

VU Research Portal

Mathematical Thinking, Learning and Performance:

Reed, H.C.

2014

document version

Publisher's PDF, also known as Version of record

[Link to publication in VU Research Portal](#)

citation for published version (APA)

Reed, H. C. (2014). *Mathematical Thinking, Learning and Performance: Insights and interventions for primary and secondary education*. [PhD-Thesis - Research and graduation internal, Vrije Universiteit Amsterdam].

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

E-mail address:

vuresearchportal.ub@vu.nl

Summary

In a world in which mathematical proficiency increasingly underpins success at school and in society, there is pressing need for research that can contribute to improving mathematics education in schools. This thesis addresses some important aspects of mathematical thinking, learning and performance signalled both by educational practitioners in the Netherlands and in international comparative studies (TIMSS, PISA). The thesis takes a multidimensional approach, covering a wide range of ages and educational challenges (both cognitive and affective), employing diverse methods, and combining learning perspectives from the educational sciences, cognitive psychology and neuropsychology. As well as providing theoretical insights, the thesis presents implications for educational practice that could help educators to improve mathematics teaching and learning in schools.

A general introduction is given in **Chapter 1**. Thereafter, the thesis is presented in two parts. Part One contains four classroom studies that focus on interventions or instruments aiming to stimulate or understand specific aspects of mathematical thinking and learning. Part Two contains three survey studies that analyse self-report data and cognitive and performance measures from large-scale, cross-sectional and longitudinal datasets.

An important prerequisite for mathematical thinking and learning is the ability to reason about causes and effects. **Chapter 2** reports on a study with preschoolers that investigates how shared picture-book storytelling could stimulate causal reasoning. Preschoolers were allocated to small peer-groups that participated in six storytelling sessions over a period of two weeks. During these sessions, the children freely generated stories from pictorial stimuli in two picture-books. Storytelling discourse was analysed in the groups that showed the lowest and the highest pre-to-post-intervention improvement on a series of causal reasoning tasks. It was found that discourse in the most-improving group was characterised by detailed interpretations of perceptual features, causal explanations and explicit justifications of statements. Discourse in the least-improving group was characterised by ‘superficial’ talk (i.e., labelling perceptual features, simple inferences, uncritical acceptance of statements and disagreements). These types of discourse could be related to time spent on storytelling. The study generates several hypotheses for future research on stimulating causal reasoning in early childhood education.

With older children, mathematical learning centres on the acquisition of conceptual, procedural and factual knowledge. Developing fluency in arithmetic facts is one of the key goals of mathematics education in primary school. **Chapter 3** presents a study that compares the effects of two practice conditions on children’s fluency in simple multiplication facts. Third and fourth graders practised in either a

SUMMARY

conventional *recall* condition - where they produced answers to problems - or a *choice* condition - where they had to choose between competing answers that included common multiplication errors. The ability to suppress interference from such 'intrusions' is important for fast, accurate fact retrieval. Practice in the choice condition was found to be faster and as accurate as recall practice but was not more beneficial to performance on speed tests of practised facts. For more experienced students (i.e., Grade 4), recall practice led to greater improvement on a conventional recall fluency test. The differential effects of practice conditions on test performance appear to be related to practice-to-test transfer demands. The relative merits of recall and choice tasks in multiplication fact learning are discussed.

Chapter 4 continues with the theme of simple multiplication. The study presented here investigates whether children's verbal reports accurately represent their thinking processes when solving simple multiplication problems. Third graders solved simple multiplication problems and retrospectively reported how they had done this. The degree to which verbal reports corresponded to known performance patterns regarding speed, accuracy, errors and strategy-use was assessed, taking account of problem characteristics and cognitive characteristics that are known to affect multiplication performance. The verbal reports were largely consistent with known patterns. The results support using verbal reports to assess multiplication strategy use. Moreover, verbal reports provide valuable information that can alert teachers and educational researchers to specific issues that students face when solving simple multiplication problems.

