Percolation, loop soups and stochastic domination
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Summary

This thesis is on probability theory, in particular on percolation, loop soups and stochastic domination. It is based on the papers [8], [53], [7] and [52], which form the basis for Chapters 2–5, respectively. Chapter 1 contains an introduction.

In Chapter 2 we study stochastic domination of conditioned Bernoulli random vectors. We consider sequences of vectors $X_n$ and $Y_n$ that each consist of $n$ independent Bernoulli random variables. We assume that $X_n$ and $Y_n$ each consist of $M$ “blocks” such that the Bernoulli random variables in block $i$ have success probability $p_i$ and $q_i$, respectively, with $p_i \leq q_i$ for all $i$. Here $M$ does not depend on $n$ and the size of each block is essentially linear in $n$. We consider the conditional laws of $X_n$ and $Y_n$, conditioned on the total number of successes being at least $k_n$, where $k_n$ is also essentially linear in $n$. In general, the conditional law of $X_n$ is not necessarily stochastically dominated by the conditional law of $Y_n$. We give a complete answer to the question with what maximal probability two such conditioned Bernoulli random vectors can be ordered in any coupling, when the length $n$ of the vectors tends to infinity.

In Chapter 3 we study the random connection model, which is a model in continuum percolation (see [39]) defined as follows. Take a Poisson point process $X$ on $\mathbb{R}^d$ of density $\lambda$ and connect each pair of points $x$ and $y$ in $X$ with probability $g(|x-y|)$, independently of other pairs of points, independently of the point process $X$. Here $g$ is a connection function, which is a non-increasing function from the positive reals to $[0,1]$. We consider a sequence of random connection models $X_n$, where $X_n$ is a Poisson point process on $\mathbb{R}^d$ of density $\lambda_n$ such that $\lambda_n/n^d \to \lambda > 0$. The points of $X_n$ are connected according to the connection function $g_n$ defined by $g_n(x) = g(nx)$, for some connection function $g$. Let $I_n$ be the number of isolated points in the random connection model $X_n$ in some bounded set $K$. The main result in [44] by Roy and Sarkar is a central limit theorem for $I_n$. Although the statement of this result is correct, the proof in [44] has errors. We explain what went wrong in the proof, and how this can be corrected. We also prove an extension to components larger than a single point in case the connection function has bounded support.

In Chapter 4 we study two variations on the fractal percolation model introduced by Mandelbrot [38]. The first variation is $k$-fractal percolation, defined as follows. Divide the $d$-dimensional unit cube in $N^d$ equal subcubes and retain $k$ of them in a uniform way while the others are removed. Then iterate the procedure inside the retained subcubes at all smaller scales. We prove that the (properly rescaled) percolation critical value of the model converges to the critical value of ordinary site percolation on a particular $d$-dimensional lattice as $N$ tends to infinity. This is analo-
gous to the result of Falconer and Grimmett [24] that the critical value of Mandelbrot
fractal percolation converges to the critical value of site percolation on the same d-
dimensional lattice. The second model we study is fat fractal percolation. In this
model subcubes are retained with probability $p_n$ at iteration step $n$ of the construc-
tion, where $p_n$ is non-decreasing in $n$ such that $\prod_n p_n > 0$. The Lebesgue measure
of the limit set is positive a.s. given non-extinction. We prove that either the set of
connected components larger than one point has Lebesgue measure zero a.s. or its
complement in the limit set has Lebesgue measure zero a.s.

In Chapter 5 we study the random walk loop soup, which is a Poissonian collection
of lattice loops. It has been extensively studied because of its connections to
the discrete Gaussian free field [33], but was originally introduced by Lawler and
Trujillo Ferreras [31] as a discrete version of the Brownian loop soup of Lawler and
Werner [32], a conformally invariant Poissonian collection of planar loops with deep
connections to conformal loop ensembles (CLE) [48] and the Schramm-Loewner evo-
lution (SLE). Lawler and Trujillo Ferreras [31] showed that, roughly speaking, in
the continuum scaling limit, “large” lattice loops from the random walk loop soup
converge to “large” loops from the Brownian loop soup. Their results, however, do
not extend to clusters of loops, which are interesting because the connection between
Brownian loop soup and CLE goes via cluster boundaries. We study the scaling limit
of clusters of “large” lattice loops, showing that they converge to Brownian loop soup
clusters. In particular, our results imply that the collection of outer boundaries of
outermost clusters composed of “large” lattice loops converges to CLE.