

VU Research Portal

Morse-Conley-Floer Homology

Rot, T.O.

2014

document version

Publisher's PDF, also known as Version of record

[Link to publication in VU Research Portal](#)

citation for published version (APA)

Rot, T. O. (2014). *Morse-Conley-Floer Homology*.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal ?

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

E-mail address:

vuresearchportal.ub@vu.nl

Contents

1	Introduction	1
1.1	Dynamics and Topology	1
1.2	Classical Morse theory, the half-space approach	4
1.3	Morse homology	7
1.4	Conley theory	11
1.5	Local Morse homology	16
1.6	Morse-Conley-Floer homology	18
1.7	Functoriality for flow maps in Morse-Conley-Floer homology	21
1.8	Duality under time reversal	24
1.9	A spectral sequence in Morse-Conley-Floer homology	24
1.10	Morse-Conley-Floer homology in infinite dimensional systems	26
1.11	The Weinstein conjecture and symplectic geometry	30
1.12	Closed characteristics on non-compact hypersurfaces	33
I	Morse-Conley-Floer homology	41
2	Morse-Conley-Floer homology	43
2.1	Introduction	43
2.2	Isolating blocks and Lyapunov functions	47
2.3	Gradient flows, Morse functions and Morse-Smale flows	52
2.4	Morse homology	57
2.5	Morse-Conley-Floer homology	67
2.6	Morse decompositions and connection matrices	70
2.7	Relative homology of blocks	76
3	Functoriality	81
3.1	Introduction	81

CONTENTS

3.2	Chain maps in Morse homology on closed manifolds	89
3.3	Homotopy induced chain homotopies	100
3.4	Composition induced chain homotopies	102
3.5	Isolation properties of maps	106
3.6	Local Morse homology	111
3.7	Morse-Conley-Floer homology	116
3.8	Transverse maps are generic	118
4	Duality	121
4.1	Introduction	121
4.2	The dual complex	121
4.3	Morse cohomology and Poincaré duality	122
4.4	Duality in local Morse homology	123
4.5	Duality in Morse-Conley-Floer homology	124
4.6	Maps in cohomology	124
4.7	Shriek maps in Morse-Conley-Floer homology	126
4.8	Other algebraic structures	126
4.9	Projection formula	128
5	Spectral sequence	133
5.1	The birth of a spectral sequence	133
5.2	The Morse-Conley relations via the spectral sequence	136
5.3	Interpretation of the spectral sequence for Morse-Bott functions	138
5.4	A counter example to misstated Morse-Bott inequalities.	140
II	Periodic orbits on non-compact hypersurfaces	143
6	Periodic orbits	145
6.1	Introduction	145
6.2	Contact type conditions	148
6.3	The variational setting	154
6.4	The Palais-Smale condition	157
6.5	Minimax characterization	164
6.6	Some remarks on the geometry and topology of the loop space	167
6.7	The topology of the hypersurface and its shadow	170
6.8	The link	177
6.9	Estimates	188
6.10	Proof of the main theorem	190

6.11 Appendix: construction of the hedgehog	190
6.12 Appendix: local computations	199
Nederlandse samenvatting: Morse-Conley-Floer homologie	203
Overzicht	203
Deel I: Morse-Conley-Floer homologie	204
Deel II: Periodieke banen op niet compacte hyperoppervlakken . . .	205
Acknowledgments	207
Bibliography	209