\[ q_i \frac{k}{\rho_s} - q_i \left[ \frac{\gamma}{\rho_i} + b_i (\varphi - \alpha_i) \right] - \lambda_i \left[ \frac{\gamma}{\rho_i} + b_i (\varphi - \alpha_i) \right] = 0; \quad (3.54) \]

\[ q_i \left[ \frac{P}{\rho_i} - \alpha_i + \frac{b_i^2}{2} \right] + \lambda_i \left[ \frac{P}{\rho_i} - \alpha_i + \frac{b_i^2}{2} \right] = 0; \quad (3.55) \]

\[-tq_i + \lambda_i = 0. \quad (3.56)\]

Hence, for (3.54) we have \( \frac{k}{\rho_s} = (1 + t) \left[ \frac{\gamma}{\rho_i} + b_i (\varphi - \alpha_i) \right] \). That means that

\[ \frac{k}{\rho_s(1+t)} = G = \frac{\gamma}{\rho_i} + b_i (\varphi - \alpha_i) \]

and hence

\[ b_i^* = \frac{\rho_i G - \gamma}{\varphi - \alpha_i^*}. \quad (3.57) \]

Next, for (3.55) we have \( (1 + t)q_i \left[ \frac{P}{\rho_i} - \alpha_i + \frac{b_i^2}{2} \right] = 0 \), and hence

\[ \alpha_i^* = \frac{P}{\rho_i} + \frac{(b_i^*)^2}{2}. \quad (3.58) \]

(3.57) and (3.58) are exactly equal to (3.17) and (3.21). The proof of the latter is easy. Rewriting (3.57) we have \( \rho_i b_i^* (\varphi - \alpha_i^*) = \rho_i G - \gamma \). Using (3.58), this becomes \( \rho_i G - \gamma = \rho_i b_i^* \left( \varphi - \frac{P}{\rho_i} - \frac{(b_i^*)^2}{2} \right) \). Dividing by \( \rho_i \) we have

\[ G - \frac{\gamma}{\rho_i} - b_i^* \left( \varphi - \frac{P}{\rho_i} \right) = -\frac{(b_i^*)^3}{2}. \quad (3.59) \]

3.A.6 Proof of Proposition 3.4

The Lagrangian of the government’s optimization problem (3.4) is the following:
\[ L = \sum_{i \in \{P,I\}} \frac{q_i b_i}{\rho_i} + \sum_{i \in \{P,I\}} q_i \left[ \frac{p^2}{2\rho_i^2} - \frac{b^2_i}{\rho_i} - \frac{y b_i}{\rho_i} - \frac{b^2_i}{\rho_i^2} \left( \frac{p - b_i}{\rho_i} - \frac{b_i}{\rho_i} \right) \right] - t \sum_{i \in \{P,I\}} q_i s_i + \sum_{i \in \{P,I\}} \lambda_i \left[ s_i - \frac{y b_i}{\rho_i} - \frac{b_i^2}{2} \right] \left( \frac{p - b_i^2}{2\rho_i} + \frac{b_i^2}{2} \right) + \sum_{i \in \{P,I\}} \mu_i \left[ s_i - \frac{y b_i}{\rho_i} - \frac{b_i^2}{2} + \frac{p b_i^2}{2\rho_i} + \frac{b_i^4}{8} - s_i + \frac{y b_i}{\rho_i} + \frac{p b_i^2}{2\rho_i} - \frac{b_i^2}{2} \right]. \]  

