A distinctive feature of the Standard Model (SM) is the existence of a quantum number that distinguishes the “flavour” of elementary particles. Particles with different flavours (e.g. $u$ and $d$ quarks) can be interchanged without affecting the physics of electromagnetic and strong interactions, as photons and gluons only couple quarks and antiquarks of the same flavour (QED and QCD conserve flavour). This does not hold for weak interactions, a historical example being nuclear $\beta$-decay, in which a neutron can decay into a proton, emitting an electron and an electron anti-neutrino.

In the development of the SM theory of weak interactions, this peculiar picture of flavour has constantly played a central role in shaping the theory and in providing predictive power that has led to key experimental discoveries.

The first steps towards a unified description of the electromagnetic and weak interactions were set in the thirties, with the Pauli proposal for a neutrino “emission” in $\beta$-decays and the Fermi formulation of a $\beta$-decay field theory [1]. Up until 1956 the general feeling was that all physical processes would conserve parity (P). A large number of particles were discovered in the fifties, though, and experimental observations of their decay modes (e.g. the “$\theta - \tau$ puzzle” [2,3]) suggested that parity could be violated in weak transitions, the first evidence of such violation coming not too long after, from the Wu experiment [4]. In the same year, the idea of a vector-boson mediated weak interaction was proposed by Schwinger [5], Lee and Yang [6], opening the way to the V-A theory of weak interactions by Feynman and Gell-Mann [7]. From the observed suppression of weak decays involving a change of strangeness, the Cabibbo rotation angle was introduced [8], hence inaugurating the era of flavour mixing investigation.

Up to the sixties, CP-symmetry was thought to be a valid symmetry, closely coupled to the notion of time-reversal invariance through the CPT theorem [9–11]. However, the study of kaon decays soon revealed the first experimental evidences of CP violation in weak processes [12], a discovery saluted with surprise by the physics community. Although the concept of CP violation was then accepted rather quickly, considerable experimental and theoretical work was devoted to understand its occurrence, particularly in the context of the Standard Model. On the other side, the lack of experimental observation of processes mediated by Flavour Changing Neutral Currents (FCNC) –i.e. involving transitions between different quarks with the same charge– led to the formulation of the GIM mechanism by Glashow, Iliopoulos and Maiani [13]. They realised that there was no way to explain the absence of FCNCs with only $u,d,$ and $s$ quarks and conjectured a fourth quark (the charm quark): with this, the neutral weak current would become diagonal in flavour space. The first charmed particle, the $J/\Psi$ meson, was discovered in 1974 during the “November Revolution”, at SLAC [14] and BNL [15]. In the meanwhile, a number of suggestions had extended the quark model to include a third generation and with the Kobayashi-Maskawa theory [16] CP violation was accommodated in the Standard Model with 6 flavours of quarks. Both third generation quarks, the bottom and the top quarks, were indeed observed at Fermilab [17–19].
The current description of weak interactions [20–22] is accommodated in what is now known as the Standard Model of fundamental interactions, the result of the effort of numerous theoretical and experimental physicists over a long period of time. The SM is a quantum-field theory of electro-weak and strong interactions, in which an interaction is described as the result of a fundamental symmetry of nature. The imposed symmetry is a local invariance of the Lagrangian under transformations of the gauge group

\[ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y . \]  

The symmetry, or gauge invariance, is guaranteed by introducing gauge boson fields for each symmetry into the Lagrangian. The elementary particles in the resulting model are the massless matter fermions (quarks and leptons) and the force carrying gauge bosons (photon \( \gamma \), \( W \) and \( Z \) bosons, gluons \( G \)). It turns out nature includes three observed generations of leptons and quarks, as illustrated in Fig. 1, precisely the minimum number required for CP-violation in the SM.

In order to describe non-zero masses of the \( W \), \( Z \) bosons, the electroweak symmetry is spontaneously broken by adding a gauge invariant Higgs field potential [23], with a non-zero vacuum expectation value, into the Lagrangian. Additionally, an interaction term of the Higgs field with the fermion fields (the Yukawa interaction) provides the means to explain the non-zero masses of the fermions. The resulting Lagrangian can be parametrised as:

\[ \mathcal{L} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{EWK}} + \mathcal{L}_H + \mathcal{L}_{\text{YU}} . \]  

**Figure 1**

Sketch illustrating the Standard Model elementary particles.
The first two terms describe the kinetic energy of the fermions and gauge bosons and include their interactions. The quantum chromodynamics Lagrangian $\mathcal{L}_{\text{QCD}}$ describes the color interaction between quarks and gluons, with $SU(3)$ symmetry:

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{q}^f \gamma^\mu \left( i \partial_\mu - g_s \frac{1}{2} \vec{\alpha} \cdot \vec{G}_\mu \right) q^f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

where $q_f$ is a color triplet quark field (the index $f$ runs over the 6 flavours), $G^a_{\mu\nu}$ are the strength tensors of the 8 gluon fields $\vec{G}_\mu$ and $\mu\nu$ and the components of $\vec{\alpha}$ are the generators of the $SU(3)$ symmetry group.

The electroweak Lagrangian $\mathcal{L}_{\text{EWK}}$ describes the interactions between fermions and vector bosons, through the symmetry groups $SU(2)_L \otimes U(1)_Y$:

$$\mathcal{L}_{\text{EWK}} = \sum_i \bar{\psi}^i \gamma^\mu \left( i \partial_\mu - g' \frac{1}{2} Y_W B_\mu - g \frac{1}{2} \vec{\tau} \cdot \vec{W}_\mu \right) \psi_i - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} ,$$

where $B_\mu$ is the $U(1)$ gauge field and $Y_W$ is the weak hypercharge, $\vec{W}_\mu$ are the $SU(2)$ gauge fields and the components of $\vec{\tau}$ are the Pauli matrices (whose eigenvalues give the weak isospin). The strength tensors of the gauge fields are $W^i_{\mu\nu}$ and $B_{\mu\nu}$. In Eq. 4 $\psi_i$ represents a Dirac fermion field (quark field or lepton field) and the subscript $i$ runs over the three generations. For each generation of quarks, $\psi$ represents left-handed doublets ($\psi \equiv Q_L \equiv (u_L, d_L)$) and right-handed singlets ($\psi \equiv u_R$ or $d_R$), and similarly for leptons. Singlets and doublets are defined in the weak-eigenstate basis. The interaction terms of the electroweak Lagrangian are of the form:

$$e J^\mu_{\text{em}} A_\mu + g (J^+_{\mu} W^+_\mu + J^-_{\mu} W^-_\mu) + g Z J^\mu_Z Z_\mu ,$$

where $J^\pm_{\text{em}}$ are the left-handed charged currents carried by the physical bosons $W^\pm$, while $J^\pm_Z$ are the neutral currents carried by the photon and $Z$ boson, respectively.

