Mathematical Thinking, Learning and Performance:

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We envision a world where everyone is enthused about mathematics, sees the value and beauty of mathematics, and is empowered by the opportunities mathematics affords

National Council of Teachers of Mathematics

1

Introduction

INTRODUCTION

Mathematical proficiency is essential for success at school and for participation in society. The capacity to understand mathematical concepts, apply mathematical reasoning and use mathematical tools underlies study in many school subjects and further education. Moreover, the increasing dependence of everyday and professional life on science and technology also requires proficiency in these kinds of skills, particularly in the higher-paying scientific and technical professions (Mullis, Martin, Foy, & Arora, 2012; NCTM, 2000; NMAP, 2008; OECD, 2013a).

Because of its importance, the mathematical proficiency of young people is subject to rigorous scrutiny worldwide. This is further fuelled by international comparative assessments: the Trends in International Mathematics and Science Study (TIMSS; a periodic international comparative study into educational achievement in mathematics and science in Grades 4 and 8) and the OECD Programme for International Student Assessment (PISA; a periodic international comparative study of 15-year-olds' competencies in mathematics, science and reading). In many Western nations, the results of these assessments give cause for concern to educators and educational researchers. Taking the Netherlands as an example, the most recent TIMSS cycle shows that it has dropped out of the top 10 best-performing countries at Grade 4, and that it lags behind many Western European countries and the United States on international benchmarks of mathematics achievement (Mullis et al., 2012). For older students, the latest PISA assessment reveals a decreasing trend in Dutch 15-year-olds' ability to apply mathematical knowledge to solve real-life problems. Where the Netherlands occupied 4th place in international rankings in 2003, it had dropped to 10th place in 2012 (OECD, 2013a). Worryingly, although the Netherlands is still one of the highest ranked European countries, it is one of only 14 OECD countries (out of 64 with trend data up to 2012) whose performance has deteriorated between 2003 and 2012, and shows one of the largest performance decreases.

There is, therefore, pressing need for relevant, high quality research on mathematical thinking, learning and performance that can inform educational policy and practice and thereby contribute to improving learning and teaching in schools. This need is felt in many countries worldwide. In the United States, the final report of the National Mathematics Advisory Panel calls for more research on how children learn mathematics and on effective classroom practices that can inform policy and practice (NMAP, 2008). In the United Kingdom, the Advisory Committee on Mathematics Education calls for research on teaching and learning mathematics to inform teacher professional development (ACME, 2011). In the Netherlands, the Royal Netherlands Academy of Arts and Sciences advocates more extensive, more varied, multidisciplinary research into mathematics teaching and learning, including analyses of data from large-scale national and international studies as well as studies on the role and impact of information technology (KNAW, 2009).

This thesis presents research addressing a number of issues in mathematical thinking, learning and performance that have been identified in dialogue with teachers, schools and school organisations. Real-life challenges are, of course, complex and multi-faceted; consequently, the presented research takes a multidimensional approach, involving diverse age groups and methods, and combining learning perspectives from the educational sciences, cognitive psychology and neuropsychology. As well as providing insights into particular phenomena, the studies in this thesis present implications for educational practice that could contribute to improving mathematics learning and teaching in schools.

The presented research incorporates multiple perspectives on the learning child and adolescent that connect brain, cognition, development and education. This approach stems from the recognition that cognitive and emotional development during childhood and adolescence is determined by mutual interactions between biological, psychological and contextual factors (Crone & Dahl, 2012; Halpern, 2012). For example, boys and girls differ with respect to the dynamics of development of certain cortical structures and functions that are essential to learning (Giedd, Raznahan, Mills, & Lenroot, 2012). These biological differences may be further strengthened by contextual influences such as sex-stereotypical socialisation at home and school (Halpern, 2012; Wood & Eagley, 2012) and social-affective experiences (e.g., threats, rewards, risk-taking) that differentially affect motivation, self-concept, personal interests, etcetera (Crone & Dahl, 2012; Peper & Dahl, 2013). Consequently, girls and boys show differences in cognition, affect and behaviour that can impact the effectiveness of their educational experiences. This example illustrates the importance of understanding how interactions between biological, psychological and contextual factors influence individuals' thinking, learning and performance. This understanding should, in turn, facilitate tailoring of teaching and instructional approaches to take account of differences between students.

