Summary

The research described in the thesis concerns stochastic modeling and statistical analysis of brain dynamics. The main goal of the research was to use mathematical modeling and statistical analysis to identify which regions in the brain become active and cooperate in different situations and under different conditions, and how they do this, based on co-registered EEG and fMRI data. These two types of data contain complementary information about the activity of the brain. The idea was to build an integrated model for EEG and fMRI data, so that a statistical analysis could take advantage of the high temporal resolution of EEG as well as of the high spatial resolution of fMRI.

Before actual data analysis was addressed, several mathematical problems had to be tackled. First, a general mathematical framework for certain constrained minimization problems was established. This is described in Chapter 2. Theoretical analyses of these problems with respect to existence and uniqueness of a solution were carried out. The results of the analyses were needed for proving properties of the maximum likelihood estimators that are considered in Chapters 3 and 4.

In Chapter 3 the modeling and estimation of the covariance structure of the EEG data are investigated. Since the EEG signal is generally modeled as a multivariate normal distribution for which the covariance matrix is a Kronecker product of two matrices, one corresponding to the time domain and one corresponding to the space domain, existence and uniqueness of maximum likelihood estimators of such structured covariance matrices were studied. Several new results have been found, and it was shown that some existing, frequently used, results were in fact not correct.

Next, from preliminary data analysis it turned out that the two-domain EEG-covariance model was not appropriate for an integrated analysis of EEG data together with fMRI data, and that a third domain that would represent the fMRI data collection procedure, had to be added. Moreover, the time-domain component needed to be further structured as a Toeplitz matrix. However, this meant that also the estimation procedure had to be adapted. Especially the estimation of the Toeplitz component was no longer straightforward. A way to approach this estimation problem and an accompanying computer algorithm was developed. The three-domain model not only serves to model the covariance structure of EEG in integrated EEG-fMRI modeling but also the covariance structure of MEG data under certain experimental conditions. In Chapter 4 the new model and the estimation procedure are described and evaluated, and examples of their use for actual data analysis are given.
Because some aspects of the overall model that was used for integrated EEG-fMRI analysis, a linear regression model in which fMRI data serve as response variables and functions of the EEG data as explanatory variables, were not completely satisfying, a new integrated model which improves EEG-fMRI data analysis results was sought for. For that purpose, a class of graphical models was investigated. In particular, in Chapter 5 of the thesis it is proposed to use a hierarchical Gaussian graphical model with two layers, one layer for both types of observed random variables and one hidden layer for unobserved random variables that represent the neuronal and hemodynamic processes that underlie the observed phenomena. This model takes into account covariance structure of EEG, autocorrelation of fMRI, and models the relationship between the hidden variables of these two data modalities. A corresponding numerical iterative estimation procedure based on maximum likelihood and the graphical Lasso was derived and its performance investigated on simulated data and its use is illustrated on experimental co-registered EEG-fMRI data.