The Higgs term of the Lagrangian has the form

$$\mathcal{L}_H = \left( \partial_\mu + g' \frac{i}{2} Y_W B_\mu + g \frac{i}{2} \vec{\tau} \cdot \vec{W}_\mu \right) \phi \bigg| \phi \bigg|^2 - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 ,$$

including a dynamical Higgs term and the Higgs potential. The field $\phi$ is a complex spinor (doublet) representing the Higgs field, while the Higgs particle mass is given as $m_H = \sqrt{2\lambda^2\mu^2} = \sqrt{-\mu^2}$. When $\phi$ acquires a non-zero vacuum expectation value, the $W$ and $Z$ bosons acquire mass.

The Yukawa term describes the interactions of the Higgs field with the matter fields. For quarks, it is of the form

$$\mathcal{L}_{\text{YU}} = \sum_{i,j} Y_{i,j}^u \bar{Q}^i_L \phi u^j_R + \sum_{i,j} Y_{i,j}^d \bar{Q}^i_L \phi d^j_R + h.c. ,$$

where $\phi = i \sigma_2 \phi$ and the Yukawa matrices $Y_{i,j}^u,d$ are arbitrary (to be determined experimentally) $3 \times 3$ complex matrices for the three generations $(i,j)$ of quarks. The Yukawa matrices can be diagonalised through “rotations” of the fields in generation space, by means of four unitary matrices $V_{R,L}^{u,d}$. After electroweak symmetry breaking, $\phi$ acquires a non-zero vacuum
expectation value and the diagonalised Yukawa terms yield mass terms for the quarks, for instance as

$$M^u = \frac{v}{\sqrt{2}} V^u_L V^u_R.$$  \hspace{1cm} (8)

The charged current $W^\pm$ interactions hence couple to the physical mass eigenstates of the fermions ($u'^L_L, d'^L_L$)

$$L_{CC} = -\frac{g}{\sqrt{2}} (\bar{u}'_L \bar{c}'_L \bar{b}'_L) \gamma^\mu V_{CKM}(\bar{d}'_L \bar{s}'_L \bar{b}'_L) + h.c.,$$  \hspace{1cm} (9)

through generation-changing couplings which are proportional to the elements of the so-called $V_{CKM}$ matrix:

$$V_{CKM} \equiv V^u_L V^d_R.$$  \hspace{1cm} (10)

The mass eigenstates of the fermions are thus different from the weak interaction eigenstates, and the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_{CKM}$ connects the two, determining the flavor mixing of the down-type quarks:

$$\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix} = V_{CKM} \begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix} = \begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix} \begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}.$$  \hspace{1cm} (11)

As the CKM-matrix is not diagonal, transitions between different quark generations (Flavour Changing Currents) are allowed. However, flavour changes between quarks with the same charge (Flavour Changing Neutral Currents, FCNC) are absent in the Lagrangian and can thus only manifest themselves at higher order diagrams.

Decades of experimental results, culminating with the LHC-run1 outcome and the much-acclaimed Higgs-like discovery [25, 26], have repeatedly confirmed the success of the Standard Model description. Despite this, the model is not completely satisfactory for both experimental and theoretical reasons. The observation of neutrino oscillations implies the presence of Flavour Changing Neutral Currents in the lepton sector. The evidence of Dark Matter, as indirectly observed from galactic rotation curves and from gravitational lensing of colliding galaxies, could be explained by yet undiscovered subatomic particles. Theoretical models indicate that the amount of CP-violation in the quark sector is insufficient to explain the matter-antimatter asymmetry via the electroweak phase transition. These shortcomings, together with the theoretical lack of “naturalness”\(^2\) of a low mass of the Higgs boson, conjure to indicate that the SM is an effective “low-energy” theory. New Physics (NP) models have been theorised including various extensions of the Standard Model, for instance a larger Higgs sector, a new symmetry as in SuperSymmetric models, or a fourth generation of quarks and leptons. In some models, the GIM mechanism may cease to be a general property and FCNC processes could already take place at the tree level [27].

New Physics can be probed in two ways: through direct and indirect searches, while direct searches are limited by the experimentally available centre-of-mass energy, indirect

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1 The choice of rotating (mixing) down- rather than up-type quarks is historical, and it is just a convention: only the relative orientation of up- and down-quark mass eigenstates matters.

2 A theory is considered “natural” if its free parameters are of about the same order of magnitude and do not require a very precise adjustment (“fine-tuning”) to match the experimental data.
searches allow us to access new particles produced virtually in loop processes. In indirect searches, flavour observables play a key-role to explore New Physics at higher energy scales.

This chapter is devoted to the theoretical description of rare processes involving FCNCs, with particular attention to the $B^0 \rightarrow \mu^+\mu^-$ and $B^0_s \rightarrow \mu^+\mu^-$ decays. The search for such rare decays ultimately aims at testing the Standard Model of particle interactions and eventually uncovering New Physics beyond the Standard Model.

### 2.1 Flavour Changing Neutral Currents

Flavour Changing Neutral Currents are absent at the tree level in the Standard Model. Charged currents mediated by $W^\pm$ bosons can instead violate flavour, therefore one can use a $W$ boson in a loop to create an overall Flavour Changing Neutral process: FCNC processes are thus possible at higher orders. The diagrams in 2 represent decay amplitudes at the level of elementary particles (quarks, leptons, bosons).

