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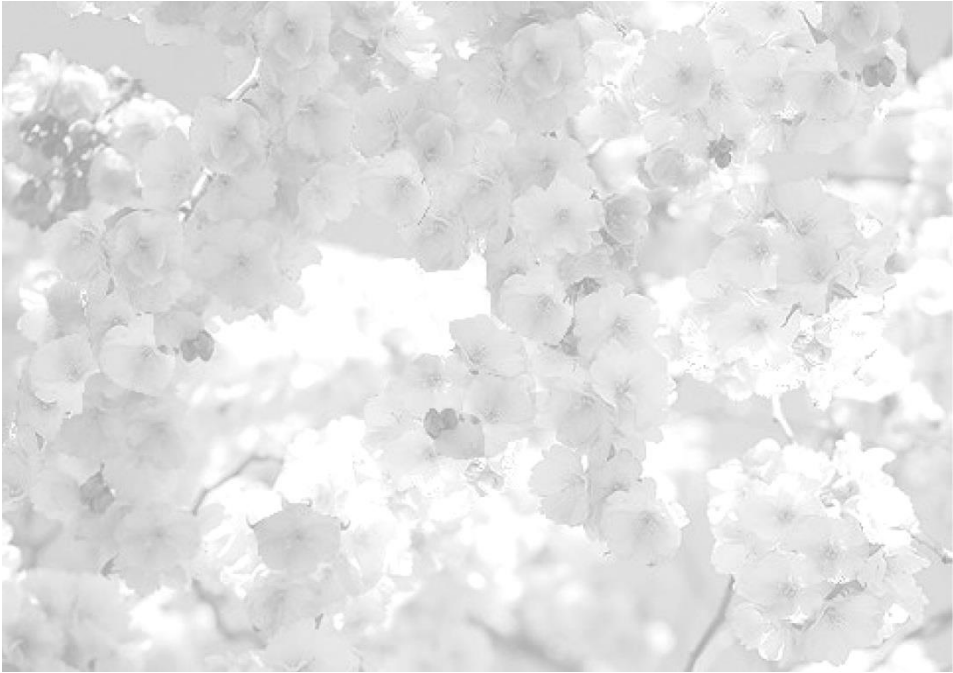
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## Concluding Remarks





## INTRODUCTION

This thesis presents research addressing a number of issues in mathematical thinking, learning and performance signalled both by educational practitioners in the Netherlands and in international comparative studies (TIMSS, PISA). The thesis takes a multidimensional approach to tackling these issues, involving a wide range of age groups and educational settings, diverse methods, and combining theories and insights on brain, cognition, development and education from different scientific disciplines. The thesis addresses cognitive challenges in mathematical thinking and learning as well as affective and motivational factors that affect how students interact with mathematics education. Furthermore, it recognises the influence of individual differences in sex and socio-economic background as well as characteristics of the Dutch educational system (e.g., educational tracking).

The thesis has two main objectives. First, to contribute to theories and insights regarding mathematical thinking, learning and performance in school children. Second, to translate its findings into implications for educational practice and policy that could help improve mathematical teaching and learning in schools. This final chapter reflects on the reported research in terms of the thesis approach and objectives.

## ON CROSS-POLLINATION OF PERSPECTIVES ON LEARNING

The presented research seeks to understand mathematical thinking, learning and performance from multiple perspectives on the learning child and adolescent. A distinguishing feature of the thesis is that it combines insights from the educational sciences (e.g., socio-cultural theory, Chapter 2; learning behaviours, Chapter 5), cognitive psychology (e.g., organisation of arithmetic facts in long-term memory, Chapter 3; multiplication strategy-use, Chapter 4) and neuropsychology (e.g., individual characteristics, various Chapters; affective and motivational factors, Chapters 5, 8; developmental differences, Chapters 3, 6, 7; brain maturation and functioning, Chapters 3, 4, 7).

For example, insights from neuroimaging research were essential for interpreting children's behaviour and verbal reports in the context of arithmetic learning (Chapters 3 and 4). Knowledge of functional changes in brain activity during arithmetic fact learning (Ischebeck, Zamarian, Egger, Schocke, & Delazer, 2007) allowed informed inferences to be drawn from behavioural measures regarding how choice tasks could contribute to multiplication fact learning. Similarly, neuroimaging studies that reveal the specific brain regions involved in mental arithmetic and arithmetic fact retrieval (Dehaene, 2011; De Smedt, Taylor, Archibald, & Ansari, 2010) allowed us to understand why children with higher phonological skills perform

procedural strategies more quickly and accurately than children with lower phonological skills. As these examples illustrate, multiple perspectives on the 'learner' allow a more complete understanding of the processes underlying mathematical thinking, learning and performance.