THE SCOPE OF THE THESIS

Mathematics in primary through to secondary school comprises several substantive domains (e.g., arithmetic, algebra, geometry, measurement, data analysis, probability) and requires acquisition of conceptual, procedural and factual knowledge (Delazer, 2003; NCTM, 2000; NMAP, 2008). In addition, it requires particular thinking and reasoning skills, including the ability to understand and construct causal chains of facts or events, to handle mathematical abstractions, and to reason about relationships between mathematical objects (Devlin, 2000; Epstein, 2013; O'Neill, Pearce, & Pick, 2004; Richland, Stigler, & Holyoak, 2012). Nowadays, the ability to use mathematical tools, such as graphical calculators and instructional software is also part and parcel of school mathematics (NCTM, 2000; NMAP, 2008; OECD, 2013a).

Within this context, the thesis takes a broad view of mathematical thinking, learning and performance. First, it recognises the importance of the long lines in mathematics education, where the effectiveness of later mathematical learning depends on the strength of earlier acquired knowledge and skills (KNAW, 2009; NMAP, 2008). Earlier experiences and acquired knowledge lead to development and functional maturation of brain networks and thus facilitate acquisition of deeper insights and more complex learning at a later stage (Menon, 2013). To this end, the thesis covers a wide range of ages and commensurate educational challenges, from preschool to lower secondary school. In preschool, it investigates how to stimulate causal reasoning skills, considered a prerequisite for mathematical thinking and learning. In primary school, it investigates the acquisition of fluent factual knowledge in simple multiplication and how children report solving simple multiplication problems. In lower secondary school, it examines the relationship of mathematics performance to the development of higher-order thinking skills, as well as to performance in other academic domains.

Second, the thesis recognises the central role of affective and motivational factors that affect how students interact with mathematics education. Specifically, it examines the influence of attitudes and learning behaviours when using mathematical computer tools, and the role of self-beliefs in mediating mathematical performance between the end of primary school and the end of lower secondary school.

Third, the thesis recognises the influence of individual differences and contextual factors such as social settings and educational system characteristics on learning processes and outcomes. Thus, the presented research examines the effects of sex, socio-economic background and educational track, as well as the effects of working in peer groups and dyads.

THEORETICAL FRAMEWORK

Casual reasoning: A prerequisite for mathematical thinking and learning

Early childhood education (e.g., preschool) plays an important role in preparing children for formal schooling, particularly for children whose home background may offer few opportunities for cognitive and socio-emotional development. Most preschool research in the domain of mathematics focuses on early numeracy, yet mathematics thinking and learning also crucially depend on reasoning about causes and effects and the ability to follow a causal chain of facts or events (Bills, 2002; Devlin, 2000; O'Neill et al., 2004). Indeed, causal thinking of preschool children is indicated as a significant predictor of later mathematical skills (O'Neill et al., 2004).

There are important individual differences between children with regard to their causal reasoning abilities, however, stemming from differences in knowledge and experience that are primarily related to variability in home context (Wellman & Liu, 2007). In consequence, some young children's causal reasoning competence lags behind that of their peers from more stimulating home environments. Given the importance of causal reasoning for mathematical thinking (as well as thinking in other school subjects), it is necessary to provide preschoolers with experiences that can stimulate their causal reasoning skills even before they enter formal education.

