are efficient. Finally, for high values of s, the Nash and market-maker equilibria differ, and only the latter is efficient.

JEL classification: C78; D83

Keywords: Simultaneous search; Competitive search; Market makers; Efficiency

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1. Introduction

In competitive search models, capacity-constrained agents on one side of the market post and commit to terms of trade. Agents on the other side of the market, after observing all posted terms of trade, decide where to direct their search. Consider, for example, a product market application of competitive search with S sellers, each with one unit of a homogeneous good to sell, and B buyers, each wanting to purchase one unit of the good.[1] Each seller posts a price, and buyers, after observing all posted prices, direct their search. That is, each buyer chooses a seller from whom he or she tries to purchase the good. When setting its price, each seller faces a tradeoff. The higher the price, the greater is the payoff the seller receives if the good is sold but the lower is the probability of sale. The seller chooses a price to maximize its expected payoff taking this tradeoff into account through a market utility constraint; i.e., the seller realizes that any buyers its posted price might attract must expect a payoff no lower than is available elsewhere in the market. In competitive search equilibrium, each seller takes the buyer expected payoff that is available elsewhere in the market, i.e., the "market utility," as given. This, of course, requires B and S sufficiently large, i.e., competitive search equilibrium is a large-market concept.

The literature offers two common interpretations of competitive search equilibrium. The first views competitive search equilibrium as the limit of a sequence of Nash equilibria as the numbers of players on the two sides of a market get arbitrarily large. Consider again the above product market example. For fixed B and S, one can compute the symmetric Nash equilibrium of this game. Letting $B, S \to \infty$ while holding $\lambda = B/S$ fixed, we can compute the sequence of Nash equilibria and take the limit. This gives the competitive search equilibrium. This first interpretation is the one presented in Peters (1991, 2000), Burdett et al. (2001), and Galenianos and Kircher (2012) among others.

A second interpretation of competitive search equilibrium uses the concept of "market maker." Again imagine an arbitrarily large market, i.e., $B, S \to \infty$ with λ fixed. Suppose all sellers post the same price, p, so that buyers randomize their search and each seller expects λ buyers. Then (p, λ) is a symmetric competitive search equilibrium if there is no profitable possibility for a market maker to set up a "submarket" in which sellers are promised an arrival rate $\widetilde{\lambda}$ of buyers in return for posting a price of $\widetilde{p}$. This is the interpretation of competitive search equilibrium presented in Moen (1997) and Mortensen and Wright (2002) among others.

When each buyer can contact only one seller, these two interpretations lead to the same equilibrium allocation. This can be shown by comparing the analyses of, e.g., Burdett et al. (2001) and Mortensen and Wright (2002). However, in settings in which buyers make multiple visits, the equivalence can break down. A deviating market maker can attract multiple sellers giving buyers the option to pay more than one visit to the deviant submarket, but under the Nash interpretation, only one seller deviates so each buyer can pay at most one visit to a deviant seller. This difference can change the equilibrium outcome.

The purpose of our paper is to examine the Nash and market-maker equilibrium concepts in a competitive search environment in which buyers direct their search to multiple sellers or in which workers apply to more than one job. We are interested in determining which market characteristics lead to equivalence, i.e., when do the Nash and market-maker approaches yield the same equilibrium outcomes, and when do they differ? Under what circumstances do the two approaches generate equilibrium outcomes that are constrained efficient? Of course, if it

[1] Another standard example is a labor market in which v firms, each with one vacancy, post and commit to wages; then u unemployed workers direct their search after observing all posted wages.

is optimal for buyers to pay exactly one visit to the deviant submarket in the market-maker approach, then the two approaches yield the same outcome on and off the equilibrium path. However, if buyers find it optimal to pay more than one visit to the deviant submarket, then the two equilibria can differ.

We consider a simple static environment. Buyers are homogeneous, and so too are sellers, each of which has one unit of a homogeneous good for sale. Every buyer visits two sellers, and we assume that buyer value is (weakly) increasing in the number of units purchased. Specifically, we assume that the buyer value of one unit is $s \in [0, 1]$ while the buyer value of two units is 1. We allow the value of purchasing one unit to vary from $s = 0$ (perfect complements) to $s = 1$ (perfect substitutes). The case of perfect substitutes is clearly relevant for both the product market and the labor market (buyers who need only one unit of a good, firms that open a vacancy to hire a single worker). Regarding perfect complements, a simple example is one in which firms require two workers to complete a task; absent the second worker, nothing gets done.[2] In any case, most products (and most workers) lie somewhere between perfect complements and perfect substitutes. The competitive search literature has so far only considered the case of $s = 1$, that is, the case in which each buyer wants to buy only one unit of the good or each firm wants to hire only one worker. By allowing s to vary, we generalize the model to deal with the case in which buyers may want to buy more than one unit of the good (or firms want to hire more than one worker). In these cases, the value that the seller offers the buyer may depend on what other sellers offer so that the buyers face a portfolio problem. This suggests that even though we consider a particular trade arrangement in this paper, the underlying logic for our results likely carries over to other settings where buyers face similar portfolio problems.[3]

To attract buyer visits, every seller posts and commits to a maximum price for its unit. A seller may then be visited by more than one buyer. In this case, since buyers are ex ante identical, the seller selects one of its prospective buyers at random and attempts to transact with that buyer. Once a seller has selected its buyer, we assume the seller can observe whether its chosen buyer has also been selected by the other seller its buyer visited. Given this information, the seller may then choose to reduce its offered price to try to consummate the transaction. Finally, we also assume no recall; that is, if a seller fails to transact with its chosen buyer, we do not allow the seller to move on to a different buyer (if present). We discuss these assumptions below.

The relationship between the Nash and market-maker equilibria depends on the value of s, i.e., on the extent to which the two units are complements/substitutes. When $s = 0$, i.e., when the units are perfect complements, the two approaches yield the same equilibrium allocation, and that common allocation is not constrained efficient. The same is true when s is sufficiently close to zero, i.e., when the goods are "strong complements." It is noteworthy that the market-maker approach fails to yield the constrained efficient outcome when the units are strong complements. Suppose a deviant market maker could enforce the following *exclusive participation rule*: buyers must either pay both visits to the deviant submarket or none. In this case, we could think of the

[2] For the case of heterogeneous labor, Roth and Sotomayor (1990, p. 173) give the example of an American football team, which wants to employ one player who can throw long passes and one who can catch them. Left shoes and right shoes are considered perfect complements by some, but since they are never sold separately, we consider them and other such goods, e.g., gloves, as single products.

[3] Even though the competitive search literature has only considered the case in which buyers demand exactly one unit, the traditional matching/general equilibrium literature has studied extensively the case where buyers may demand multiple units of indivisible goods. Kelso and Crawford (1982) found that goods being "gross substitutes" is important for the existence of competitive equilibrium. Our paper can also serve as an interesting example, where with search frictions, decentralized market equilibrium exists even when the goods are complements.

deviant market maker as purchasing a pair of buyer visits. Then, following the logic of competitive search models with single visits, the equilibrium would be constrained efficient. However, in the absence of such an exclusive participation rule, the number of visits per buyer to the deviant submarket is chosen optimally by the buyers, and when the units are strong complements, any buyer who visits a deviant submarket chooses to pay only one visit there. The reason, as we show below, is that buyer visits are strategic substitutes when the units are strong complements.

When the units are perfect substitutes ($s = 1$), the Nash and market-maker equilibria are no longer the same. Only the market-maker allocation is constrained efficient. The same is true when s is sufficiently close to one, i.e., when the units are "strong substitutes." In the Nash equilibrium, sellers understand that buyers want to receive two offers since a buyer with two offers can anticipate that his or her sellers will lower their prices via Bertrand competition. This leads sellers to post a price that is inefficiently high. In contrast, in the market-maker equilibrium, when the units are strong substitutes, buyer visits are strategic complements; i.e., a buyer who visits the deviant submarket chooses to pay both of his or her visits there. Since buyers who visit a deviant submarket pay both of their visits there, the market maker can solve the sellers' coordination problem.

The last case occurs when the two units are neither strong complements nor strong substitutes. Over this intermediate range of s, the Nash and market-maker equilibria coincide, and the common allocation is constrained efficient. The intuition is easily seen for the case of $s = 1/2$. When $s = 1/2$, a buyer's marginal value for the first unit is the same as his or her marginal value for the second unit. It is as if there were $2B$ buyers, each of whom pays a single visit to one of the S sellers. In this case, the standard competitive search logic applies, sellers post a price between the competitive ($p = 0$) and monopoly ($p = 1/2$) levels, and the equilibrium is constrained efficient, exactly as, for example, in Burdett et al. (2001). In this case, the posted price is a *fixed price*; i.e., a seller has no incentive to lower its price whether or not its selected buyer has another offer. A similar logic – sellers post fixed prices, the Nash and market-maker equilibria coincide, and the common equilibrium allocation is constrained efficient – applies for s sufficiently close to $1/2$.

Finally, we have studied the relation between the two concepts of competitive search equilibrium using a particular selling mechanism. In an online appendix (Appendix B.1), we consider a different selling mechanism in which sellers collect an application fee and then offer the good for free to a randomly selected buyer. Although fee posting is very rare, we consider it nevertheless because it helps us understand what forces are at work and how they interact with the type of good, i.e., substitute versus complement. We find that with fees, both the Nash equilibrium and the market-maker equilibrium are constrained efficient. The difference between the two approaches arises only off equilibrium in the case of substitutes.

1.1. Related literature

Although there is a large and growing literature on competitive search,[4] only a few papers in this literature allow for simultaneous search, i.e., assume that buyers can approach multiple sellers or that job seekers can make multiple applications. All of these papers treat the case of perfect substitutes. In an early paper in this literature, Albrecht et al. (2006), allow workers to apply simultaneously to multiple firms and show that when firms Bertrand compete for workers with multiple offers, that unlike the case in which workers make only a single application, the

[4] See Wright et al. (2021) for a recent review of the competitive search literature.

competitive search Nash equilibrium is not constrained efficient. Albrecht et al. (2020) extend this work and show that the above conclusion depends on the choice set of firms. If firms can post both a wage floor and a non-refundable application fee, then there exists a continuum of competitive search Nash equilibria, only one of which is constrained efficient. As a complement to these results, Albrecht et al. (2020) add an appendix in which they characterize the equilibrium outcome when a measure zero of firms deviates "collectively." In this case, the unique equilibrium is again constrained efficient.

Galenianos and Kircher (2009) and Kircher (2009) also allow for multiple applications in the labor market. In these papers, firms post fixed wages, and workers with multiple offers select the one with the highest wage. Since a worker only needs one job and wages are fixed, a second high-wage offer has no value. A second application to a low-wage job serves as insurance in the event that the worker doesn't receive the high-wage offer. Job applications are therefore strategic substitutes. The Nash and market-maker approaches are equivalent in these two papers, and the issues we consider here do not arise.[5]

The Galenianos and Kircher (2009) approach can be related to our assumption of perfect information. One way to motivate their assumption that firms post fixed wages, i.e., that a firm cannot adjust its wage ex post if its selected applicant has another offer, is to argue that workers cannot credibly show firms that they have other offers. That is, their approach is tantamount to an assumption of *imperfect* information. In contrast, our approach relies on the idea that a seller can adjust its price offer once it learns if its selected buyer has another offer.[6]

Finally, Albrecht et al. (2006) offers some insight regarding our no-recall assumption. They partially relax this assumption by considering "shortlists" or limited recall in the sense that a firm can offer its job to a second applicant (if it has one) if its first offer is rejected. Allowing for limited recall does not change the qualitative results of that paper.

In the next section, we present our model of competitive search. Each buyer directs his or her search to two sellers, and we allow the units sold by any two sellers to range from perfect complements to perfect substitutes. Section 3 characterizes and compares the competitive search Nash and market-maker equilibria, and compares the equilibrium allocations to the social planner optimum. Concluding remarks are offered in Section 4.

2. The model

2.1. Setup

We consider a model of competitive search with a continuum of identical buyers with endogenous measure B and a continuum of identical sellers with exogenous measure S. Each seller has one unit of an indivisible good to sell, and each buyer visits two sellers. The value for a buyer who purchases two units is normalized to 1, while the value of purchasing one unit is s, where $0 \leq s \leq 1$. The value of no trade for a seller or a buyer is normalized to zero. When $0 \leq s < 1/2$,

5 Galenianos and Kircher (2009) consider a third concept of equilibrium. To evaluate a deviant seller's expected payoff, they let a measure zero of the sellers tremble and offer any mechanism from a given mechanism space. The number of submarkets is then the same as the number of elements in the mechanism space. If a worker who visits a deviant submarket finds it optimal to pay all of his or her other visits there, then this equilibrium concept reduces to the market-maker equilibrium concept. This is the case in Galenianos and Kircher (2009) but need not be the case in other models.

6 Another source of private information can be worker productivity, as in Auster et al. (2021). They show that multiple job applications make screening and hence a fully separating equilibrium more difficult to realize.

the units are complements (perfect complements when $s = 0$); when $1/2 < s \leq 1$, the units are substitutes (perfect substitutes when $s = 1$).[7]

Each seller posts and commits to a maximum price that it will charge for its unit of the good. Then, after observing all posted prices, buyers direct their search. Since buyers are ex ante identical, a seller who is visited by more than one buyer selects one of those buyers at random. The seller then learns whether its selected buyer has also been selected by the other seller that the buyer visited. Depending on the outcome of the selected buyer's other visit, the seller may find it worthwhile to offer a lower price in order to facilitate trade. We do not allow for recall. That is, if a seller and its selected buyer do not reach a deal, we do not allow the seller to select another buyer (if any others have visited that seller).

We allow for a general constant returns to scale meeting technology. Consider a seller with expected queue length λ, i.e., λ is the expected number of buyers contacting the seller. The probability that the seller meets at least one buyer is $m(\lambda)$.[8] We assume that $m(\lambda)$ is strictly increasing and strictly concave, which implies that the elasticity $\varepsilon_m(\lambda) \equiv \lambda m'(\lambda)/m(\lambda)$ is always between 0 and 1.[9] Furthermore, we assume that $m'(0) = 1$, which implies that as the number of buyers goes to zero, each buyer visit leads to a match. By an accounting identity, the probability that a buyer's visit leads to an offer is then $q(\lambda) \equiv m(\lambda)/\lambda$, since the seller treats all buyers symmetrically. Note that $\lim_{\lambda \to 0} q(\lambda) = m'(0) = 1$ (L'Hôpital's rule), and $q(\lambda)$ is strictly decreasing since $\varepsilon_m(\lambda) < 1$. If all sellers face the same expected queue length, then that common expected queue length is $\lambda = 2B/S$, but in general, the expected queue length depends on the posted price and may vary from seller to seller.[10]

We further impose the following regularity assumption on the meeting technology to ensure that the surplus function satisfies the concavity condition needed for Lemma 1 below.

Assumption 1. $q(\lambda)^{-1}$ or equivalently $\lambda/m(\lambda)$ is convex.