![Feynman diagrams of the SM processes contributing to $B^0 \rightarrow \mu^+\mu^-$ decays, involving top quarks and $W$ bosons: $Z^0$-penguin diagrams on the left and box diagram on the right. Self energy (gluonic) corrections and Higgs contributions are here not considered.](image)

To actually calculate a decay rate, one needs to account for the fact that quarks are confined inside hadrons, bound by the exchange of soft gluons. The case of the $B^0_{s(d)} \rightarrow \mu^+\mu^-$ decay is the cleanest possible exclusive $B$-decay: due to the purely leptonic final state, all non-perturbative effects can be confined to a single parameter, the $B$-meson decay constant, defined via the axial-vector current matrix element [28]:

$$
\langle 0 | \bar{q} \gamma_\mu \gamma_5 b | B_q (p) \rangle = i p_\mu F_{B_q} ,
$$

where $p_\mu$ is the four-momentum of the initial $B$-meson and $q$ represents the $d$ or $s$ quark.

Theoretical calculations of hadronic decay rates are based on effective Hamiltonians of the type [29]:

$$
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum \frac{C_i(\mu) Q_i(\mu)}{m_\mu} .
$$

and the decay amplitude for a meson $|M\rangle$ (e.g. $K, D, B$) into a final state $|F\rangle$ (e.g. $\pi\pi, \mu\mu$), is given by

$$
\mathcal{A}(M \rightarrow F) = \langle F | \mathcal{H}_{\text{eff}} | M \rangle = \frac{G_F}{\sqrt{2}} \sum \frac{C_i(\mu) \langle F | Q_i(\mu) | M \rangle}{m_\mu} .
$$
In Eq. 13, $\mu$ is a renormalisation scale, $G_F$ is the Fermi constant, $Q_i$ are local operators whose matrix elements describe the long-distance effects, and the Wilson coefficients $C_i$ play the role of effective coupling constants, “weighting” the strength of each operator.

Equation 12 can be regarded as a generalisation of the Fermi theory of interactions [30]: taking the example of the $\beta$-decay, the diagram in Fig. 3 (left) with a $W$-propagator – depicting the situation at short-distance scales – can be represented in the effective theory as in Fig. 3 (right), with the effective point-like vertices described by local operators of the type $\bar{u}\gamma_\mu(1 - \gamma_5)d \otimes \bar{e}\gamma_\mu(1 - \gamma_5)\nu_e$ (product of V-A currents) and the corresponding Wilson coefficient equal to 1. Such coefficient can be seen as the coupling constant associated to the effective vertices. Accounting for the proper $V_{\text{CKM}}$ elements:

$$H_{\beta\text{-decay}}^\text{eff} = \frac{G_F}{\sqrt{2}} \cdot V_{ud} \left[ \bar{u}\gamma_\mu(1 - \gamma_5)d \otimes \bar{e}\gamma_\mu(1 - \gamma_5)\nu_e \right] .$$

![Feynman diagrams](image)

By means of a technique known as Operator Product Expansion (OPE), $H_{\text{eff}}$ can be written as a series of operators multiplied by effective coupling constants, with the scale $\mu$ separating short- and long-distance contributions. The scale can be chosen arbitrarily and allows a perturbative calculation of the Wilson coefficients $C_i$. These compactly collect the effects of short distance physics, that is physics contribution from scales higher than $\mu$ (as the top-quark, $Z$, $W$ and heavy new particles contributions). The matrix elements of the local operators contain the long distance physics and are evaluated at the scale $\mu$. One could say that the OPE “moves” the physics contributions at scales higher than $\mu$ from the operators $Q_i$ to the Wilson coefficient $C_i$.

The effective vertices have to be renormalised when quantum corrections (QCD or electroweak) are taken into account, hence the Wilson coefficients depend on the renormalization scheme beyond leading order (the scale-dependence of the couplings $C_i(\mu)$ has to cancel the scale-dependence of the hadronic matrix elements $\langle F | Q_i(\mu) | M \rangle$, since the measurable amplitude cannot depend on the scale).
The Wilson coefficients for the FCNC diagrams in Fig. 2 depend on the masses \( m_k \) of the particles in the loops. They can be expressed as linear combinations of the corresponding CKM-matrix elements times basic universal functions \( F(x_k) \) \[31\]:

\[ C_i \propto \sum_k \Gamma_{i,k} F_i(x_k), \quad (15) \]

where

\[ x_k = \frac{m_k^2}{m_W^2}, \quad k = u, c, t. \quad (16) \]

For a vertex involving a \( B_s \) meson, \( \Gamma \) will be of the type \( V_{k^k} V_{k^s} \), with \( k = u, c, t \).

The dependence on \( m_k \), together with that on the corresponding \( V_{CKM} \) elements, determines the strength of the vertex itself. The GIM mechanism, beside preventing tree-level FCNC’s, determines also the hierarchy of FCNC transitions: in fact, the unitarity of the CKM matrix implies \( \Gamma_{i,u} + \Gamma_{i,c} + \Gamma_{i,t} = 0 \) and would lead to vanishing \( C_i \) if \( x_u = x_c = x_t \), i.e. when the masses of internal quarks of a given charge in loop diagrams are equal. The hierarchy of FCNC transitions thus depends on the masses involved in the internal loops. This is why the GIM mechanism is less effective in suppressing processes governed by top quarks contributions (\( K \) and \( B \) decays) than those in which only \( d, s \) and \( b \) quarks enter internal loops (\( D \) decays). Moreover, the size of FCNC transitions depends on the behaviour of the basic functions after QCD corrections and on the \( V_{CKM} \) elements. For instance, \( B_s \) decays are less suppressed than \( B_d \) decays, due to the different sizes of the CKM matrix element involved.

### 2.2 The \( B_{s(d)}^0 \rightarrow \mu^+\mu^- \) decay in the SM

The fact that in the SM FCNC’s take place only at higher order makes them useful to test physics beyond the SM. Particularly interesting are purely leptonic decays of \( B_s^0 \) and \( B_d^0 \) mesons, due to their SM suppression and to angular conservation (helicity suppression). In the SM, only \( Z^0 \) penguins and box diagrams involving the top quark contribute to \( B_{s(d)}^0 \rightarrow \mu^+\mu^- \) decays (see Fig. 2) –the \( \gamma \)-penguin does not contribute because it is charge conjugation violating. The branching fraction \( B(B_{s(d)}^0 \rightarrow \mu^+\mu^-) \) is expected to be very small and not easy to access experimentally. Nevertheless, the theoretical calculation of such processes in the SM is rather clean. Typically, the main theoretical uncertainty is related to the \( B \)-meson decay constant \( F_{B_s^0} \).