Such experiences could, for example, take the form of narrative construction (i.e., storytelling), one of the most fundamental ways in which children apply causal reasoning to understand their daily experiences (Nelson, 2007; Paris & Paris, 2003; Schick & Melzi, 2010). Furthermore, while it is known that older children and adolescents can benefit from peer interaction during learning tasks (Howe & Mercer, 2010; Rojas-Drummond, Pérez, Vélez, Gómez, & Mendoza, 2003; Wegerif, Mercer, & Dawes, 1999), the full potential of peer-group learning in early childhood settings has not yet been realised. Taken together, research that examines how shared storytelling within a peer-group setting could stimulate causal reasoning in preschoolers could usefully extend the repertoire of early childhood educators for stimulating young children's thinking skills.

How could shared storytelling within a peer-group setting stimulate causal reasoning in preschool children? (Chapter 2)

Factual knowledge and solution strategies in simple multiplication

Mathematical learning places considerable demands on a learner's limited information processing capacity (Kilpatrick, Swafford, & Findell, 2001). When basic arithmetic facts and simple operations are quickly and accurately available from memory, cognitive resources are freed for more complex learning (LeFevre, DeStefano, Coleman, & Shanahan, 2005; Tronsky, 2005). Quick and accurate retrieval of arithmetic facts is, therefore, an important goal of mathematics education (NCTM, 2000; NMAP, 2008). Unfortunately, many children - both typically developing and those with mathematics difficulties - have difficulty achieving arithmetic fact fluency (Geary, Hoard, & Bailey, 2012; Jordan, Hanich, & Kaplan, 2003; Mullis et al., 2012).

Previous research has established that arithmetic fact fluency can be improved through systematic practice (e.g., Wong & Evans, 2007; Woodward, 2006). However, there is no clear evidence as to which practice methods and materials are most effective (MinOCW, 2011). To address this, research is needed that takes account of how the brain learns arithmetic facts. It is generally agreed that such facts are organised in associative networks comprising connections between problems and answers in long-term memory; unfortunately, connections also exist to incorrect answers (such as close operand-related answers) that compete with and thus interfere with the correct response (Campbell, 1995; Galfano, Rusconi, & Umiltà, 2003; Verguts & Fias, 2005). Being able to quickly and accurately retrieve arithmetic facts thus depends to a degree on the ability to inhibit such interference (Barrouillet, Fayol, & Lathulière, 1997; Geary et al., 2012). Consequently, there could be benefit in investigating the effects of practice materials that train children to inhibit competing answers that represent these kinds of errors.

Can multiplication fact fluency be improved by learning to suppress interference from competing answers? (Chapter 3)

Many children who have difficulty achieving arithmetic fact fluency continue to use time-consuming and error-sensitive procedural methods to solve simple arithmetic problems (Geary et al., 2012; Jordan et al., 2003). If teachers could identify children who consistently use immature strategies, they could provide more timely and individuated instruction (Gersten, Jordan, & Flojo, 2005). One way for teachers to assess strategy-use would be to ask children how they solve arithmetic problems. Verbal reports can provide important information about cognitive processing (Fox, Ericsson, & Best, 2011; Robinson, 2001; Taylor & Dionne, 2000). However, it is not certain whether what children report about how they solve problems accurately represents their thinking processes (i.e., veridicality). Thus, research is needed that investigates the veridicality of children's verbal reports of strategy-use during arithmetic problem-solving.

How veridical are children's verbal reports of multiplication strategy-use? (Chapter 4)

Cross-domain patterns of school performance in academic tracks

In the Netherlands, as in several other countries (e.g., Belgium, Germany, Singapore), secondary education is stratified according to general scholastic ability, with high differentiation between academic and vocational education. Students are placed in designated educational tracks from early in secondary school and subsequent movement between tracks depends on ongoing performance in the first years of secondary school. Low performers in academic tracks often face being moved 'down' to a vocational track. The far-reaching consequences of such a move (Andersen & Van de Werfhorst, 2010; Hanushek & Wößmann, 2006; Schütz, Ursprung, & Wößmann, 2008; Shavit & Müller, 2000; Van de Werfhorst & Mijs, 2010) make it important to understand as much as possible about these students. It may then be possible to take specific measures to improve their performance and keep them within academic education. Important insights into this issue could be gained by examining how competence levels in different domains - including mathematics - are related at different stages of development, and which individual and educational factors differentiate low performing from better performing students.