The above assumption can be thought as a requirement that $m(\lambda)$ be "sufficiently" concave. By taking the second derivative of $\lambda/m(\lambda)$, the above assumption is equivalent to $2(1 - \varepsilon_m(\lambda)) \leq -\varepsilon_2(\lambda)$, where $\varepsilon_2(\lambda) \equiv \lambda m''(\lambda)/m'(\lambda)$ is the elasticity of $m'(\lambda)$. Concavity of $m(\lambda)$ is equivalent to $\varepsilon_2(\lambda)$ being negative, whereas the above assumption further requires that $\varepsilon_2(\lambda)$ must be less than $-2(1 - \varepsilon_m(\lambda))$. Note that $m'(\lambda)$ denotes the marginal contribution of a buyer to the matching process. When λ is small so that $\varepsilon_m(\lambda)$ is close to 1, the percentage drop of $m'(\lambda)$,

[7] In an online appendix we allow buyers to visit $a \geq 2$ buyers. To maintain tractability, we assume units of the good are either perfect substitutes or perfect complements. Compared with the case of $a = 2$, both the analysis and the conclusions are virtually the same.

[8] A common microfoundation for $m(\lambda)$ for the Nash approach in a model that moves from finite numbers of buyers and sellers to a continuum of each has been presented in a number of papers. (See, for example, Burdett et al. (2001).) Suppose there are B (a finite integer) buyers. If each buyer visits a given seller with probability γ, then the expected queue length for that seller is $B\gamma$. If $B \to \infty$ and $B\gamma \to \lambda$, then the number of buyers that the seller meets follows a Poisson distribution with mean λ so that $m(\lambda) = 1 - e^{-\lambda}$, the urn-ball meeting function. While the realized number of buyers that visit a particular seller must be an integer, the expected queue, λ, can take any non-negative value. Of course, different microfoundations can be used to derive other meeting functions. This is the microfoundation for the meeting function when the Nash approach is viewed as the limit of Nash equilibria in finite economies. The market-maker approach to equilibrium in these models takes the meeting function for the limiting economy as a primitive.

[9] Since $m'(\lambda) > 0$, $\varepsilon_m(\lambda) > 0$. Since $m''(\lambda) < 0$, $m(\lambda) - \lambda m'(\lambda)$ is strictly increasing and hence strictly positive, the latter implies $\varepsilon_m(\lambda) < 1$ when $\lambda > 0$.

[10] Since no buyer pays both visits to the same seller, for an individual seller, λ denotes the expected number of (different) buyers the seller meets. In the aggregate, $\lambda = 2B/S$ is total buyer visits per seller since each buyer visits two sellers.

measured by its elasticity $\varepsilon_2(\lambda)$, can be very small (close to zero) according to the above assumption; when λ increases, the above assumption requires that the corresponding percentage drop of $m'(\lambda)$ must be sufficiently large, with the lower bound being $2(1-\varepsilon_m(\lambda))$. Finally, note that Assumption 1 also implies that $\varepsilon_m(\lambda)$ is strictly decreasing, since the latter is equivalent to $1-\varepsilon_m(\lambda) < -\varepsilon_2(\lambda)$.[11] Common meeting technologies that satisfy this assumption include the urn ball, $m(\lambda) = 1 - e^{-\lambda}$, and the geometric, $m(\lambda) = \lambda/(1+\lambda)$.[12]

Finally, we assume that the number of buyers is determined by free entry. We suppose there is a large measure of potential buyers and that each must pay a fixed cost K to participate, where $0 < K < 1$. The measure of sellers in the market is given exogenously. By endogenizing the number of buyers we can investigate whether the market equilibrium is efficient by contrasting equilibrium tightness with the social planner's market tightness.

2.2. The social Planner's problem

As a benchmark, we begin by characterizing the social planner allocation. We calculate total surplus and the marginal contributions to surplus of sellers and buyers. We do this first for the cases of perfect complements and perfect substitutes and then generalize.

When the units are perfect complements, surplus is generated if and only if a buyer is selected by 2 sellers. When the units are perfect substitutes, surplus is generated if and only if a buyer is selected by at least one seller. Since there are constant returns to scale in the meeting technology, surplus per seller depends only on market tightness $\lambda = 2B/S$ and can be written as

$$y(\lambda) = \begin{cases} y_{pc}(\lambda) = \frac{\lambda}{2} q(\lambda)^2 = \frac{B}{S} q(\frac{2B}{S})^2 & \text{perfect complements} \\ y_{ps}(\lambda) = \frac{\lambda}{2}\left(1-(1-q(\lambda))^2\right) = \frac{B}{S}\left(1-(1-q(\frac{2B}{S}))^2\right) & \text{perfect substitutes} \end{cases} \tag{1}$$

since $\lambda/2$ is the number of buyers per seller. Total surplus is then simply

$$V(B,S) = Sy\left(\frac{2B}{S}\right).$$

By direct computation, the marginal contribution to surplus of sellers, $V^S(\lambda) \equiv \partial V(B,S)/\partial S$ or equivalently $y(\lambda) - \lambda y'(\lambda)$, is

$$V^S(\lambda) = \begin{cases} V^S_{pc}(\lambda) = m(\lambda) q(\lambda)\left(1-\varepsilon_m(\lambda)\right) & \text{perfect complements} \\ V^S_{ps}(\lambda) = m(\lambda)\left(1-q(\lambda)\right)\left(1-\varepsilon_m(\lambda)\right) & \text{perfect substitutes} \end{cases} \tag{2}$$

When the units are perfect complements, the marginal seller contributes to surplus if it meets at least one buyer, which happens with probability $m(\lambda)$, and its chosen buyer has another offer, which happens with probability $q(\lambda)$. In the efficient allocation, the seller should receive a share of the surplus equal to its marginal contribution to the matching process. The latter equals the elasticity of the total number of matches with respect to the number of sellers, which is $1-\varepsilon_m(\lambda)$.

[11] Eeckhout and Kircher (2010) also assume that $1/q(\lambda)$ is convex, although for a different purpose. In their model, the assumption is used to show that square-root supermodularity of the production function is sufficient for positive assortative matching.

[12] With an urn-ball meeting technology, the probability that a seller meets exactly n buyers is given by $e^{-\lambda}\frac{\lambda^n}{n!}$ for $n = 0, 1, \ldots$. With a geometric meeting technology, the corresponding probability is $\frac{1}{1+\lambda}\left(\frac{\lambda}{1+\lambda}\right)^n$.

When the units are perfect substitutes, the marginal seller contributes to surplus when it meets at least one buyer, which happens with probability $m(\lambda)$, and when the buyer it selects does not have another offer, which happens with probability $1 - q(\lambda)$. Again when a seller contributes to surplus, it should receive a share equal to $1 - \varepsilon_m(\lambda)$, which is the seller's contribution to the meeting.

The marginal contribution to surplus by buyers, $V^B(\lambda) \equiv \partial V(B, S)/\partial B$ or equivalently $2y'(\lambda)$ is then

$$V^B(\lambda) = \begin{cases} V^B_{pc}(\lambda) = q(\lambda)^2 \left(1 - 2(1 - \varepsilon_m(\lambda))\right), & \text{perfect complements} \\ V^B_{ps}(\lambda) = q(\lambda)^2 + 2(1 - q(\lambda))q(\lambda)\varepsilon_m(\lambda), & \text{perfect substitutes} \end{cases} \tag{3}$$

When the units are perfect complements, a buyer contributes to surplus if he or she receives offers from 2 sellers, which happens with probability $q(\lambda)^2$. In this scenario, the buyer should receive the residual part of the surplus, which equals $1 - 2(1 - \varepsilon_m(\lambda))$ since each of the sellers receives $1 - \varepsilon_m(\lambda)$. Note that $\lim_{\lambda \to 0} V^B_{pc}(\lambda) = 1$ since at $\lambda = 0$, $q(0) = 1$ and $\varepsilon_m(0) = 1$. As λ increases, Lemma 1 below shows that $V^B_{pc}(\lambda)$ strictly decreases until it reaches zero at a point $\Lambda(0)$ where $\varepsilon_m(\Lambda(0)) = 1/2$ (see equation (3) above). Since we assumed that $\varepsilon_m(\lambda)$ is strictly decreasing, when λ is larger than $\Lambda(0)$, $V^B_{pc}(\lambda)$ stays negative. Furthermore, $\lim_{\lambda \to \infty} V^B_{pc}(\lambda) = 0$ since $\lim_{\lambda \to \infty} q(\lambda) = 0$. When the units are perfect complements, buyers impose a negative externality. If more than one buyer visits a particular seller, then the buyers who are not chosen by that seller will be unwilling to transact with the other sellers they meet. Hence when the number of buyers is sufficiently large, their marginal contribution turns negative.

When the units are perfect substitutes and a buyer has two offers, which happens with probability $q(\lambda)^2$, the buyer's contribution to surplus is 1; when the buyer has exactly one offer, which happens with probability $2q(\lambda)(1 - q(\lambda))$, his or her contribution to surplus is $\varepsilon_m(\lambda)$. As in the case when the goods are perfect complements, we have $\lim_{\lambda \to 0} V^B_{ps}(\lambda) = 1$ and $\lim_{\lambda \to \infty} V^B_{ps}(\lambda) = 0$. Furthermore, by Lemma 1 below, $V^B_{ps}(\lambda)$ is always strictly decreasing. When the units are perfect substitutes, a buyer who is not selected by one seller still has the possibility to trade with a second seller so the marginal contribution to surplus of buyers is always strictly positive in contrast to the case when the goods are perfect complements.

We now consider the general case. Surplus per seller is

$$y(\lambda) = \frac{\lambda}{2}\left[2q(\lambda)(1 - q(\lambda))s + q(\lambda)^2\right]. \tag{4}$$

The first term in the brackets represents the case in which a buyer is selected by a single seller and hence surplus s is generated, while the second term represents the case in which a buyer has two offers and hence a surplus of 1 is generated. Since $y(\lambda)$ is linear in s, it can be rewritten as $(1 - s)y_{pc}(\lambda) + sy_{ps}(\lambda)$, where $y_{pc}(\lambda)$ is surplus per seller for perfect complements and $y_{ps}(\lambda)$ for perfect substitutes, as given by (1).

The marginal contribution to surplus of buyers, $V^B(\lambda) = 2y'(\lambda)$, is given by,

$$V^B(\lambda) = (1 - s)V^B_{pc}(\lambda) + sV^B_{ps}(\lambda) \tag{5}$$

where $V^B_{pc}(\lambda)$ and $V^B_{ps}(\lambda)$ are the marginal contribution to surplus by buyers in the case of perfect complements and of perfect substitutes, respectively, which were given by (3). The same logic applies to the sellers' marginal contribution to surplus.

The social planner chooses the number of buyers to maximize net output,

$$\max_{B \geq 0} V(B, S) - KB$$

The socially optimal number of buyers satisfies the first-order condition $V^B(\lambda) = K$. By Lemma 1 below, this condition is also sufficient.

Lemma 1. *Given Assumption 1, for each s there exists some $\Lambda(s)$, which may be infinite, such that $V^B(\lambda)$ is strictly positive for $\lambda < \Lambda(s)$ and strictly negative for $\lambda > \Lambda(s)$. Furthermore, $V^B(\lambda)$ is strictly decreasing in λ when $\lambda < \Lambda(s)$. When $s \geq 1/2$ (the case of substitutes), $\Lambda(s)$ is infinite. When $\Lambda(s)$ is finite, $\Lambda(s)$ is strictly increasing. Finally, $\varepsilon_m(\Lambda(0)) = 1/2$.*

Proof. See Appendix A.1. □

Note that the exact range of s such that $\Lambda(s)$ is infinite depends on the specific meeting technology. When $m(\lambda) = 1 - e^{-\lambda}$, $\Lambda(s) = \infty$ if and only if $s \geq 1/2$; when $m(\lambda) = \lambda/(1+\lambda)$, $\Lambda(s) = \infty$ if and only if $s \geq 1/4$.[13]

2.3. Definitions of decentralized market equilibrium

We look for a symmetric, pure-strategy equilibrium in which all sellers post the same price p. In this case, buyers simply randomize their visits. To establish that a price p is an equilibrium, we need to show that no profitable deviation exists. However, since buyers can visit multiple sellers simultaneously, single-seller deviations and market-maker deviations, which have been used interchangeably in the literature, may yield different outcomes.

To start, we define some terms. Non-deviant sellers post price p and can expect queue length λ. Deviant sellers post price $\widetilde{p}$ with corresponding expected queue length $\widetilde{\lambda}$. Using the Nash concept of competitive search equilibrium, we only consider single-seller deviations. A deviant market maker, however, can attract multiple deviant sellers simultaneously. The expected payoff of a buyer who visits a deviant seller posting $\widetilde{p}$ and a non-deviant seller posting p is $U(\widetilde{p}, \widetilde{\lambda}, p, \lambda)$. The expected value of visiting two deviant sellers is then $U(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda})$, while the expected value of visiting two non-deviants sellers is $U(p, \lambda, p, \lambda)$. Since the role of the two markets is symmetric to the buyers, $U(\widetilde{p}, \widetilde{\lambda}, p, \lambda) = U(p, \lambda, \widetilde{p}, \widetilde{\lambda})$. For the seller side, let $\pi(\widetilde{p}, \widetilde{\lambda}, p, \lambda)$ represent the expected profit of a deviant seller whose chosen buyer's other visit is to a non-deviant seller. Note that the equilibrium seller value is then $\pi(p, \lambda, p, \lambda)$. In the market-maker approach, if all buyers who visit the deviant submarket pay both visit there, then the expected value of a deviant seller is $\pi(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda})$.

We begin with the Nash equilibrium concept in which equilibrium is defined by the absence of a profitable single-seller deviation. To establish that there is no profitable deviation, consider a deviant seller who posts a price $\widetilde{p}$ and expects queue length $\widetilde{\lambda}$. Since only that single seller deviates, buyers who decide to visit the deviant seller must pay their other visit to a non-deviant seller. As we consider a large market, a deviation by a single seller does not change the market utility of buyers, $\overline{U}$, (their *equilibrium* expected payoff), which is

$$\overline{U} = U(p, \lambda, p, \lambda), \tag{6}$$

Similarly, the *equilibrium* expected payoff of sellers is given by $\pi(p, \lambda, p, \lambda)$.

Buyer optimality implies that

$$\overline{U} = U(\widetilde{p}, \widetilde{\lambda}, p, \lambda). \tag{7}$$

[13] Equation (45) in the proof of Lemma 1 shows that $\Lambda(s) = \infty$ if and only if $s \geq 1/(2 + 2\lim_{\lambda\to\infty} \lambda^2 m'(\lambda))$.

The left-hand side denotes the buyer value of visiting only non-deviant sellers and the right-hand side denotes the buyer value of visiting one deviant and one non-deviant seller. Equation (7) is the indifference condition that determines the expected queue length at the deviant seller. The expected payoff of the deviant seller is then

$$\pi^d(\widetilde{p}) = \pi(\widetilde{p}, \widetilde{\lambda}, p, \lambda), \tag{8}$$

where $\widetilde{\lambda}$ is implicitly determined by $\widetilde{p}$ through equation (7).

