Summary

A Bayesian Audit Assurance Model with Application to the Component Materiality Problem in Group Audits

This thesis proposes a theoretically grounded yet practical method for determining optimal component materiality in group audits. The method is based on a Bayesian audit assurance model that generalizes and extends the auditing profession’s standard audit risk model. The following is a non-technical summary by chapter.

Chapter 1, The Auditing Context, explains the auditing context of component materiality and assurance in group audits within the framework of International Standards on Auditing (ISAs). ISA 600, “Special Considerations—Audits of Group Financial Statements (Including the Work of Component Auditors)”, is especially relevant.

ISA 600 requires the group auditor to determine component materiality, a factor that directly affects the amount of work performed by component auditors and hence the assurance that can be derived from their work. ISA 600 provides little guidance on how to set component materiality, merely indicating that component materiality must be lower than materiality for the group financial statements as a whole but need not be as low as an arithmetical portion of group materiality.

Not only is there a lack of authoritative guidance, there is no generally accepted theory or method for how component materiality should be determined, and prior academic research is limited. The ad hoc methods used in practice vary widely and can lead to substantial underauditing as well as overauditing and may expose investors to unnecessary information risk or excessive audit cost. This is a matter for concern because groups are dominant in global capital markets and their audited financial statements are an important source of information for investment, corporate governance, and regulation.

Auditing standards and related literature define audit assurance and its complement audit risk as well as the widely accepted and used audit risk model (ARM) in which audit risk (AR) is expressed as the product of the pre-audit risk of material misstatement (RMM) and detection risk (DR). ARM is a simplistic single-entity model that works correctly only in the presence of no anticipated or detected misstatement. Additionally, and critically for group audits, ARM has no construct for aggregating risk or assurance across components. An important aspect of this thesis is the development of a more complete audit assurance model—named GUAM, for general unified assurance and materiality. The GUAM model provides the framework for the GUAM component materiality method.
ISA 600 introduces the concept of group-wide and other group-level controls. These are central to well run groups and have an important effect on the economics of group audits. They play an important role in the GUAM model and component materiality method.

Chapter 2, Representing, Accumulating, and Aggregating Assurance, describes the GUAM audit assurance model and provides a brief primer on gamma distributions and Bayes’ rule.

The key generalization in GUAM is to replace point probabilities AR, RMM and DR in ARM with probability distributions. Because probability distributions provide a complete summary of the auditor’s assurance across the entire range of potential misstatement they are also referred to as assurance profiles. The prior and posterior probabilities RMM and AR become prior and posterior probability distributions, respectively; the probability DR becomes a likelihood distribution (a likelihood function standardized so that total probability equals 1); and the simplistic multiplication of probabilities in the ARM is supplanted by Bayes’ rule, which is essentially \[ \text{Posterior Distribution} = \text{Prior Distribution} \times \text{Likelihood Distribution} \].

The GUAM model is based on the family of gamma probability distributions. The gamma family provides a rich variety of shapes that tend to be intuitively appealing as models of audit assurance—shapes ranging from exponential to almost bell-shaped. Auditors who assess RMM under ARM can immediately translate that assessment into an appropriate exponential prior distribution (the simplest gamma distribution) via a straightforward formula.

An important feature of GUAM is that it automatically provides a construct for aggregating assurance across group components. The aggregate group assurance profile is formed by the convolution of the component assurance profiles. The group assurance profile can be closely approximated by another gamma distribution thus keeping the model within the gamma family. ARM has no construct for aggregation thus severely limiting its usefulness in group and other multilocation audits.

Chapter 3, Gamma Distributions in Auditing, expands on the relevance of gamma distributions to auditing.

If the auditor is able to make a judgment (subjectively if necessary) about most probable total misstatement and some upper limit, such as the 95th percentile, then that judgment can often be expressed as a gamma distribution.

There is a close relationship between the Poisson distribution, which is used in monetary unit sampling (MUS), and the gamma distribution. The results of MUS can be nicely interpreted in terms of a gamma distribution that peaks at most probable misstatement and has a 95th percentile (say) equal to the Poisson-based 95% upper error limit. In classical applications of MUS, the upper error limit is usually computed as a Stringer bound. In Bayesian applications an analogous process can be used to determine a Stringer likelihood distribution.