How does mathematical performance relate to performance in other domains in academic tracks? (Chapter 6)

Higher-order thinking and mathematical performance

As children pass through primary school to secondary school, there is an increased need for them to develop so-called higher-order thinking skills that allow them to deal with more abstract and more complex material. Nowhere is this so important as in mathematics, which depends crucially on understanding and reasoning with abstract concepts and with the relations between them (Devlin, 2000; Epstein, 2013; Richland et al., 2012). Neuroimaging studies show that brain activity during abstract and relational thinking tasks involves brain networks that undergo protracted maturation throughout early to mid-adolescence (e.g., Bazargani, Hillebrandt, Christoff, & Dumontheil, 2013; Crone et al., 2009; Dumontheil, Houlton, Christoff, & Blakemore, 2010; Eslinger et al., 2009). The rate of maturational change in these mediating brain areas could therefore constrain performance on mathematical tasks requiring abstract and relational thinking. This could partly explain the fact that very little progress is made in mathematical competence during the initial years of secondary school (Ryan & Williams, 2007). It is therefore important to understand the relation between abstract and relational thinking and mathematical performance in this period. This could inform educators and curriculum developers with respect to the necessity of developing these skills in school.

How do abstract and relational thinking skills relate to mathematical performance at the end of lower secondary school? (Chapter 7)

Attitudes, self-beliefs and mathematical learning and performance

Emotional and motivational phenomena also play a central role in how students engage with mathematics (Hannula, 2006; OECD, 2013c). Two important phenomena are attitudes (i.e., cognitive and affective orientations or dispositions towards an object, idea, person, situation, etcetera; Fiske & Taylor, 2008) and self-beliefs (i.e., expectations and confidence about one's capabilities and competence in accomplishing academic tasks and - in this thesis - mathematical tasks in particular; Chiu & Klassen, 2010; Skaalvik & Skaalvik, 2006; Valentine, DuBois, & Cooper, 2004).

Positive attitudes towards mathematics are often associated with higher learning outcomes, and it is likely that these are mutually reinforcing (Mullis et al., 2012; OCED, 2013c; Williams, Williams, Kastberg, & Jocelyn, 2005). One area of growing interest is how attitudes impact what students learn when using mathematical computer tools, an issue that is not yet well understood (Gravemeijer, 2005; Pierce & Stacey, 2004; Pierce, Stacey, & Barkatsas, 2007). Gaining insight into the relationships between tool use, individual psychological factors and learning could help educators and tool designers make tool-based learning arrangements and experiences more effective.

To what extent do attitudes and behaviours predict learning from mathematical computer tools? (Chapter 5)

Positive self-beliefs are widely reported to be related to higher performance, both generally and in mathematics specifically (Chiu & Klassen, 2010; Huang, 2011; Marsh & Martin, 2011; OECD, 2013c; Skaalvik & Skaalvik, 2006; Valentine et al., 2004). It is not known, however, whether self-beliefs affect the relation between students' performance in primary and in secondary school. This question is especially relevant in systems that make use of early tracking - such as the Netherlands and Germany - where a substantial degree of stability in performance between primary and secondary school is assumed. However, the stability of this relationship could be affected by the decline in self-beliefs - especially regarding mathematics - that is found between primary and secondary school (Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002; Mullis et al., 2012). It is, therefore, important to understand the extent to which self-beliefs influence the relation between mathematical performance in primary and in secondary school in early tracking educational systems. This could inform educators in developing timely interventions for managing self-beliefs that could prevent performance from dropping below levels expected in designated tracks.

To what extent do self-beliefs mediate the relation between mathematical performance at the end of primary school and the end of lower secondary school?