Definition 1. In a symmetric pure-strategy **competitive search Nash equilibrium**, sellers choose a price p and buyers receive their market utility $\overline{U}$ such that the following conditions are satisfied.[14]

1. Buyer optimality. Market utility $\overline{U}$ is given by equation (6) on and off the equilibrium path. For buyers who visit the deviant seller, the indifference condition, (7), holds.
2. Seller optimality. There exists no profitable deviation for sellers. That is, the expected payoff from a deviation, given by equation (8), does not exceed the equilibrium payoff.
3. Free entry. The expected buyer payoff equals the entry cost K.

Next, we consider the market-maker equilibrium. A market maker can open a submarket that is characterized by price $\widetilde{p}$. This submarket can potentially attract multiple buyers and multiple sellers, while taking the market utility of buyers as given. The difference relative to the competitive search Nash equilibrium case is that a buyer can visit more than one deviant seller. A market maker opens a deviant submarket in the following way. First, the market maker announces a price $\widetilde{p}$ and sellers that are part of the deviant submarket are required to post $\widetilde{p}$. In return, the market maker promises market tightness $\widetilde{\lambda}$. The market tightness $\widetilde{\lambda}$ must be such that it is worthwhile for buyers to participate in the deviant submarket. The price p is a competitive search market-maker equilibrium if no market maker has an incentive to open a submarket with a different price.

Formally, suppose that the deviant submarket has price $\widetilde{p}$ and expected queue length $\widetilde{\lambda}$. Then buyers who visit the deviant submarket choose to either send $k = 1$ or $k = 2$ visits there to maximize their expected payoff, i.e.,

$$\max\{U(\widetilde{p}, \widetilde{\lambda}, p, \lambda), U(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda})\}. \tag{9}$$

Moreover, given the buyers' optimal choice of k, $\widetilde{\lambda}$ must satisfy the buyer indifference condition,

$$\overline{U} = \max\{U(\widetilde{p}, \widetilde{\lambda}, p, \lambda), U(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda})\} \tag{10}$$

where the right-hand side is the expected payoff of a buyer who optimally chooses the number of visits he or she pays to the deviant submarket. The expected payoff of a seller who joins the deviant submarket is then

$$\pi^d(\widetilde{p}) = \begin{cases} \pi(\widetilde{p}, \widetilde{\lambda}, p, \lambda) & \text{if} \quad U(\widetilde{p}, \widetilde{\lambda}, p, \lambda) > U(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda}) \\ \pi(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda}) & \text{if} \quad U(\widetilde{p}, \widetilde{\lambda}, p, \lambda) < U(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda}) \end{cases} \tag{11}$$

[14] Here the adjective "Nash" means that we only consider single-seller deviations; it does not mean that we consider a game with finite numbers of buyers and sellers. We consider a large market and assume that the deviant seller treats the expected buyer payoff as given (the market utility approach), which has been justified as the limit of Nash equilibria of a sequence of finite games when buyers only visit one seller (see, for example, Peters (1997, 2000) and Burdett et al. (2001)).

where $\widetilde{\lambda}$ must satisfy the corresponding buyer indifference condition given by equation (10).

Definition 2. In a symmetric pure-strategy **competitive search market-maker equilibrium**, sellers choose a price p and buyers receive their market utility $\overline{U}$ such that the following conditions are satisfied.

1. Buyer optimality. Market utility $\overline{U}$ is given by equation (6) on and off the equilibrium path. Buyers who visit the deviant submarket choose k optimally, and the indifference condition, equation (10), holds.
2. Seller and market maker optimality. No market maker can create a profitable submarket. That is, the expected payoff of sellers in a deviant submarket, which is given by equation (11), is no greater than their equilibrium payoff.
3. Free entry. The expected buyer payoff equals the entry cost K.

We now reformulate the buyer optimality problem. In all our models below, $U(\widetilde{p}, \widetilde{\lambda}, p, \lambda)$ is unsurprisingly strictly decreasing in $\widetilde{\lambda}$ since all else equal buyers benefit from shorter queues. Lemma 2 below shows that buyer optimality can be reformulated as a no-arbitrage condition. Fix a price $\widetilde{p}$. For $k = 1, 2$, there is an expected queue length, $\lambda_k(\widetilde{p}, p, \lambda)$, which makes buyers indifferent between receiving the market utility and paying k visits to the deviant submarket and $2 - k$ visits to the non-deviant submarket. The no-arbitrage condition implies that the buyers' optimal k gives the larger expected queue length for the deviant submarket.

Lemma 2. *Assume that $U(\widetilde{p}, \widetilde{\lambda}, p, \lambda)$ is strictly decreasing in $\widetilde{\lambda}$. Then the number of visits that a buyer who visits the deviant submarket pays to that submarket solves*

$$\max\{\lambda_1(\widetilde{p}, p, \lambda), \lambda_2(\widetilde{p}, p, \lambda)\} \tag{12}$$

where $\lambda_1(\widetilde{p}, p, \lambda)$ and $\lambda_2(\widetilde{p}, p, \lambda)$ are the expected queue lengths, $\widetilde{\lambda}$, that solve $\overline{U} = U(\widetilde{p}, \widetilde{\lambda}, p, \lambda)$ and $\overline{U} = U(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda})$, respectively.

Proof. See Appendix A.2. □

Whenever there are too few visits to the deviant submarket (so that $\widetilde{\lambda} < \lambda_k(\widetilde{p}, p, \lambda)$ for some k), an individual buyer could obtain an expected payoff strictly above his or her market utility by visiting the deviant submarket. Hence, given buyers' optimal choice of k, only the longer queue for the deviant submarket is consistent with the market-utility constraint.

In the models that we consider below, $U(\widetilde{p}, \widetilde{\lambda}, p, \lambda)$ is also strictly decreasing in $\widetilde{p}$. Buyer optimality can then also be restated as follows. Given $\widetilde{\lambda}$, the optimal k solves

$$\max\{p_1(\widetilde{\lambda}, p, \lambda), p_2(\widetilde{\lambda}, p, \lambda)\} \tag{13}$$

where $p_1(\widetilde{\lambda}, p, \lambda)$ and $p_2(\widetilde{\lambda}, p, \lambda)$ again solve the indifference condition $\overline{U} = U(\widetilde{p}, \widetilde{\lambda}, p, \lambda)$ and $\overline{U} = U(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda})$, respectively.

In this alternative formulation, we can think of the choice variable of sellers in the deviant submarket to be $\widetilde{\lambda}$ instead of $\widetilde{p}$ since choosing the optimal price is isomorphic to choosing the optimal expected queue length that satisfies the market utility constraint. Thus as in equation (11), the expected payoff of deviant sellers can be expressed as a function of $\widetilde{\lambda}$. Below, we follow this approach since analytic expressions of $p_k(\widetilde{\lambda}, p, \lambda)$ can be obtained relatively easily.

3. The decentralized market equilibrium

In this section, we characterize the Nash and market-maker equilibria and compare the equilibrium outcomes with the social planner allocation. We begin by considering the polar cases of $s = 0$ (perfect complements) and $s = 1$ (perfect substitutes) and then generalize.

3.1. Perfect complements

Suppose the units that are offered for sale are perfect complements and sellers post price p. A buyer receives a positive payoff if and only if both of the buyer's visits lead to offers.

3.1.1. Competitive search Nash equilibrium

In the Nash approach, we consider a single-seller deviation. Consider first the buyer side. Buyers who visit the deviant seller must pay their other visit to a non-deviant seller. The expected payoff of a buyer who visits the deviant seller with posted price $\widetilde{p}$ and expected queue length $\widetilde{\lambda}$ and a non-deviant seller with posted price p and expected queue length λ is

$$U(\widetilde{p}, \widetilde{\lambda}, p, \lambda) = q(\widetilde{\lambda})q(\lambda)(1 - \widetilde{p} - p). \tag{14}$$

A buyer who receives offers from the deviant seller and a non-deviant seller receives an expected payoff of $(1 - \widetilde{p} - p)$ times the corresponding probability of receiving those offers. When the buyer receives fewer than 2 offers, then his or her payoff is zero even though the seller would lower its price to zero in this case.

Next consider the seller side. The expected payoff of the deviant seller is

$$\pi(\widetilde{p}, \widetilde{\lambda}, p, \lambda) = m(\widetilde{\lambda})q(\lambda)\widetilde{p}. \tag{15}$$

The deviant seller receives at least one visit with probability $m(\widetilde{\lambda})$ and earns profit if and only if the chosen buyer receives another offer, which happens with probability $q(\lambda)$.

Before analyzing the two versions of competitive search equilibrium, we first compute the socially efficient price which sets the buyer's payoff equal to the buyer's marginal contribution to surplus.[15] By equations (3) (the case of perfect complements) and (14), that price is

$$p^*_{pc} = 1 - \varepsilon_m(\lambda), \tag{16}$$

which is derived by setting $V^B_{pc}(\lambda) = U(p, \lambda, p, \lambda)$. When a buyer has 2 offers, from a social point of view, the 2 sellers should get their share of the surplus, which is $1 - \varepsilon_m(\lambda)$, and the buyer should get the residual $1 - 2(1 - \varepsilon_m(\lambda))$.

From the buyer indifference condition, equation (7), we can solve for the relationship between $\widetilde{p}$ and $\widetilde{\lambda}$. Substituting equation (14) into equation (7) yields

$$p_1(\widetilde{\lambda}, p, \lambda) = p + (1 - 2p)\left(1 - \frac{q(\lambda)}{q(\widetilde{\lambda})}\right), \tag{17}$$

which implies that $p_1(\widetilde{\lambda}, p, \lambda)$ is a decreasing function of $\widetilde{\lambda}$ (since $q(\widetilde{\lambda})$ is strictly decreasing in $\widetilde{\lambda}$); i.e., a higher price leads to fewer buyer visits. Given the relationship between $\widetilde{\lambda}$ and

[15] Given constant returns to scale, if the buyer's payoff equals his or her marginal contribution to surplus, then the same is true for the seller.

$p_1(\widetilde{\lambda}, p, \lambda)$, we can represent the expected payoff of the deviant seller as a function of $\widetilde{\lambda}$ alone. That is, substituting the above equation into equation (15) yields

$$\pi^d(\widetilde{\lambda}) \equiv \pi(p_1(\widetilde{\lambda}, p, \lambda), \widetilde{\lambda}, p, \lambda) = q(\lambda)\left((1-p)m(\widetilde{\lambda}) - (1-2p)\frac{m(\lambda)}{\lambda}\widetilde{\lambda}\right). \tag{18}$$

Since $m(\widetilde{\lambda})$ is strictly concave in $\widetilde{\lambda}$ and $1-p > 1-2p \geq 0$, $\pi^d(\widetilde{\lambda})$ is also strictly concave in $\widetilde{\lambda}$, so the deviant seller's first-order condition is both necessary and sufficient. Setting the derivative with respect to $\widetilde{\lambda}$ equal to 0 yields

$$(1-p)m'(\widetilde{\lambda}) = (1-2p)\frac{m(\lambda)}{\lambda}.$$

In a symmetric pure-strategy equilibrium, the first-order condition holds at $\widetilde{\lambda} = \lambda$. Thus the equilibrium price p_{pc}^N (superscript "N" for Nash and subscript "pc" for perfect complements) is given by

$$p_{pc}^N = (1-\varepsilon_m(\lambda))\left(1 - p_{pc}^N\right), \tag{19}$$

From an individual buyer's and an individual seller's point of view, their match surplus is $1 - p_{pc}^N$, given that the buyer needs to pay p_{pc}^N to the other seller; hence the equilibrium price equals this match surplus times $1 - \varepsilon_m(\lambda)$. Simplifying the above equation yields

$$p_{pc}^N = \frac{1-\varepsilon_m(\lambda)}{2-\varepsilon_m(\lambda)}. \tag{20}$$

Therefore, the equilibrium price is lower than the socially optimal price p_{pc}^* given by equation (16), which implies that the buyer's expected payoff is strictly greater than his or her marginal contribution to surplus. When the two units are perfect complements, there is excessive buyer entry in the competitive search Nash equilibrium.

3.1.2. *Competitive search market-maker equilibrium*

In this case, a market maker can set up a deviant submarket. We need to first solve the buyer portfolio problem. That is, if a buyer decides to visit the deviant submarket, what is the optimal number of sellers to approach in that submarket?

Since we have already computed $p_1(\widetilde{\lambda}, p, \lambda)$, to solve the alternative buyer problem (13), we only need to solve for $p_2(\widetilde{\lambda}, p, \lambda)$ from the buyer's indifference condition $\overline{U} = U(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda})$. By equation (14), $U(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda}) = q(\widetilde{\lambda})^2(1-2\widetilde{p})$ and we have

$$p_2(\widetilde{\lambda}, p, \lambda) = p + (1-2p)\frac{1}{2}\left(1 - \left(\frac{q(\lambda)}{q(\widetilde{\lambda})}\right)^2\right). \tag{21}$$

Comparing the above equation with equation (17) shows that $p_1(\widetilde{\lambda}, p, \lambda) > p_2(\widetilde{\lambda}, p, \lambda)$.[16] By Lemma 2, buyers who visit the deviant submarket find it optimal to pay only one visit there.

Thus buyers who decide to visit the deviant submarket find it optimal to pay exactly one visit there even though the goods are perfect complements. The reason is that buyer visits are strategic substitutes. To better understand why buyer visits are strategic substitutes, imagine a deviant submarket with a lower price and a longer expected queue than the non-deviant submarket. A

[16] To see this, note that for any $x \neq 1$, $1 - x > \frac{1}{2}(1-x^2)$. Set $x = q(\lambda)/q(\widetilde{\lambda})$ in equations (17) and (21).

buyer who receives an offer from a seller in the deviant submarket can only realize this discount if he or she receives a second offer, and the probability of that is higher in the non-deviant submarket. Thus, the buyer sets $k = 1$.

This can be shown formally as follows. Let $\boldsymbol{x} = (p, q(\lambda))$ and $\widetilde{\boldsymbol{x}} = (\widetilde{p}, q(\widetilde{\lambda}))$. A lower price implies a longer queue so that we either have $\boldsymbol{x} > \widetilde{\boldsymbol{x}}$ or $\boldsymbol{x} < \widetilde{\boldsymbol{x}}$. By equation (14) with $k = 1$, the payoff for a buyer who makes exactly one visit to the deviant submarket is $\mathcal{U}(\widetilde{\boldsymbol{x}}, \boldsymbol{x}) = q(\lambda)q(\widetilde{\lambda})(1 - p - \widetilde{p})$. In logs, this has a simple additively separable structure: $\ln\mathcal{U}(\widetilde{\boldsymbol{x}}, \boldsymbol{x}) = \ln q(\lambda) + \ln q(\widetilde{\lambda}) + \ln(1 - p - \widetilde{p})$. The cross partial derivatives of $\ln\mathcal{U}$ are: $(\ln\mathcal{U})_{13}(\widetilde{\boldsymbol{x}}, \boldsymbol{x}) = \partial^2 \ln\mathcal{U}/(\partial(p)\partial(\widetilde{p})) = -1/(1 - p - \widetilde{p})^2 < 0$ and $(\ln\mathcal{U})_{14} = (\ln\mathcal{U})_{23} = (\ln\mathcal{U})_{24} = 0$. Therefore, $\boldsymbol{x}$ and $\widetilde{\boldsymbol{x}}$ are strategic substitutes in $\ln\mathcal{U}$.[17] That is, $\frac{1}{2}\ln\mathcal{U}(\boldsymbol{x}, \boldsymbol{x}) + \frac{1}{2}\ln\mathcal{U}(\widetilde{\boldsymbol{x}}, \widetilde{\boldsymbol{x}}) < \ln\mathcal{U}(\widetilde{\boldsymbol{x}}, \boldsymbol{x})$ when $\boldsymbol{x} \neq \widetilde{\boldsymbol{x}}$. Consequently, a buyer who visits the deviant submarket will approach only one seller there and pay the other visit to the non-deviant submarket.

