Estimation of Alonso's Theory of Movements for Commuting

de Vries, J.J.

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CHAPTER 6

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

This final chapter summarizes the main conclusions from this thesis, and discusses directions for further research. Section 6.1 contains the conclusions, ordered by chapter. Section 6.2 proposes a spatial equilibrium model including prices and supply and demand functions, from which Alonso's Theory of Movements (ATM) can be derived as the reduced form for quantities. The remaining sections suggest further research on theory, estimation, and applications.

6.1 CONCLUSIONS

Spatial interaction modeling is not only about flows, but should involve true interaction between the levels of activities at origins and destinations. In the application in this thesis, residential and employment locations interact through commuting. The total outflow from an origin does not only depend on the characteristics of that origin, but also, through the system of flows, on all destinations, distances and other origins. Similarly, the total inflow at a destination does not only depend on the characteristics of that destination, but also, through the system of flows, on all origins, distances and other destinations. The bilateral flows do not only depend on the characteristics of their origin and destination and the distance between them, but also on the accessibility of the origin and the accessibility of the destination. So the interaction is system-wide, and a model should be chosen that incorporates that (nicely depicted by Bettman (1981)). *(Chapter 2)*

Several researchers have developed such a model, starting in the 1970s. The models of Hamerslag (1972), Alonso (1973), Bikker and De Vos (1980), and Hallefjord and Jörnsten (1985) have the same mathematical structure. This contains the four models of Wilson's

(1967, 1970, 1971, 1974) Family of Spatial Interaction Models as special cases, but the new models have additional features and properties. The interpretation of the model and its variables has raised a lot of discussion. Hamerslag (1972, 1975c) characterizes the model as a transportation model with elastic constraints, Alonso (1973, 1978) as a systemic model including opportunities and competition, Bikker and De Vos (1980, 1992) as an equilibrium between three components, and Hallefjord and Jörnsten (1985, 1986) as a multi-objective optimization problem. It is remarkable that from research on such different applications as commuting, migration, international trade, and hospital admissions essentially the same model arose. *(Chapter 2, Appendix 5C)*

Estimation of ATM can proceed in two stages (Ledent 1980). The first stage is similar to the doubly constrained model. The often applied doubly constrained model can be interpreted as a partial estimation of a more general model. If outflows and inflows are both fully constrained, the question arises why their sums would be equal. If they are elastic and endogenous, the equality of their sums results from the model. The first estimation stage is conditional on them, to estimate the distance effect, as well as the balancing factors (Cesario 1974). These balancing factors indicate accessibility (inversely), so they are endogenous, and therefore called “systemic variables”. In the second stage, their effect on outflows and inflows is estimated, for which simultaneous-equations estimation methods are required. De Vries et al. (2002) proposed to apply the iterative version of the Instrumental Variable estimation method described by Hausman (1983). In Chapter 5 of this thesis, this method is applied for commuting in Denmark. *(Chapter 3)*

From the empirical application on commuting in Denmark it appeared that the effect of travel cost on commuting flows is nonlinear, even after taking logs. So the well-known exponential and power specification do not adequately describe this situation. From an exploratory analysis with a spline function, it appears that the cost elasticity of the flows is largest (in absolute value) for distances of around 20 km. This specification could nicely be simplified using a logistic function (in a log-log setting). Recognizing the large variation in elasticity is important for the evalution of infrastructure projects. Moreover, a suitable specification of the distance part of the model is essential to estimate the systemic effects correctly. *(Chapter 4)*

The implementation of the Iterative Instrumental Variable estimation method required quite some programming, but the computations are fast and the iteration converges quickly. Iteration is essential, as in the first round strange results occur. The difference with alternative estimation methods, applied in the literature on ATM, appeared to be small in the empirical application on commuting in Denmark. Both systemic parameters are estimated to be small, but significantly positive. So the constrained models are rejected. The estimates for the accessibilty elasticities are 0.27 on the residential side and 0.19 on the employment side. The tentative conclusion is that jobs and people follow each other to a limited extent. *(Chapter 5)*

6.2 RESEARCH DIRECTION: INCLUDING PRICES

The tradition of ATM is primarily from geography. Economists like to include prices. For hospital admissions, prices are not the regulating mechanism, but, for residential and employment location choice, rents and wages are important. In this section we present a spatial interaction model containing supply and demand functions, with prices as an equilibrium mechanism.[1] The description of flows which results from this model is identical to that of ATM. The leading example will be commuting.