Background factors

Thinking, learning and performance in mathematics are the result of a complex interplay of individual and contextual factors. Background characteristics of students, schools and educational systems can profoundly affect learning and teaching processes and their outcomes. These characteristics include *sex*, *socio-economic background* and *educational track*, which are addressed at various points in this thesis.

Sex differences in mathematical performance in favour of boys are widely reported (Buchmann, DiPrete, & McDaniel, 2008; Driessen & Van Langen, 2010; Stoet & Geary, 2013). Yet, while the latest TIMSS report shows growing equity between boys and girls in mathematics achievement in many countries, Dutch girls in Grade 4 perform consistently below boys (Mullis et al., 2012), a finding confirmed by Dutch national assessments (Van Weerden, Hemker, Straat, & Mulder, 2013). Among older students (15-year-olds), boys outperform girls in mathematics across OECD countries and the performance gap in their favour has widened over the past decade in several countries, including the Netherlands (OECD, 2013a). In addition, even when girls perform as well as boys in mathematics, their attitudes towards mathematics and their mathematical self-beliefs are less favourable (Else-Quest, Hyde, & Linn, 2010; Meelissen & Luyten, 2008; OECD, 2013c). These findings have serious implications for enrolment in mathematics-related courses in senior secondary school and higher education and for later employment in STEM-related (science, technology, engineering and mathematics) fields (Crombie et al., 2005; Meelissen & Luyten, 2008; Van Langen, Rekers-Mombarg, & Dekkers, 2006). It is, therefore, of great importance to unravel differences between boys and girls in processes and factors underlying mathematical thinking, learning and performance that may help to narrow the gap between them.

Socio-economic differences are also related to school achievement and educational outcomes in general. Families with greater socio-economic capital provide advantages such as better schooling or extra tutoring, a stronger parental involvement with schooling, and a home environment that stimulates educational success, all of which may be absent in families with low socio-economic capital (Lamb, Markussen, Teese, Sandberg, & Polesel, 2011; Marks, Cresswell, & Ainley, 2006; Schütz et al., 2008). At school and societal levels, socio-economic differences are related to the availability and quality of educational resources (e.g., facilities such as computers and libraries) and subsequently students' educational opportunities (Causa & Chapuis, 2010; Lamb et al., 2011; Marks et al., 2006; Schütz et al., 2008). Socio-economic differences are evident in mathematics outcomes also. The latest PISA assessment reports that, across OECD countries, mathematics performance of more socio-economically disadvantaged students lags considerably behind that of more-advantaged students - the equivalent of nearly one year of schooling (OECD, 2013b).

Educational tracking is the practice of stratifying students according to general scholastic ability with the idea of creating more homogeneous learning environments. However, tracking can have considerable negative effects on student outcomes in lower tracks and when tracks are determined early in secondary school (Schütz et al., 2008). Lower tracks tend to be assigned less experienced or capable teachers and to offer less stimulating learning environments (Gamoran, 2004; Hanushek & Wößmann, 2006). Teacher expectations may be lower, which in turn impacts students' own expectations, self-beliefs, engagement and efforts (Marks et al., 2006). Furthermore, students in lower tracks are unable to benefit from being in an academically more successful milieu (Gamoran, 2004; Hanushek & Wößmann, 2006). Extensive and early tracking also appears to exacerbate sex and socio-economic inequalities, being less favourable to achievement of girls (Driessen & Van Langen, 2010) and students from disadvantaged backgrounds (Hanushek & Wößmann, 2006; Marks et al., 2006; OECD, 2013d; Schütz et al., 2008; Van de Werfhorst & Mijs, 2010). Thus, it is important to consider tracking effects on educational and - in this thesis - mathematical outcomes.

THESIS OVERVIEW

Objectives

This thesis presents research addressing a number of issues in mathematical thinking, learning and performance that have been signalled both by educational practitioners in the Netherlands and in international comparative studies (TIMSS, PISA). The thesis is informed by and contributes to theories and insights from the educational sciences, cognitive psychology and neuropsychology. A particular aim is to translate its findings into implications for educational professionals and policy makers to help improve learning and teaching mathematics in schools.