The following Proposition summarizes our results.

Proposition 1. *When the goods are perfect complements, then the outcomes, both on and off the equilibrium path, of the two versions of competitive search equilibrium (Nash and market maker) coincide. The (common) equilibrium is not constrained efficient, and the equilibrium payoff of buyers is strictly greater than their marginal contribution to surplus.*

Proof. See the above discussion. □

The inefficiency arises due to an externality that sellers impose on each other. A seller who makes an offer to a buyer benefits from short queues at other sellers because then the buyer in question is more likely to receive another offer. Individual sellers however choose socially inefficient long queues and correspondingly low prices in order to secure trade (reduce the probability of zero arrivals). Therefore, sellers receive less than their social contribution to surplus in equilibrium. One might have expected that a market maker would be able to internalize this externality, but the market maker cannot force buyers to pay both visits to a deviant submarket. It can offer sellers the opportunity to enter a deviant high-price, short-queue submarket, but sellers realize that buyers will still pay their other visits to non-deviant sellers, i.e., those with low prices and long queues.

3.2. Perfect substitutes

When the goods are perfect substitutes and a buyer has two offers, Bertrand competition among sellers implies that sellers lower the price to zero and the buyer receives the full value of the match. This case is an extension of the model presented in Albrecht et al. (2006), which was set in the labor market and assumed an urn-ball meeting technology and only considered the

[17] Consider a general payoff function $f(\boldsymbol{x}, \boldsymbol{y})$, where $\boldsymbol{x} \in \mathbb{R}^m$ and $\boldsymbol{y} \in \mathbb{R}^n$ are two variables that a decision maker has to choose. Following the literature, we call $\boldsymbol{x}$ and $\boldsymbol{y}$ strategic complements (resp. substitutes) in f if $\partial^2 f/(\partial x_i \partial y_j) \geq 0$ (resp. ≤ 0) for $i = 1, \ldots, m$ and $j = 1, \ldots, n$. Strategic complementarity is equivalent to the concept that $f(\boldsymbol{x}, \boldsymbol{y})$ has increasing differences in $(\boldsymbol{x}, \boldsymbol{y})$. That is, for any $\boldsymbol{x}' \geq \boldsymbol{x}$ and $\boldsymbol{y}' \geq \boldsymbol{y}$, we have $f(\boldsymbol{x}', \boldsymbol{y}') - f(\boldsymbol{x}, \boldsymbol{y}') \geq f(\boldsymbol{x}', \boldsymbol{y}) - f(\boldsymbol{x}, \boldsymbol{y})$. Note that in our setup, $\mathcal{U}(\boldsymbol{x}, \widetilde{\boldsymbol{x}})$ is always symmetric in $(\boldsymbol{x}, \widetilde{\boldsymbol{x}})$: $\mathcal{U}(\boldsymbol{x}, \widetilde{\boldsymbol{x}}) = \mathcal{U}(\widetilde{\boldsymbol{x}}, \boldsymbol{x})$, so that increasing differences or equivalently strategic complementarity implies that $\mathcal{U}(\boldsymbol{x}, \boldsymbol{x}) + \mathcal{U}(\widetilde{\boldsymbol{x}}, \widetilde{\boldsymbol{x}}) > 2\mathcal{U}(\boldsymbol{x}, \widetilde{\boldsymbol{x}})$, where we have a strict inequality because each component of $\boldsymbol{x}$ is strictly greater than the corresponding component of $\widetilde{\boldsymbol{x}}$ and at least for one pair (i, j), the cross derivative $\partial^2\mathcal{U}/(\partial x_i \partial \widetilde{x}_j)$ is strictly positive.

competitive search Nash equilibrium.[18] Here we have a general meeting technology, $m(\lambda)$, and, more importantly, we show that the Nash equilibrium differs from the market-maker equilibrium. Only the latter is constrained efficient.

3.2.1. Competitive search Nash equilibrium

First consider the Nash approach. As before, first consider a buyer who visits a deviant seller who posts $\widetilde{p}$ and has expected queue length $\widetilde{\lambda}$ and a non-deviant seller who posts p and has expected queue length λ. Then, the buyer receives no offers, and thus payoff zero, with probability $(1-q(\lambda))(1-q(\widetilde{\lambda}))$; one offer from a nondeviant together with no offer from a deviant, and thus payoff $1-p$, with probability $q(\lambda)(1-q(\widetilde{\lambda}))$; one offer from a deviant together with no offer from a nondeviant, and thus payoff $1-\widetilde{p}$, with probability $(1-q(\lambda))q(\widetilde{\lambda})$; and two offers, and thus payoff 1, with the complementary probability. The buyer's expected payoff can then be written as

$$U(\widetilde{p},\widetilde{\lambda},p,\lambda)=\left[1-(1-q(\lambda))(1-q(\widetilde{\lambda}))\right]-\left[q(\lambda)(1-q(\widetilde{\lambda}))p+(1-q(\lambda))q(\widetilde{\lambda})\widetilde{p}\right], \tag{22}$$

where the term in the first bracket denotes the total surplus and the term in the second bracket denotes the expected total payment.

On the seller side, the expected payoff of the deviant seller is

$$\pi(\widetilde{p},\widetilde{\lambda},p,\lambda)=(1-q(\lambda))m(\widetilde{\lambda})\widetilde{p}, \tag{23}$$

where the deviant seller receives at least one visit with probability $m(\widetilde{\lambda})$ and the selected buyer's other visit fails with probability $1-q(\lambda)$.

As before, we first compute the socially efficient price, which sets the buyer payoff equal to the buyer marginal contribution to surplus. By equations (3) (the case of perfect substitutes) and (22), the socially efficient price is

$$p^*_{ps}=1-\varepsilon_m(\lambda), \tag{24}$$

which is the same as the socially efficient price when goods are perfect complements (see equation (16)). When a buyer receives exactly one offer, the buyer should receive his or her share of surplus, which is $\varepsilon_m(\lambda)$.

We now solve for the competitive search Nash equilibrium. As before, the buyer indifference condition (7) determines the relationship between the deviant seller's posted price $\widetilde{p}$ and the expected queue length $\widetilde{\lambda}$. Substituting equation (22) into (7) yields $\widetilde{p}$ as a function of $q(\widetilde{\lambda})$ and hence of $\widetilde{\lambda}$:

$$p_1(\widetilde{\lambda},p,\lambda)=p+\frac{p(2q-1)+(1-q)}{(1-q)}\left(1-\frac{q}{\widetilde{q}}\right), \tag{25}$$

where, to simplify notation, we have replaced $q(\lambda)$ with q and $q(\widetilde{\lambda})$ with $\widetilde{q}$. Note that the coefficient in front of the large parenthesis on the right-hand side is linear in p, and it is strictly positive at $p=0$ and 1, which then implies that it is always strictly positive. Therefore, $\widetilde{p}$ is strictly increasing in $\widetilde{q}$ and thus decreasing in $\widetilde{\lambda}$. That is, a higher price $\widetilde{p}$ leads to fewer buyer visits in expectation, i.e., a smaller $\widetilde{\lambda}$.

[18] In Albrecht et al. (2006), workers send out multiple job applications, but they can only work for one firm, which corresponds to the case in which the goods are perfect substitutes.

After substituting equation (25) into (23), we can write the expected profit of the deviant seller as a function of $\widetilde{\lambda}$ alone,

$$\pi^d(\widetilde{\lambda}) = \pi(p_1(\widetilde{\lambda}, p, \lambda), \widetilde{\lambda}, p, \lambda) = q\left[\left(p + \frac{1-q}{q}\right)m(\widetilde{\lambda}) - (p - (2p-1)(1-q))\widetilde{\lambda}\right].$$

The above equation is strictly concave in $\widetilde{\lambda}$, since $m(\cdot)$ is strictly concave. The first-order condition is then both necessary and sufficient, and is given by,

$$\frac{d\pi^d(\widetilde{\lambda})}{d\widetilde{\lambda}}\Big|_{\widetilde{\lambda}=\lambda} = (1-q)q\left[p\left(1 + \frac{\lambda q'}{1-q}\right) + \frac{\lambda q'}{q}\right],$$

where we replaced $q'(\lambda)$ with q' and used the fact that $m'(\lambda) = q(\lambda) + \lambda q'(\lambda)$. Note that the right-hand side of the above equation is linear in p and at $p = 0$ it is strictly negative. If the derivative at $p = 1$ it is strictly positive, then we have the interior solution. Otherwise, if at $p = 1$ the derivative is negative, then we have a corner solution. Therefore, the competitive search Nash equilibrium price p^N_{ps} (subscript "1" for perfect substitutes) is given by

$$p^N_{ps} = \begin{cases} 1 & \text{if} \quad 1 + \frac{\lambda q'}{q(1-q)} \leq 0 \\ -\frac{\lambda q'}{q}/(1 + \frac{\lambda q'}{1-q}) & \text{otherwise.} \end{cases} \tag{26}$$

Note that for common meeting technologies such as the urn ball and geometric, we have

$$1 + \frac{\lambda q'}{q(1-q)} \leq 0, \tag{27}$$

which is equivalent to $\varepsilon_m(\lambda) \leq q(\lambda)$ since $\lambda q'/q = -(1 - \varepsilon_m(\lambda))$. Given inequality (27), the interior solution never occurs. That is, in equilibrium we always have $p^N_{ps} = 1$. This generalizes Albrecht et al. (2006) by allowing for a more general class of meeting technologies.

Since $1 - \varepsilon_m(\lambda) = -\lambda q'/q$, we have

$$p^N_{ps} > p^*_{ps}, \tag{28}$$

which holds in general, i.e., irrespective of whether or not equation (26) has an interior solution. Thus the equilibrium p^N_{ps} is too high and a seller's equilibrium payoff is higher than its marginal contribution to surplus. This occurs because if all sellers charged the socially efficient price, each seller would have an incentive to deviate to a higher price. A higher price reduces the expected queue length less than it would in the case of a = 1, since when a = 2, buyers have an incentive to get multiple offers which would give them a price of zero. Buyers are therefore less deterred by a higher price. For the social planner, however, there is no value in multiple offers. That is, once a buyer has a first offer, a second offer adds nothing to total surplus; indeed, a second offer to one buyer makes a first offer to another buyer less likely. In short, the possibility of multiple offers means that the volume-margin tradeoff that the social planner faces is not the same as the one faced by buyers and sellers.

3.2.2. *Competitive search market-maker equilibrium*

We now analyze the competitive search market-maker equilibrium by following the same procedure as before, i.e., by comparing $p_1(\widetilde{\lambda}, p, \lambda)$ and $p_2(\widetilde{\lambda}, p, \lambda)$. Consider the buyer indifference condition, $U(p, \lambda, p, \lambda) = U(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda})$. This is a linear equation in $\widetilde{p}$, from which we solve for $\widetilde{p}$ as a function of $\widetilde{\lambda}$:

$$p_2(\widetilde{\lambda}, p, \lambda) = \frac{q}{1-q}\left(\frac{1-\widetilde{q}}{\widetilde{q}}\right)p - \frac{1-q+2pq}{1-q}\left(\frac{1-\widetilde{q}}{\widetilde{q}}\right)\frac{1}{2}\left(1-\left(\frac{1-q}{1-\widetilde{q}}\right)^2\right), \tag{29}$$

where we have again replaced $q(\lambda)$ by q and $q(\widetilde{\lambda})$ by $\widetilde{q}$. The following lemma shows that $p_2(\widetilde{\lambda}, p, \lambda)$ is always greater than $p_1(\widetilde{\lambda}, p, \lambda)$. Hence buyers who visit the deviant submarket pay both their visits there.

Lemma 3. *When the goods are perfect substitutes in a competitive search market-maker equilibrium, buyers who visit the deviant submarket pay all their visits to the deviant submarket.*

Proof. Rewrite $p_1(\widetilde{\lambda}, p, \lambda)$ in equation (25) as

$$p_1(\widetilde{\lambda}, p, \lambda) = \frac{q}{1-q}\left(\frac{1-\widetilde{q}}{\widetilde{q}}\right)p - \frac{1-q+2pq}{1-q}\left(\frac{1-\widetilde{q}}{\widetilde{q}}\right)\left(1-\left(\frac{1-q}{1-\widetilde{q}}\right)\right).$$

Define $x = (1-q)/(1-\widetilde{q})$ in both the above equation and equation (29). Then, since $\frac{1}{2}(1-x^2) < 1-x$ for any $x \neq 1$, we have $p_2(\widetilde{\lambda}, p, \lambda) > p_1(\widetilde{\lambda}, p, \lambda)$. Hence by Lemma 2, buyers who visit the deviant submarket pay all their visits to the deviant submarket. □

Even though the goods are perfect substitutes, a buyer's visits are complementary to each other because the buyer receives the full surplus with two offers. To better understand why buyer visits are strategic complements in this case, note that buyers need only one successful offer, but they get the full surplus when they receive two offers. Imagine a deviant submarket with a higher price and a shorter expected queue than the non-deviant submarket. It is optimal for a buyer to pay a second visit to the deviant submarket with the shorter expected queue if he or she pays the first visit there. It is as if a visit to the high-price, low-expected-queue submarket is an investment in which the buyer accepts the chance of paying a higher price if he or she gets only one offer. Paying a second visit to this submarket raises the probability that the buyer will not have to pay the higher price but will rather get the whole surplus. This is why the visits are strategic complements.

To see formally that the visits are indeed strategic complements, let $\zeta = q(1-p)$, $\widetilde{\zeta} = \widetilde{q}(1-\widetilde{p})$, $\boldsymbol{x} = (\zeta, 1-q)$, and $\widetilde{\boldsymbol{x}} = (\widetilde{\zeta}, 1-\widetilde{q})$. By equation (22) with $k=1$, the buyer payoff of paying only one visit to the deviant submarket is $\mathcal{U}(\boldsymbol{x}, \widetilde{\boldsymbol{x}}) = q\widetilde{q} + (1-q)\widetilde{\zeta} + (1-\widetilde{q})\zeta$. Note that $\widetilde{\zeta}$ is the buyer's expected value of visiting the deviant submarket conditional on the visit to the non-deviant submarket failing to generate an offer. Since buyers always prefer short queues (high $\widetilde{q}$) and high $\widetilde{\zeta}$, $1-\widetilde{q} > 1-q$ is always associated with $\widetilde{\zeta} > \zeta$ and vice versa. This implies that either $\boldsymbol{x} > \widetilde{\boldsymbol{x}}$ or $\boldsymbol{x} < \widetilde{\boldsymbol{x}}$. The cross partial derivatives of $\mathcal{U}$ are: $\mathcal{U}_{14} = \mathcal{U}_{23} = \mathcal{U}_{24} = 1$ and $\mathcal{U}_{13} = 0$. Therefore, even though the goods are perfect substitutes, $\boldsymbol{x}$ and $\widetilde{\boldsymbol{x}}$ are strategic complements in $\mathcal{U}$, and a buyer who visits the deviant submarket will pay both visits there.