The effective Hamiltonian generating \( B_{s(d)}^0 \rightarrow \mu^+\mu^- \) decays in the SM is \[28\]:

\[ 
H_{\text{eff}}(B_{s(d)}^0 \rightarrow \mu^+\mu^-) = - \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi\sin^2\theta_W} \left\{ \bar{V}_{tb} V_{tb}^* c_{10} Q_{10} + \text{h.c.} \right\}. \quad (17)
\]

The \( Q_{10} \) is a V-A operator:

\[ Q_{10} = (\bar{q} \gamma^\mu \frac{1 - \gamma^5}{2} b) (\bar{l} \gamma^\mu \gamma_5 l), \quad (18) \]
where \( q, b \) are the quarks on the “external legs” of the Feynman diagram (i.e. \( q = d, s \)) and \( l \) are the final state leptons (i.e. \( l = \mu \)). The Wilson coefficient \( c_{10} \) gets the largest contributions in the SM from a \( Z^0 \)-penguin top loop (\( \sim 75\% \)) and a \( W^\pm \)-box diagram (\( \sim 24\% \)). It can be written in terms of loop functions \( F(x_i) \) that describe the dependence on the internal up-type quark masses (see Eq. 16). For \( B^0_{s(d)} \rightarrow \mu^+\mu^- \) decays, the top contribution to the effective coupling is dominant on the charm one and such decays are governed by the Inami-Lim function [31]

\[
Y_0(x_t) = C_0(x_t) - B_0(x_t),
\]

(19)

where \( C \) and \( B \) correspond to \( Z^0 \) penguin and box-type contributions. The expression of \( C_0(x_t) \) and \( B_0(x_t) \) can be found in the Appendix of Ref. [28]. When QCD next-to-leading order (NLO) corrections are included, gluon exchanges between the quarks are taken into account using renormalisation methods. Equation 19 is modified into [32]

\[
Y(x_t) = Y_0(x_t) + \frac{\alpha_s}{4\pi} Y_1(x_t) = \eta_Y Y_0(x_t),
\]

(20)

\[
\eta_Y = 1.026 \pm 0.006 .
\]

(21)

Accordingly, the SM effective Hamiltonian for \( B^0_s \rightarrow \mu^+\mu^- \) following from Eq. 17 and including QCD NLO corrections reads

\[
H_{\text{eff}}(B^0_s \rightarrow \mu^+\mu^-) = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi \sin^2 \theta_W} (V^\ast_{tb} V_{ts}) Y(x_t) (\bar{s} \gamma^\mu \frac{1-\gamma_5}{2} b)(\bar{\mu} \gamma^\mu \gamma_5 \mu).
\]

(22)

The same expression is valid for the \( B^0_d \rightarrow \mu^+\mu^- \) decay, where the \( s \)-quark is replaced by a \( d \)-quark.

The branching fraction of \( B^0_s \rightarrow \mu^+\mu^- \) decays derived from the Hamiltonian in Eq. 22 can be written as [32]

\[
B_{\text{SM}}(B^0_s \rightarrow \mu^+\mu^-) = \frac{G_F^2}{\pi} \left( \frac{\alpha}{4\pi \sin^2 \theta_W} \right)^2 \tau_{B^0_s} F^2_{B^0_s} m_{B^0_s}^2 m_\mu^2 \sqrt{1 - \frac{4m_\mu^2}{m_{B^0_s}^2}} \cdot |V^\ast_{tb} V_{ts}|^2 Y^2(x_t),
\]

(23)

where \( \tau_{B^0_s} \) is the \( B^0_s \) mean life-time and \( F_{B^0_s} \) is its decay constant. The factor \( m_\mu \) reflects the decay helicity suppression within the SM: due to angular momentum conservation, both muons must have the same helicity since the \( B \) meson is spinless [33].

This expression has been used to provide predictions based on electroweak calculations up to the leading order, QCD calculations at NLO and in the absence of soft photon corrections (see e.g. Refs. [34] and references therein).

**Electroweak corrections**

If non-SM effects modify the \( B(B^0_{s(d)} \rightarrow \mu^+\mu^-) \) by a large amount, the leading order electroweak calculation can be considered sufficient, since an NLO electroweak contribution would change the leading order estimate by a few percent. Considering the more recent
experimental results (see Sec. 2.4), though, these corrections become relevant. A recent evaluation of NLO electroweak corrections leads to the prediction \[ B_{SM}(B_d^0 \to \mu^+\mu^-) = (1.07 \pm 0.10) \times 10^{-10}, \] where the error contribution due to the theoretical calculation of \( F_{B_d^0} \) represents the dominant source of systematic uncertainty. The result also depends on the choice of the renormalisation scheme \[ 35 \].

In the case of the \( B_s^0 \) decay, a recent lattice calculation of \( F_{B_s^0} \) \[ 36 \] yields the prediction \[ B_{SM}(B_s^0 \to \mu^+\mu^-) = (3.25 \pm 0.17) \times 10^{-9}, \] where now the main source of uncertainty lies in the CKM factor \( |V_{tb}V_{ts}^*| \).

### 2.3 Theory and experiment

To allow a direct comparison with the experimental results, the correspondence between initial and final states experimentally detected and those used in the theoretical prediction has to be addressed. This implies dealing with two effects: the electromagnetic radiation and the lifetime difference of the mass eigenstates. In fact, the experimental \( B \) measurement relies on the extraction of the number of candidates \( N_{B_s^0(d) \to \mu^+\mu^-} \), calculated regardless of the mesons decay time, by means of a normalisation technique (see Sec. 8.10):

\[
B_{B_s^0(d) \to \mu^+\mu^-} = B_{\text{norm}} \cdot \frac{N_{B_s^0(d) \to \mu^+\mu^-}}{N_{\text{norm}}} \cdot \frac{\epsilon_{\text{norm}}}{\epsilon_{B_s^0(d) \to \mu^+\mu^-}} \cdot \frac{f_{\text{norm}}}{f_{B_s^0(d)}},
\]

where \( N_{\text{norm}} \) is the number of events for a given normalisation channel measured in data and \( \epsilon_{B_s^0(d) \to \mu^+\mu^-} \) and \( \epsilon_{\text{norm}} \) are the total selection efficiencies for signal and normalisation channels, respectively. The probability that a \( b \)-quark fragments into a \( B \)-hadron relevant for the signal (normalisation channel) enters the formula as \( f_{B_s^0(d)}/f_{B_s^0} \). For the analysis presented in this dissertation, the recent LHCb evaluation \( f_s/f_d = 0.256 \pm 0.020 \) \[ 38 \] is used.

