At the closing part of the dissertation are concluding summaries. Chapter 2 studies an asset pricing model that can simultaneously explain the three important features in the asset price data, namely, the high equity premium, the low risk free rate, and the positive slope of the term structure of the real interest rates at average. Many important asset pricing models can explain one or two of the features, but not all three. The model considers a hidden Markov economy in which the dynamics of the fundamental economic structure is subject to the regime of the economy. The regime that is invisible to investors switches among a finite number of states, and one of the states represents the great depression. By considering robust investors who are wary of the rational expectation estimation of the underlying state, and alternatively assign a greater probability weight to the depression state, the model generates a higher risk premium than in the standard rational expectations models. As robust investors are more pessimistic of the outlook of the economic growth in the next period, the short term real interest rate is low. However, in the long run the state of the economy is still decided by the stationary distribution of the fundamental state, rather than the investors’ belief of the current state. As a result, the long term real rate is much less affected by the robust belief than the short term rate. Therefore the model also produces a positive real term structure.

Chapter 3 discovers a pricing anomaly between S&P500 options and the VIX futures. On the one hand, by combining S&P500 option prices at different maturities and strike prices, one can infer the expected variance in the risk neutral measure up to the option maturities. On the other hand, prices of futures contracts written on the volatility index VIX measures the expected volatility in the risk neutral measure over the thirty day horizon after the futures maturities. If options and futures markets are integrated, the same pricing kernel should apply to both, and the option-implied variance and futures-implied volatility can be combined to give the risk-neutral variance of the VIX. But taking them together often implies a negative measurement of the variance of the VIX, a mathematical impossibility. The recurrent inconsistencies are not simply due to possible measurement errors, because simple trading strategies designed to exploit these inconsistencies produce very high Sharpe ratios. The analysis suggests an anomaly between the variances implied from options and volatilities implied from VIX futures.

Chapter 4 examines the affine stochastic volatility models with the volatility index VIX and variance swap rates synthesized by S&P500 options. In affine volatility models, the expected variance implied by the VIX square is linear with the instantaneous variance. Based on this implication we carry out two exercises. First, we regress the

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VIX square on the instantaneous variance and find that there is a quasi linear relationship between the VIX square and the instantaneous variance, and that the perfect linear relationship is ruined by volatility persistence. Second, we form a vega neutral portfolio that should have a constant value representing the long run average of the variance under the risk neutral measure. However, the volatility hedging by the affine models is not successful and the portfolio is still subject to volatility risk. These findings indicate the inefficiency of using affine volatility models to price and hedge variance swaps.

This dissertation opens up a few potential topics for future research. Compared to previous studies on aggregate equity returns whose focus is on matching empirical moments, the model in chapter 2 is able to produce a more plausible description of the historical time series of equity prices. However, the model time series generated by robust preference is more volatile than the historical counterpart. This feature suggests to search for a smoothing modification of robustness preferences. One possibility is to have a dynamic structure of forming robustness that smooths the divergence from rational expectations in respect to large shocks. It can be some kind of stickiness in the adjustment of the agent’s formulation of robustness. Another observation as a byproduct in chapter 2 is that corporate earnings do not always move in the same direction as market returns. This suggests that either we should look for alternative proxies of aggregate cash flows, or that we should add more state variables that matter for pricing and can change differently than earnings which, however, introduces more complications.

The empirical study of volatility derivatives can be extended with the results that have already been obtained. For example, it would be interesting to compare the implied volatility of the VIX with the Black-Scholes volatility of VIX options, as both of them represent the volatility of the VIX under the risk neutral measure. Possibly we can search for a certain pattern in their relationship, and try to build volatility forecast models. Another follow-up research that can be attempted is to link the implied volatility of the VIX to the modeling of realized variance, as the implied volatilities are forward looking and therefore informative.