(3.60)

where \( \lambda_i \geq 0 \) Kuhn–Tucker multiplier associated with type \( i \)'s participation constraint \( (i = \{P,I\}) \); see ((3.20)), and \( \mu_i \geq 0 \) is the multiplier associated with the incentive compatibility constraint of type \( i = \{P,I\} \). The corresponding Kuhn–Tucker conditions are:

\[
\frac{\partial L}{\partial b_p} = q_p k - (q_p + \lambda_p) \left[ \frac{y}{\rho_p} + b_p \left( \frac{p - b_p}{\rho_p} - \frac{b_p^2}{2} \right) \right] - \mu_p \left[ \frac{y}{\rho_p} + b_p \left( \frac{p - b_p}{\rho_p} - \frac{b_p^2}{2} \right) \right] + \mu_i \left[ \frac{y}{\rho_i} + b_i \left( \frac{p - b_i}{\rho_i} - \frac{b_i^2}{2} \right) \right] = 0; \tag{3.61}
\]

\[
\frac{\partial L}{\partial b_i} = q_i k - (q_i + \lambda_i) \left[ \frac{y}{\rho_i} + b_i \left( \frac{p - b_i}{\rho_i} - \frac{b_i^2}{2} \right) \right] + \mu_p \left[ \frac{y}{\rho_p} + b_p \left( \frac{p - b_p}{\rho_p} - \frac{b_p^2}{2} \right) \right] - \mu_i \left[ \frac{y}{\rho_i} + b_i \left( \frac{p - b_i}{\rho_i} - \frac{b_i^2}{2} \right) \right] = 0; \tag{3.62}
\]

\[
\frac{\partial L}{\partial s_p} = -t q_p + \lambda_p + \mu_p - \mu_i = 0; \tag{3.63}
\]

\[
\frac{\partial L}{\partial s_i} = -t q_i + \lambda_i - \mu_p + \mu_i = 0; \tag{3.64}
\]

\[
\lambda_p \left[ S_p - \frac{y b_p}{\rho_p} - \frac{b_p^2}{2} + \frac{p b_p^2}{2\rho_p} + \frac{b_i^4}{8} \right] = 0; \quad \lambda_p \geq 0; \quad S_p - \frac{y b_p}{\rho_p} - \frac{b_p^2}{2} + \frac{p b_p^2}{2\rho_p} + \frac{b_i^4}{8} \geq 0; \tag{3.65}
\]

\[
\lambda_i \left[ S_i - \frac{y b_i}{\rho_i} - \frac{b_i^2}{2} + \frac{p b_i^2}{2\rho_i} + \frac{b_i^4}{8} \right] = 0; \quad \lambda_i \geq 0; \quad S_i - \frac{y b_i}{\rho_i} - \frac{b_i^2}{2} + \frac{p b_i^2}{2\rho_i} + \frac{b_i^4}{8} \geq 0; \tag{3.66}
\]

\[
\mu_p \left[ S_p - \frac{y b_p}{\rho_p} - \frac{b_p^2}{2} + \frac{p b_p^2}{2\rho_p} + \frac{b_i^4}{8} - S_i - \frac{y b_i}{\rho_i} - \frac{b_i^2}{2} + \frac{p b_i^2}{2\rho_i} - \frac{b_i^4}{8} \right] = 0; \quad \mu_p \geq 0; \tag{3.67}
\]

\[
\mu_i \left[ S_i - \frac{y b_i}{\rho_i} - \frac{b_i^2}{2} + \frac{p b_i^2}{2\rho_i} + \frac{b_i^4}{8} - \frac{p b_i^2}{2\rho_i} - \frac{b_i^4}{8} \right] = 0; \quad \mu_i \geq 0; \tag{3.68}
\]

\[
S_i - \frac{y b_i}{\rho_i} - \frac{b_i^2}{2} + \frac{p b_i^2}{2\rho_i} + \frac{b_i^4}{8} - \frac{p b_i^2}{2\rho_i} - \frac{b_i^4}{8} \geq 0; \tag{3.69}
\]
In the complete information case, we can ignore Kuhn–Tucker conditions (3.67) and (3.68) and set $\mu_p = \mu_I = 0$. Now, (3.63) and (3.64) imply that $\lambda_i = tq_i \geq 0$ so that $S_i^{*E} = \frac{y b_i^{*E}}{\rho_i} + \frac{\varphi (b_i^{*E})^2}{2} - \frac{p (b_i^{*E})^2}{2 \rho_i} - \frac{(b_i^{*E})^4}{8}$ (see (3.65) and (3.66)). Next, the optimal conservation level is implicitly defined by $G - \frac{Y}{\rho_i} - b_i^{*E} \left( \varphi - \frac{p}{\rho_i} \right) = -\frac{(b_i^{*E})^3}{2}$ where $G \equiv (\rho_S(1 + t))^{-1} k$, as desired.