Since buyers who visit the deviant submarket pay both their visits there and must receive an expected payoff equal to their market utility, the expected profit of a deviant seller is the difference between total surplus per seller and the expected payoffs to the buyers, that is,

$$\pi^d(\widetilde{\lambda}) = y(\widetilde{\lambda}) - \frac{\widetilde{\lambda}}{2}\overline{U} \tag{30}$$

where total surplus per seller, $y(\widetilde{\lambda})$, is defined in equation (1), $\widetilde{\lambda}/2$ is the number of buyers per seller in the deviant submarket and $\overline{U}$ is the expected payoff that each buyer receives.[19]

By Lemma 1, to maximize $\pi^d(\widetilde{\lambda})$, the deviant seller expected profit, the first-order condition is both necessary and sufficient. In a symmetric pure-strategy equilibrium, the first-order condition holds at $\widetilde{\lambda} = \lambda$. Therefore, $2y'(\lambda) = \overline{U}$, or equivalently, the buyer marginal contribution to surplus equals his or her market utility, and hence the decentralized equilibrium is constrained efficient. Thus the competitive search market-maker equilibrium price p_{ps}^{MM} equals $p_{ps}^* = 1 - \varepsilon_m(\lambda)$ (see equation (24)).

The logic is the same as in the familiar case in which a buyer can only visit one seller. A deviant seller's problem is analogous to one where the deviant seller can "buy" queues directly from a competitive market where the price for the expected queue length is $\overline{U}/2$.

The following proposition summarizes the results for the case of perfect substitutes.

Proposition 2. *When the goods are perfect substitutes, the competitive search market-maker equilibrium is constrained efficient, whereas in the competitive search Nash equilibrium, the equilibrium payoff of buyers is strictly less than their marginal contribution to surplus.*

The key to this non-equivalence result has to do with the options available to buyers under the two interpretations of competitive search equilibrium. Interpreting competitive search equilibrium as the limit of a sequence of Nash equilibria means that no single seller can profit by posting a non-equilibrium price. In this case, a buyer can pay at most one visit to a deviant seller. Using the market-maker interpretation, no profitable deviation means that a competing submarket, in which multiple sellers post a non-equilibrium price, cannot be profitably established. In this case, a buyer can choose any number of visits from $k = 0$, 1 or 2 to pay to deviant sellers. This expansion of buyer choice matters when there are interactions among a buyer's visits; that is, when the value of any one visit depends on the outcomes associated with his or her other visit.[20]

The intuition for why the competitive search market-maker equilibrium is constrained efficient is that the market maker enables sellers to coordinate on p_{ps}^*. Each seller in a submarket where this price is posted knows that all other sellers in that submarket are posting the same price and that any buyer who visits this submarket is not visiting sellers in other submarkets. By joining the submarket with price p_{ps}^*, sellers are implicitly agreeing to cooperate with one another, i.e., to not raise p above the social planner value. Relative to the Nash case, a seller in the submarket

[19] Alternatively, we can use the buyer indifference condition $U(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda}) = \overline{U}$ to solve for $\widetilde{p}$ as a function of $\widetilde{\lambda}$ and then substitute it into $\pi(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda})$. This gives the expected payoff of a deviant seller:

$$\pi^d(\widetilde{\lambda}) = (1-\widetilde{q})m(\widetilde{\lambda})\frac{1-(1-\widetilde{q})^2-\overline{U}}{2(1-\widetilde{q})\widetilde{q}} = y(\widetilde{\lambda}) - \frac{\widetilde{\lambda}}{2}\overline{U}.$$

.

[20] The non-equivalence of the two approaches (Nash and market-maker) depends on the ex-post-Bertrand-competition assumption, which leads to lower posted prices and to interactions among buyer visits. If instead sellers post entry fees, then as we show in Proposition 5 of the online Appendix, the two approaches are equivalent (both on and off the equilibrium path), and the common equilibrium is constrained efficient. Another possible alternative trade arrangement is fixed prices as considered in Galenianos and Kircher (2009), which shows that the Nash and market-maker approaches are equivalent (both on and off the equilibrium path) and that the common equilibrium is not constrained efficient. In these two alternative trade arrangements, there are no interactions among buyers' multiple offers.

with price p^*_{ps} receives a lower price when his or her buyer has no other offer, but this is more than compensated for by a longer expected queue.

3.3. *General case:* $0 \leq s \leq 1$

We now consider the general case in which s can take on any value between 0 and 1. Our results are illustrated in Fig. 1, where we set $m(\lambda) = 1 - e^{-\lambda}$ and $\lambda = 1$ and analyze how the equilibrium varies with s. The red solid line plots the socially efficient price as a function of s. The blue dashed line plots the competitive search Nash equilibrium price, and the green dashed-dotted line represents the competitive search market-maker equilibrium price. All three curves are piece-wise linear, with the same domain cutoff points $\sigma_0(\lambda)$ and $\sigma_1(\lambda)$ (to be specified later). As can be seen in the figure, between the two cutoff points, all three lines coincide, i.e., the competitive search Nash equilibrium and the competitive search market-maker equilibrium generate the same allocation and it is socially efficient. Below $\sigma_0(\lambda)$, when units are strong complements, the competitive search Nash equilibrium and the competitive search market-maker equilibria coincide, but they are not socially efficient. Finally, above $\sigma_1(\lambda)$, the case of strong substitutes, the two equilibria are different and only the competitive search market-maker equilibrium is socially efficient.

To understand Fig. 1, suppose the cost of buyer entry, K, is such that the efficient level of market tightness when $s = 1/2$ is $\lambda = 1$. At $s = 1/2$, the marginal values for a buyer of a first and a second offer are the same, so the model in which B buyers each visit 2 sellers is equivalent to one in which $2B$ buyers each visit one seller. In this case, sellers post a fixed price; i.e., there is no incentive for a seller to lower its price depending on whether or not its chosen buyer received another offer. Further, as is well known in the literature, when $2B$ buyers each visit one seller (equivalently at $s = 1/2$ when B buyers each visit 2 sellers), the Nash and market-maker equilibria are the same, and that equilibrium is constrained efficient. The equilibrium and socially efficient price are both equal to $(1 - \varepsilon_m(\lambda))/2 < 1/2$.

Next, suppose s is close to $1/2$ (either smaller or larger) and that we vary K so that the efficient level of market tightness is still $\lambda = 1$.[21] The socially efficient price will still be less than the marginal value of an offer, so a seller will not lower its price depending on whether its chosen buyer has another offer; i.e., the posted price is still a fixed price. Again, the Nash and market-maker equilibria coincide, and the equilibrium is efficient. However, when s differs sufficiently from $1/2$ ($s < \sigma_0(\lambda)$ or $s > \sigma_1(\lambda)$ in the figure), the difference between the marginal values of the first and second offer are sufficiently large that seller prices are no longer fixed. If $s < \sigma_0(\lambda)$, a seller has the incentive to lower its price when its chosen buyer only has that one offer. In this case, the situation is similar to the case of perfect complements, and the same forces that drive inefficiency of equilibrium when $s = 0$ apply. When $s > \sigma_1(\lambda)$, a seller has the incentive to lower its price when its chosen buyer has a second offer. In this case, the situation is similar to the case of perfect substitutes.

3.3.1. *Payoff functions*

We begin by writing down the expected payoffs for buyers and sellers, both on and off the equilibrium path. We then derive the socially efficient price, which sets the expected payoffs of buyers and sellers equal their marginal contributions to surplus. As before, we assume that

[21] Equivalently, we could fix K and allow the efficient value of λ to vary with s.

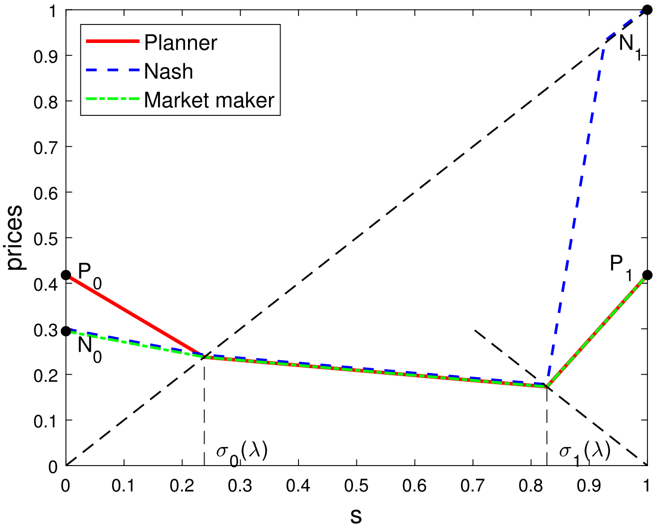

Fig. 1. The socially efficient prices versus the prices under the two equilibrium concepts. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

the sellers can perfectly price discriminate between buyers with and without other offers but, nonetheless, efficiency does not always obtain and the Nash and market-maker equilibria do not always coincide.

Complements: $s \leq 1/2$ Consider first the case where the goods are complements, $s \leq 1/2$. Again, each seller posts and commits to a maximum price p. After selecting its buyer, a seller observes whether or not its buyer has another offer. If not, the seller can lower its price to s, the maximum price that a buyer with no other offer is willing to accept.

The expected payoff of a buyer who visits a deviant seller posting $\widetilde{p}$ and a non-deviant seller posting p is

$$U(\widetilde{p}, \widetilde{\lambda}, p, \lambda) = (1 - q(\lambda))q(\widetilde{\lambda})(s - \min\{\widetilde{p}, s\}) + q(\lambda)(1 - q(\widetilde{\lambda}))(s - \min\{p, s\}) + q(\lambda)q(\widetilde{\lambda})(1 - p - \widetilde{p}). \quad (31)$$

The first two terms on the right-hand side represent the cases in which the buyer obtains only one offer. In these cases, if the posted price of the seller who has selected the buyer is greater than s, then that seller will lower its price to s. The last term represents the case in which the buyer obtains two offers. Note that the expected value of visiting two deviant sellers is $U(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda})$, while the expected value of visiting two non-deviants sellers is $U(p, \lambda, p, \lambda)$.

The profit of a deviant seller whose chosen buyer's other visit is to a non-deviant seller is

$$\pi(\widetilde{p}, \widetilde{\lambda}, p, \lambda) = m(\widetilde{\lambda})\left((1 - q(\lambda))\min\{\widetilde{p}, s\} + q(\lambda)\widetilde{p}\right). \quad (32)$$

With probability $m(\widetilde{\lambda})$, the seller meets at least one buyer, and conditional on successfully meeting a buyer, with probability $1 - q(\lambda)$ the seller's chosen buyer does not have a second offer and hence the resulting transaction price is $\min\{\widetilde{p}, s\}$. Otherwise, the buyer has two offers and the deviant seller has no incentive to lower the price. Note that the equilibrium seller value is then

$\pi(p, \lambda, p, \lambda)$. In the market-maker approach, if all buyers who visit the deviant submarket pay both visits there, then the expected value of a deviant seller is $\pi(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda})$.

Substitutes: $s \geq 1/2$ Next, suppose the products are substitutes ($s \geq 1/2$). It is possible that a seller's posted price is too high to trade for a buyer who has another offer. The seller can then lower its price ex post to $1 - s$, which is the additional surplus generated by a second offer and is the maximum price that its buyer is willing to pay for a second unit of the good.

As before, consider the expected payoff of a buyer who visits two sellers p and $\widetilde{p}$.

$$U(\widetilde{p}, \widetilde{\lambda}, p, \lambda) = (1 - q(\lambda))q(\widetilde{\lambda})(s - \widetilde{p}) + (1 - q(\widetilde{\lambda}))q(\lambda)(s - p) + q(\lambda)q(\widetilde{\lambda})(1 - \min\{p, 1 - s\} - \min\{\widetilde{p}, 1 - s\}). \quad (33)$$

The first two terms on the right-hand side represent the cases in which the buyer obtains exactly one offer, while the last term represents the case in which the buyer obtains two offers. In that case, if a seller's price is higher than $1 - s$, then the seller will lower its price to $1 - s$.

The expected profit of a deviant seller is

$$\pi(\widetilde{p}, \widetilde{\lambda}, p, \lambda) = m(\widetilde{\lambda}) \left((1 - q(\lambda))\widetilde{p} + q(\lambda) \min\{\widetilde{p}, 1 - s\}\right). \quad (34)$$

The interpretation of the above equation is similar to that of (32). The difference is that if the buyer has one offer, then the seller has no incentive to lower its price; if the buyer has two offers, then the deviant seller has to lower its price to $1 - s$ if $\widetilde{p} > 1 - s$.

3.3.2. The socially efficient prices

We now derive the socially efficient price that sets a buyer's and seller's expect payoffs equal to their marginal contributions to surplus. This socially efficient price will be used to contrast the planner's solution with the two notions of competitive search equilibrium.

Intermediate s Consider first the special case $s = 1/2$, where the value of one offer does not depend on whether buyers have a second offer or not. This case is equivalent to one where there are twice as many buyers and each buyer visits one seller, since the outcome of one visit does not interfere with that of the other visit. Thus, posted prices become fixed prices, and as is well known from the competitive search literature, the efficient price is $(1 - \varepsilon_m(\lambda))/2$, where the coefficient $1/2$ is the surplus of one offer.

More generally, when s is close to $1/2$ (either smaller or larger), the socially efficient price will be different but still smaller than s and $1 - s$, which implies that sellers will not lower the price no matter whether the chosen buyer has one offer or two offers (the posted price becomes a fixed price). Hence the efficient price is determined by the following equation:

$$m(\lambda) \cdot p = V^S(\lambda) = (1 - s)V^S_{pc}(\lambda) + sV^S_{ps}(\lambda)$$

where the left-hand side denotes the expected payoff of sellers, and the right-hand side their marginal contribution to surplus. Plugging the expressions for $V^S_{pc}(\lambda)$ and $V^S_{ps}(\lambda)$ from equation (2) into the above equation yields the social planner's solution of p, assuming that s is close to $1/2$,

$$\frac{p^*_M(\lambda, s)}{(1 - q(\lambda))s + q(\lambda)(1 - s)} = 1 - \varepsilon_m(\lambda), \quad (35)$$

where the subscript M in $p_M^*(\lambda, s)$ represents the fact that the above expression is only valid for intermediate values of s given λ (more on this below). Conditional on a seller meeting at least one buyer, with probability $1-q(\lambda)$, the seller's chosen buyer has no other offer, in which case the surplus brought by the seller is s, and with probability $q(\lambda)$, the buyer has a second offer, in which case the surplus created by the seller is $1-s$. Hence the left-hand side denotes the ratio between sellers' payoff and their marginal contribution to surplus, conditional on meeting a buyer. The right-hand side $1-\varepsilon_m(\lambda)$ is sellers' marginal contribution to the matching process.