With **Chapter 5**, the focus of the thesis moves to lower secondary school. The chapter presents a mixed-methods study into the effects of student attitudes and behaviours on the outcomes of learning mathematics with computer tools. A computer tool was used to help seventh and eighth graders develop the mathematical concept of function, and learning outcomes (i.e., conceptual understanding and tool mastery) were assessed. Student attitudes were found to account for 34% difference in test scores between individuals. General attitude towards mathematics positively predicted test scores. However, higher ability students who were well-disposed towards mathematical computer tools achieved lower scores. Self-reported behaviours were unrelated to test scores. Detailed observation of a small number of students revealed that positive attitudes towards mathematics and mathematical computer tools augmented positive learning behaviours, and that both a positive attitude to mathematical computer tools and positive learning behaviours benefited tool mastery. Although tool mastery and test scores are intimately related, reflective processes appear to mediate this relationship. It is argued that promoting learning with mathematical computer tools needs to take several factors into account, including improving student attitudes, raising levels of learning behaviours, and

giving sufficient opportunity for constructing new mathematical knowledge within meaningful mathematical discourse.

The three survey studies presented in Chapters 6 to 8 comprise Part Two of the thesis. **Chapter 6** reports on a large-scale cross-sectional study that investigates performance patterns in academic tracks in the first three years of secondary school. By identifying patterns that reveal how competence levels in different domains are related at different stages of development and by comparing low performers with other students, the study sheds light on individual and educational aspects that could underlie poor performance in academic tracks. School performance was found to reflect three domains - *languages*, *social studies* and *science and math* - that appear to interact in a process of co-construction. General language skills were robustly related to performance in other domains throughout the period studied. By comparison, proficiency specifically in the language of schooling was less strongly related to *social studies* and *science and math* performance after the first year. Suggestions are given as to how educators and curriculum developers could use these insights to accommodate individual and developmental differences and to develop learning materials that may help low performers keep on track.

In secondary school, mathematical thinking and learning depend increasingly on the ability to think about abstract concepts ('abstract thinking') and reason about relationships between them ('relational thinking'). **Chapter 7** investigates in a large-scale cohort study how these abilities relate to mathematical performance at the end of lower secondary school and whether sex, socio-economic background, educational track and delay in educational progression affect this relation. Abstract and relational thinking were found to jointly explain 39% variance and 18% unique variance in mathematical performance, but unique variance drops to 5% when educational track is accounted for. Students in higher tracks have higher mathematics performance and better abstract and relational thinking skills. Sex and delay have additional small effects, but socio-economic background is overshadowed by tracking effects. Compared to age-typical controls, age-delayed students have lower mathematical performance but equivalent abstract and relational thinking skills. The findings support paying attention to developing abstract and relational thinking skills in secondary school, particularly for students in lower tracks and those at risk of having to repeat a grade.

The large-scale longitudinal cohort study presented in **Chapter 8** examines the extent to which mathematics self-concept and academic self-efficacy mediate the relation between mathematical performance at the end of primary school (Grade 6) and the end of lower secondary school (Grade 9) in an early tracking educational system. It was found that the relation in question is uniquely mediated by both mathematics self-concept in Grade 9 and academic self-efficacy in Grade 6, but in

SUMMARY

opposing directions. Mathematics self-concept is the most influential mediator, explaining nearly a quarter of this relation. High academic self-efficacy in Grade 6 has a negative influence on Grade 9 mathematical performance. This suggests that self-efficacy needs to be actively managed when students move to secondary school. Findings were similar for both sexes and all educational tracks.

Finally, **Chapter 9** reflects on the reported research in terms of the thesis approach and objectives. It emphasises the importance of combining multiple perspectives on learning, of promoting thinking skills throughout the long lines in mathematics education, of listening to what children say about what they think and learn, and of striving to understand the sources of individual differences in mathematical thinking, learning and performance. In all of this, the thesis provides concrete pointers for educational policy and practice that could be extended in further applied research aiming to improve mathematical teaching and learning in schools.