Externalities and Cooperation in Algorithmic Game Theory

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Summary

This Ph.D. thesis discusses research in algorithmic game theory, carried out by the author and various collarborators. Algorithmic game theory is an area of research that lies in the intersection of economics, mathematics and computer science. The field encompasses the combination of game theory (the study of strategic decision making and models of conflict) with theoretical computer science. The central object of study in the thesis is a *game*, which is an abstraction of a situation in which multiple autonomous entities (called *players*) participate by making autonomous decisions.

The topics discussed are roughly centered around the notions of cooperation, externalities, and in general "other-regarding behavior". The study of these topics is motivated by the observation that the usual assumption that players act perfect rational, and in isolation (classically assumed in strategy game theory) is often inaccurate: in many realistic scenarios, players are embedded into some sort of social context and/or are able to cooperate with eachother.

In this thesis we investigate models that include (or allow for the inclusion of) the aspects of cooperation and externalities in games. We will be interested in the impact of this inclusion to various established notions and results in algorithmic game theory; most prominently the *price of anarchy*, which measures the worst-case quality of an *equilibrium* (i.e., roughly a "stable outcome") of a game. Another prominent theme is the study of the consequences to algorithmic problems, that the presence of externalities induces. Some related topics are studied as well: various problems in cooperative game theory in the final chapters of the thesis, and the price of anarchy of multi-unit auctions in the second chapter of the thesis.

The most important results and insights obtained in this thesis that are related are summarized as follows:

- Lower bounds and upper bounds on the price of anarchy of multi-unit auctions. The upper bounds are shown to be optimal with respect to the currently known techniques.

- An asymptotic characterization of the price of anarchy of *linear bottleneck congestion game* when cooperation among the players can occur. Linear bottleneck congestion games form a class of games that model e.g. load balancing scenarios in which multiple autonomous participants have to schedule their jobs on a set of machines.

- The price of anarchy of various games may deteriorate when players behave

more altruistically, and it is possible to derive good upper bounds on the price of anarchy of some important classes of games, by extending a known technique.

- The same ideas may be used for a more general scenario in which players have particular, player-specific, attidudes towards other players. For many such *social contexts*, some broad classes of games can be shown to have a low price of anarchy.
- It is possible to characterize the set of equilibria and the price of anarchy of procurement auctions in a precise way, in the presence of players that behave spitefully.
- There exist efficient algorithms that find a good-quality profile of strategies for congestion games in which players have positive externalities for sets of other players, from which they may derive additional utility. Congestion games form a class of games of central importance, and model all sorts of situations in which there are resources of which the performance depends on the set of entities making use of it.
- For the setting of housing markets, there exists a broad and simple family of mechanisms that compute allocations of houses to players, and satisfy a large set of desirable properties. Moreover, there exists a mechanism that has been previously proposed and falls within this class, but unfortunately does not compute an allocation in an satisfying amount of time.
- Computing the best structure for a set of players to cooperate is difficult in general, but easy for a number of special cases. Most importantly, there exists an efficient algorithm for this problem in case the number of types of the players is bounded and it is known which player has which type.
- The hardness of computing the Shapley value in a matching game can be completely characterized. The Shapley value provides an important method for dividing a value among a set of cooperating players. Matching games form a class of cooperative games that help understand and model auctions and assignments.