Note that for fixed λ, $p_M^*(\lambda, s)$ is linear in s; at $s = 1/2$, $p_M^*(\lambda, s) < s$, at $s = 0$, $p_M^*(\lambda, s) > s$, and at $s = 1$, $p_M^*(\lambda, s) > 1-s$. Therefore, there exists a unique $s \in (0, 1/2)$ such that

$$s = p_M^*(\lambda, s) \Leftrightarrow s = \sigma_0(\lambda) \tag{36}$$

Similarly, there exists a unique $s \in (1/2, 1)$ such that

$$1 - s = p_M^*(\lambda, s) \Leftrightarrow s = \sigma_1(\lambda). \tag{37}$$

For completeness, the exact expressions for $\sigma_0(\lambda)$ and $\sigma_1(\lambda)$ are given in Appendix A.3 by equations (46) and (47), respectively. Hence, the socially efficient price is given by equation (35) when $\sigma_0(\lambda) \le s \le \sigma_1(\lambda)$. In Fig. 1, it is represented by the red solid line segment between $\sigma_0(\lambda)$ and $\sigma_1(\lambda)$, and is smaller than s (the diagonal line) and $1-s$ (the other black dashed line originating from the bottom right corner). Note that when $s = \sigma_0(\lambda)$, the efficient price equals s, and when $s = \sigma_1(\lambda)$, it equals $1-s$.

Low s Next, consider the case of strong complements, i.e., $s < \sigma_0(\lambda)$. In this case, the efficient price is greater than s and is determined by

$$m(\lambda)\left((1-q(\lambda))s + q(\lambda)p\right) = (1-s)V_{pc}^S(\lambda) + sV_{ps}^S(\lambda).$$

The left-hand side denotes the expected payoff of a seller. When a seller meets at least one buyer, which happens with probability $m(\lambda)$, with probability $1-q(\lambda)$ the buyer has no other offer so the seller has to lower its price from p to s, while with probability $q(\lambda)$ the buyer has another offer and the seller does not need to lower its price. Dividing both sides of the above equation by $m(\lambda)$ and using the result from equation (35) yields the efficient price $p_L^*(\lambda, s)$ when the units are strong complements,

$$\frac{(1-q(\lambda))s + q(\lambda)p_L^*(\lambda, s)}{(1-q(\lambda))s + q(\lambda)(1-s)} = 1 - \varepsilon_m(\lambda), \tag{38}$$

where, as in equation (35), the left-hand side is the ratio between a seller's payoff and its marginal contribution to surplus, conditional on having met a buyer.[22] In Fig. 1, the efficient price when the units are strong complements is represented by the red solid line segment between 0 and $\sigma_0(\lambda)$. Note that when $s = 0$ (perfect complements), the efficient price is $1-\varepsilon_m(\lambda)$ and is represented by point P_0.

[22] Note that the numerator on the left-hand side is simply the expression for $p_M^*(\lambda, s)$, which is greater than s when $s < \sigma_0(\lambda)$. Hence indeed we have $p_L^*(\lambda, s) > s$. Similar logic implies that $p_H^*(\lambda, s)$ in equation (39) is greater than $1-s$ when $s > \sigma_1(\lambda)$.

High s Finally, consider the case of strong substitutes, i.e., $s > \sigma_1(\lambda)$. In this case, the efficient price is greater than $1-s$ and is determined by

$$m(\lambda)\left((1-q(\lambda))p + q(\lambda)(1-s)\right) = (1-s)V^S_{pc}(\lambda) + sV^S_{ps}(\lambda).$$

The left-hand side denotes the expected payoff of sellers: when the buyer has two offers, the seller has to lower the price from p to $1-s$. Again, dividing both sides of the above equation by $m(\lambda)$ yields the efficient price $p^*_H(\lambda, s)$,

$$\frac{(1-q(\lambda))p^*_H(\lambda,s) + q(\lambda)(1-s)}{(1-q(\lambda))s + q(\lambda)(1-s)} = 1 - \varepsilon_m(\lambda). \tag{39}$$

The interpretation of the above equation is similar to that of (35) and (38). In Fig. 1, the efficient price when the units are strong substitutes is represented by the red solid line segment between $\sigma_1(\lambda)$ and 1. Note when $s = 1$ (perfect substitutes), the efficient price is $1 - \varepsilon_m(\lambda)$ and is represented by point P_1.

The results can be summarized as follows: i) If $s \le \sigma_0(\lambda)$, the efficient price is given by $p^*_L(\lambda, s)$ in equation (38). In this case, the price will be lowered to s when buyers have only one offer; ii) If $\sigma_0(\lambda) \le s \le \sigma_1(\lambda)$, the efficient price is given by $p^*_M(\lambda, s)$ in equation (35). In this case, the posted price becomes a fixed price. iii) If $s \ge \sigma_1(\lambda)$, the efficient price is given by $p^*_H(\lambda, s)$ in equation (39). In this case, the price will be lowered when buyers have two offers.

Thus for s close to 1/2, the results are qualitatively similar to the case in which $s = 1/2$. Each visit creates approximately the same expected payoff and there is no portfolio choice of visits. Therefore, the model with B buyers in which each buyer pays 2 visits is in that case very similar to the standard case in which $2B$ buyers each pay one visit. The complements case (s close to 0) is qualitatively similar to the case of perfect complements while the case of substitutes (s close to 1) is qualitatively similar to the case of perfect substitutes that we analyzed in the previous section.

3.3.3. Competitive search Nash equilibrium

We again look for a symmetric pure-strategy equilibrium where all sellers post price p.

The buyer indifference condition in the Nash approach is $U(p, \lambda, p, \lambda) = U(\widetilde{p}, \widetilde{\lambda}, p, \lambda)$, where U is either given by equation (31) (the case of complements) or by equation (33) (the case of substitutes). The indifference condition is linear in $\widetilde{p}$. Thus one can easily solve for $p_1(\widetilde{\lambda}, p, \lambda)$ and then substitute this expression into the deviant seller's expected profit, which is either given by equation (32) (the case of complements) or by equation (34) (the case of substitutes). The deviant seller's expected profit can thus be written as a function of $\widetilde{\lambda}$ alone: $\pi^d(\widetilde{\lambda}) = \pi(p_1(\widetilde{\lambda}, p, \lambda), \widetilde{\lambda}, p, \lambda)$.

First, consider the case of complements. The above procedure yields[23]

$$\pi^d(\widetilde{\lambda}) = m(\widetilde{\lambda})\left[q(\lambda)\left(1 - \max\{p, s\}\right) + (1-q(\lambda))s\right] - \widetilde{\lambda}\left(\overline{U} - q(\lambda)(s - \min\{p, s\})\right) \quad \text{if } s \le \frac{1}{2} \tag{40}$$

[23] One can solve for $p_1(\widetilde{\lambda}, p, \lambda)$ by considering separately two cases: $\widetilde{p} \ge s$ and $\widetilde{p} \le s$. However, both cases generate the same expression (40) for the deviant seller's expected profit as a function of $\widetilde{\lambda}$. To save space, we suppress the expression of $p_1(\widetilde{\lambda}, p, \lambda)$ here. Similarly, for the case of substitutes one can solve for $p_1(\widetilde{\lambda}, p, \lambda)$ by considering separately two cases: $\widetilde{p} \ge 1-s$ and $\widetilde{p} \le 1-s$. They generate the same expression (41) for the deviant seller's expected profit.

The deviant seller's expected profit is again strictly concave in $\widetilde{\lambda}$, which implies that the first-order condition is both necessary and sufficient.

Similarly, for the case of substitutes we have

$$\pi^d(\widetilde{\lambda}) = m(\widetilde{\lambda})\,[q(\lambda)\max\{p, 1-s\} + (1-q(\lambda))s] - \widetilde{\lambda}(\overline{U} - q(\lambda)(s-p)) \quad \text{if } s \geq \frac{1}{2}. \tag{41}$$

Note that the above equation is again strictly concave in $\widetilde{\lambda}$. Below we only describe the main results and relegate the formal analysis to Proposition 3 in Appendix A.3.

Intermediate s When s is close to $1/2$, the model is similar to the standard case in which $2B$ buyers each visit one seller. Thus, when the goods are complements ($s \leq 1/2$), the equilibrium price $p \leq s$; when the goods are substitutes ($s \geq 1/2$), the equilibrium price $p \leq 1-s$.

Both equation (40) (assuming $p \leq s$) and equation (41) (assuming $p \leq 1-s$) reduce to the same expression:

$$\pi^d(\widetilde{\lambda}) = m(\widetilde{\lambda})\,[(1-q(\lambda))s + q(\lambda)(1-s)] - \widetilde{\lambda}(\overline{U} - q(\lambda)(s-p)). \tag{42}$$

The first term on the right-hand side denotes the surplus created by the deviant seller: the probability of meeting a buyer is $m(\widetilde{\lambda})$; with probability $1-q(\lambda)$ the buyer has no other offer, in which case the deviant seller creates a surplus of s; with probability $q(\lambda)$ the buyer has another offer, then the deviant seller creates a surplus of $1-s$. The second term denotes the expected payoff that the deviant seller must offer to the visiting buyers. With probability $q(\lambda)$ each buyer receives a payoff $s-p$ from other non-deviant sellers. Since buyers must receive their market utility, $\overline{U}$, the deviant seller must offer the difference $(\overline{U} - q(\lambda)(s-p))$. From the deviant seller's perspective, this is fixed and independent from its choice of $\widetilde{\lambda}$. The above observations imply that the deviant seller is the residual claimant of the surplus that this seller creates. Following the standard logic in the literature, the Nash equilibrium is constrained efficient. Thus the equilibrium price coincides with the socially efficient price $p_M^*(\lambda, s)$ given by (35). Since the above analysis is based on the assumption that the competitive search Nash equilibrium price is smaller than s and $p_M^N(\lambda, s) = p_M^*(\lambda, s)$, from the planner's problem, we know that this is equivalent to $s \in [\sigma_0(\lambda), \sigma_1(\lambda)]$.

Low s Next we consider the case $s < \sigma_0(\lambda)$ or equivalently that the equilibrium price $p > s$. The deviant seller's problem (40) now becomes

$$\pi^d(\widetilde{\lambda}) = m(\widetilde{\lambda})\,[(1-q(\lambda))s + q(\lambda)(1-s)] - \widetilde{\lambda}\left[\overline{U} - q(\lambda)q(\widetilde{\lambda})(s-p)\right]. \tag{43}$$

Comparing the above equation with (42) shows that the deviant seller now needs to offer more to its applicants, since applying to the deviant seller is risky: Without an offer from the deviant seller, the applicants would receive a zero payoff. Hence in this case the equilibrium payoff of sellers is less than their marginal contribution to surplus for the same reason that we discussed in the case with *perfect* complements.

High s Finally, we consider the case $s > \sigma_1(\lambda)$, or equivalently that the equilibrium price $p > 1-s$. The deviant seller's expected profit, (41), then becomes

$$\pi^d(\widetilde{\lambda}) = m(\widetilde{\lambda})\,[(1-q(\lambda))s + q(\lambda)(1-s)] - \widetilde{\lambda}\left[\overline{U} - q(\lambda)(s-p-q(\widetilde{\lambda})(1-s-p))\right] \tag{44}$$

which is the difference between the surplus created by the deviant seller (the first term on the right-hand side) and the expected payoff that the deviant seller must offer to the visiting buyers (the second term). Note that the second term in the bracket is the expected payoff that buyers receive from the non-deviant sellers: With probability $q(\lambda)$ a buyer obtains an offer from a non-deviant seller; with probability $q(\widetilde{\lambda})$ the buyer has an offer from the deviant seller, in which case the price paid to the non-deviant seller is lowered by an amount $p-(1-s)>0$.

Comparing the above equation with (42) shows that the deviant seller now offers a lower expected payoff to its applicants since it can induce non-deviant sellers to lower their price. Hence in this case the equilibrium payoff of sellers is more than their marginal contribution to surplus, and the equilibrium price is greater than the socially efficient price. Note that the equilibrium price cannot exceed s. In Fig. 1, this constraint ($p \leq s$) is binding when s is close to 1, i.e., the blue dashed line coincides with the diagonal line when s is close to 1.

3.3.4. Competitive search market-maker equilibrium

First, consider the special case $s=1/2$, where the buyer value of one offer does not depend on the outcome of his or her other visit. Thus $U(p, \lambda, \widetilde{p}, \widetilde{\lambda})$, the expected buyer value of visiting two buyers with (p, λ) and $(\widetilde{p}, \widetilde{\lambda})$ respectively, is $q(\lambda)(1/2-p)+q(\widetilde{\lambda})(1/2-\widetilde{p})$. If a buyer sends exactly one visit to the deviant submarket, then his or her indifference condition $U(p, \lambda, p, \lambda)=U(p, \lambda, \widetilde{p}, \widetilde{\lambda})$ reduces to $q(\lambda)(1/2-p)=q(\widetilde{\lambda})(1/2-\widetilde{p})$; if a buyer pays exactly two visits to the deviant submarket, then his or her indifference condition $U(p, \lambda, p, \lambda)=U(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda})$ reduces to exactly the same condition. Hence, in the market-maker approach even though the queue length $\widetilde{\lambda}$ is uniquely determined by $\widetilde{p}$, buyers' optimal number of visits to the deviant submarket is indeterminate. This indeterminancy does not affect the deviant seller's expected payoff, and the competitive search market-maker equilibrium coincides with the corresponding Nash equilibrium and is also socially efficient.

Compared with the special cases of $s=0$ or 1, the analysis of market-maker equilibrium for general s is more complicated because the optimal number of visits to the deviant submarket by a buyer who decides to pay at least one visit there typically depends on the price, $\widetilde{p}$, that is posted in that submarket as well as on the non-deviant price, p. Lemma 4 in Appendix A.3 characterizes the solution to the buyer portfolio problem when $\widetilde{p}$ and p both lie in particular intervals. This lemma is sufficient to characterize the market-maker equilibrium because, as we show in Appendix A.3, a market-maker will not have the incentive to create a submarket with price $\widetilde{p}$ lying outside these intervals. Below we provide a summary of the results; see Proposition 4 in Appendix A.3 for a formal presentation.

Intermediate s When s is close to $1/2$, the market-maker equilibrium price coincides with the Nash equilibrium price, both of which are socially efficient. To see this, consider a deviant submarket with price $\widetilde{p}$. Suppose that $\widetilde{p}$ is close to p. If buyers find it optimal to pay one visit there, then the market-maker equilibrium price must coincide with the Nash equilibrium price; if buyers find it optimal to pay both visits there, then the expected profit of a deviant seller is by equation (30), i.e., $\pi^d(\widetilde{\lambda})=y(\widetilde{\lambda})-\frac{\widetilde{\lambda}}{2}\overline{U}$, which again implies that the market-maker equilibrium is constrained efficient (for the same logic see Proposition 2) and thus coincides with the Nash equilibrium. As before, the relevant domain for this case is $s \in [\sigma_0(\lambda), \sigma_1(\lambda)]$.