#### 2.3.1 Radiative corrections: the photons treatment

Equation 23 does not include the effect of electromagnetic interactions, i.e. \( B_{SM} \) refers to the “non-radiative” branching fraction, \( B_{SM}^{(0)} \). In reality photon emissions inevitably occur. Two types of radiation can be identified: Bremsstrahlung radiation of the final state muons (final state radiation photons, or FSR) and direct emission of photons (initial state radiation, ISR). The interference between the two further complicates matters. Typically the FSR is identified with a “soft” contribution –soft-Bremsstrahlung approximation– and ISR with a “hard” one. While \( B_s^0 \to \mu^+\mu^-\gamma \) decays with ISR photons should not be taken into account in the \( B_{s(d)}^0 \to \mu^+\mu^- \) branching fraction, \( B_s^0 \to \mu^+\mu^-\gamma \) decays with FSR by the final-state muons are an integral part of the \( B_{s(d)}^0 \to \mu^+\mu^- \) branching fraction.
Experimentally, the eventual loss of $B^0_{s(d)} \to \mu^+\mu^-$ events due to FSR –missing invariant mass taken away by the photon– needs to be accounted for in the experimental efficiency (see Sec. 8.10.1), while spurious contributions by $B^0_s \to \mu^+\mu^-\gamma$ events with ISR represent a background for the computation of the $B^0_{s(d)} \to \mu^+\mu^-$ branching fraction (see Sec. 8.8.6). A recent discussion of the theoretical treatment of the electromagnetic radiative effects in $B^0_{s(d)} \to \mu^+\mu^-$ can be found in Ref. [35]. In particular, the branching fraction including an arbitrary number of undetected FSR photons with total energy lower than $E_{\text{max}}$ can be related to the non-radiative $B^{(0)}$ through a correction factor:

$$\mathcal{B}^{\text{phys}}(E_{\text{max}}) = \omega(E_{\text{max}}) \cdot B^{(0)}. \quad (27)$$

For instance, in the soft-Bremsstrahlung photon approximation with $E_{\text{max}} \sim 60$ MeV, the correction factor leads to an enhancement of the branching fraction of about 11%. In the limit of $E_{\text{max}} \to m_B^2/2$, $\omega(E_{\text{max}}) \to 1$ and $B^{(0)}$ can be identified with the $B$ fully inclusive of Bremsstrahlung. Experimentally, this correction factor is accounted for when estimating the generation efficiency on signal MonteCarlo (see Sec. 8.10.1). This efficiency is computed using simulated $B^0_{s(d)} \to \mu^+\mu^-$ events, where the package PHOTOS [39] simulate QED radiative corrections in the decay.

### 2.3.2 Lifetime effects

The branching fraction measurements presented in this dissertation (see Chaps. 8-9) are obtained from the total yields of decays detected, irrespectively of their decay time. However, neutral $B^0_s$ mesons “mix”, i.e. they quantum-mechanically oscillate between $B^0_s$ and $\bar{B}^0_s$ states. Therefore, the measured “untagged” decay rate is actually the sum of two exponentials [40]:

$$< \Gamma(B^0_s(t) \to f) > = \Gamma(B^0_s(t) \to f) + \Gamma(\bar{B}^0_s(t) \to f) \quad (28)$$

$$= R^f_H e^{-\Gamma_H t} + R^f_L e^{-\Gamma_L t}, \quad (29)$$

where $\Gamma^f_H$ and $\Gamma^f_L$ are the lifetimes of the heavy and light mass eigenstates respectively. Therefore, the measured branching fraction $B^{\text{EXP}}$ is a time-integrated flavour-averaged rate:

$$B(B^0_s \to f)_{\text{EXP}} = \frac{1}{2} \int_0^\infty < \Gamma(B^0_s(t) \to f) > \, dt. \quad (30)$$

However, the branching fraction calculations presented in the previous section are based on the decay rate at the initial time $t = 0$, $< \Gamma(B^0_s(t) \to f) > |_{t=0}$, and lead to a branching fraction that does not depend on the $B^0_s - \bar{B}^0_s$ mixing, but only on the $B^0_s$ mean lifetime $\tau$ [40]:

$$B(B^0_s \to f)_{\text{TH}} = \frac{\tau}{2} < \Gamma(B^0_s(t) \to f) > |_{t=0}. \quad (31)$$

The relation between $B^{\text{EXP}}$ and $B^{\text{TH}}$ for the $B^0_s \to \mu^+\mu^-$ decay has been studied in detail in Ref. [41], where it was shown that

$$B(B^0_s \to \mu^+\mu^-)_{\text{TH}} = \left[ \frac{1 - \frac{\gamma_5}{1 + \gamma_5 A_{\Delta t}}}{1 + \gamma_5 A_{\Delta t}} \right] B(B^0_s \to \mu^+\mu^-)_{\text{EXP}}. \quad (32)$$
The mixing parameter \( y_s \) is the decay width asymmetry, which is sizeable in the \( B_0^s \) system:

\[
y_s \equiv \frac{\Gamma_L^s - \Gamma_H^s}{\Gamma_L^s + \Gamma_H^s} = 0.088 \pm 0.014 ,
\]

from the LHCb measurement in Ref. [42]. In the SM, \( A_{\Delta\Gamma} = 1 \), but depends sensitively on NP and is essentially unknown. Therefore, in the Standard Model

\[
B(B_0^s \to \mu^+\mu^-)_{\text{EXP, SM}} = \left( \frac{1}{1 - y_s} \right) B(B_0^s \to \mu^+\mu^-)_{\text{TH, SM}}, \tag{34}
\]

leading to the \( B_0^s \to \mu^+\mu^- \) branching fraction prediction [37]:

\[
B_{\text{EXP, SM}}(B_0^s \to \mu^+\mu^-) = (3.56 \pm 0.18) \times 10^{-9}. \tag{35}
\]

In the \( B_d \) system the decay width asymmetry is instead negligible (\( \sim 10^{-3} \) [43]), hence the theoretical prediction can be directly compared with the experiment.