Conversely, combining multiple perspectives can generate new research hypotheses. For example, the finding that brain activity during abstract and relational thinking tasks - such as underlie secondary school mathematics - involves brain networks that undergo protracted maturation (e.g., Bazargani, Hillebrandt, Christoff, & Dumontheil, 2013; Crone et al., 2009; Eslinger et al., 2009) suggested that a mismatch between educational demands and the rate of maturational change in these brain areas could impact mathematical competence during the initial years of secondary school. This led us to examine the relationship between abstract and relational thinking and mathematical performance in lower secondary school (Chapter 7).

Thus, combining multiple perspectives on learning leads to a fruitful cross-pollination of ideas. We believe that this kind of multidisciplinary approach has much added value. Nonetheless, there are risks of a too precipitous mixing of ideas from different disciplines if the underlying theories, perspectives and methodologies are not well understood. For this reason, it is important that research continues to investigate where common ground lies (Ansari, Coch, & De Smedt, 2011; De Smedt & Verschaffel, 2010; Ferrari, 2011; Howard-Jones, 2011), and that both researchers and teachers learn how to look beyond disciplinary boundaries.

## **ON PROMOTING THINKING SKILLS**

A theme of this thesis is the promotion of thinking skills throughout the long lines in mathematics education, where the effectiveness of later learning depends on the strength of earlier acquired knowledge and skills (KNAW, 2009; NMAP, 2008). While much research, policy and practice focus on substantive domains of mathematics, there is surprisingly little attention in schools and curricula for timely development of key skills as causal reasoning, higher-order thinking (e.g., reflection, abstract thinking, relational thinking) and the acquisition of academic language (i.e., complex, abstract, cognitively demanding language underlying communication at school; Laghzaoui, 2011; Schleppegrell, 2004).

This thesis contributes to redressing this imbalance by according a central role to these kinds of thinking skills. In Chapter 2, we investigated causal reasoning in preschoolers and found that young children can be stimulated to interact in ways that are conducive to causal thinking. In Chapter 5, we found that the extent to which students learn from mathematical computer tools requires them to reflect on the understanding acquired about tool techniques and their relationship to mathematical

tasks. In Chapter 6, we concluded that students' ability to use academic language to access content areas significantly influences their performance at the beginning of secondary school. In Chapter 7, we found that abstract and relational thinking skills predict mathematical performance in lower secondary school.

Together, these studies underscore the importance of these kinds of skills for mathematical thinking, learning and performance. In addition, the studies highlight the need for continuing classroom research in these areas and give concrete recommendations for improving educational practice and the professional knowledge and skills of teachers. For example, we described how teachers could help students reflect on new understandings and connect them to mathematical concepts (Chapter 5), and we discussed instructional practices that give students opportunities to reason about the connections and relations between mathematical ideas (Chapter 7).

### **ON LISTENING TO WHAT CHILDREN SAY**

Three studies in this thesis analysed children's talk during various learning tasks. In Chapter 2, group discourse was analysed when preschoolers told stories together based on picture-book stimuli. In Chapter 4, children were interviewed about how they solve simple multiplication problems. In Chapter 5, communication in dyads was analysed during learning with a computer tool. These studies underscore that what children say about what they think and do may provide one of the most informative pictures of their cognitive processing. On the one hand, this can help to build and test more complete theories of cognitive development (Robinson, 2001; Taylor & Dionne, 2000; Sherin & Fuson, 2005). On the other, it allows individual differences in how children think and learn to be explored.

For teachers, listening to what children say can help identify difficulties they encounter during learning. To that end, teachers need to acquire skills for soliciting and listening to children's talk in an unbiased way. For instance, in soliciting children's verbal reports of multiplication strategy use (Chapter 4), we found that it was necessary to strictly proscribe the adult's questions and prompts and that considerable practice was required to avoid influencing what children said. While this approach produces more reliable and generalisable data, it proved insufficient to fully unravel individual differences in children's thinking. Thus, more research is needed to ascertain how this kind of information can be reliably and validly obtained. At the same time, teachers need to increase their knowledge of problems children encounter when learning mathematics, so that they are well-equipped to respond adequately to difficulties that they uncover in this way.

## ON INDIVIDUAL DIFFERENCES

The large-scale survey studies reported in Part Two of the thesis examined the influence of individual differences in sex and socio-economic background on mathematical thinking, learning and performance in lower secondary school. The studies confirm the general finding that boys have higher mathematical performance and self-beliefs than girls (Buchmann, DiPrete, & McDaniel, 2008; Else-Quest, Hyde, & Linn, 2010; Herbert & Stipek, 2005; Ireson & Hallam, 2009; Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002; Meelissen & Luyten, 2008; OECD, 2013a, 2013c; Stoet & Geary, 2013). As noted, this can have far-reaching consequences for female participation in important fields of higher education and employment (Crombie et al., 2005; Van Langen, Rekers-Mombarg, & Dekkers, 2006). It is, therefore, crucial to keep the issue of sex differences in mathematics high on the agenda of researchers, teachers and parents. For instance, teachers and parents need to be made aware of the socialisation impact of sex-stereotypical views on students' mathematics self-concept (Herbert & Stipek, 2005).