As for the optimal number of buyer visits to the deviant submarkets, Lemma 4 in Appendix A.3 shows when $p, \widetilde{p} \leq s < 1/2$ (strict complements), then it is strictly optimal for the buyers to send both visits to the deviant submarket; when $p, \widetilde{p} \leq 1-s < 1/2$ (strict substitutes), then it is strictly optimal for the buyers to pay exactly one visit there.

Low s Next, suppose $s < \sigma_0(\lambda)$ or equivalently $p > s$. Lemma 4 in Appendix A.3 shows that buyers who visit the deviant will pay exactly one visit there when $\widetilde{p} \geq s$. The expected profit of a deviant seller in this case is the same as that in the Nash approach. The first-order condition implies that the market-maker approach coincides with the Nash approach in this case. Deviations with $\widetilde{p} < s$ are not profitable.

High s Finally, suppose $s > \sigma_1(\lambda)$ or equivalently the equilibrium price $p > 1 - s$. Lemma 4 in Appendix A.3 shows that buyers who visit the deviant submarket will pay both visits there when $\widetilde{p} \geq 1 - s$. In this case, the expected profit of a deviant seller is again given by equation (30). As before, the first-order condition implies that the market-maker equilibrium is constrained efficient, and the equilibrium price coincides with the socially efficient price. Deviations with $\widetilde{p} < 1 - s$ are not profitable.

Summary When $s < \sigma_0(\lambda)$, $p^N = p^{MM} < p^*$; i.e., the Nash equilibrium price coincides with the market-maker equilibrium price and both are less than the socially efficient price. When $\sigma_0(\lambda) \leq s \leq \sigma_1(\lambda)$, $p^N = p^{MM} = p^*$; i.e., all three prices are the same. When $s > \sigma_1(\lambda)$, $p^N > p^{MM} = p^*$; i.e., the market-maker equilibrium price coincides with the socially efficient price, and both are less than the Nash equilibrium price.

4. Final remarks

In this paper, we have explored two interpretations of competitive search equilibrium, one based on a Nash foundation and the other on a market-maker foundation. In the baseline model of competitive search in which each buyer directs his or her search to a single seller (or each job seeker applies to a single firm), the two approaches are equivalent, and the common equilibrium allocation is constrained efficient. However with simultaneous search, i.e., when each buyer visits multiple sellers or each worker sends out multiple applications, the Nash and market-maker equilibria can differ, and the constrained efficiency of competitive search equilibrium is no longer assured.

We demonstrate this point in a model of a product market in which each buyer visits two sellers. Every seller has one unit of a homogeneous good, and buyer value is (weakly) increasing in the number of units he or she is able to purchase. This allows us to consider a variety of cases, ranging from perfect complements (the buyer gets no value from purchasing only one unit but full value from purchasing two units) to perfect substitutes (the buyer gets full value from purchasing one unit; purchasing a second unit adds no value). In this setting, we show that whether the Nash and market-maker equilibria coincide and whether equilibrium is constrained efficient depends on the extent of substitutability between the two units. Specifically, we show that if the two units are "strong complements," the Nash and market-maker equilibria coincide, but that common equilibrium is not constrained efficient. If the two units are "strong substitutes," the Nash and market-maker equilibria are not the same, and only the market-maker equilibrium is constrained efficient. Finally, if the units are neither strong complements nor strong substitutes, the two equilibria coincide and are constrained efficient.

The bottom line of our paper is that while the Nash and market-maker approaches are interchangeable when buyers visit one seller or workers visit one vacancy, introducing simultaneous search can break the equivalence. With simultaneous search, the choice of equilibrium concept is more than a matter of convenience. Researchers need to justify the equilibrium concept that they use in this setting since it may affect the equilibrium allocation and its efficiency.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Appendix A. Proofs

A.1. Proof of Lemma 1

Consider first the case of perfect complements, where the buyer's marginal contribution $V_{pc}^B(\lambda) > 0$ is strictly positive if and only if $\varepsilon_m(\lambda) > 1/2$. Furthermore, its derivative is

$$\frac{dV_{pc}^B(\lambda)}{d\lambda} = \frac{2q(\lambda)^2}{\lambda}\left[(1-\varepsilon_m(\lambda))^2 + \varepsilon_m(\lambda)\varepsilon_2(\lambda)\right],$$

where $\varepsilon_2(\lambda) = \lambda m''(\lambda)/m'(\lambda)$. Note that $0 > 1-\varepsilon_m(\lambda)+\varepsilon_2(\lambda) > 1-\varepsilon_m(\lambda)+\varepsilon_2(\lambda)\varepsilon_m(\lambda)/(1-\varepsilon_m(\lambda))$, where the first inequality follows from our assumption, and the second from $\varepsilon_m(\lambda) > 1/2$. Therefore, the term in the bracket and hence the above second derivative is strictly negative when $V_{pc}^B(\lambda)$ is positive.

Next, consider the case of perfect substitutes where $V_{ps}^B(\lambda)$ is always strictly positive. Furthermore, its derivative is

$$\frac{dV_{ps}^B(\lambda)}{d\lambda} = \frac{2q(\lambda)}{\lambda}\left[(1-q(\lambda))\varepsilon_m(\lambda)\varepsilon_2(\lambda) - q(\lambda)(1-\varepsilon_m(\lambda))^2\right],$$

which is strictly negative since $\varepsilon_2(\lambda) < 0$.

For the general case we have $V^B(\lambda) = (1-s)V_{pc}^B(\lambda) + sV_{ps}^B(\lambda)$. Note that $V^B(\lambda)$ and its derivative are both linear in s. When $\varepsilon_m(\lambda) > 1/2$, $V_{pc}^B(\lambda) > 0$ and hence $V^B(\lambda) > 0$ since $V_{ps}^B(\lambda) > 0$ for any λ. In this case, $dV^B(\lambda)/d\lambda < 0$ because at both $s = 0$ and 1, its value is strictly negative.

Consider λ with $\varepsilon_m(\lambda) \le 1/2$ or equivalently $V_{pc}^B(\lambda) \le 0$. Given such a λ, the value of s such that $V^B(\lambda) = (1-s)V_{pc}^B(\lambda) + sV_{ps}^B(\lambda)$ equals zero is given by,

$$s = \Delta(\lambda) \equiv \frac{-V_{pc}^B(\lambda)}{-V_{pc}^B(\lambda) + V_{ps}^B(\lambda)} = \frac{1}{2}\frac{q(\lambda)(1-2\varepsilon_m(\lambda))}{\varepsilon_m(\lambda) + q(\lambda)(1-2\varepsilon_m(\lambda))}. \tag{45}$$

Because $q(\lambda)$ is between 0 and 1, the denominator of the far right-hand side is between $1-\varepsilon_m(\lambda)$ and $\varepsilon_m(\lambda)$, which implies that it is strictly positive. It is also easy to see that $\Delta(\lambda) < 1/2$ since $\varepsilon_m(\lambda) > 0$. Note that $s = \Delta(\lambda)$ and $\lambda = \Lambda(s)$ are inverse functions of each other. Thus when $s \ge 1/2$, $\Lambda(s) = \infty$.

Again consider λ with $\varepsilon_m(\lambda) \le 1/2$. Since $dV^B(\lambda)/d\lambda$ is linear in s, to show $dV^B(\lambda)/d\lambda < 0$ we only need to show that it holds at $s = 1$ and $s = \Delta(\lambda)$. Recall that $V^B(\lambda) > 0$ if and only if $s > \Delta(\lambda)$. From the above analysis we know that at $s = 1$, we have $dV^B(\lambda)/d\lambda < 0$. When $s = \Delta(\lambda)$, we have

$$\frac{dV^B(\lambda)}{d\lambda}\Big|_{s=\Delta(\lambda)} = \frac{\varepsilon_m(\lambda)q(\lambda)^2}{\lambda\left(\varepsilon_m(\lambda)+q(\lambda)(1-2\varepsilon_m(\lambda))\right)}\left[\varepsilon_2(\lambda)+2(1-\varepsilon_m(\lambda))^2\right]$$

Note that the term in the bracket is strictly negative, because $\varepsilon_2(\lambda)+2(1-\varepsilon_m(\lambda))^2 < \varepsilon_2(\lambda)+2(1-\varepsilon_m(\lambda)) < 0$, where the last inequality follows from our assumption.

Finally, the derivative of $\Delta(\lambda)$ above is given by,

$$\Delta'(\lambda) = -\frac{\varepsilon_m(\lambda)q(\lambda)}{\lambda\left(\varepsilon_m(\lambda)+q(\lambda)(1-2\varepsilon_m(\lambda))\right)^2}\left[\varepsilon_2(\lambda)+2(1-\varepsilon_m(\lambda))^2\right]$$

As we argued above, the term in the bracket is strictly negative, which implies that $\Delta(\lambda)$ and hence $\Lambda(s)$ are strictly increasing. □

A.2. *Proof of Lemma 2*

Suppose otherwise that k_1 is the equilibrium solution and $\lambda_{k_1}(\widetilde{p}, p, \lambda) < \lambda_{k_2}(\widetilde{p}, p, \lambda)$ for some k_2. In that case, the no-arbitrage condition would be violated because a buyer could then pay k_2 visits to the deviant submarket $(\widetilde{p}, \lambda_{k_1}(\widetilde{p}, p, \lambda))$ and obtain an expected payoff that exceeds the market utility: $\overline{U} = U_{k_2}(\widetilde{p}, \lambda_{k_2}(\widetilde{p}, p, \lambda), p, \lambda) < U_{k_2}(\widetilde{p}, \lambda_{k_1}(\widetilde{p}, p, \lambda), p, \lambda)$, where the first equality follows from the definition of the function $\lambda_k(\widetilde{p}, p, \lambda)$, and the second inequality holds because $U_{k_2}(\widetilde{p}, \widetilde{\lambda}, p, \lambda)$ is strictly decreasing in $\widetilde{\lambda}$. □

A.3. *Analysis of equilibrium for general s*

Before moving to analyze the two versions of competitive search equilibrium, we first write out explicitly the expressions of $\sigma_0(\lambda)$ and $\sigma_1(\lambda)$ from the planner's solution. Combining equations (35) and (36) yields

$$\sigma_0(\lambda) = \frac{q(\lambda)(1-\varepsilon_m(\lambda))}{2q(\lambda)(1-\varepsilon_m(\lambda))+\varepsilon_m(\lambda)}. \tag{46}$$

Similarly, combining equations (35) and (37) yields

$$\sigma_1(\lambda) = \frac{1-q(\lambda)(1-\varepsilon_m(\lambda))}{2-\varepsilon_m(\lambda)-2q(\lambda)(1-\varepsilon_m(\lambda))}. \tag{47}$$

Consider first the competitive search Nash equilibrium. The results are summarized by the following proposition.

Proposition 3. *Consider the Nash approach.*

1. *If $s < \sigma_0(\lambda)$, then the equilibrium price $p_L^N(\lambda, s) > s$ and is given by*

$$p_L^N(\lambda, s) = \frac{q(\lambda)(1-\varepsilon_m(\lambda)) - s(1-q(\lambda))\varepsilon_m(\lambda)}{q(\lambda)(2-\varepsilon_m(\lambda))} < p_L^*(\lambda, s) \tag{48}$$

   The Nash equilibrium is not constrained efficient; the expected payoff of buyers is greater than their marginal contribution to surplus.
2. *If $\sigma_0(\lambda) \le s \le \sigma_1(\lambda)$, then the equilibrium price $p_M^N(\lambda, s) = p_M^*(\lambda, s)$, which is given by equation (35). The Nash equilibrium is constrained efficient. In the special case $s = 1/2$, $p_M^N = (1-\varepsilon_m(\lambda))/2$.*
3. *If $s > \sigma_1(\lambda)$, then the equilibrium price $p_H^N(\lambda, s) > 1 - s$.*

i) *If* $\varepsilon_m(\lambda) > q(\lambda)(2s-1)/s$, *it is given by*

$$p_H^N(\lambda, s) = \frac{s(1-q(\lambda))(1-\varepsilon_m(\lambda)) - q(\lambda)(1-s)}{(1-q(\lambda)) - q(\lambda)(1-\varepsilon_m(\lambda))} \tag{49}$$

ii) *Otherwise,* $p_H^N(\lambda, s) = s$.

Furthermore, $p_H^N(\lambda, s) > p_H^*(\lambda, s)$. *The Nash equilibrium is not constrained efficient; the expected payoff of buyers is less than their marginal contribution to surplus.*

Proof. First, consider the case that s is close to $1/2$, where $p \le s$ if $s \le 1/2$ or $p \le 1-s$ when $s \ge 1/2$. As we discussed in the main text, the sellers' problem reduces to equation (42). Evaluating the first-order condition of (42) at $\widetilde{\lambda} = \lambda$ yields

$$\frac{d\pi^d(\widetilde{\lambda})}{d\widetilde{\lambda}}\Big|_{\widetilde{\lambda}=\lambda} = m'(\lambda)\left[(1-q(\lambda))s + q(\lambda)(1-s)\right] - \left(\overline{U} - q(\lambda)(s-p)\right),$$

Since buyers' market utility is $\overline{U} = 2q(\lambda)(1-q(\lambda))(s-p) + q(\lambda)^2(1-p)$, from the above equation we can solve for the Nash equilibrium price $p = p_M^*(\lambda, s)$. Since we require $p \le \min\{s, 1-s\}$, from the analysis of the socially efficient price we know that $s \in [\sigma_0(\lambda), \sigma_1(\lambda)]$.

Next, consider the case $s < \sigma_0(\lambda)$ or equivalently the equilibrium price $p > s$. The sellers' problem reduces to equation (43). Evaluating the first-order condition of (43) at $\widetilde{\lambda} = \lambda$ yields

$$\frac{d\pi^d(\widetilde{\lambda})}{d\widetilde{\lambda}}\Big|_{\widetilde{\lambda}=\lambda} = m'(\lambda)\left[(1-q(\lambda))s + q(\lambda)(1-p)\right] - \overline{U}$$

By (31), buyers' market utility $\overline{U} = q(\lambda)^2(1-2p)$. Solving the above equation then yields the equilibrium price, which is given by equation (48). Note that at $s = 0$, $p_L^N(\lambda, s)$ reduces to p_{pc}^N given by equation (20), which is strictly smaller than the corresponding socially efficient price p_{pc}^* given by equation (16). At $s = \sigma_0(\lambda)$, $p_L^N(\lambda, s) = s = p_L^*(\lambda, s)$. Since both $p_L^N(\lambda, s)$ and $p_L^*(\lambda, s)$ are linear in s, $p_L^N(\lambda, s) < p_L^*(\lambda, s)$ when $s < \sigma_0(\lambda)$.