The model includes two sets of spatially separated markets (residential areas and employment areas), and the interactions between these. The basis is the simultaneous choice of residential and employment location, depending on housing prices, wages and commuting costs. The model contains equations for housing supply and labor demand, along with equilibrium conditions on the housing and labor markets. We demonstrate that, if the prices are eliminated, the model for quantities is the same as ATM. As could be

[1] This section is based on research with Aart F. De Vos.

expected, the balancing factors in ATM are related to the prices in the structural model. This relationship is not straightforward, however, as also exogenous variables from the supply and demand functions are involved.

The supply-demand model has implications for the specification of the exogenous parts in ATM. The equations for outflows and inflows contain combinations of the exogenous variables from the supply and demand functions. The distinction between reduced-form coefficients and structural-form coefficients has consequences for the interpretation and possible restrictions. The model presented in this section provides a further rationale for ATM, and a link to economics. The usual assumptions on the signs of price elasticities imply that the systemic parameters lie between zero and one. Various special cases, including the four models of Wilson's Family of Spatial Interaction Models, result from zero price elasticities.

In the model, a labor and a housing market are discerned, defined for spatial areas which may differ for housing and labor. The interaction between the labor and the housing markets is caused by the fact that the suppliers on the labor market are demanders on the housing market. The strong relationship between the behavior of people in supplying labor and in demanding housing is caused by the role of distance between the residential and the employment location. We can think of the model as describing the action of three groups of economic agents. Each employment region can be seen as one large firm, which searches for a certain number of employees. Each residential region can be seen as a local authority, which offers housing to a certain number of working people. The third group consists of a large number of individuals who choose simultaneously a residential and an employment region.

We model this situation at an aggregate level. The model explains the number of people working in some region and living in some (other) region. The data to be modeled can be represented in a table, where the rows correspond to residential regions and the columns to employment regions. Each cell contains the number of people living in a specified residential region and working in a particular employment region.

The model can be described in five (vector) equations. These include two equilibrium conditions (for both sets of markets), and three behavioral equations. One of these concerns the individuals, who are suppliers in the labor market and demanders in the housing market. The other two concern the demand function which they face on the labor market and the supply function they face on the housing market. The five equations are: the demand function for labor; the supply function for housing; an equation for the aggregated choices of the individuals for a specific combination of a region to supply labor and a region to demand housing; an equilibrium condition for the labor market; and an equilibrium condition for the housing market. As the model contains three behavioral equations, it is called the *Three Component Model* (3CM).

We describe the choice of residential and employment location as a simultaneous choice. A person[2] chooses a job and a house, taking into account the attractivity of the various jobs (including the wage), the attractivity of the various houses (including house price), and factors such as the distance between them. He/she may also choose not to work at all. The model describes the interaction between the aggregated choices of individuals and the capacity constraints on the various housing and labor markets. In the structural model, this interaction occurs through prices. It is essentially a simultaneous equations model. The resulting picture will be something like the following.

If too many people want to work in the same region, there will not be enough jobs available. This will either drive wages down, or, if wages are fixed, decrease the chance to find a job. Some people will choose another region to work in, others will remain unemployed. Some of the other regions will not have enough jobs either, so the process continues. On the other hand, in some employment regions there is a shortage of labor, and firms will hire people from other regions. This continues until in each employment region

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[2] We ignore the extra complications of multi-earner households (Rouwendal and Van der Straaten 2004).

there is an equilibrium between the number of jobs available and the number of people preferring to work there.

In the meantime, a similar process occurs within the residential regions. The capacity to absorb (employed) inhabitants is flexible, but limited. People who cannot find a house will choose to live in another region. This continues until there is equilibrium in all residential regions.

Both processes are interrelated: employment and residential location are chosen simultaneously. For example, if there is a shortage of houses in a region, some people may choose not only another residential region, but also another employment region than originally intended. This is a logical consequence of the general equilibrium character of the model.

Our primary goal is to explain the 3CM, and not to analyze the labor and housing markets. The 3CM is an econometric model for cross-section data; it is a static model. Thus, we will treat the situation on the labor and housing market as being in long-term equilibrium. Further we assume (if necessary) that in both markets a price mechanism exists with perfect competition. The purpose of this stylized case is to explain the working of the model. But, as we discuss later, the model is also suited to describe many situations where these assumptions are violated.