The exponential distribution (the simplest member of the gamma family) links the GUAM and ARM audit assurance models. In fact, ARM works correctly if and only if the probabilities RMM and DR are derived from exponential distributions, which occurs when no misstatement is
anticipated or indicated. This makes ARM a special case of GUAM. Thus GUAM generalizes a model that auditors are familiar with and which has been used for audit planning since the 1970s. Importantly, GUAM works correctly in more complex situations where ARM does not. Where ARM is used in the presence of anticipated or indicated misstatement, it tends to result in an understatement of the true audit risk.

The exponential distribution is important where there is no anticipation or indication of misstatement as it provides the most conservative representation of the auditor’s assurance in the sense of making the least assumptions. This characteristic arises from the exponential distribution’s status as a *maximum entropy* distribution.

While GUAM is based on gamma distributions, an alternative is possible based on the beta distribution. Such a model could potentially be used as the basis for an alternative component materiality method, but it would likely be more complex than GUAM.

**Chapter 4, The GUAM Method for Determining Component Materiality**, derives the GUAM component materiality method and algorithm.

The process starts with the group auditor using professional judgment to establish the overall group assurance objective—for example, achieving 95% assurance that total group misstatement does not exceed group materiality of $100,000. This defines the target to be achieved by the posterior group assurance profile.

Because most of the group audit is conducted at the component level, the trick is to disaggregate the target group posterior into appropriate target component posteriors. This is accomplished by assigning weights to the components, such that the weights sum to 1, and then plugging those weights and the group target into an algorithm that finds appropriate component posteriors. The algorithm sets component materiality at an amount that will drive just enough audit work at the component level to achieve the target group posterior (assuming the audits go as planned). The algorithm uses Bayes’ rule to inflate component materiality to account for the group auditor’s prior component assurance, if any. This reduces the assurance required from the component auditor and the extent of the component audit. If component audits go as planned, the target component posteriors are achieved and they aggregate to achieve the target group posterior.

The GUAM algorithm works correctly for any set of initial component weights that are assigned provided they sum to 1. This leaves the group auditor free to choose weights that also achieve secondary objectives. If that objective is to minimize group audit costs and there are no other constraints, then it is optimal to weight components in proportion to the square root of their size.

**Chapter 5, Comparison of Component Materiality Methods**, describes the following alternative component materiality methods used in practice today and compares them with the GUAM method (my labels):

- MACM: Allocates *maximum aggregate component materiality* (a tabulated multiple of group materiality) to the components in proportion to the square root of size.
• SQRT: Sets component materiality equal to group materiality times the square root of the relative size of the component.
• PROP: Allocates group materiality to components in proportion to their size, the lower limit suggested by ISA 600.
• HALF: Sets component materiality equal to half group materiality regardless of the number of components or their size.
• FULL: Sets component materiality equal to group materiality for all components, the upper limit allowed by ISA 600.

Different methods produce different component materiality amounts, have different effects on total variable audit cost, and achieve different levels of group audit assurance. Measures of cost and achieved group assurance provide a way to compare the alternative methods with GUAM. In order to eliminate extraneous factors the main numerical comparison is done for groups consisting of two to ten equally-sized components for which it is assumed that the group auditor has no prior assurance. Methods are also compared for an illustrative example in which component sizes vary significantly.

The comparison shows that GUAM produces smaller component materiality amounts than the other methods except for PROP. This is also reflected in greater relative total cost for GUAM, with only PROP being greater. On the other hand GUAM consistently achieves the desired level of group assurance (assumed to be 95%), having been specifically designed to do just that, while PROP achieves more than is required and the other methods less.

The MACM and SQRT methods, which, like GUAM, have probabilistic rationales, are analyzed in detail.

Chapter 6, Approximations and Optimizations, elaborates key technical results that are used but glossed over in previous chapters.