### 2.4 Survey of experimental \( B_0^{0(d)} \to \mu^+\mu^- \) measurements

The enormous theoretical interest for the \( B_0^{0(d)} \to \mu^+\mu^- \) decay illustrated in the previous sections reflected in a continuous experimental effort to search for these decays. In this section we summarise the most recent results preceding the analysis presented in this dissertation.

The Tevatron experiments (CDF and D0) have set the first upper limits on the \( B_0^{0(d)} \to \mu^+\mu^- \) branching fractions. The latest CDF result, based on \( 7 \text{ fb}^{-1} \) collected at CDF II [44], is:

\[
B(B_d^0 \to \mu^+\mu^-) < 6.0 \times 10^{-9}, \quad B(B_s^0 \to \mu^+\mu^-) < 3.9 \times 10^{-8}, \tag{36}
\]

at 95% confidence level. The CDF analysis has observed an excess of \( B_d^0 \) candidates, compatible with a \( B(B_d^0 \to \mu^+\mu^-) = 1.8^{+1.1}_{-0.9} \times 10^{-8} \) and with a probability to be only due to background fluctuations of 0.27%.

The latest result published by the D0 experiment on the base of \( 10.4 \text{ fb}^{-1} \) is [45]

\[
B(B_s^0 \to \mu^+\mu^-) < 1.5 \times 10^{-8}, \tag{37}
\]

at 95% confidence level.

In 2009, also the LHC experiments (LHCb, ATLAS and CMS) started to take part in the \( B_0^{0(d)} \to \mu^+\mu^- \) search. In particular, LHCb has published several results with 37 pb\(^{-1}\) of \( pp \) collisions at \( \sqrt{s} = 7 \text{ TeV} \) collected in 2010 [46], 370 pb\(^{-1}\) collected in 2011 [47] and 1 fb\(^{-1}\) collected in 2011 [48]. The upper limits at 95% (resp. 90%) C.L. reported in Ref. [48] are:

\[
B(B_d^0 \to \mu^+\mu^-) < 1.0(0.81) \times 10^{-9}, \quad B(B_s^0 \to \mu^+\mu^-) < 4.5(3.8) \times 10^{-9}. \tag{38}
\]
Analogous searches have been conducted by the CMS \cite{49} and ATLAS experiments \cite{50}. The LHC 2012 results combined\(^3\) led to the most stringent upper exclusion limits before the publication presented in this dissertation \cite{51}:

\[
B(B^0_d \rightarrow \mu^+ \mu^-) < 8.1 \times 10^{-10}, \quad B(B^0_s \rightarrow \mu^+ \mu^-) < 4.2 \times 10^{-9}, \quad (39)
\]
at 95\% confidence level.

### 2.5 The \(B^0_{s(d)} \rightarrow \mu^+ \mu^-\) decay: beyond the SM

Much of the present interest for the \(B^0_{s(d)} \rightarrow \mu^+ \mu^-\) decay is due to the realisation of its potential to uncover NP beyond the SM.

To describe NP effects, two main strategies can be adopted: build an explicit model and specify which are the new fields beyond the SM ones, or analyse the NP effects using a generic effective-theory approach. The first approach is more predictive and more model dependent; we will review some examples in Sec. 2.5.2. The second strategy, illustrated in the following section, is less predictive but also more general \cite{52}.

#### 2.5.1 Effective description (model independent)

In models beyond the SM, the effective Hamiltonian in Eq. 17 acquires new terms. New particles can mediate the \(B^0_{s(d)} \rightarrow \mu^+ \mu^-\) decays (even at tree-level), hence New Physics can enter the already existing SM Wilson coefficients, but also introduce new operators, with corresponding coefficients. \(B^0_{s(d)} \rightarrow \mu^+ \mu^-\) decays are in principle sensitive to a large set of new operators (see e.g. Refs. \cite{35, 37, 53, 54}), particularly in the scalar and pseudo-scalar sectors. The general Hamiltonian for \(b \rightarrow s l^+ l^-\) transitions is given in Ref. \cite{54}. For \(B^0_{s(d)} \rightarrow \mu^+ \mu^-\) decays the general model-independent low-energy effective Hamiltonian reduces to \cite{28, 37}:

\[
H_{\text{eff}}(B^0_q \rightarrow \mu^+ \mu^-) = -\frac{G_F \alpha}{\sqrt{2} \pi \sin^2 \theta_W} \left\{ V_{tb} V_{tq}^* \sum_{k}^{10, S, P} (c_k Q_k + c'_k Q'_k) + \text{h.c.} \right\}, \quad (40)
\]

where \(q = d, s\) and

\[
\sum_{k}^{10, S, P} (c_k Q_k + c'_k Q'_k) = c_{10} Q_{10} + c'_{10} Q'_{10} + c_S Q_S + c'_S Q'_S + c_P Q_P + c'_P Q'_P. \quad (41)
\]

---

\(^3\) The results are obtained with 2.4 fb\(^{-1}\), 5 fb\(^{-1}\) and 1.0 fb\(^{-1}\) of \(pp\) collisions at \(\sqrt{s} = 7\) TeV, collected by ATLAS, CMS and LHCb, respectively.
The relative effective operators $Q_{S,P}$ are labelled according to their scalar ($S$) or pseudoscalar ($P$) form and the primed terms are the chirality flipped ones\footnote{In the literature, sometimes $Q'_{S}$ is obtained from $Q_S$ only by replacing $P_L \leftrightarrow P_R$, without applying $m_b \leftrightarrow m_q$. This choice could be motivated by the assumed “model independence” of the approach\cite{55}.} \cite{37}:

$$Q_S^{(l)} = m_b (\bar{q} P_{R(L)} b)(\bar{l} l),$$

$$Q_P^{(l)} = m_b (\bar{q} P_{R(L)} b)(\bar{l} \gamma_5 l),$$

$$Q_{10}^{(l)} = (\bar{q} \gamma^\mu P_R b)(\bar{l} \gamma_\mu \gamma_5 l),$$

$$P_{L,R} = (1 \mp \gamma_5)/2.$$  \hspace{1cm} (42) \hspace{1cm} (43) \hspace{1cm} (44) \hspace{1cm} (45)

As already anticipated in Sec. 2.2, $c_{10}$ is the only non null and real coefficient in the SM. The $S,P$ contributions are dominated by NP, since in the SM the $Z^0$, a vector boson, does not contribute to scalar and pseudoscalar operators and the SM Higgs penguin contributions are suppressed \cite{28}. In fact, in the SM the couplings of the neutral Higgs $H^0$ to fermions are proportional to $m_f/m_W$, where $m_f$ is the fermion mass. In the flavour-changing SM Higgs vertex, the coupling (hence the corresponding term in the effective Lagrangian) is thus proportional to $m_b(s)/m_W$ and its contribution to the total $B_{s(d)}^0 \to \mu^+ \mu^-$ amplitude is $m_b/m_W$ smaller than the $Z$-penguin contribution \cite{56}. This suppression is mitigated in models in which the couplings of the Higgs to down-type quarks is modified by the parameter $\tan \beta$, as mentioned later in this section, or worked around in models without GIM constraints.

