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Bolzano's Notion of Grounding and the Classical Model of Science

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2014

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citation for published version (APA)

Roski, S. P. (2014). *Bolzano's Notion of Grounding and the Classical Model of Science*. [PhD-Thesis - Research and graduation internal, Vrije Universiteit Amsterdam].

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Conclusion

The aim of this thesis was a comprehensive analysis of Bolzano's theory of grounding against the background of Betti and de Jong's Classical Model of Science ([17]). A general hypothesis guiding my analysis was that grounding is Bolzano's construal of an explanatory consequence relation—a sense in which the theorems of a proper science may be said to *follow from*, and *be explained by*, its axioms.¹ In other words, it is Bolzano's construal of condition (3b) of the Classical Model of Science. That axioms and theorems of a proper science are connected by some kind of explanatory relation, or that they should follow an objective explanatory order, was a widespread assumption among adherents of the Classical Model of Science. Bolzano's views represent, in a sense, the peak of this tradition. In his mature writings, he was in possession of a highly general, technically sophisticated account of a consequence relation defined in terms of truth-preservation (deducibility), which anticipates the sense of 'following from' that was to become central to axiomatics in the 20th century. At the same time, though, Bolzano still fully embraced the view that *this* sense of 'following from' does not appropriately capture the sense pertinent in the context of a proper science. For, that truth is preserved from the premises to the conclusions of an argument does not guarantee that these premises also *explain* the conclusion. Against the background of his definition of deducibility, Bolzano could make perfectly clear *why* he thought this is the case. In pointing out what distinguishes these two senses of 'following from' and how they relate, Bolzano relied on many traditional methodological ideas, most of which can already be found in, or traced back to, Aristotle's **APo**.

In the first chapter this thesis, I have, in particular, mentioned *six claims* con-

¹ In the course of this summary I will use the term 'axiom' in the same sense as *starting point of a science*.

cerning grounding that come to the fore, in one way or another, throughout Bolzano's entire work (cf. Section I.2.2):

- (I) Grounding proceeds from more to less general truths.
- (II) Grounding proceeds from simple to more complex truths.
- (III) The grounds of a given consequence are uniquely determined.
- (IV) There are ungrounded truths.
- (V) Grounding is an asymmetric relation.
- (VI) Grounding gives rise to deductive economy.

In the remainder of this chapter, in which I will review the highlights of the previous chapters, I will frequently come back to Bolzano's take on these claims according to my interpretation.

I have investigated the roots of Bolzano's mature theory of grounding in his early **BD** (Chapter 2). The **BD** contain the ambitious attempt to capture an explanatory notion of *following from* in terms of a small set of simple formal rules (formal in roughly the sense in which syllogistic rules are formal) (Section 2.3). These rules are supposed to assure certain methodological desiderata for explanatory proofs and proper scientific presentations, which, I have argued, can be straightforwardly related to certain assumptions concerning explanations (Section 2.4). (*That* they can be so related, I have further argued, explains *why* Bolzano takes his rules to capture an explanatory sense of 'following from'.) Bolzano's desiderata reveal that by the time of the **BD**, he assumes that grounding proceeds from more to less general [(I)] and from less to more complex truths [(II)]. He further assumes that explanatory proofs must not invoke superfluous premises or cross to other kinds [\approx (VI)], and that there is no explanatory overdetermination (i.e. the ground for any given grounded truth is uniquely determined) [(III)]. My investigation of Bolzano's arguments to the effect that his small set of formal rules in fact captures these desiderata (in Sections 2.4.1, 2.4.2 and 2.4.3), revealed further assumptions of him that remained central to his mature theory of grounding: on the one hand, the assumption that grounding is asymmetric [(V)], and, on the other hand, the assumption that some truths are ungrounded [(IV)]. While most of the main *ideas* around which Bolzano's views on grounding center are thus already in place in the **BD**, I have shown that he is not very successful in spelling them out in an expedient way. Still mostly relying on the mainstream logics of the 18th century, he often runs into technical and conceptual problems resulting from an imprecise conception of, for instance, the relation between content and extension of concepts. The major problem of the **BD** is, however, the lack of a clear distinction between an explanatory and a merely truth-preserving sense of 'following from'. Only against the background of his rigorous explication of the latter in the **WL**, he managed to properly clarify his ideas on the former in a satisfactory way.