An important feature of the GUAM model is its ability to represent the group assurance profile as the aggregate (the convolution) of the component gamma distributions. In general, the convolution of gamma distributions is a complicated non-gamma probability distribution. Nevertheless, we approximate it with a gamma distribution and the approximation is good enough for any group that might be encountered in auditing practice.

The GUAM component materiality algorithm involves assigning weights to components and plugging those weights into a formula to derive target component posteriors and hence component materiality amounts. This method results in achieving the group assurance objectives regardless of the weights that are assigned. This chapter analyses the simulations that support the approach.

When weights are assigned to components in the GUAM component materiality algorithm, total group audit costs will be minimized when those weights are proportional to the square root of component size. The square root formula is derived as the solution to a classic constrained optimization problem.
While group auditor prior assurance about a component reduces component audit cost, there is a cost to the group auditor who must expend effort in establishing and supporting that prior assurance. There is a tradeoff between group audit and component auditor cost, which is illustrated in an example.

**Chapter 7, Optimizing for Group Context, Constraints, and Structure**, derives further optimizations for practical group contexts, constraints, and structure.

Factors other than component size often arise in group audit practice and lead to different optimal component materiality combinations. For example, component materiality may be constrained by statutory audit requirements, audit costs may vary significantly between components, or a component may be easy to audit “one-hundred percent” thus effectively reducing component audit risk to zero.

The GUAM method delivers component materiality combinations that meet the group audit assurance objective regardless of how the component are weighted, just as long as the weights sum to 1. Because there is a continuum of such weight combinations there is a corresponding continuum of component materiality combinations that meet the group audit assurance objective. That continuum is defined to be the efficient materiality frontier. Adjusting component materiality for practical constraints amounts to locating the point on the efficient materiality frontier that is optimal given those constraints.

To this point it has been assumed that groups are comprised of stand-alone components that are separately managed. However, some groups assemble subgroups of components that have similar business activities, processes, controls, and risks, and that, while separately managed, share subgroup-level resources and oversight that provide a degree of cohesiveness. Such a subgroup is referred to as a cluster.

In some circumstances the group auditor can treat a cluster as a virtual single “component” for group audit planning and implementation. It is shown that when that is the case the group auditor can use cluster materiality for each component in the cluster, which can reduce audit cost significantly. This essentially provides theoretical support for current practice, recognized in ISA 600, that the group auditor may work at a certain level of aggregation rather than at the level of individual components.

**Chapter 8, Software Implementation**, presents an Excel-based software implementation of the GUAM method including a detailed step-by-step algorithm. The algorithm is described in pseudocode to facilitate porting to any software platform.

Besides providing a practical solution to a specific problem, the software illustrates a wider point, which is that quite complex, computationally-demanding Bayesian methods can be implemented in forms that are within reach of any auditor. This is a far cry from the days not so long ago when scholars and practitioners could describe the possibilities of Bayesian modeling but lacked the necessary tools.
Chapter 9, Conclusions, summarizes the contributions made by the thesis and points to further research and applications of the GUAM model.

Three contributions to the theory and practice of auditing are noted:

- A generalization and extension of the profession’s standard audit risk model
- A theoretically-grounded solution to the determination of component materiality
- An algorithm and specific software implementation for determining component materiality.

These contributions are potentially of interest to various constituents including auditing standards setters, partners in accounting firms responsible for determining audit policies and methodologies, individual practitioners, regulators and practice inspectors, scholars engaged in audit research, and others concerned with effective and efficient group audits.

The thesis touches on several topics that deserve further research. In general, the GUAM model has the potential to contribute to the understanding and solution of assurance-related problems in auditing where progress has been stymied for want of a suitable analytical or computational framework. Specific avenues for research include group audit practice and theory, the quantification of professional judgment, and enhancements to the assurance model beyond GUAM. There are also certain direct applications of the GUAM assurance model that are worth exploring. These include sample design and interpretation; applying the component materiality method to single-entity audits where the components are financial statement line items; designing and evaluating stratified, risk-based MUS; and auditing shared-services entities.

My hope is that the GUAM model’s application to the component materiality problem in group audits is not only valuable in itself but will be seen as an example of how Bayesian methods provide a framework for logical reasoning and solving practical problems in auditing.