Finally, consider the case $s > \sigma_1(\lambda)$ or equivalently the equilibrium price $p > 1-s$. The seller problem reduces to equation (44). Evaluating the first-order condition of (44) at $\widetilde{\lambda} = \lambda$ yields

$$\begin{aligned}\frac{d\pi(p_1(\widetilde{\lambda}, p, \lambda), \widetilde{\lambda}, p, \lambda)}{d\widetilde{\lambda}}\Big|_{\widetilde{\lambda}=\lambda} &= m'(\lambda)\left[(1-q(\lambda))s + q(\lambda)p\right] - (\overline{U} - q(\lambda)(s-p)) \\ &= (1-q)q\left[p\left(1+\frac{\lambda q'}{1-q}\right) + (1-s)\frac{q}{1-q} + s\frac{\lambda q'}{q}\right],\end{aligned} \tag{50}$$

where for later use we have written out $\pi^d(\widetilde{\lambda})$ as $\pi(p_1(\widetilde{\lambda}, p, \lambda), \widetilde{\lambda}, p, \lambda)$ and for the second equality, we used the fact that by (33), buyers' market utility $\overline{U} = 2q(\lambda)(1-q(\lambda))(s-p) + q(\lambda)^2(1-2(1-s))$.

Note that the right-hand side of equation (50) is strictly negative at $p = 1-s$. To see this, set $p = 1-s$; the right-hand side of equation (50) then becomes linear in s. At $s = 1$, the term in the bracket reduces to $\lambda q'/q = -(1-\varepsilon_m(\lambda)) < 0$; at $s = \sigma_1(\lambda)$, we know $p = 1-s$ is the equilibrium price so that the term in the bracket equals zero. Hence for $s \in (\sigma_1(\lambda), 1]$, the right-hand side of equation (50) is strictly negative at $p = 1-s$.

If the right-hand side of equation (50) is negative at $p = s$, then we have a corner solution $p^N = s$ since the equilibrium price cannot be higher than s (buyers will not accept any price higher than s). If at $p = s$, the right-hand side of this equation is strictly positive, which is equivalent to $\varepsilon_m(\lambda) > q(\lambda)(2s-1)/s$, then we have an interior solution $p^N \in (1-s, s)$ given by (49).

Suppose $1 + \lambda q'/(1-q) > 0$ or equivalently the elasticity of $1-q$ is less than 1, a sufficient condition for which is that $q(\lambda)$ is strictly convex, which is satisfied for most common meeting technologies. The term in brackets on the right-hand side of (50) is strictly increasing in p; hence it is strictly positive when $p = s$ and s is slightly above $\sigma_1(\lambda)$, which implies we have an interior solution given by (49) when s is slightly above $\sigma_1(\lambda)$. If at $\varepsilon_m(\lambda) \geq q(\lambda)$, then at $s = 1$, $p_H^N(\lambda, 1)$ is given by (49), which is greater than the socially efficient price $p_H^*(\lambda, 1)$, as we showed for the case of perfect substitutes in the main text. Note that at $s = \sigma_1(\lambda)$, $p_H^N(\lambda, \sigma_1(\lambda)) = p_H^*(\lambda, \sigma_1(\lambda))$. For $s \in (\sigma_1(\lambda), 1]$, $p_H^N(\lambda, s) > p_H^*(\lambda, s)$ because both prices are linear in s. If, on the other hand, $\varepsilon_m(\lambda) > q(\lambda)$, then there exists some cutoff value $s^* \in (\sigma_1(\lambda), 1)$, such that when $s \in (\sigma_1(\lambda), s^*)$, $p_H^N(\lambda, s) \in (1-s, s)$, i.e., it is interior, and when $s \geq s^*$, $p_H^N(\lambda, s) = s$. Hence again for $s \in (\sigma_1(\lambda), 1]$, $p_H^N(\lambda, s) > p_H^*(\lambda, s)$, because $p_H^*(\lambda, s)$ is linear in s and $p_H^*(\lambda, s) < s$.

If $1 + \lambda q'/(1-q) \leq 0$, then the term in the bracket on the right-hand side of (50) is decreasing in p, which implies that it is always strictly negative because we already showed that it is strictly negative at $p = 1-s$. Hence for $s > \sigma_1(\lambda)$, we always have the corner solution $p^N = s$, which is greater than the socially efficient price $p_H^*(\lambda, s)$. □

Next, we consider the market-maker equilibrium. As before, we first need to analyze buyers' optimal number of visits to the deviant submarket, the results of which are given by the following lemma.

Lemma 4. *Consider the buyers who visit the deviant submarket.*

Suppose that the goods are strict complements ($s < 1/2$). When the equilibrium price $p \leq s$ and also the deviation price $\widetilde{p} \leq s$, then it is strictly optimal for the buyers to send both visits to the deviant submarket; when $p, \widetilde{p} \geq s$, then it is strictly optimal for the buyers to send only one visit there.

Suppose that the goods are strict substitutes ($s > 1/2$). When $p, \widetilde{p} \leq 1-s$, then it is strictly optimal for the buyers to pay exactly one visit there; when $p, \widetilde{p} \geq 1-s$, then it is strictly optimal for the buyers to pay both visits there.

Proof. In each of the four cases, we compare $p_1(\widetilde{\lambda}, p, \lambda)$, the solution of $\widetilde{p}$ to the equation $U(\widetilde{p}, \widetilde{\lambda}, p, \lambda) = U(p, \lambda, p, \lambda)$, and $p_2(\widetilde{\lambda}, p, \lambda)$, the solution of $\widetilde{p}$ to the equation $U(\widetilde{p}, \widetilde{\lambda}, \widetilde{p}, \widetilde{\lambda}) = U(p, \lambda, p, \lambda)$, where depending on whether the goods are complements or substitutes, $U(\widetilde{p}, \widetilde{\lambda}, p, \lambda)$ is given by equation (31) and (33), respectively. Note that $p_k(\widetilde{\lambda}, p, \lambda)$, $k = 1, 2$, is easy to calculate, since the buyers' expected payoffs are always linear in $\widetilde{p}$. We suppress the expressions of $p_k(\widetilde{\lambda}, p, \lambda)$, $k = 1, 2$, here to save space since they are complicated. By Lemma 2, the buyers strictly prefer to send one (resp. two) visit(s) to the deviant submarket if and only if $p_1(\widetilde{\lambda}, p, \lambda) > p_2(\widetilde{\lambda}, p, \lambda)$ (resp. $p_1(\widetilde{\lambda}, p, \lambda) < p_2(\widetilde{\lambda}, p, \lambda)$).

1) Suppose $s < 1/2$, $p \leq s$, and $\widetilde{p} \leq s$. Then straightforward calculation yields $p_2(\widetilde{\lambda}, p, \lambda) - p_1(\widetilde{\lambda}, p, \lambda) = (1-2s)(q-\widetilde{q})^2/(2\widetilde{q}) > 0$.

2) Suppose $s < 1/2$, $p \geq s$, and $\widetilde{p} \geq s$. Then we have $p_2(\widetilde{\lambda}, p, \lambda) - p_1(\widetilde{\lambda}, p, \lambda) = -(1 - 2p)(q-\widetilde{q})^2/(2\widetilde{q}^2) < 0$.

3) Suppose $s > 1/2$, $p \leq 1-s$, and $\widetilde{p} \leq 1-s$. Then we have $p_2(\widetilde{\lambda}, p, \lambda) - p_1(\widetilde{\lambda}, p, \lambda) = -(2s-1)(q-\widetilde{q})^2/(2\widetilde{q}) < 0$.

4) Suppose $s > 1/2$, $p \geq 1-s$, and $\widetilde{p} \geq 1-s$. Then we have

$$p_2(\widetilde{\lambda}, p, \lambda) - p_1(\widetilde{\lambda}, p, \lambda) = \frac{(q-\widetilde{q})^2}{2\widetilde{q}(1-\widetilde{q})}\left[2s - 1 - q + 2pq\right].$$

Note that $p \leq s$ since buyers will not accept any price higher than s, which implies that $1-s \leq p \leq s$. Since the term in brackets is linear in p, when $p = s$ it equals $(2s-1)(1+q) > 0$, and when $p = 1-s$ it equals $(2s-1)(1-q) > 0$, it is always strictly positive and we have $p_2(\widetilde{\lambda}, p, \lambda) > p_1(\widetilde{\lambda}, p, \lambda)$. □

Note that the above lemma does not specify all the possible cases. Consider, for example, the case of complements with $s < 1/2$. The buyers' best response when p and $\widetilde{p}$ lie on different sides of s (one is greater than s and the other is smaller than s) is indeterminate.[24] For our purpose the above lemma is enough since we will show that when the equilibrium price is p, it is not profitable for sellers to join a deviant submarket with $\widetilde{p}$ lying on the other side of s no matter whether buyers send one visit or two visits there.

Our proposition below further shows that when $s < \sigma_0(\lambda)$, as in the case of perfect complements, the market-maker equilibrium coincides with the Nash equilibrium; when $s > \sigma_1(\lambda)$, similar to the case of perfect substitutes the market-maker equilibrium is socially efficient.

Proposition 4. *Consider the market maker approach.*

1. *If $s \leq \sigma_0(\lambda)$, then the equilibrium price p^{MM} coincides with the Nash equilibrium price.*
2. *If $s \geq \sigma_0(\lambda)$, then the equilibrium price p^{MM} coincides with the socially efficient price.*

Proof. We have already discussed the special case of $s = 1/2$. Consider now the case of strict complements ($s < 1/2$) and the equilibrium price $p < s$. Lemma 4 above implies that for $\widetilde{p} \leq s$, buyers find it optimal to send both visits to the deviant submarket. Thus when $\widetilde{p} \leq s$, the deviant sellers' expected payoff is $\pi(p_2(\widetilde{\lambda}, p, \lambda), \widetilde{\lambda}, p, \lambda)$, which is simply $y(\widetilde{\lambda}) - \frac{\widetilde{\lambda}}{2}\overline{U}$. Evaluating the first-order condition at $\widetilde{p} = p$ or equivalently $\widetilde{\lambda} = \lambda$ implies that the market-maker equilibrium, if it exists, must be socially efficient. To verify that the market-maker equilibrium exists, we need show that there exist no profitable deviations. Consider a given $\widetilde{p}$. If buyers find it optimal to pay exactly two visits to the deviant submarket, which is the case when $\widetilde{p} \leq s$, then due to the concavity of $y(\lambda)$ (see Lemma 1), the deviation is not profitable. Thus we only need to consider the remaining possibility, buyers pay exactly one visit to the deviant submarket, and $\widetilde{p} > s$. Since $p^{MM} = p^N$ (both prices are socially efficient), any deviations where buyers pay exactly one visit to the deviant submarket are not profitable due to the construction of Nash equilibrium. Since $p^{MM} = p^N = p^*$ we know from the planner's solution that $s \in [\sigma_0(\lambda), 1/2)$ for this case.

Other cases are analyzed similarly. Consider next the case of strict substitutes ($s > 1/2$) and the equilibrium price $p < 1-s$. Lemma 4 above implies that for $\widetilde{p} \leq 1-s$, buyers find it optimal to send exactly one visit to the deviant submarket. If the market-equilibrium exists, then it must coincide with the Nash equilibrium, which is efficient in this case ($s \in (1/2, \sigma_1(\lambda)]$). To show there are no profitable deviations, we need to consider the case where $\widetilde{p} > 1-s$ and

[24] For example, set $m(\lambda) = 1 - e^{-\lambda}$, $s = 0.45$, and $\lambda = 4$. Furthermore, set $p = 0.439$, which, as we show below, is the equilibrium price in both the Nash approach and the market-maker approach. When $\widetilde{p} = 0.47$, then the optimal number is two; when $\widetilde{p} = 0.48$, then the optimal number is one.

buyers pay exactly two visits to the deviant submarket. However, assuming that the market-maker equilibrium is socially efficient, any deviations where buyers pay exactly two visits are not profitable.

Next, suppose that $s \le \sigma_0(\lambda)$, or equivalently the equilibrium price $p \ge s$. Lemma 4 above shows that if $\widetilde{p} \ge s$, buyers find it optimal to send exactly one visit to the deviant submarket. If the market-equilibrium exists, then p^{MM} must coincide with the Nash equilibrium price p^N in this case. To show there exists no profitable deviations, we need to consider the case where $\widetilde{p} \le s$ and buyers pay exactly two visits to the deviant submarket. In this case, the expected payoff of deviant sellers is again given by $y(\widetilde{\lambda}) - \frac{\widetilde{\lambda}}{2}\overline{U}$. Recall by Proposition 3, that the market utility of buyers under the candidate equilibrium ($p^{MM} = p^N$) is greater than their marginal contribution to surplus. Therefore, at $\widetilde{\lambda} = \lambda$, $y'(\widetilde{\lambda}) < \overline{U}/2$. Hence, $\pi(p, \lambda, p, \lambda) > \pi(p_2(\widetilde{\lambda}, p, \lambda), \widetilde{\lambda}, p, \lambda) = y(\widetilde{\lambda}) - \frac{\widetilde{\lambda}}{2}\overline{U}$ for any $\widetilde{\lambda} > \lambda$, since the surplus function $y(\cdot)$ is strictly concave when $y'(\cdot) > 0$ (see Lemma 1). Since a lower price $\widetilde{p}$ implies a longer queue $\widetilde{\lambda}$, any deviations with $\widetilde{p} \le p$ and hence $\widetilde{p} \le s$ are not profitable.

Finally, suppose that $s \ge \sigma_1(\lambda)$, or equivalently that the equilibrium price $p \ge 1 - s$. Lemma 4 shows that if $\widetilde{p} \ge 1 - s$, buyers find it optimal to pay both visits to the deviant submarket. If the market-equilibrium exists, then p^{MM} must coincide p^*, the socially efficient price. To show there are no profitable deviations, we need to consider the case where $\widetilde{p} \le 1 - s$ and buyers pay exactly one visit to the deviant submarket. The expected payoff of deviant sellers $\pi(p_1(\widetilde{\lambda}, p, \lambda), \widetilde{\lambda}, p, \lambda)$ in this case is given by equation (41). From the analysis of the seller problem in equation (50), we know that for any $p \in (1 - s, p_H^N(\lambda, s))$,

$$\frac{d\pi(p_1(\widetilde{\lambda}, p, \lambda), \widetilde{\lambda}, p, \lambda)}{d\widetilde{\lambda}}\Big|_{\widetilde{\lambda}=\lambda} < 0$$

Since in our candidate equilibrium $p_H^{MM}(\lambda, s) = p_H^*(\lambda, s) < p_H^N(\lambda, s)$, the above equation is strictly negative at $p = p_H^{MM}(\lambda, s)$. Recall that $\pi(p_1(\widetilde{\lambda}, p, \lambda), \widetilde{\lambda}, p, \lambda)$ is strictly concave in $\widetilde{\lambda}$. If the above derivative is strictly negative, then it is strictly negative for all $\widetilde{\lambda} \ge \lambda$. Hence $\pi(p_1(\widetilde{\lambda}, p, \lambda), \widetilde{\lambda}, p, \lambda) < \pi(p, \lambda, p, \lambda)$. Since a lower price $\widetilde{p}$ implies a longer queue $\widetilde{\lambda}$, any deviations with $\widetilde{p} \le p$ and hence $\widetilde{p} \le 1 - s$ are not profitable. □

Appendix B. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2023.105605.

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