Having emancipated himself from the relatively poor logical framework of the

BD, Bolzano pursues a different strategy of clarifying the notion of grounding in his mature writings. Against the background of the powerful logical apparatus of the WL (for which cf. Chapter 3), he identifies highly general characteristics an explanatory notion of *following from* ought to possess according to him, and by which such a notion can be precisely distinguished from, and related to, notions such as deducibility and causality (Chapters 4 and 5). I have distinguished two kinds of principles by which he captures these characteristics: on the one hand, principles that apply to each case of grounding irrespective of any properties of the respective relata, and, on the other hand, more specific principles that are formulated in terms of complexity, content or extension of the relata. I have called the collection of the former principles the *pure* logic of grounding and the collection of the latter the *impure* logic of grounding.

Two central principles of Bolzano's pure logic of grounding are to the effect that grounding is asymmetric [(V)] and that some truths do not have a ground [(IV)] (Section 4.4). On my interpretation, a third crucial principle of this part of Bolzano's theory is that for each truth, there is at most one complete ground [(III)] (Section 4.4.2). While Bolzano is admittedly not fully clear on this point, and while the claim is responsible for some difficulties I have discussed throughout this thesis, it seems that he relies on it far too often at far too crucial places in order not to ascribe it to him. I have shown that from the aforementioned three claims, a significant part of various other principles that belong to Bolzano's pure logic of grounding can be derived (Section 4.5). A further claim of Bolzano's pure logic of grounding that is independent of this part is to the effect that grounding is a serial relation, i.e. that every truth and every collection of truths is the ground of some other truths. While this claim seems harmless at first, I have shown that it reveals severe tensions between Bolzano's pure and the impure logic of grounding.

In Bolzano's impure logic of grounding, the ideas expressed by claims (I) and (VI) above take center stage. Bolzano argues that in the realm of conceptual truths, grounds are never more complex than their consequences (Section 5.2.1). He shows that his particular formulation of this claim, together with the assumption that there are only finitely many simple concepts (and some less controversial assumptions), entails that each conceptual truth is dependent on at most finitely many other conceptual truths (Section 5.2.2). In the realm of conceptual truths, every explanation must thus ultimately come to an end at some fundamental truths. Bolzano further argues for a certain contentual relation between grounds and consequences (Section 5.2.3). I have highlighted a number of passages in which he suggests that the grounds of a given conceptual truth ψ are not only at most as complex as ψ , but also do not contain any occurrence of simple concepts that do not also occur in ψ as well. I have argued that this claim can be very naturally related to a certain ideal of *purity* of explanatory proofs. It is perhaps only due to the fact that Bolzano still thought that many such proofs have the form of a syllogism that he did not fully embrace this idea and refrained from stating it as a generally valid principle.

A traditional assumption that Bolzano still defended in the **BD** is rejected in the **WL** in its most straightforward version: Bolzano denies that grounding must proceed from more to less general truths. Hence, claim (I) above is not part of Bolzano's mature views on grounding (Section 5.4). What Bolzano argues instead is that if a deductively valid argument is explanatory, it contains the most general premises from which the conclusion is deducible (each of these premises may be less general than the conclusion, though). I have pointed out, however, that it is not fully clear how to spell out this particular claim in the logical framework of the **WL**. Another claim on grounding that relates to the notion of generality is that explanatory arguments are *generalizable* in a certain sense. Bolzano argues that, under certain restrictions, variants of explanatory arguments are explanatory themselves (Section 5.5.5).