**3.A.7 Proof of Proposition 3.5**

The complete information solution can be implemented if $\mu_p = \mu_I = 0$ does not yield a contradiction in (3.61)-(3.68). Note that if $\mu_p = \mu_I = 0$ it holds that $\lambda_i = tq_i \geq 0$, and thus $S_i^{*E} = \frac{y b_i^{*E}}{\rho_i} + \frac{\varphi (b_i^{*E})^2}{2} - \frac{p (b_i^{*E})^2}{2 \rho_i} - \frac{(b_i^{*E})^4}{8}$ for $i = \{P, I\}$ which does not yield a contradiction in (3.61)-(3.68). Hence, inserting (3.22) into (3.25) and dividing by $b_j^{*E}$, we have

$$0 \geq \frac{Y}{\rho_j} - \frac{P b_j^{*E}}{2 \rho_j} - \frac{Y}{\rho_i} + \frac{P b_j^{*E}}{2 \rho_i}. \quad (3.69)$$

Rearranging terms yields

$$\frac{Y}{\rho_i} - \frac{Y}{\rho_j} \geq \frac{b_j^{*E}}{2} \left( \frac{P}{2 \rho_i} - \frac{P}{2 \rho_j} \right). \quad (3.70)$$

Substituting $i, j = \{I, P\}$ with $i \neq j$ in (3.70), we have

$$b_i^{*E} \leq \frac{2Y}{P} \leq b_p^{*E}. \quad (3.71)$$

**3.A.8 Incentive compatibility of the complete information solution when there are more than two farmer types and when land quality is malleable at the time of the policy launch**

Since land quality is endogenous it holds that $\alpha_i = \alpha_i^{E} = \frac{P (1 - b_i)}{\rho_i} + \frac{b_i^2}{2}$. Similar to the analysis in Appendix 3.A.3, assume that farmers of type $\{1, 2, \ldots, \eta\}$ should participate, but now $\bar{\rho}$ is defined by
\[
\rho_i > \frac{\gamma + 0.5P(b_i^{*N})^2 + \sqrt{\left[\gamma + 0.5P(b_i^{*N})^2\right]^2 + 4GP^2(1 - b_i^{*N})}}{2G} = \bar{\rho}.
\]  

(3.72)

In this case, the Lagrangian of the government’s optimization problem (3.4) is the following:

\[
\begin{align*}
L &= \sum_{e \in [1,2,\ldots,n]} \frac{q_i b_i^k}{\rho_i} + \sum_{e \in [1,2,\ldots,n]} \kappa \left[ \frac{P^2(1 - b_i^2)}{2\rho_i^2} - \frac{b_i^2}{8} - \frac{\gamma b_i}{\rho_i} - \frac{b_i^2}{2} \left( \frac{P(1 - b_i)}{\rho_i} - \frac{b_i^2}{2} \right) \right] \\
&\quad - \sum_{e \in [1,2,\ldots,n]} \kappa S_i + \sum_{e \in [1,2,\ldots,n]} \mu_i \left[ S_i - \frac{P^2 b_i^2 (2 - b_i)}{2\rho_i^2} + \frac{b_i^4}{8} - \frac{\gamma b_i^2}{\rho_i} - \frac{\rho b_i^2}{2} - S_j + \frac{P^2 b_j^2 (2 - b_j)}{2\rho_j^2} \right] \\
&\quad + \sum_{e \in [1,2,\ldots,n]} \sum_{e \in [1,2,\ldots,n]} \mu_{ij} \left[ S_i - \frac{P^2 b_i^2 (2 - b_i)}{2\rho_i^2} + \frac{b_i^4}{8} - \frac{\gamma b_i^2}{\rho_i} - \frac{\rho b_i^2}{2} - S_j + \frac{P^2 b_j^2 (2 - b_j)}{2\rho_j^2} \right]
\end{align*}
\]

(3.73)

Where \( \lambda_i \geq 0 \) is the Kuhn–Tucker multiplier associated with type \( i \)'s participation constraint, and \( \mu_{ij} \geq 0 \) is the multiplier associated with the incentive compatibility constraint of type \( i, j = \{1,2,\ldots,n\}, j \neq i \).