As the model is intended to be econometrically useful, we will introduce many restrictive assumptions. From the viewpoint of mathematical economics a more general specification would be preferred, but then the way to an estimable econometric model would be too long. In principle one could ask beyond the demand and supply functions, and construct a framework of utility or profit maximization. This is, however, outside the scope of this section. We primarily intend to give a basis for a sensible specification of an econometric model.

We specify behavioral equations multiplicatively. To be able to estimate the model from reasonably-sized data sets, a functional form for these equations has to be chosen. A multiplicative specification has appeared to be suitable and is indeed often used. The reason is that often the size of the effect on a flow of a change in an explanatory variable is roughly proportional to the size of the flow concerned. This is modeled well by a multiplicative specification, as then elasticities are constant. Other functional forms could be chosen, and we think that in many cases a substantial part of the analysis will essentially be unaffected. Incorporating more functional forms would complicate the notation, so we will restrict ourselves here to a multiplicative specification.

We have a set of n regional housing markets, indicated by the subscript i, and a set of m regional labor markets, indicated by the subscript j. Geographically, these regions can be the same, but this is not required. The variable to be explained by the model is $\mathbf{T} = \lfloor T_{ij} \rfloor$, the n by m matrix whose ij - element is the number of people living in residential region i and working in employment region j. From this are derived:

$$O_i = \sum_{j=1}^{m} T_{ij} , \tag{6.1}$$

the total number of working people living in residential region (origin) i, and

$$D_j = \sum_{i=1}^{n} T_{ij} , \tag{6.2}$$

the number of people working in employment region (destination) j. These should not be confused with the total number of people living in a region (including the unemployed and those not in the labor force), or the total number of jobs available (including vacancies).

The model contains three behavioral equations (6.3 - 6.5). The number of working people living in a residential region is related to the price of housing. A high rent r_i will increase housing supply, drive away non-working people, and stimulate higher occupation rates. So we formulate a kind of supply function:

$$O_i = r_i^{\mu} H_i , \tag{6.3}$$

where μ is a parameter (price elasticity of supply, assumed to be positive), and H_i is a submodel containing exogenous factors (for example, the number of houses in a base year). Labor demand is modeled as:

$$D_j = w_j^{\nu} J_j , \tag{6.4}$$

where w_j is the wage, ν is a parameter (wage elasticity of labor demand, assumed to be negative), and J_j is a submodel containing exogenous factors (for example, the available area for business). The crucial equation is simultaneously a demand function on the housing market and a supply function on the labor market, and describes the number of people living in i and working in j:

$$T_{ij} = r_i^{-\kappa} w_j^{-\lambda} R_i L_j F_{ij}\,. \tag{6.5}$$

This equation contains both the rent and the wage, with parameters κ (assumed to be positive, while price elasticity of demand is negative), and λ (assumed to be negative, while wage elasticity of labor supply is positive). R_i is a submodel of exogenous factors making residential region i attractive to live in, and L_j is a submodel of exogenous factors making employment region j attractive to work in. F_{ij} is a submodel of exogenous factors related to the commuting from i to j, containing a distance-deterrence function as its primary part.

The endogenous variables in this model are the quantities $\mathbf{O}$, $\mathbf{D}$ and $\mathbf{T}$, and the prices $\mathbf{r}$ and $\mathbf{w}$. The exogenous variables, possibly including parameters to be estimated, and disturbance terms, are summarized in the five submodels. The structure of the model is essentially symmetric between the origin and destination side. The only difference is the sign of the coefficients of wage compared with that of rent. The explanation for this is that the agents act as demanders on the housing markets and as suppliers on the labor market.

Rearranging each of the equations of the structural model, we have from (6.3), (6.4) and (6.5):

$$O_i = r_i^{\mu} H_i = \left(r_i^{-(\mu+\kappa)} R_i H_i^{-1}\right)^{-\frac{\mu}{\mu+\kappa}} \left(R_i^{\frac{\mu}{\mu+\kappa}} H_i^{1-\frac{\mu}{\mu+\kappa}}\right); \tag{6.6}$$

$$D_j = w_j^{\nu} J_j = \left(w_j^{-(\nu+\lambda)} L_j J_j^{-1}\right)^{-\frac{\nu}{\nu+\lambda}} \left(L_j^{\frac{\nu}{\nu+\lambda}} J_j^{1-\frac{\nu}{\nu+\lambda}}\right); \tag{6.7}$$

$$T_{ij} = r_i^{-\kappa} w_j^{-\lambda} R_i L_j F_{ij} = \left(r_i^{-(\mu+\kappa)} R_i H_i^{-1}\right)\left(r_i^{\mu} H_i\right)\left(w_j^{-(\nu+\lambda)} L_j J_j^{-1}\right)\left(w_j^{\nu} J_j\right) F_{ij}\,. \tag{6.8}$$