The $B_{s(d)}^0 \to \mu^+ \mu^-$ branching fraction generated by the Hamiltonian in Eq. 40 is related to that in Eq. 23 through

$$\frac{\mathcal{B}(B_{s(d)}^0 \to \mu^+ \mu^-)_{NP}}{\mathcal{B}(B_{s(d)}^0 \to \mu^+ \mu^-)_{SM}} = |S|^2 + |P|^2,$$  \hspace{1cm} (46)

where $S$ and $P$ are combinations of the Wilson coefficients defined as \cite{37,41}

$$P \equiv \frac{c_{10} - c'_{10}}{c_{10}^{SM}} + \frac{m_{P_{R(L)}^0}^2}{2 m_{\mu}} \left( \frac{m_b}{m_b + m_q} \right) \left( \frac{c_{P} - c'_{P}}{c_{10}^{SM}} \right),$$

$$S \equiv \sqrt{1 - \frac{4 m_{\mu}^2}{m_{P_{R(L)}^0}^2} \frac{m_{P_{R(L)}^0}^2}{2 m_{\mu}} \left( \frac{m_b}{m_b + m_q} \right) \left( \frac{c_{S} - c'_{S}}{c_{10}^{SM}} \right)}.$$  \hspace{1cm} (47) \hspace{1cm} (48)

In the SM, $c_{10} = C_{10}^{SM}$, $P_{SM} = 1$ and $S_{SM} = 0$.

Experimental measurements put stringent constraints on $c_{10}$ \cite{57}, hence, in models where only the Wilson coefficient $c_{10}$ receives NP contributions, an order-of-magnitude enhancement of the branching ratio is disfavoured \cite{54,58}; this is only possible with sizeable contributions from scalar or pseudoscalar operators. This renders $B_{s(d)}^0 \to \mu^+ \mu^-$ decays particularly suitable probes of NP models with extended scalar and pseudoscalar sectors.
2.5.2 $B_{s(d)}^0 \to \mu^+\mu^-$ and NP models (2HDM-II and CMSSM)

Among the models with an extended scalar and pseudoscalar sector, we consider here two specific models: 2HDM-II and CMSSM. For a more extended review, see e.g. Refs. [37, 59] and references therein.

The 2HDM models theorise a larger Higgs sector having two Higgs doublets $\phi_{a,b}$ with non null vacuum expectation values [60]:

$$
\phi_a = \left( \begin{array}{c} 
\phi_a^- \\
\phi_a^0 
\end{array} \right), \langle \phi_a \rangle_0 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 
0 \\
v_a 
\end{array} \right), \quad \phi_b = \left( \begin{array}{c} 
\phi_b^+ \\
\phi_b^0 
\end{array} \right), \langle \phi_b \rangle_0 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 
0 \\
v_b 
\end{array} \right). \tag{49}
$$

Rotating the fields by an angle $\beta \equiv \arcsin(v_b/\sqrt{v_a^2 + v_b^2})$:

$$
\phi_1 = \sin \beta \phi_b + \cos \beta \phi_a, \quad \phi_2 = -\sin \beta \phi_a + \cos \beta \phi_b; \tag{50}
$$

the physical particles predicted by 2HDM are two neutral scalar Higgs, $H^0$ and $h^0$. They are obtained as linear combinations of the real part of $\phi_{1,2}^0$, a neutral pseudoscalar $A^0$ (proportional to the imaginary part of $\phi_2^0$) and two charged scalars $H^\pm = \phi_2^\pm$. The ratio of the vacuum expectation values of the two Higgs fields, $\tan \beta \equiv v_b/v_a$, is an important free parameter of the theory and the contribution given in this model to the $B_{s(d)}^0 \to \mu^+\mu^-$ decay is particularly sensitive to $\tan \beta$. Depending on how the quark masses are obtained, 2HDM models are divided into Type-I, where both up and down quarks acquire mass from the same Higgs field, and Type-II, where the masses are acquired from different fields, depending on their charge.

SuperSymmetric models (SUSY), required to be invariant under the transformation of fermions to bosons and vice versa, predict the existence of new particles, each one a supersymmetric partner of a SM particle: for every SM fermion a “sfermion”, and for every boson a corresponding SUSY partner, labelled with the suffix “-ino”. If the SUSY symmetry was unbroken, SUSY particles would have masses equal to the SM partners and would have been observed in accelerators. Since it did not happen so far, supersymmetry must be broken. It is exactly the SUSY-breaking mechanism that differentiates the various SUSY species currently discussed. In particular, the Minimal Supersymmetric Standard Model (MSSM) has a Higgs structure essentially analogous to that of the 2HDM-II: two Higgs doublets generate mass for both up- and down-type quarks and charged leptons. $\tan \beta$ is again a free parameter of the model, which makes the $B_{s(d)}^0 \to \mu^+\mu^-$ decay a suitable probe of the MSSM.

Models with two Higgs doublets, such as 2HDM or MSSM, may provide sizeable contributions to the scalar and pseudoscalar Wilson coefficients, that do not suffer from the helicity suppression. In fact, the helicity suppression may be lifted in the factor $(m_\mu/M_W)\tan \beta$ stemming from the Higgs Yukawa couplings to the final state leptons, and thus the new contributions may be similar in size to those of the SM [61].