At the center of my discussion of Bolzano's impure logic of grounding have been a number of principles concerning the relation between grounding and deducibility (Section 5.3). A fruitful perspective on these principles, I have maintained, is to read them as partial answers to the question under what conditions a deductively valid argument is explanatory. For reasons having to do with the intricacies of Bolzano's notion of deducibility, I have proposed that one should in fact better read them as answers to the question under what conditions a *logically* valid argument is explanatory (Section 5.3.1). On what I took to be the most charitable reading of them, Bolzano's principles essentially boil down to the claim that a logically valid argument is explanatory, *only if* there is no argument with fewer premises, for none of the premises there is a logically equivalent proposition that is simpler, and none of the premises is more complex than the conclusion (Sections 5.3.2 and 5.3.4). These principles, I have maintained, can be considered to be an explication of the idea that every premise and every concept in an explanatory argument is deductively relevant. More importantly, they can be considered to be an explication of the idea that explanations ought to go in hand with some kind of theoretical economy. This last point becomes particularly explicit when Bolzano formulates a claim that is very close to the one just mentioned on a more global level. He argues that an ordering of certain arbitrary large collections of conceptual truths coincides with grounding, *only if* this ordering results in the deductively most efficient partition into truths that are taken as primitive and truths that are logically deducible from them, where no truth is deduced from premises more complex than itself (Section 5.3.6).

As I have further shown, Bolzano's economy principles have interesting consequences concerning the explanatory (in)significance of logically analytic truths that are congenial to a number of remarks Bolzano makes in this direction (Section 5.3.3). On my interpretation, they entail, in particular, that logically analytic truths do not figure as partial grounds of synthetic truths. Unfortunately, the principles also reveal some severe tensions with two other claims Bolzano makes about grounding, namely: a) the claim that any truth and any collection of truths is the ground of some other truths, and b) the claim that there is an explanatory hierarchy also in

the realm of logically equivalent truths (Section 5.3.5). I have argued that there are reasons to reject claim a) from Bolzano's perspective, but that it is not fully clear how the tension between b) and the remainder of Bolzano's impure logic of grounding is to be resolved.

In spite of the aforementioned tensions, I have continued to investigate Bolzano's ideas on grounding and deductive economy, as they seem to be of the highest importance to Bolzano's thought about grounding. This, I have argued, is witnessed, on the one hand, by many back-references Bolzano makes to the respective passages in the **WL** and in other late writings. On the other hand, it is witnessed by two attempts of Bolzano to strengthen the 'only if' in the aforementioned economy principles into an 'if and only if'. Bolzano suggests, in particular, that a logically valid argument for a conceptual truth is explanatory *if and only if* its premises consist of the *smallest* collection of premises for its conclusion, each premise (and the conclusion) is the *simplest* among all logically equivalent ones, and no premise is more complex than the conclusion (Section 5.5). This seems as though he suggests that conditions which are each necessary for a logically valid argument to be explanatory are also jointly sufficient. As interesting as this proposal is, I have argued that on what I took to be the most charitable reading of it, it either rests on highly implausible assumptions concerning the notion of deducibility, or it only captures conditions that might be viewed as jointly sufficient for a given logically valid argument to be explanatory, but not as necessary.

A second interesting idea that goes in the same direction as the one just mentioned is Bolzano's tentative definition of grounding. On my interpretation, this definition amounts to the claim that a given ordering of certain collections of conceptual truths coincides with grounding *if and only if* this ordering results in the deductively most efficient partition into simple truths that are taken as primitive and remaining truths that are logically deducible from them, where no truth is deduced from premises more complex than itself (Section 5.6). I have discussed various technical and conceptual problems of the proposal. The conceptual problems concern the fact that the definition seems to presuppose that deducibility is a necessary condition for grounding, which Bolzano denies elsewhere. Next to this, the definition arguably has a narrower scope than explicitly stated by Bolzano: given what he has said about grounding earlier, the definition only applies to grounding insofar it obtains among conceptual truths (and I have stated it accordingly above). Concerning the technical problems, I have observed that Bolzano's definition captures at best the relation of mediate grounding among conceptual truths, which raises the question of whether the notion of immediate grounding, which is at the center of Bolzano's theory of grounding, can be defined in terms of it. I have argued that it is at least not inconceivable how this might be done. In the end, however, the interesting aspect of Bolzano's proposal for a definition of grounding, as well as his proposal for necessary and jointly sufficient conditions for a logically valid argument to be explanatory, is not so much the technical side of things. Rather,