If we now define:

$$\alpha = \frac{\mu}{\mu+\kappa}; \tag{6.9}$$

$$\beta = \frac{\nu}{\nu+\lambda}; \tag{6.10}$$

$$A_i = \left(r_i^{-(\mu+\kappa)} R_i H_i^{-1} \right); \qquad (6.11)$$

$$B_j = \left(w_j^{-(\nu+\lambda)} L_j J_j^{-1} \right); \qquad (6.12)$$

$$V_i = \left(R_i^{\frac{\mu}{\mu+\kappa}} H_i^{1-\frac{\mu}{\mu+\kappa}} \right) = R_i^{\alpha} H_i^{1-\alpha}; \qquad (6.13)$$

$$W_j = \left(L_j^{\frac{\nu}{\nu+\lambda}} J_j^{1-\frac{\nu}{\nu+\lambda}} \right) = L_j^{\beta} J_j^{1-\beta}, \qquad (6.14)$$

we have the equations of ATM.

The auxiliary variables *A* and *B* more or less replace the prices. They are related to the prices *r* and *w* through the simple equations (6.11) and (6.12). Under the reasonable assumptions of positive supply elasticities and negative demand elasticities, we have $\kappa\mu > 0$ and $\lambda\nu < 0$. This is true independent of the question whether (6.3) is a demand and (6.4) is a supply function, or both are demand functions, or both supply functions, or (6.3) a supply and (6.4) a demand function. From these we have $0 < \alpha < 1$ and $0 < \beta < 1$ as restrictions for the parameters of the quantities model. If one of the price elasticities is zero, then α or β becomes either zero or one. If both on the housing and the labor market one of the price elasticities is zero, the four cases of Wilson's (1967) Family of Spatial Interaction Models result. The submodels *V* and *W* are weighted geometric averages of submodels of demand and supply factors. The weights are given by the systemic variables. This has implications for the interpretation of parameter estimates and for parameter restrictions. If, for theoretical reasons, a parameter is restricted to one in the structural model, the relevant restriction in the reduced-form model contains a systemic variable (De Vos and Bikker 1989).

In some applications the equilibrium is not attained through prices, but through other mechanisms. For example, in the hospital case (De Vos and Bikker 1984, 1986, 1989), waiting lists and admission policies play that role. Also in the example of the labor and housing markets, one could imagine that prices are fixed, due to government regulations. The equilibrium is then attained through other variables. These situations can be captured by the 3CM by calling the equilibrium-bringing variable "price", and applying the model. Probably the specification of the demand and supply functions will change, but the general structure remains. The characteristics of the variables that can be treated as prices are that they are endogenous variables which affect the quantity demanded or the quantity supplied, or both.

In this section we have presented a structural model, based on two sets of markets. It contains two sets of prices, and is essentially symmetric in its structure. It is possible to eliminate the prices from this model. The resulting model for quantities has the structure of ATM. So this derivation gives an economic basis for ATM. In the explanation of the model, we use the example of living and working, but it is more general. The relevance of the structural model, even in the case that prices are unobserved, is that it provides an interpretation of parameters in the quantities model. Without this, reduced-form coefficients are easily erroneously interpreted as structural-form coefficients. It has important implications for various issues such as choice of explanatory variables, interpretation of parameters, and choice of restrictions.

6.3 RESEARCH DIRECTION: THEORY

The approach sketched in the previous section could be elaborated on. The supply of housing and the demand for labor could be modeled further, by including land use, along the lines of Fortuijn (1975, 1975, 1982) and Geurs and Ritsema van Eck (2001, 2003). Also for practical applications a commuting model following ATM could be integrated in a more comprehensive model such as a *Land Use Transport Interaction model* (LUTI).

It would also be interesting to relate Alonso’s (1978) Theory of Movements to the *Spatial Computable General Equibibrium* (SCGE) modeling by Bröcker (1998a, 1998b). This would clarify the role of the systemic variables in the modeling of interregional trade. In interregional trade the role of prices is different compared with that in commuting. Import and export prices are closely related by transport costs.