Figure 4 shows the main $B_s^0 \to \mu^+\mu^-$ diagrams in 2HDM-II that give a contribution to the scalar and pseudoscalar Wilson coefficients proportional to $\tan^2 \beta$ [61]:

$$
c_s^{2\text{HDM-II}} = c_p^{2\text{HDM-II}} = \frac{m_\mu}{2m_W^2} \tan^2 \beta \log \left( \frac{M_W^2}{M_\mu^2} \right) \frac{M_\mu^2}{M_\tau^2} \frac{M_\ell^2}{M_\ell^2}; \tag{51}
$$
This will lead to an enhancement factor $\tan^4 \beta$ in the branching fraction.

In the framework of MSSM models, new contributing diagrams are obtained by exchanging loop particles with their SUSY partners. The leading contribution at high $\tan \beta$ comes from the self-energy corrections in diagrams where the Higgs propagators are attached to the external quark legs, as shown in the left diagram in Fig. 5. Additional contributions are given by diagrams involving quartic coupling with sparticles, as that in Fig. 5 (right). Diagrams like those in Fig. 5 will give additional contributions to the SM $Z^0$ penguin and box diagrams, as well as to the otherwise suppressed Higgs penguin. These diagrams can lead to an enhancement in the decay branching fraction, with a $\tan^6 \beta$ dependence.

However, it is worth noticing that the SUSY impact on the $B^0_s \rightarrow \mu^+ \mu^-$ decay can also be “hidden”, leading to an MSSM branching fraction close to the SM expectation. In fact, $R$-parity is conserved; the $R$ quantum number is defined as $R = (-1)^{3B+L+2S}$, where $B$ is the baryonic number, $L$ the leptonic number and $S$ the spin.

Moreover, $R$-parity is conserved; the $R$ quantum number is defined as $R = (-1)^{3B+L+2S}$, where $B$ is the baryonic number, $L$ the leptonic number and $S$ the spin.
the MSSM contribution may well be suppressed in regions of intermediate tan \( \beta \) values or of large masses of the physical Higgs boson \( M_A \). As an example, a Standard Model-like \( \mathcal{B}(B_s^0 \to \mu^+\mu^-) \) is expected in the Constrained MSSM (CMSSM), as discussed in Ref. [63].

**B\(_s(d)\) \to \mu^+\mu^-** measurements and NP

As discussed in the previous section, the \( B_{s(d)}^0 \to \mu^+\mu^- \) branching fraction is a powerful probe of New Physics models. Setting limits on \( B(B_{s(d)}^0 \to \mu^+\mu^-) \) or, even better, measuring it– allows to constrain numerous well-motivated scenarios and to test directly the Standard Model predictions. In particular, the ratio of the \( B_s^0 \) and the \( B_d^0 \) branching fractions represents a stringent test for Minimal Flavour Violation (MFV)

\[
\frac{\mathcal{B}(B_s^0 \to \mu^+\mu^-)}{\mathcal{B}(B_d^0 \to \mu^+\mu^-)} = \frac{\tau_{B_s^0} f_{B_s^0}^2 m_{B_s^0} |V_{ls}|^2}{\tau_{B_d^0} f_{B_d^0}^2 m_{B_d^0} |V_{td}|^2}.
\]

Any result incompatible with this prediction would rule out MFV models.

Figure 6 shows the correlation between the branching ratios of \( B_s^0 \to \mu^+\mu^- \) and \( B_d^0 \to \mu^+\mu^- \) in different models; the SM expectation is represented by the star-point, MFV scenarios are along the red line and various non-MFV models are also considered. The plot also shows in grey the areas experimentally ruled out before the result presented in this dissertation [48–51].

In Ref. [63], various experimental measurements are combined through a global fit in a frequentist analysis of the parameters space of different NP models. In Fig. 7 (a) the results of the fit to the \( B_s^0 \to \mu^+\mu^- \) and other experimental results are indicated by solid lines in the \( (M_A, \tan \beta) \) plane of the CMSSM model. In Fig. 7 (b) is shown the \( \chi^2 \) likelihood function for \( B(B_s^0 \to \mu^+\mu^-) \) in the CMSSM, for a global fit to experimental data, where the solid (dashed) curve is obtained including (excluding) the last combined branching fraction results. This shows the strong impact of the \( B_s^0 \to \mu^+\mu^- \) result, which restricts the \( \mathcal{B}(B_s^0 \to \mu^+\mu^-)_{\text{CMSSM}} \) to the range \( \mathcal{B}(B_s^0 \to \mu^+\mu^-)_{\text{SM}}, 1.4 \times \mathcal{B}(B_s^0 \to \mu^+\mu^-)_{\text{SM}} \) at 90% C.L. To narrow further the CMSSM parameters space would require a significant improvement of the experimental measurement (a measurement of \( \mathcal{B}(B_s^0 \to \mu^+\mu^-) \) at the level of 30% precision, according to the authors of Ref. [63]). The impact of the new measurements presented in this dissertation will be discussed in Sec. 9.3.

---

6 \( M_A \) is one of the free parameters of the model; quartic squark couplings in neutral Higgs penguins produce terms proportional to \( \tan^3 \beta m_{\tilde{q}_L} / M_W M_{A_h} \) [62].

7 CMSSM is a MSSM model where SUSY breaking is mediated by gravity and the number of free parameters is reduced by means of a set of universality conditions, although less restrictive than those of the similar mSUGRA model.

8 MFV scenarios identify in the Yukawa SM couplings the only source of flavour symmetry breaking.
2.5 The $B^{0}_{s(d)} \rightarrow \mu^{+}\mu^{-}$ decay: beyond the SM

Correlation between the branching ratios of $B^{0}_{s} \rightarrow \mu^{+}\mu^{-}$ and $B^{0}_{d} \rightarrow \mu^{+}\mu^{-}$ in the SM (marked by a star), in MFV scenarios (along the red line) and various non-MFV models. The plot shows in grey the areas already ruled out by the measurements $[48-51]$ existing before the results presented in this dissertation. Figure from Ref. $[53]$.

On the left, ($M_{A}$, tan $\beta$) plane in the CMSSM, including several experimental results (see Sec. 2.4). The results of the current fits are indicated by solid lines and filled stars, while older fits based on a smaller LHC data sample are indicated by dashed lines and open stars. The blue (red) lines denote 68\% (95\%) CL contours. On the right, $\Delta \chi^{2}$ from the global fits to the same data sample. The solid (dashed) line corresponds to the fit including (excluding) the combined branching fraction results.