Solving the corresponding Kuhn–Tucker conditions, we have the optimal conservation levels implicitly defined by \( G - \frac{\gamma}{\rho_i} - b_i^{*N} \left( \varphi - \frac{P}{\rho_i} \right) - \frac{p^2(1-b_i^{*N})^2}{\rho_i^2} = -\frac{(b_i^{*N})^3}{2} \) and \( S_i^{*N} = \frac{p^2 b_i^{*N} (2-b_i^{*N})}{2\rho_i^2} - \frac{P (b_i^{*N})^2}{2\rho_i} + \frac{\gamma b_i^{*N}}{\rho_i} + \frac{\varphi (b_i^{*N})^2}{2} - \frac{b_i^{*N}}{8} \) as the complete information menu of subsidies when land quality is endogenous.

To find when this solution is incentive compatible, we solve \( \mu_{ij} = 0 \) for all \( i, j \in \{1,2,\ldots,n\}, j \neq i \). This yields \( \lambda_i = t q_i > 0 \), and thus \( S_i^{*N} = \frac{p^2 b_i^{*N} (2-b_i^{*N})}{2\rho_i^2} - \frac{P (b_i^{*N})^2}{2\rho_i} + \frac{\gamma b_i^{*N}}{\rho_i} + \frac{\varphi (b_i^{*N})^2}{2} - \frac{b_i^{*N}}{8} \). Substituting \( S_i^{*N} \) and \( \pi_i^{*N} \) in (3.20) and cancelling terms, we find that the complete information solution is incentive compatible if and only if, for all \( \{j,i\}, j < i, P > \gamma \), we have \( b_i^{*N} \leq \bar{b}_i^{N} (\rho_i, \rho_j) \leq b_j^{*N} \) with \( \bar{b}_i^{N} (\rho_i, \rho_j) = \left[ \frac{2\gamma}{P} + 2P \left( \frac{1}{\rho_i} + \frac{1}{\rho_j} \right) \right] / \left[ 1 + P \left( \frac{1}{\rho_i} + \frac{1}{\rho_j} \right) \right] \).

Note that if \( j < i \), we have \( \rho_j > \rho_i \) due to the ordering of types. Now, let us define \( v_{ij} \equiv \rho_j - \rho_i \). In this case, we have \( \bar{b}_i^{N} (\rho_i, \rho_j) = \left[ \frac{2\gamma}{P} + 2P \left( \frac{1}{\rho_i} + \frac{1}{\rho_j} \right) + \frac{1}{\rho_i + v_{ij}} \right] / \left[ 1 + P \left( \frac{1}{\rho_i} + \frac{1}{\rho_i + v_{ij}} \right) \right] \), and if \( P > \gamma \) we have \( \frac{\partial \bar{b}_i^{N}}{\partial \rho_i} = \left[ 1 + P \left( \frac{1}{\rho_i} + \frac{1}{\rho_i + v_{ij}} \right) \right] \).
\[ \left( \frac{1}{\rho_{i+\nu_{ij}}} \right)^{-2} \left[ 2 \left( \frac{1}{\rho_{i}^2} + \frac{1}{(\rho_{i+\nu_{ij}})^2} \right) (\gamma - P) \right] < 0. \] Hence, if \( P > \gamma \), the intersection point of the cost functions of farmers \( i \) and \( j \) is closer to the origin the more impatient the reference farmer type is. Having established the type-specific intersection point, necessary conditions for \( b_i^N \leq b_i^N(\rho_i, \rho_j) \leq b_i^N \) to be non-empty are that \( \frac{\partial b_i^N}{\partial \rho_i} < 0 \) and \( P > \gamma \).