The thesis also generated new insights into the mechanisms by which socio-economic background could affect school outcomes and mathematics performance. When we examined the role of the language of schooling on school performance (Chapter 6) and the effects of understanding abstract concepts on mathematical performance (Chapter 7), we concluded that traditional instructional materials, which rely heavily on the ability to understand complex and abstract language, disadvantage students who lack proficiency in the language of schooling (e.g., from homes where the language of schooling is not the first language or from environments that offer little language stimulation). We proposed two ways in which the educational field could tackle this problem. First, curriculum developers and educational publishers could develop alternative methods of instruction (e.g., using visuospatial materials) to help these students acquire initial knowledge of content areas. Our results suggest that acquisition of initial knowledge may bootstrap further knowledge acquisition, independent of language proficiency. Second, we proposed that specific instructional attention should be given to helping students to understand the meanings of abstract mathematical concepts (e.g., 'differential', 'exponential', 'permutation') before expecting them to master their use.

The thesis also revealed other important sources of individual differences that affect mathematical thinking, learning and performance. In Chapter 5, it was found that students with positive attitudes towards mathematics profited more from using a mathematical computer tool, while higher ability students with positive attitudes towards computer tools learned less. In Chapter 8, we found a negative effect of high self-efficacy at the end of primary school on mathematical performance in lower secondary school, relative to classmates in the same educational track. When positive

attitudes and self-beliefs influence performance in a negative sense, it is imperative to understand why and to develop measures to prevent this happening. In these chapters, we proposed ways to manage attitudes and self-beliefs when these are dysfunctional with respect to particular educational challenges.

### **ON EARLY EDUCATIONAL TRACKING**

The thesis produced some important findings that contribute to ongoing debate as to the advantages and disadvantages of highly differentiated early tracking systems. When we examined how competence levels in different domains are related at different stages of lower secondary school (Chapter 6), we found that no general factor underlies performance in all domains, and that competence in science and mathematics appears to develop along a different path to competence in other domains. These findings argue against early stratification on the basis of 'general ability' and support a more modular approach that recognises *intra*-individual differences. Specifically, students should be offered instruction at their own level of competence in specific domains or content areas, for example through ability groupings specific to content areas. We also noted that tracks differ with regard to content, coverage and level of intellectual complexity (Chapter 7). Students in lower tracks have reduced exposure to material that requires - and could stimulate - higher-level thinking skills. Thus, differential educational experiences of students in different tracks may accentuate or even create and then perpetuate differences in skill levels.

Furthermore, negative effects of early tracking with respect to exacerbation of social-economic inequalities are being increasingly recognised (Hanushek & Wößmann, 2006; Schütz, Ursprung, & Wößmann, 2008; Shavit & Müller, 2000; Van de Werfhorst & Mijs, 2010). We, too, found that students with lower parental level of education were overrepresented in lower tracks and underrepresented in higher tracks (Chapter 7). When students are stratified early, the impact of parental resources is disproportionately high compared with stratification later in their school careers. Parents with socio-economic advantages are in a better position to directly or indirectly promote their children's chances than disadvantaged ones (Lamb, Markussen, Teese, Sandberg, & Polesel, 2011; Marks, Cresswell, & Ainley, 2006; OECD, 2013d; Schütz et al., 2008).

It is also important to realise that cortical networks that are essential for learning continue to mature throughout adolescence (Crone & Dahl, 2012; Menon, 2013). This makes it highly likely that assessment of scholastic ability in early adolescence underestimates the potential of large numbers of students who have yet to achieve more mature levels of functioning. Taken together, these arguments support the view



that highly differentiated early tracking in secondary school may curb the intellectual potential of many students.

## **CONCLUSIONS**

It is clear that a wide range of factors relating to educational structures, instructional methods and materials, individual characteristics, and experiences within the classroom can affect students' mathematical thinking, learning and performance. This thesis helps to connect some of these issues and improve understanding of them. It underlines the importance of combining multiple perspectives on learning, of promoting thinking skills, of listening to what children say about what they think and learn, and of striving to understand the sources of individual differences. In all of this, the thesis provides concrete pointers for educational policy and practice that could be extended in further applied research (e.g., evidence-based, evidence-supported, evidence-informed) aiming to improve mathematical teaching and learning in schools. The presented research thus makes a contribution to bridging the much observed and commented on gap between educational research and practice (Broekkamp & Van Hout-Wolters, 2007; Burkhardt & Schoenfeld, 2003; Coonen & Nijssen, 2011; KNAW, 2009). A particular strength of the thesis is that it approaches this gap from both sides: on the one hand by translating its findings into implications for educational policy and practice, and on the other by enhancing scientific insights into issues that are signalled by and relevant to educational professionals.