The structural model presented in the previous section contains a choice function which is simultaneously a demand function on the housing market and a supply function on the labor market. The specification is simple and chosen so that ATM results. The simultaneous choice of residential and employment location, including the required commuting, could be modeled from a utility framework. The utility of a choice depends on the attractiveness of the residential region, the attractiveness of the employment region, and the disposable income. The disposable income is the wage minus rent minus commuting cost. The characteristics of dwellings and jobs can be seen as corrections to rent and wages. The choice between regions is discrete, and a logit model could be used to relate this to utility, acknowledging the variety of people's preferences . The resulting functional form might well differ[3] from that used in Section 6.2, and that would have implications for the functional form of the model for flows. Equations (3.1), (3.4) and (3.5), which occur in the doubly constrained model, could remain unchanged, but the way in which O depends on A, and D depends on B, might differ from (3.2) and (3.3). This would affect the concept of macro-elasticities, but the general approach to the estimation still holds.

Section 2.4 of this thesis demonstrates that the optimization program of Hallefjord and Jörnsten (1985) results in Alonso's (1978) Theory of Movements. The constraints (2.26) and (2.27) were substituted in the objective function. An alternative approach would be to use Lagrange multipliers. The role of those multipliers would be quite comparable to that of the systemic variables A and B. Investigation of the exact relationship could help in the interpretation.

The model as presented in this thesis is an equilibrium model. As residential and employment locations change slowly, it is useful to consider mutations from a starting situation. People can be looking for a job given their residential location, or they can search for a job and find a new residential location afterwards. Change of dwelling or job occurs usually only after several years. Matching theory might help to model this (Pissarides 2000; Petrolongo and Pissarides 2001; Gautier and Teulings 2009; Xiao 2014). The supply of housing and the demand for labor change even more slowly, as real estate is

[3] Indirect utility depends on w - r - c; such an expression is not found in the multiplicative specification in Section 6.2.

involved. Reformulating the model as a dynamic model might improve the application. This might also provide a theoretical justification for the inclusion of delayed variables in the specification in Chapter 5 of this thesis.

6.4 RESEARCH DIRECTION: ESTIMATION

This thesis is the first application of the Iterative Instrumental Variables estimation method on ATM, as proposed in De Vries et al. (2002). In the application on commuting in Denmark (Chapter 5 of this thesis), the difference with OLS appeared to be small. However, OLS is theoretically known to be inconsistent in this model. The size of the bias might well depend on the application, the range of the systemic variables, and the variances of the error terms. It would be useful to compare the estimation methods on different data sets, and/or in a simulation.

Further, it would be interesting to apply the Full Information Maximum Likelihood (FIML) estimator of De Vos and De Vries (1990) on the Danish data and other applications. De Vos and De Vries (1990) applied it to hospital admissions aggregated to provinces to demonstrate the method.

If prices are included in the model, and observations are available, the estimation methods change somewhat. In the first stage, A and B are still estimated. In the second stage, there are four equations instead of two:

- Outflow O, dependent on price r, equation (6.3);
- Inflow D, dependent on price w, equation (6.4);
- The product OA, dependent on r, from equation (6.11) times (6.3);
- The product DB, dependent on w, from equation (6.12) times (6.4).

The price variables r and w should be instrumented with their predictions which arise from solving the model.

Applied models are often large and complex. In the kernel of regional models that involve interaction often something like ATM is present (Bröcker 2014), or would be desirable. Such a structure can also occur repeatedly in the same model: for commuting, for migration, and for trade flows. In principle, it would be possible to extend the estimation method applied in this thesis to such composed models. That would require all parameters to be estimated simultaneously, using instruments, and to solve the complete model in each iteration. In practice, this might be difficult or undesirable. Alternative methods like the Generalized Method of Moments might be useful, if exogenous variables are available as instruments.

6.5 RESEARCH DIRECTION: APPLICATION

This thesis has applied Alonso's Theory of Movements to commuting in Denmark, with the available data. The estimation of the distance-decay function was complicated because of the geographical structure of Denmark. It would be interesting to perform this analysis for other countries. The model could be improved by including the time cost of a commute, and data on relevant factors for firm location.

The estimation method could also be applied to other spatial-interaction situations than commuting. A first candidate is migration (Ledent 1980; Poot 1986).

6.6 THE END

In this final chapter of this thesis, first, the conclusions were summarized. A brief explanation of how prices could be introduced in the model was presented, and possibilities for further theoretical research, estimation and applications were discussed. With regard to this potentially fruitful research, this thesis is a start.