these proposals are interesting because of what they reveal about Bolzano's views on grounding on a more general level. In this proposal for a definition we find, in effect, the ideas corresponding to claims (II), (IV), (V) and (VI) above combined. *That* Bolzano suggests a definition in terms of conditions that correspond to *these* ideas underscores their centrality to his views on grounding and, ultimately, with respect to his views on explanation (cf. Betti [15]). In this respect, his theory appears remarkably modern in that it tries to capture a highly general notion of explanation in terms of notions such as deductive economy and simplicity. I have mentioned that this idea exhibits certain similarities with the unificatory theory of explanation. To explore these similarities is a topic for future research.

I have not only investigated Bolzano's notion of grounding against the background of its role in the context of a classical, axiomatic conception of science, but have also tried to take a closer look on what he actually has to say on the concept of science and on scientific methodology, and what role grounding occupies in these remarks precisely. In the **BD**, the picture is relatively simple (Section 2.5). Sciences consist of collections of truths ordered by grounding. Some of these truths are axioms (*Grundsätze*), which are conceived of as ungrounded truths, and some theorems, which are either grounded in the axioms or in truths from superordinate sciences. Each axiom is an absolutely simple truth, *viz.* a truth that does not contain defined concepts. The domain of a science is conceived to be a realm of objects, and the demarcation of sciences is determined by objective properties of the truths of the given science. The presentation of a science in the sense of the **BD** must, in the first instance, make the objective connection of the truths of the science explicit. Explanatory proofs are thus essentially the only proofs that Bolzano acknowledges in the **BD**.

The conception of science that Bolzano defends in the **WL** can be considered as an attempt to adjust the basic ideas of such a classical, axiomatic conception of science to the actual needs of scientific practice as Bolzano conceived of it—without, however, departing from the general axiomatic framework (Chapter 6). I have pointed out that Bolzano argues for a flexible and pragmatic conception of the demarcation of scientific disciplines, and that he incorporates various ethical and pragmatic concerns in his reflections on science. Partially as a result of this, he revises his conception of the starting points of a science (Section 6.3). He rejects the idea that the starting points of every science are *absolute* starting points. In other words, starting points of a science (*Grundsätze*) need not be ungrounded truths (*Grundwahrheiten*). Bolzano also rejects the idea that the starting points of a science have to contain only absolutely simple (undefined) concepts. Further, even though he still finds objective demarcation criteria for sciences and their domains to be desirable, and even though he at least toys with suggesting one such criterion in terms of simple concepts occurring in the science's truths, he exhibits an overall skepticism with respect to this idea (Section 6.4). I have argued that Bolzano's skepticism on objective demarcation criteria for sciences, and his less restrictive views

on the starting points of a science, cannot only be explained by the fact that he pays attention to a much broader range of sciences than he did in his early writings. Also the mathematical disciplines cannot be objectively demarcated from each other, and also in the mathematical disciplines not every starting point (*Grundsatz*) is an absolutely fundamental truth (a *Grundwahrheit*) containing undefined concepts.

Even though Bolzano thus changes his take on conditions (1), (3a) and (2a) of the Classical Model of Science relative to his earlier conception, he continues to adhere to a rather traditional construal of conditions (3b) and (6): the connection between the starting points of a science and the remaining truths is provided by grounding, and proofs in *rigorous* scientific presentations ought to be explanatory proofs (Section 6.5). Further, Bolzano's skepticism concerning objective demarcation criteria for sciences notwithstanding, he still acknowledges an objective hierarchy of the scientific disciplines in terms of grounding.