Approach

To achieve these objectives, four studies were carried out that focused on interventions or instruments aiming to stimulate or understand specific aspects of mathematical thinking and learning in the classroom. The studies involved preschoolers (on causal reasoning), primary school students (on factual knowledge and solution strategies in simple multiplication in Grades 3 and 4) and lower secondary school students (on attitudes and behaviours when learning mathematics with computer tools in Grades 7 and 8). The classroom studies were performed at primary and secondary schools in the Netherlands. Three further studies made use of self-report data and cognitive and performance measures from large-scale, cross-sectional and longitudinal datasets for lower secondary school students

(Grades 7 to 9) in the Netherlands. These studies investigate how mathematical performance relates to performance in other academic areas, higher-order thinking skills, and self-beliefs.

Chapter outline

The research is presented in two parts: Part One comprises the four classroom studies and Part Two the three survey studies. Concluding remarks are presented in the final chapter.

Part One (Classroom Studies)

Chapter 2 presents a cross-case comparison descriptive study examining how shared storytelling within a peer-group setting could stimulate causal reasoning in preschool children. Children were allocated to one of several groups that participated in a picture-book based storytelling intervention. Storytelling discourse was analysed in the groups that showed the lowest and the highest pre-to-post-intervention improvement on a series of causal reasoning tasks. The findings are discussed in relation to research on and implications for stimulating causal reasoning in early childhood education.

Chapter 3 investigates whether training children to inhibit interference from incorrect competing answers improves fluency in simple multiplication facts. Children in Grades 3 and 4 were placed in either a conventional *recall* practice condition - where they produced answers to problems - or a *choice* practice condition - where they had to choose between competing answers that represent commonly-made errors. Differential effects of practice conditions on performance on fluency tests are explained in terms of transfer demands and the relative merits of recall and choice tasks in multiplication fact learning are discussed.

Chapter 4 investigates the extent to which third graders' verbal reports accurately represent their thinking processes when solving simple multiplication problems. The aim is to determine whether soliciting verbal reports could help teachers in mainstream schools identify children who consistently use more immature strategies than their age-level peers. This is done by assessing the degree to which verbal reports correspond to known patterns in children's multiplication performance.

Chapter 5 presents a mixed methods study that investigates the effects of attitudes and behaviours on the outcomes of learning mathematics with computer tools for students in Grades 7 and 8. Relationships between attitudes towards mathematics, learning behaviours and learning outcomes (i.e., conceptual understanding and tool mastery) are explored and recommendations are given for promoting learning with mathematical computer tools.

Part Two (Survey Studies)

Chapter 6 reports on a large-scale, cross-sectional study that investigates performance patterns in academic tracks in the first three years of secondary school (Grades 7 to 9) within a stratified educational system. By identifying patterns that reveal how competence levels in different domains are related at different stages of development and by comparing low performers with other students, the study sheds light on individual and educational aspects that may be amenable to intervention and help low performers keep on track.

Chapter 7 investigates the relationship between abstract and relational thinking skills and mathematical performance at the end of lower secondary school (Grade 9) in a large-scale cohort study. The findings support paying attention to abstract and relational thinking skills in secondary school as a means to improve students' mathematics performance. The study also provides insights into the influence of specific individual factors that are relevant in current debate on the determinants of educational progression.

Chapter 8 reports on a large-scale longitudinal cohort study that examines whether students' self-beliefs mediate the relation between mathematics performance at the end of primary school (Grade 6) and the end of lower secondary school (Grade 9) in an early tracking educational system. The role of individual differences in sex and educational track is also investigated. Suggestions are made for managing self-beliefs to help safeguard performance levels expected in designated tracks.

Finally, ***Chapter 9*** presents the general conclusions of the thesis and reflects on the findings of the preceding chapters in terms of the thesis approach and objectives.