### 3.A.9 Derivation of the second-best solution when land quality is endogenous

We first derive the second-best policy when \( b_i^E > \frac{2\gamma}{p} \). The complete information solution with \( \mu_p = \mu_j = 0 \) cannot be implemented because \( \mu_p = 0 \) now yields a contradiction in (3.67). Therefore, we are left with the possibility that \( \mu_p > 0 \) and \( \mu_i = 0 \). Now, (3.64) reads \( \lambda_i = \mu_p + t q_i > 0 \), so

\[ \lambda_p \geq 0 \text{ and } \lambda_i > 0. \]

Now, \( \lambda_p > 0 \) and \( \lambda_i > 0 \) would imply that

\[ S_i = \frac{y b_i}{\rho_i} + \frac{p b_i^2}{2} - \frac{p b_i^2 + n b_i}{2 \rho_i} - \frac{n \lambda_i}{8} \text{ with } i = \{P, I\}; \]

see (3.65) and (3.66). However, since \( \mu_p > 0 \) this implies that

\[ S_i = \frac{y b_i}{\rho_i} + \frac{p b_i^2}{2} - \frac{p b_i^2 + n b_i}{2 \rho_i} - \frac{n \lambda_i}{8} \text{ with } i = \{P, I\} \]

in the latter, and rearranging terms, yields \( b_i = \frac{2\gamma}{p} \) which contradicts \( b_i^E > \frac{2\gamma}{p} \). Hence, if \( \mu_p > 0 \) and \( \mu_i = 0 \) it needs to hold that \( \lambda_p = 0 \) and \( \lambda_i > 0 \). In this case the second-best solution reads:

\[
\begin{align*}
\frac{\gamma}{\rho_p} + b_p \left( \varphi - \frac{P}{\rho_p} \right) - \frac{b_p^3}{2} &= G; \\
\frac{\gamma}{\rho_p} + b_p \left( \varphi - \frac{P}{\rho_p} \right) - \frac{b_p^3}{2} &= G - \frac{t}{1 + t} + \frac{q_p}{q_i} (b_i P - \gamma) \left( \frac{1}{\rho_p} - \frac{1}{\rho_i} \right); \\
S_p - \frac{y b_p}{\rho_p} - \frac{p b_i^2}{2} - \frac{p b_i^2 + n b_i}{8} &= S_i + \frac{y b_i}{\rho_i} + \frac{p b_i^2}{2} - \frac{p b_i^2 + n b_i}{8} > 0; \\
S_i - \frac{y b_i}{\rho_i} - \frac{p b_i^2}{2} - \frac{p b_i^2 + n b_i}{8} &= 0.
\end{align*}
\]

The same analysis for \( b_i^E < \frac{2\gamma}{p} \) yields that the only feasible set of Kuhn-Tucker multipliers is \( \mu_p = 0, \mu_i > 0, \lambda_p > 0, \) and \( \lambda_i = 0 \). Hence, in this case the second-best solution also reads (3.74)-(3.77) except that all subscripts \( P \)
should now read \( I \), and vice versa. (3.74) states that the level of conservation provided by patient farmers should be such that the marginal costs of conservation equal the social marginal benefits of conservation. Note that since \( b_i^E > \frac{2\gamma}{p} \), \( b_i P - \gamma > 0 \) in (3.75). Hence, the level of conservation that is provided by impatient farmers is smaller than the level that would equate marginal conservation cost to the social marginal benefits of conservation. Furthermore, (3.76) indicates that patient farmers should be indifferent between contracts, while (3.77) shows that impatient farmers receive compensation that is exactly equal to the conservation costs incurred. Hence, in this case, patient farmers receive informational rents for the conservation they provide, while impatient farmers provide less conservation than is socially desirable.